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1.0 Introduction

Beyond Meat (BM) is a plant-based meat manufacturing company, with the plant-based burger patty being their signature product. BM products have attracted vegetarians and meat-eaters alike, providing a healthier alternative to meat products. Major food institutions in Canada like A&W and Tim Hortons have tapped into the booming market for BM based menus, by introducing BM burgers and sandwiches. This report outlines an analysis performed to design an optimal market research strategy and determine the expected market share for a Burger King BM Whopper, if introduced into the current market.

The analysis focuses on the Burger King at the University of Waterloo location (150 University Ave W), studying a hypothetical scenario, where a decision must be made if the restaurant should launch a BM Whopper burger as **a four-month trial program**. This decision will be made by conducting a series of surveys and building a decision tree based on the survey results. The decision tree will then be solved through expected value and expected utility analysis to determine profit and market share for the burger. Using the results, an effective marketing strategy will be developed to maximize expected profit.

2.0 Data Synthesis and Initial Analysis

2.1 Retail Price: $5.20

Research was conducted to determine the retail cost of plant-based food items at fast food restaurants. Typically, retailers sell plant-based items for approximately $1 more than their real meat counterparts. For reference, Tim Hortons BM Sausage Egg & Cheese retails for $4.39 [1], while their regular Sausage Egg & Cheese Biscuit retails for $3.29 [1]. This difference in price is around $1. As such, since the Whopper at Burger King has a retail price of $4.19, the retail price of its BM counterpart can be approximated as $5.20.

2.2 Cost Price: $2.60

Typically, a fast food restaurant in Canada has a profit margin of approximately 100% to 150% on burgers/sandwiches. As a result, since the retail price of the BM burger is approximated as $5.20, it will cost Burger King approximately $2.60 to make the burger (100% profit margin).

2.3 Developing a Prior Market-Share Probability Distribution

Burger King at the Waterloo Plaza introduces new menu items with customer reactions that can be classified as either good, average, or bad. A good customer reaction to a product in the food industry leads to a market share of 30%, an average customer reaction leads to a market share of 20%, and a bad customer reaction leads to a market share of 10%, with a market-size of the total number of people that walk into the restaurant per day. These were estimated by speaking to Mr. Nicholas Clauser, one of the managers at the Burger King in Waterloo [2]. In addition, the probabilities of a good, average, and bad customer reactions are estimated to be 25%, 50%, 25% respectively. This was estimated after speaking to several workers in industry, being made aware that the probabilities of good customer reactions and poor customer reactions were equally likely, and lower than an average reaction.

Through this analysis, we would like to estimate how many of Waterloo Burger King's regular burger sales would be converted to BM burger sales (market-share). Waterloo Burger King sees, on average, 250 customers per day and sells 400 burgers each day (1.6 burgers per person) [2]. Therefore, for a **four-month trial marketing timeline (120 days),** it is estimated that the total market size is 250 *120 = 30,000 people or 400*120 = 48,000 burgers.

Assuming that Burger King prepares to sell enough BM burgers per day to meet the maximum market share of 30% discussed earlier, 120 (30% of 400) BM burgers will be prepared each day. This gives a total marketing cost of $2.6*120 = $312/day, or $37,440 over four months.

The value of perfect information for this strategy VPI = EVPI – EVWI.

VPI = (Profit Margin)(Market Size)(30% Market Share – Expected market share)

= ($2.6)(48000)[0.3 – ((0.1)(0.25) + (0.2)(0.5) + (0.3)(0.25))] = $12,480

2.4 Effect of Randomness in Data

It is important to address the effect of randomness on this analysis. Randomness related to the cost and selling price of the product will directly affect profit margins. Differing profit margins will affect decisions made in a decision tree as higher profit margins would allow for a greater opportunity to market the product. In addition, randomness in market share values and the probabilities of these market shares occurring will also affect profit leading to different decisions made on the tree; In general, larger market shares, or a larger market share with greater probabilities of occurring, would result in greater profits and thus welcome more opportunities to be marketed.

