

EECE 7205-Project 1

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1 Problem Description

1.1 Input

A one dimensional array, A.

1.2 Problem

Partition A into M groups, where each group is given by G[1...M] (i.e 1st group consists of G[1], 2nd group consists of G[2] etc.). in such a way that we maximize the minimum of the sum of these groups.

1.3 Output

The optimal indices at which A could be split into M groups so that we have the maximized the minimum of the group sums.

2 Pseudo Codes

3 Asymptotic Analysis of the Running Time

Analyzing the program we have:

Construction of the C Matrix, with the formula:

$$\max_{(j-1 <= k < i)} [\min(C[k; j-1], \sum_{m=k+1}^i A[m])]$$

we can see that the program requires two loops that iterate *splits* - M times and *data_size* - N times. for calculating the M x N elements

i.e, Worst case time complexity contributed by the outer loops = M * N

Now, for calculating every element, there is an inner iteration, which computes the maximum of a group of elements. This is marked by variable *k* in the code. In worst case the value of *k* will be N-1.

In addition for calculating the *min* part of the formula, there is yet another inner loop (for the second section), marked by the variable *m*, which in the worst case iterates N-1 times.

So, considering these steps together, we can say that the total time complexity of the fragment in the worst case is:

$$M * N * (N - 1) * (N - 1) \approx M * N * N * N \approx M * N^3$$

Hence, the time complexity of the code written is: $M * N^3$

4 Results

4.1 Run 1

```
~/code/explore-algorithms-cpp/src/project1/src $ ./a.out
```

Enter the number of elements: 12

Enter the array: 3 9 7 8 2 6 5 10 1 7 6 5

Enter number of arrays to split into: 3

C Matrix

| | | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|----|
| 3 | 12 | 19 | 27 | 29 | 35 | 40 | 50 | 51 | 58 | 64 | 69 |
| 0 | 3 | 7 | 12 | 12 | 16 | 19 | 23 | 24 | 29 | 29 | 34 |
| 0 | 0 | 3 | 7 | 7 | 8 | 12 | 15 | 16 | 18 | 19 | 19 |

Parent_k Matrix

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 5 | 5 | 5 |
| 0 | 0 | 2 | 3 | 3 | 3 | 4 | 5 | 6 | 6 | 7 | 7 |

Finding the number of elements to be present in each group

3 4 5

Minimum of the sum of groups: 19

4.2 Run 2

```
~/code/explore-algorithms-cpp/src/project1/src $ ./a.out
```

Enter the number of elements: 12

Enter the array: 3 9 7 8 2 6 5 10 1 7 6 5

Enter number of arrays to split into: 4

C Matrix

| | | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|----|
| 3 | 12 | 19 | 27 | 29 | 35 | 40 | 50 | 51 | 58 | 64 | 69 |
| 0 | 3 | 7 | 12 | 12 | 16 | 19 | 23 | 24 | 29 | 29 | 34 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|----|----|----|----|----|----|
| 0 | 0 | 3 | 7 | 7 | 8 | 12 | 15 | 16 | 18 | 19 | 19 |
| 0 | 0 | 0 | 3 | 3 | 7 | 7 | 10 | 11 | 12 | 14 | 16 |

Parent_k Matrix

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 5 | 5 | 5 |
| 0 | 0 | 2 | 3 | 3 | 3 | 4 | 5 | 6 | 6 | 7 | 7 |
| 0 | 0 | 0 | 3 | 3 | 4 | 4 | 6 | 6 | 7 | 7 | 9 |

Finding the number of elements to be present in each group
3 3 3 3

Minimum of the sum of groups: 16

4.3 Run 3

Enter the number of elements: 7

Enter the array: 1 2 3 4 5 6 7

Enter number of arrays to split into:4

```
C Matrix
1 3 6 10 15 21 28
0 1 3 4 6 10 13
0 0 1 3 4 6 7
0 0 0 1 3 4 6
```

```
Parent_k Matrix
0 0 0 0 0 0 0
0 1 2 2 3 4 4
0 0 2 3 4 5 5
0 0 0 3 4 5 6
```

