Maximum likelihood estimation of normal Distributions

Process

- $L(\mu, \sigma \mid x_1, x_2, x_3 \ldots x_n) = L(\mu, \sigma \mid x_1) * L(\mu, \sigma \mid x_2) * L(\mu, \sigma \mid x_n)$

Take 'ln' on both sides

$$\ln(L(\mu, \sigma \mid x_1, x_2, x_3 \ldots x_n)) = \ln(L(\mu, \sigma \mid x_1)) + \ln(L(\mu, \sigma \mid x_2)) + \cdots \ln(L(\mu, \sigma \mid x_n))$$

$$\ln(L(\mu, \sigma \mid x_1)) = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right)$$

$$\Rightarrow \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) + -\frac{(x_1-\mu)^2}{2\sigma^2} \ln(e)$$

$$\Rightarrow \ln\left((2\pi\sigma^2)^{-1/2}\right) + -\frac{(x_1-\mu)^2}{2\sigma^2} \ln(e)$$

$$\Rightarrow -\frac{1}{2} \ln(2\pi\sigma^2) - \frac{(x_1-\mu^2)}{2\sigma^2}$$

$$\Rightarrow -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln(\sigma^2) - \frac{(x_1-\mu)^2}{2\sigma^2}$$

$$\Rightarrow -\frac{1}{2} \ln(2\pi) - \ln(\sigma)^2 - \frac{(x_1-\mu)^2}{2\sigma^2}$$

~~Now take derivaten~~

For all terms :-

$$\frac{d}{}\quad \underbrace{-\left[\frac{n}{2}\ln(2\pi)\right]}_{1}\ \underbrace{-\left[n\ln\sigma\right]}_{2}\ \underbrace{-\frac{(x_1-\mu)^2}{2\sigma^2}-\ldots-\frac{(x_n-\mu)^2}{2\sigma^2}}_{3}$$

Now take derivatives :-

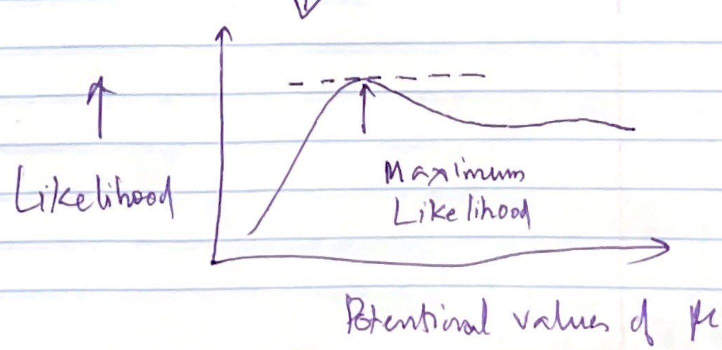
$$\frac{\partial}{\partial\mu}\ln\left[L(\mu,\sigma\,|\,x_1, x_2 \ldots x_n)\right]$$

$$\frac{\partial}{\partial\mu}(1) = 0 \qquad \frac{d}{\partial\mu}(2) = 0$$

$$\frac{\partial}{\partial\mu}(3) = \frac{2(x_1-\mu)}{2\sigma^2} + \frac{(x_2-\mu)}{\sigma^2} + \ldots \frac{(x_n-\mu)}{\sigma^2}$$

$$\left[\frac{\partial L}{\partial\mu} = \frac{1}{\sigma^2}\left[(x_1+x_2+\ldots x_n) - n\mu\right]\right]$$

Similarly find $\left[\frac{\partial L}{\partial\sigma}\right]$



$$\frac{\partial}{\partial \mu} \ln [L(\mu, \sigma \mid x_1, x_2 \dots x_n)]$$

$$= \frac{1}{\sigma^2} [(x_1 + x_2 + \dots x_n) - n\mu] \quad \text{—(1)}$$

$$\frac{\partial}{\partial \sigma} \ln [L(\mu, \sigma \mid x_1, x_2 \dots x_n)]$$

$$= \frac{-n}{\sigma} + \frac{1}{\sigma^3} [(x_1 - \mu)^2 + \dots + (x_n - \mu)^2] \quad \text{—(2)}$$

At the peak of the likelihood curve, slope = 0.

From (1)

$$\frac{1}{\sigma^2} [(x_1 + x_2 + \dots x_n) - n\mu] = 0$$

$$\left[\frac{x_1 + x_2 + \dots x_n}{n} = \mu \right]$$

From (2)

$$\frac{-n}{\sigma} + \frac{1}{\sigma^3} [(x_1 - \mu)^2 + \dots (x_n - \mu)^2] = 0$$

$$[(x_1 - \mu)^2 + \dots + (x_n - \mu)^2] = n\sigma^2$$

$$\left[\sigma^2 = \frac{(x_1 - \mu)^2 + \dots (x_n - \mu)^2}{n} \right]$$