2.4 Market Surveys

In order to conduct market surveys, a food truck will be set up for Toronto's Food Truck Festival that runs for three days from August 2nd -5th 2019. The cost of the survey is $2.60 (required to produce each BM burger), plus a fixed cost of $200 per day to rent the food truck and permit for parking at the festival. At the food festival, several other Burger King menu items will be sold, but for every 5th customer, the Burger King food truck will give away a Beyond Meat burger for free. We assume that a Burger King food truck at the festival will attract about 250-300 customers per day. Using an average of 275 customers leads to a survey size of 55 customers being given out a free BM burger sample. Every customer that receives a free Beyond Meat burger will be asked to try the burger and to vote if they would prefer the BM burger over other burgers on the Burger King menu.

Considering each day that the truck is set up to be one phase of the survey, the maximum number of surveys is limited by the number of days that the festival will run for (3 days). This three day marketing survey will cost (on a daily basis) $2.60 (cost per burger) *55 people per day + a fixed cost of $200 to rent a food truck and obtain a permit.

2.5 Analysis Input Data Summary

Table 1: Summary of Input Data for the Model

Trial Period	120 days (4 months)
Market size	(400*120 = 48,000) burgers ≡ (250*120 = 30,000) people

Initial Market Shares and Corresponding Probabilities	• 10% (p=25%) • 20% (p=50%) • 30% (p=25%)
Cost Price per Burger	$2.60
Selling Price per Burger	$5.19
Cost of Marketing	$37,440
Survey Size per Phase	55 people
Max. Number of Survey Phases	3 phases
Fixed Cost per Phase	$200
Cost per Person	$2.60
Total Cost per Phase	$2.6*55 + $200 = $343

2.6 Utility Function

The utility function for a new college graduate was developed by offering various gambles with two equiprobable outcomes to a group member and obtaining their certainty equivalence. A curve of best fit for the resulting data points was found to be $U(x) = y = 0.01004136 + 0.00001644439x + 1.067261*10^{-10}x^2$ plotted in red in Figure 1 below. The expected value plot is also overlaid, in blue.

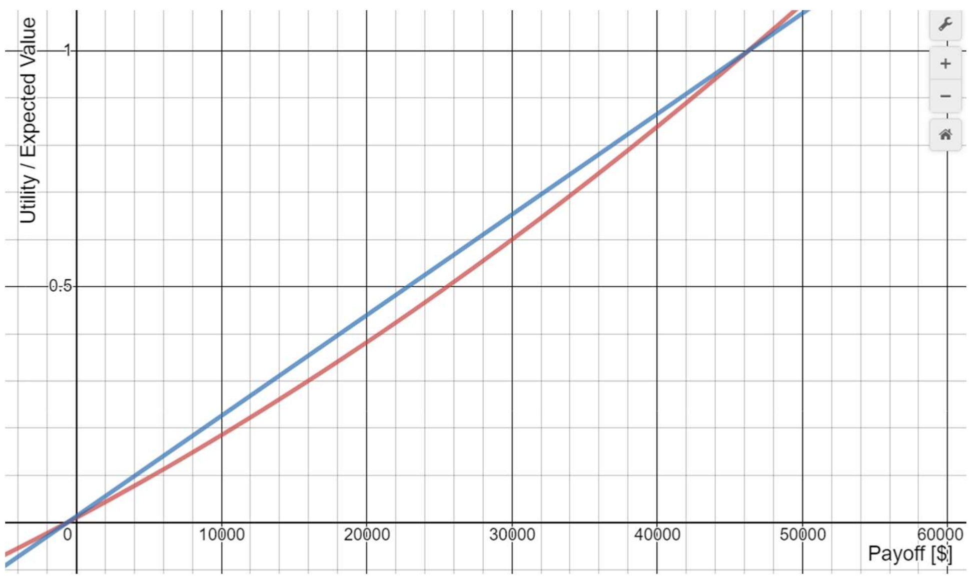

Figure 1: Plot of utility function (in red) and expected value (in blue).

As visible from the plots, the utility function is continuous and is increasing in the domain (positive first derivative). Therefore, this function is a valid, coherent utility function. Moreover, the second derivative of this function is negative, implying the decision maker is risk-seeking.