Finding the number of elements to be present in each group
4 1 1 1

Minimum of the sum of groups: 6

5 Source Code

```
#include <iostream>
#include <algorithm>
#include <vector>

/**
 * Find the index of an element in a vector
 */
int find_index(std::vector<int> max_group, int max_element){
    int index;
    for(int i=0;i<max_group.size();i++){
        if(max_group[i] == max_element){
            index = i;
            break;
        }
    }
    return index;
}

int main(){
```

```

int splits, data_size;

/**
 * Collect the inputs from the user
 */
std::cout << "Enter the number of elements: ";
std::cin >> data_size;

int data[data_size];
std::cout << "\n" << "Enter the array: ";

for (int idx=0; idx < data_size; idx++)
    std::cin >> data[idx];

std::cout << "Enter number of arrays to split into: ";
std::cin >> splits;

/**
 * Construction of the double array C[i][j]
 * and the matrix to which it
 */
int C[splits][data_size];
int parent_k[splits][data_size];

for(int i=0; i<splits; i++){
    for(int j=0; j<data_size; j++){
        C[i][j] = 0;
        parent_k[i][j] = 0;
    }
}

/**
 * Constructing the first row of the matrix
 * as the sum of elements in the data array
 * until the current index (including it)
 */
C[0][0] = data[0];
for(int k=1; k<data_size; k++){
    C[0][k] = C[0][k-1] + data[k];
}

/**
 * Initialize the first column of every row
 * in the C Matrix to 0
 */
for(int k=1; k<splits; k++){
    C[k][0] = 0;
}

```

```

/**
 * Calculation for other elements
 *
 */
for(int j=1; j<splits; j++){

    for(int i=j; i<data_size; i++){

        /**
         * Store the min values from every k
         */
        std::vector<int> max_group;

        std::vector<int> c_group;
        std::vector<int> sum_group;

        for(int k=j-1; k<i; k++){

            /**
             * First element in the
             * min part of the formula
             */
            int element1 = C[j-1][k];

            /**
             * Second element in the
             * min part of the formula
             */
            int sum_element = 0;
            for(int m=k+1; m<=i; m++)
                sum_element += data[m];

            /**
             * Choose the minimal element from the two elements
             */
            c_group.push_back(element1);
            sum_group.push_back(sum_element);

            int min_element = element1 < sum_element ? element1 : sum_element;

            /**
             * Push back the value into the vector
             */
            max_group.push_back(min_element);
        }

    }

}

```

```

    * Maximum element in this iteration
    */
    int max_element = *std::max_element(max_group.begin(), max_group.end());

    /**
    * Find the index of the maximum element and figure out
    * which element contributed to it.
    *
    * if it is c[k;j-1], use k which brought the largest element as value
    * of parent_k[j][i]
    *
    * if it is the sum element, use k-1 as value for parent_k[j][i]
    */
    int _index = find_index(max_group, max_element);
    int index;
    if(c_group[_index] == max_element){
        index = _index;
    } else {
        index = _index - 1;
    }

    max_group.clear();
    C[j][i] = max_element;
    parent_k[j][i] = index + j;

}
}

std::cout<<"\n C Matrix \n";
for(int x=0;x<splits;x++){
    for(int y=0;y<data_size;y++){
        std::cout<<C[x][y]<<"\t";
    }
    std::cout<<"\n";
}

std::cout<<"\n Parent_k Matrix \n";
for(int x=0;x<splits;x++){
    for(int y=0;y<data_size;y++){
        std::cout<<parent_k[x][y]<<"\t";
    }
    std::cout<<"\n";
}

std::cout<<"\n"<<"Finding the number of elements to be present in each group\n";

std::vector<int> increments_in_groups;
increments_in_groups.push_back(data_size);

```

```

int tmp_index = data_size-1;
for(int k=splits-1; k>0; k--){
    int next_index = parent_k[k][tmp_index];
    increments_in_groups.push_back(next_index);
    tmp_index = next_index;
}

std::reverse(increments_in_groups.begin(),increments_in_groups.end());

std::vector<int> groups;
groups.push_back(increments_in_groups[0]);
for(int i=1;i<increments_in_groups.size();i++){
    groups.push_back(increments_in_groups[i] - increments_in_groups[i-1]);
}

/**
 * Number of elements in each group
 */
for(int i=0;i<groups.size();i++){
    std::cout<<groups[i]<<"\t";
}
std::cout<<"\n";

/**
 * Minimum of the sum of the groups
 */
std::cout<<"Minimum of the sum of groups: "<<C[splits - 1][data_size - 1]<<"\n";

```