EECE 7205-Assignment1

Sreejith Sreekumar: 001277209

October 7, 2018

1 Coding

1.1 Insertion Sort

```
#include <iostream>
#include <time.h>


int *get_input(int limit){

   /**
    * Generating input - Worst case happens when the array is sorted in
    * reverse order. Sending numbers from 1000 to limit
    * decrementing by 1 for every element.
    */

  int* input = new int[limit];

  int j = 0;
  for(int i=1000; i>1000-limit; i--){
    input[j] = i;
    j+=1;
  }
  return input;
}


int *insertion_sort(int input[], int limit){

   /**
    * Insertion sort algorithm
    */

  int i, j, temp;
  for (i=0; i< limit; i++){
    for (j=i ;j>=0; j--){
      if(input[j] < input[j-1]){
```

```
        temp = input[j];
        input[j] = input[j-1];
        input[j-1] = temp;
      }
    }
  }
  return input;
}


int main(){

  int limit, i;
  std :: cout <<"Enter limit: ";
  std :: cin>> limit;

  int* input = get_input(limit);

  /*
   * Setting clocks before and after the sorting
   */
  clock_t start = clock();
  int* sorted_array = insertion_sort(input, limit);
  clock_t end = clock();

  double cpu_time_used = ((double) (end - start)) / CLOCKS_PER_SEC;

  std::cout<< "Insertion Sort took %f seconds to finish \n" << cpu_time_used;

}
```

1.2 Merge Sort

```
#include <iostream>
#include <time.h>

int *get_input(int limit){

   /**
    * Generating input - Worst case - Sending numbers from 1000 to limit
    * decrementing by 1 for every element.
    */

  int* input = new int[limit];

  int j = 0;
  for(int i=1000; i>1000-limit; i--){
    input[j] = i;
```

```
    j+=1;
  }
  return input;
}


void copy_merged_array_into_original(int original[], int merged[], int length, int left_low) {

    for (int i=0; i< length; ++i)
        original[left_low++] = merged[i];

}


void merge(int input[], int left_low, int left_high, int right_low, int right_high) {

    int length = right_high-left_low+1;
    int merged_array[length];
    int left = left_low;
    int right = right_low;

    auto left_array_exhausted = [&left, &left_high]() { return left > left_high;};
    auto right_array_exhausted = [&right, &right_high]() { return right > right_high;};

    for (int i = 0; i < length; ++i) {
        if (left_array_exhausted())
            merged_array[i] = input[right++];
        else if (right_array_exhausted())
            merged_array[i] = input[left++];
        else if (input[left] <= input[right])
            merged_array[i] = input[left++];
        else
            merged_array[i] = input[right++];
    }
    copy_merged_array_into_original(input,  merged_array, length, left_low);

}


void merge_sort(int numbers[], int low, int high) {

    if (low >= high)
        return;

    else {

        /*
         * Recursive sorting and merging parts
```

```
         */

        int mid = (low + high)/2;
        merge_sort(numbers, low, mid);
        merge_sort(numbers, mid+1, high);
        merge(numbers, low, mid, mid+1, high);
    }

}


int main(){


    int limit;

    std :: cout << "Enter limit: ";
    std :: cin >> limit;

    int* input = get_input(limit);
    std :: cout << "Sorting using mergesort: ";

  /*
   * Setting clocks before and after the sorting
   */
    clock_t start = clock();
    merge_sort(input, 0, limit-1);
    clock_t end = clock();

    double cpu_time_used = ((double) (end - start)) / CLOCKS_PER_SEC;
    cout << "Merge Sort took %f seconds to finish \n" << cpu_time_used;

}
```