2.7 Model Assumptions and Shortcomings

- This analysis measures how many of the regular burger sales at Burger King UW will be converted to BM burger sales. As a result, although the product is marketed to the public, the analysis interprets market-size as the number of people who are already customers of Burger King, ignoring the effect of increased number of customers due to the introduction of the new burger. A more comprehensive analysis would consider the entirety of the intended population to whom the product is marketed to. In this example, the number of students studying at the University of Waterloo would provide for a more accurate analysis.

- It was assumed that BK will buy enough BM burger patties to meet the maximum market-share of 30%, and this does not change through the course of the 4-month trial period. However, when the burger is marketed, a more practical approach would be to vary the number of burger patties that are bought based on the current observed market-share. This will make sure that losses due to a decreased market-share is avoided.
- Discrete market share assumption: Market share was assumed to be three discrete values (10%, 20% and 30%). Consequently, the analysis does not allow for market shares less than 10% and greater than 30% and fractional values in between the assumed discrete values. In reality, market share is continuously distributed over 0% and 100%.
- Prior market-share probability distribution: The initial market share probabilities were assumed to be 0.25(10%), 0.5(20%) and 0.25(30%). These are not accurate and will change depending on the product that is marketed. A better market-share distribution could be one that is obtained from restaurants that have already adopted Beyond Meat based menu items. In order to minimize the inaccuracies due to the assumed prior probability distribution, a large sample size is used per survey (55 people).
- During the surveys, people are asked to vote either 'Yes' or 'No', leading to a binomial distribution of survey results. This is because, when neutral options such as "neither agree nor disagree" are included in the survey, people who may only slightly lean towards a favorable or unfavorable response tend to pick the neutral option. This is also called the Error of Central Tendency [3]. A 'yes' or 'no' survey was picked to avoid such bias.
- Assume that the crowd at the food festival where the survey is conducted is representative of the market at the Burger King Waterloo location. This may not be true. Surveys conducted on the local population will yield more accurate results. However, the food festival does allow for easy access to a large sample space.

3.0 Marketing Strategy Design (Basic Modelling)

3.1 Market Share Estimation by Bayesian Inference

In order to calculate expected market share for the new product, a decision tree was formulated. After every survey phase, the results of the survey are used to update the prior market share probabilities and obtain a posterior probability of market share distribution. In this analysis, market share is the ratio number of people who would buy a BM burger over a regular burger (meat or vegetarian), to the total number of people who buy burgers. Therefore, this ratio is also equal to the number of BM burgers sold, to the total number of burgers sold. Hence, market size = total burgers sold = 400*120 = 48,000 as discussed in 2.3 Developing a Prior Market-Share Probability Distribution.

Let the number of people who vote 'yes' in phase $S_x = y_x$.

Given the results of k surveys $S_1, S_2, \ldots, S_{k-1}, S_k = y_1, y_2, \ldots, y_{k-1}, y_k$ respectively, with survey size $= n$, and a prior probability of market-share $m_i = P(M = m_i \mid S_1, S_2, \ldots, S_{k-1})$, the posterior probability of the market share after survey S_k can be calculated as follows:

$$P(M = m_i \mid S_1, S_2, \ldots, S_{k-1}, S_k) = \frac{P(M = m_i|S_1, S_2, \ldots, S_{k-1})P(S_k = y_k|M = m_i, S_1, S_2, \ldots, S_{k-1})}{\sum_j P(S_k = y_k|M = m_i, S_1, S_2, \ldots, S_{k-1})P\left(M = m_j\middle|S_1, S_2, \ldots, S_{k-1}\right)}$$

$$= \frac{P(M = m_i|S_1, S_2, \ldots, S_{k-1})\, {}^{n}C_{y_k} m_i^{y_k}(1 - m_i)^{n-y_k}}{\sum_j P\left(M = m_j\middle|S_1, S_2, \ldots, S_{k-1}\right) {}^{n}C_{y_k} m_j^{y_k}(1 - m_j)^{n-y_k}}$$

The market share probability distribution when no survey results are available (that is, the prior for Survey 1), is outlined in 2.3 Developing a Prior Market-Share Probability Distribution. The resulting decision tree is shown in Figure 2 below. y_1, y_2 *and* y_3 is the number of 'yes' votes in surveys S_1, S_2 *and* S_3.

The expected profit when the product is marketed is calculated using the updated market share probabilities. Given the results of the first *k* surveys $S_1, S_2, \ldots, S_k = y_1, y_2, \ldots, y_{k-1}, y_k$,

- Expected market share $EV(m) = \sum_j (m_j) P\left(M = m_j\middle|S_1, S_2, \ldots, S_k\right)$
- Expected profit of marketing = $EV(m)(\$5.19)(market\ size) - \$37{,}440 - Cost\ of\ surveys$, where \$37,440 is the cost of marketing calculated in 2.3 Developing a Prior Market-Share Probability Distribution. Market size = 400*120 = 48,000.
- Cost of terminating = Total number of surveys conducted x Total cost per survey = *\$343k*

Given the results of every survey and the corresponding probabilities, a decision tree can be developed, which considers all possible combinations for values of y_1, y_2 *and* y_3. The expected profit or loss can be calculated at each probability node and the profit can be maximized at the decision nodes. After every survey S_i, given that y_i out of *n* people vote 'yes', the tree will be used to decide between marketing the product, terminating the product or conducting survey S_{i+1}. After the last survey, only two options remain: terminate or market the product.

3.2 Example Calculations

This section uses a scenario where $y_1 = 10, y_2 = 8$ and $y_3 = 20$ as an example, to solve for the market-share probabilities after every survey.

- With no new information, the underlying probability is shown in Table 2.

Table 2: Market share probabilities with no new information.

Market Share $M = m$	$P(M = m)$
10%	0.25
20%	0.50
30%	0.25

- Let 10/55 people vote 'yes' in Phase 1. The market share probabilities are updated as shown below.

Table 3: Market share probabilities given results of Survey 1.

Given $S_1 = 10$				
A	B	C	D	E

Market Share $M = m$	Prior Probability $P(M = m)$	Likelihood $P(S_1 = 10\|M)$	Joint Probability $P(S_1 = 10, M = m)$ B x C	Posterior Probability $P(M = m\|S_1)$ D ÷ ∑D
10%	0.25	${}^{55}C_{10}0.1^{10}0.9^{55-10} = 0.026$	0.0065	0.086
20%	0.50	${}^{55}C_{10}0.2^{10}0.8^{55-10} = 0.130$	0.0650	0.860
30%	0.25	${}^{55}C_{10}0.3^{10}0.7^{55-10} = 0.018$	0.0045	0.059

- Let 8/55 people vote 'yes' in Phase 2. The market share probabilities are updated as shown below.

Table 4: Market share probabilities given results of Survey 2.

Given $S_1 = 10, S_2 = 8$				
A	B	C	D	E
Market Share $M = m$	Prior Probability $P(M = m\|S_1)$	Likelihood $P(S_2 = 8\|S_1, M) = P(S_2 = 8\|M)$ (Conditional Independence)	Joint Probability $P(S_2 = 8, M = m\|S_1)$ B x C	Posterior Probability $P(M = m\|S_1, S_2)$ D ÷ ∑D
10%	0.086	${}^{55}C_{8}0.1^{8}0.9^{55-8} = 0.086$	0.0074	0.090
20%	0.860	${}^{55}C_{8}0.2^{8}0.8^{55-8} = 0.087$	0.0748	0.907
30%	0.059	${}^{55}C_{8}0.3^{8}0.7^{55-8} = 0.0042$	0.00025	0.003

- Let 20 people vote 'yes' in Phase 3. The market share probabilities are updated as shown below.

Table 5: Market share probabilities given results of Survey 3.