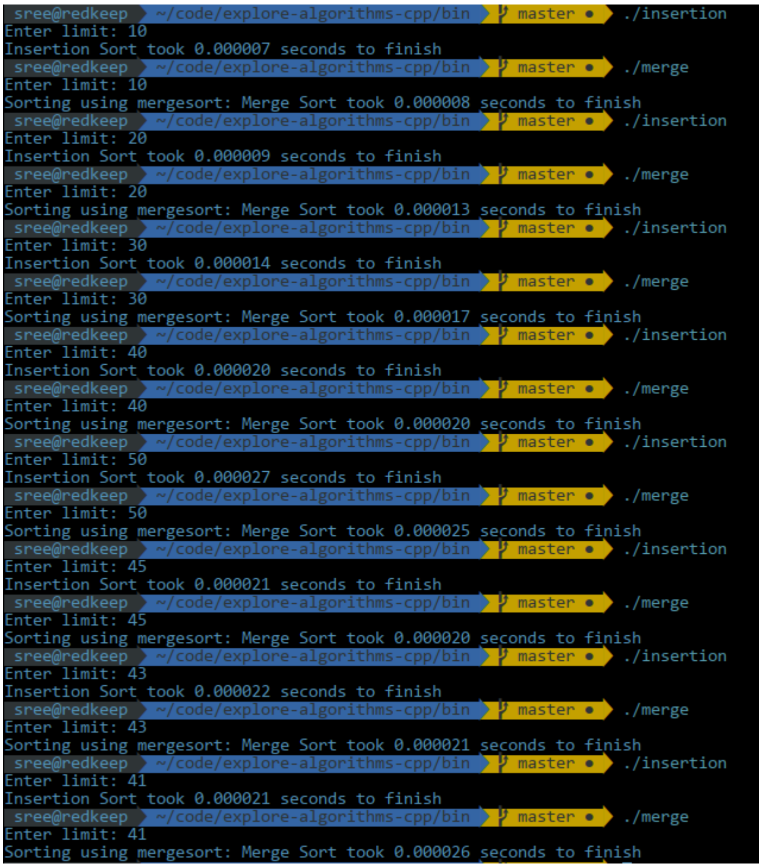


Figure 1: Sorting Algorithms - Running time experiments

1.3 Worst Case Running Times

Experimenting using the above code fragments which sorts using insertion and mergesort for different values of 'n' at their worst cases, we have: at **n=43**, mergesort starts to beat insertion sort (ref. Figure 1).

2 Arrangement of Elements during Sorting

2.1 Insertion Sort

```
Input: {10, 5, 7, 9, 8, 3}
Iteration 1: {5, 10, 7, 9, 8, 3}
Iteration 2: {5, 7, 10, 9, 8, 3}
Iteration 3: {5, 7, 9, 10, 8, 3}
Iteration 4: {5, 7, 9, 8, 10, 3}
             {5, 7, 8, 9, 10, 3}
Iteration 5: {5, 7, 8, 9, 3, 10}
             {5, 7, 8, 3, 9, 10}
             {5, 7, 3, 8, 9, 10}
```

```
           {5, 3, 7, 8, 9, 10}
           {3, 5, 7, 8, 9, 10}

Final    : {3, 5, 7, 8, 9, 10}
```

2.2 Quicksort

Input: 10, 5, 7, 9, 8, 3

```
Pivot: 10

Iteration 1: {->10, 5, 7, 9, 8, 3}
             Exchange Pivot
              {3, 5, 7, 9, 8, 10}
Iteration 2: {->3, 5, 7, 9, 8, 10}
Iteration 3: {3, ->5, 7, 9, 8, 10}
Iteration 4: {3, ->5, 7, 9, 8, 10}
Iteration 5: {3, 5, ->7, 9, 8, 10}
Iteration 6: {3, 5, 7, ->9, 8, 10}
             Exchange Pivot
             {3, 5, 7, 8, 9, 10}

Final      : {3, 5, 7, 8, 9, 10}
```

3 True or False

- n + 3 $\in \Omega$(n) - True
- n + 3 $\in \mathcal{O}(n^2)$ - True
- n + 3 $\in \Theta(n^2)$ - False
- $2^{n+1} \in \mathcal{O}$(n + 1) - False
- $2^{n+1} \in \Theta(2^n)$ - True

4 Master Method

- T(n) = 8T($\frac{n}{2}$) + n

  Here a = 8, b = 2 and f(n) = n
  Considering Case I of the Master Method, i.e $\mathcal{O}(n^{\log_b a-\varepsilon})$ and substituting for a and b, we have:

  f(n) = n = $\mathcal{O}(n^{\log_2 8-\varepsilon}) = \mathcal{O}(n^{3-\varepsilon})$

  For $\varepsilon = 1$, we have f(n) = $\mathcal{O}(n^2)$, which satisfies Case I

  $\therefore$ T(n) = $\Theta(n^{\log_b a}) = \Theta(n^3)$

- T(n) = 8T($\frac{n}{2}$) + n^2

  Here a = 8, b = 2 and f(n) = n^2
  Considering Case I of the Master Method, i.e $\mathcal{O}(n^{\log_b a - \varepsilon})$ and substituting for a and b, we have:

  f(n) = n^2 = $\mathcal{O}(n^{\log_2 8 - \varepsilon})$ = $\mathcal{O}(n^{3-\varepsilon})$, which satisfies Case I