Given $S_1 = 10, S_2 = 8, S_3 = 20$				
A	B	C	D	E
Market Share $M = m$	Prior Probability $P(M = m\|S_1, S_2)$	Likelihood $P(S_3 = 20\|S_1, S_2, M) = P(S_3 = 20\|M)$ (Conditional Independence)	Joint Probability $P(S_3 = 20, M = m\|S_1, S_2)$ B x C	Posterior Probability $P(M = m\|S_1, S_2, S_3)$ D ÷ ∑D
10%	0.090	${}^{55}C_{20}0.1^{20}0.9^{55-20} = 0.0000$	0.00000	0.000
20%	0.907	${}^{55}C_{20}0.2^{20}0.8^{55-20} = 0.0021$	0.00195	0.904
30%	0.003	${}^{55}C_{20}0.3^{20}0.7^{55-20} = 0.0667$	0.00021	0.096

Table 6 shows the expected market share, the expected profit if product is marketed, and the cost of terminating after each phase.

Table 6: Cost of marketing and terminating at each phase.

	Expected Market Share (%)	Expected Profit if Marketed ($)	Loss if Terminated ($)
Before Phase 1	10%(0.25)+20%(0.5)+30%(0.25) = 20%	(0.2)($5.19)(48000)-$37440-0 =$12,284	$0

After Phase 1	10%(0.086)+20%(0.86)+30%(0.059) = 19.83%	(0.1983)($5.19)(48000)-$37440-$200 =$11,760.5	$200
After Phase 2	10%(0.09)+20%(0.907)+30%(0.003) = 19.13%	(0.1913)($5.19)(48000)-$37440-$400 =$9,816.7	$400
After Phase 3	10%(0.0)+20%(0.904)+30%(0.096) = 20.96%	(0.2096)($5.19)(48000)-$37440-$600 =$14,175.5	$600

3.3 Decision Tree

The decision tree used for the analysis is shown in Figure 2 below. The tree includes all possible outcomes and probabilities with corresponding profits. Expected values/utilities are calculated to provide the user a decision that will maximize their profit or utility, and thus providing a viable decision route they should use, given they accept the randomness of variables involved in constructing the decision tree.

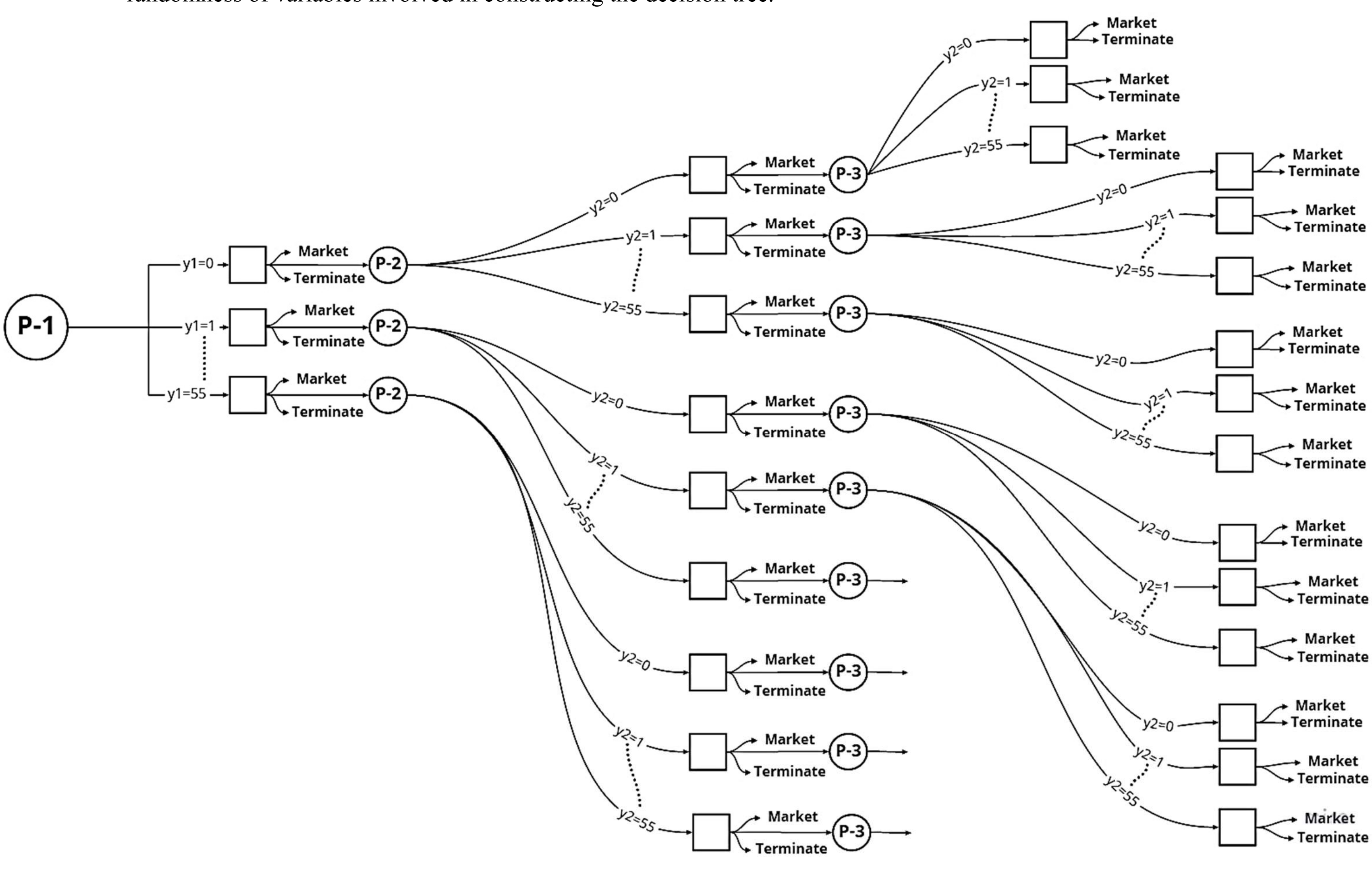

Figure 2: Decision Tree for the Problem

3.4 Solving the Decision Tree

The decision tree was solved to maximize expected value and expected utility at every decision node using Matlab. The Matlab code and the output data can be found in the Appendix.

4.0 Policy Decision Making

Strategies for maximizing expected value and expected utility were derived from the output data in the Appendix.

4.1 Maximizing Expected Value

After Phase 1 surveys:

- Terminate the product if fewer than 5 people vote ‘yes’.
- Continue with Phase 2 surveys if at least 5 people and fewer than 11 people vote ‘yes’.
- Market the product if at least 11 people vote ‘yes’.

After Phase 2 surveys:

- **Terminate the product if:**
  - 5 people voted ‘yes’ in Phase 1 and fewer than 9 people vote ‘yes’ in Phase 2.
  - 6 people voted ‘yes’ in Phase 1 and fewer than 8 people vote ‘yes’ in Phase 2.
  - 7 people voted ‘yes’ in Phase 1 and fewer than 7 people vote ‘yes’ in Phase 2.
  - 8 people voted ‘yes’ in Phase 1 and fewer than 6 people vote ‘yes’ in Phase 2.
  - 9 people voted ‘yes’ in Phase 1 and fewer than 5 people vote ‘yes’ in Phase 2.
  - 10 people voted ‘yes’ in Phase 1 and fewer than 4 people vote ‘yes’ in Phase 2.

- **Continue with Phase 3 surveys if:**
  - 5 people voted ‘yes’ in Phase 1 and at least 9 and fewer than 14 people vote ‘yes’ in Phase 2.
  - 6 people voted ‘yes’ in Phase 1 and at least 8 and fewer than 13 people vote ‘yes’ in Phase 2.
  - 7 people voted ‘yes’ in Phase 1 and at least 7 and fewer than 12 people vote ‘yes’ in Phase 2.
  - 8 people voted ‘yes’ in Phase 1 and at least 6 and fewer than 11 people vote ‘yes’ in Phase 2.
  - 9 people voted ‘yes’ in Phase 1 and at least 5 and fewer than 10 people vote ‘yes’ in Phase 2.
  - 10 people voted ‘yes’ in Phase 1 and at least 4 and fewer than 9 people vote ‘yes’ in Phase 2.