  $\therefore$ T(n) = $\Theta(n^{\log_b a})$ = $\Theta(n^3)$

- T(n) = 8T($\frac{n}{2}$) + n^3

  Here a = 8, b = 2 and f(n) = n^3
  Considering Case II of the Master Method, i.e $\Theta(n^{\log_b a})$ and substituting for a and b, we have:

  f(n) = n^3 = $\Theta(n^{\log_2 8})$ = $\Theta(n^3)$, which satisfies Case II

  $\therefore$ T(n) = $\Theta(n^{\log_b a}.logn)$ = $\Theta(n^3 logn)$

- T(n) = 8T($\frac{n}{2}$) + n^4

  Here a = 8, b = 2 and f(n) = n^4
  Considering Case III of the Master Method, i,e $\Omega(n^{\log_b a + \varepsilon})$ and substituting for a and b, we have:

  $$f(n) = n^4 = \Omega(n^{\log_2 8 + \varepsilon}) = \Omega(n^{3+\varepsilon}) \tag{1}$$

  and,

  $$8.f(\frac{n}{2}) \leq (1 - \varepsilon').n^4 \tag{2}$$

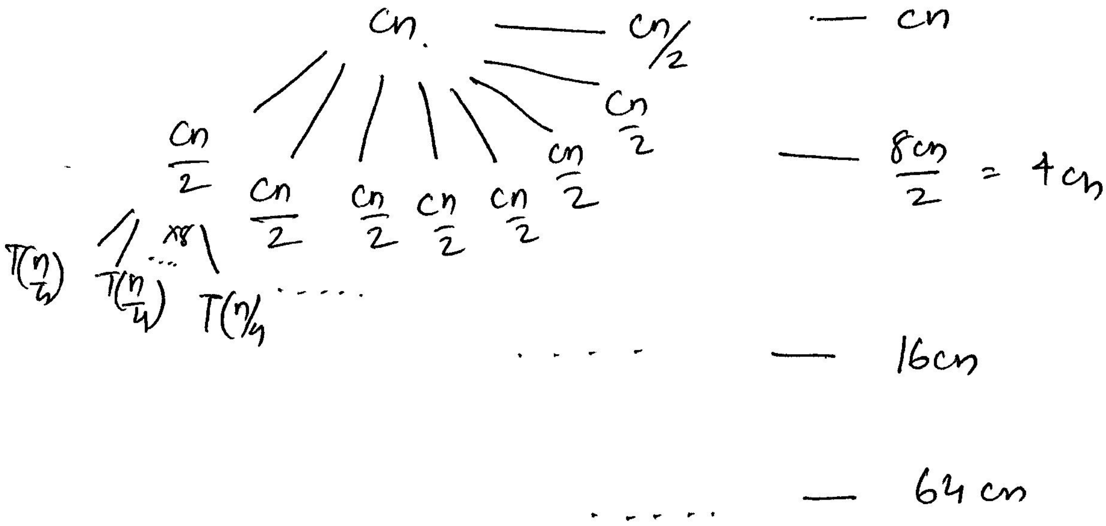
  $$\implies 8.\frac{n^4}{16} \leq (1 - \varepsilon').n^4 \tag{3}$$

  $$\implies \frac{n^4}{2} \leq (1 - \varepsilon').n^4 \tag{4}$$

  (1) is true for many $\varepsilon' s > 1$ and (4) is true for $0 < \varepsilon' < 0.5$
  From (1) and (4) Master Method Case III is satisfied.
  $\therefore$ T(n) = Θ(f(n)) = $\Theta(n^4)$

(Happy Halloween)

5 Recursion Tree



Height of the tree = $\log_2 n$

Leaves of the recursion tree = $n^{\log_2 8}$

$= \underline{\underline{n^3}}$

General expression for the recursion tree

$$= \left[1 + 4 + 4^2 + \dots 4^{\log n}\right] + n^3$$

$$= \underline{\underline{\frac{1 - 4^{\log_2 n}}{3}}} + n^3 = \underline{\underline{O(n^3)}}$$

Proof by substitution method:-

Assume $T(n) = O(n^3)$

$$T(n) \leq c_1 . k^3.$$

$$T(n) = 8T\left(\frac{n}{2}\right) + n.$$

$$\leq 8\cancel{c}\frac{k^3}{8} + n.$$

$$\leq ck^3 + n$$

$$\leq cn^3 + n.$$

Since the condition holds for $c=1$, we can say the time complexity is upper bounded by $O(n^3)$