- **Market the product if:**
  - 5 people voted ‘yes’ in Phase 1 and at least 14 people vote ‘yes’ in Phase 2.
  - 6 people voted ‘yes’ in Phase 1 and at least 13 people vote ‘yes’ in Phase 2.
  - 7 people voted ‘yes’ in Phase 1 and at least 12 people vote ‘yes’ in Phase 2.
  - 8 people voted ‘yes’ in Phase 1 and at least 11 people vote ‘yes’ in Phase 2.
  - 9 people voted ‘yes’ in Phase 1 and at least 10 people vote ‘yes’ in Phase 2.
  - 10 people voted ‘yes’ in Phase 1 and at least 9 people vote ‘yes’ in Phase 2.

After Phase 3 surveys:

- **Terminate the product if:**
  - 5 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 10 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 11 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 12 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 12 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 13 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
  - 6 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.

- 6 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
- 6 people voted ‘yes’ in Phase 1, 10 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
- 6 people voted ‘yes’ in Phase 1, 11 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
- 6 people voted ‘yes’ in Phase 1, 12 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
- 7 people voted ‘yes’ in Phase 1, 7 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
- 7 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
- 7 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
- 7 people voted ‘yes’ in Phase 1, 10 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
- 7 people voted ‘yes’ in Phase 1, 11 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
- 8 people voted ‘yes’ in Phase 1, 6 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
- 8 people voted ‘yes’ in Phase 1, 7 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
- 8 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
- 8 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
- 8 people voted ‘yes’ in Phase 1, 10 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
- 9 people voted ‘yes’ in Phase 1, 5 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
- 9 people voted ‘yes’ in Phase 1, 6 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
- 9 people voted ‘yes’ in Phase 1, 7 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
- 9 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
- 9 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
- 10 people voted ‘yes’ in Phase 1, 4 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
- 10 people voted ‘yes’ in Phase 1, 5 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
- 10 people voted ‘yes’ in Phase 1, 6 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
- 10 people voted ‘yes’ in Phase 1, 7 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
- 10 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.

- After Phase 3 surveys, if the results do not fall under the above termination conditions, market the product.

4.2 Maximizing Expected Utility

After Phase 1 surveys:

- Terminate the product if fewer than 5 people vote ‘yes’.
- Continue with Phase 2 surveys if at least 5 people and fewer than 12 people vote ‘yes’.
- Market the product if at least 12 people vote ‘yes’.

After Phase 2 surveys:

- **Terminate the product if:**
- 5 people voted ‘yes’ in Phase 1 and fewer than 9 people vote ‘yes’ in Phase 2.
- 6 people voted ‘yes’ in Phase 1 and fewer than 8 people vote ‘yes’ in Phase 2.
- 7 people voted ‘yes’ in Phase 1 and fewer than 7 people vote ‘yes’ in Phase 2.
- 8 people voted ‘yes’ in Phase 1 and fewer than 6 people vote ‘yes’ in Phase 2.
- 9 people voted ‘yes’ in Phase 1 and fewer than 5 people vote ‘yes’ in Phase 2.
- 10 people voted ‘yes’ in Phase 1 and fewer than 4 people vote ‘yes’ in Phase 2.
- 11 people voted ‘yes’ in Phase 1 and fewer than 3 people vote ‘yes’ in Phase 2.

- **Continue with Phase 3 surveys if:**
- 5 people voted ‘yes’ in Phase 1 and at least 9 and fewer than 14 people vote ‘yes’ in Phase 2.
- 6 people voted ‘yes’ in Phase 1 and at least 8 and fewer than 13 people vote ‘yes’ in Phase 2.
- 7 people voted ‘yes’ in Phase 1 and at least 7 and fewer than 12 people vote ‘yes’ in Phase 2.
- 8 people voted ‘yes’ in Phase 1 and at least 6 and fewer than 11 people vote ‘yes’ in Phase 2.
- 9 people voted ‘yes’ in Phase 1 and at least 5 and fewer than 10 people vote ‘yes’ in Phase 2.
- 10 people voted ‘yes’ in Phase 1 and at least 4 and fewer than 9 people vote ‘yes’ in Phase 2.

- **Market the product if:**
  - 5 people voted ‘yes’ in Phase 1 and at least 14 people vote ‘yes’ in Phase 2.
  - 6 people voted ‘yes’ in Phase 1 and at least 13 people vote ‘yes’ in Phase 2.
  - 7 people voted ‘yes’ in Phase 1 and at least 12 people vote ‘yes’ in Phase 2.
  - 8 people voted ‘yes’ in Phase 1 and at least 11 people vote ‘yes’ in Phase 2.
  - 9 people voted ‘yes’ in Phase 1 and at least 10 people vote ‘yes’ in Phase 2.
  - 10 people voted ‘yes’ in Phase 1 and at least 9 people vote ‘yes’ in Phase 2.

After Phase 3 surveys:

- **Terminate the product if:**
  - 5 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 10 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 11 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 12 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 12 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 5 people voted ‘yes’ in Phase 1, 13 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
  - 6 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
  - 6 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
  - 6 people voted ‘yes’ in Phase 1, 10 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
  - 6 people voted ‘yes’ in Phase 1, 11 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 6 people voted ‘yes’ in Phase 1, 12 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
  - 7 people voted ‘yes’ in Phase 1, 7 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
  - 7 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
  - 7 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
  - 7 people voted ‘yes’ in Phase 1, 10 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 7 people voted ‘yes’ in Phase 1, 11 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
  - 8 people voted ‘yes’ in Phase 1, 6 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
  - 8 people voted ‘yes’ in Phase 1, 7 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
  - 8 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
  - 8 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 8 people voted ‘yes’ in Phase 1, 10 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
  - 9 people voted ‘yes’ in Phase 1, 5 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
  - 9 people voted ‘yes’ in Phase 1, 6 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
  - 9 people voted ‘yes’ in Phase 1, 7 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
  - 9 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 9 people voted ‘yes’ in Phase 1, 9 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
  - 10 people voted ‘yes’ in Phase 1, 4 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
  - 10 people voted ‘yes’ in Phase 1, 5 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
  - 10 people voted ‘yes’ in Phase 1, 6 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
  - 10 people voted ‘yes’ in Phase 1, 7 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 10 people voted ‘yes’ in Phase 1, 8 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.
  - 11 people voted ‘yes’ in Phase 1, 3 people voted ‘yes’ in Phase 2 and fewer than 11 people vote ‘yes’ in Phase 3.
  - 11 people voted ‘yes’ in Phase 1, 4 people voted ‘yes’ in Phase 2 and fewer than 10 people vote ‘yes’ in Phase 3.
  - 11 people voted ‘yes’ in Phase 1, 5 people voted ‘yes’ in Phase 2 and fewer than 9 people vote ‘yes’ in Phase 3.
  - 11 people voted ‘yes’ in Phase 1, 6 people voted ‘yes’ in Phase 2 and fewer than 8 people vote ‘yes’ in Phase 3.
  - 11 people voted ‘yes’ in Phase 1, 7 people voted ‘yes’ in Phase 2 and fewer than 7 people vote ‘yes’ in Phase 3.

- After Phase 3 surveys, if the results do not fall under the above termination conditions, market the product.

4.3 Utilizing Natural Conjugates

The model's accuracy can be improved if instead of assuming a discrete market share distribution, a continuous normal distribution is used based on the known mean and variance of the discrete distribution. A beta distribution approximation to the normal distribution can then be used as a prior for the market share distribution.

Since the beta distribution is a natural conjugate to the binomial distribution, the posterior probabilities are also beta distributions with updated parameters. This can be used to automatize the calculations as follows:

Let the prior distribution with no information = $\beta\ (r', n')$

Given the results of the first k surveys $S_1, S_2, \ldots, S_k = y_1, y_2, \ldots, y_{k-1}, y_k$, where y_i is the number of 'yes' votes in Phase *i,* and the size of each survey = *n,* the posterior beta distribution can be calculated as follows:

Market share distribution after the k^{th} survey = $\beta\ (r' + y_1 + y_2 + \cdots + y_{k-1} + y_k\ ,\ n' + k * n)$

Therefore, by keeping track of the number of surveys and the number of favorable outcomes per survey, the posterior probability at any branch of the decision tree can be calculated.

5.0 References

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