Coin Toss - Bayesian Inference

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1 Objective

Objective: Identify the shift in probability distribution of random variable in a coin toss experiment before and after including information from the experiment.

2 The beginning

We do not have any prior information on the "biasness" of the coin used. So let us assume that there is equal probability of having any real number between 0 and 1 as the probability that the coin lands a head.

It has unknown probability of coming up heads. We express the ignorance in prior by a uniform distribution The probability function is U[0,1]

$$f(\theta) = I_{[0<=\theta<=1]}$$

3 Modeling

From Bayes Rule:

$$f(\theta|y) = \frac{f(y|\theta)f(\theta)}{\int f(y|\theta)f(\theta)d\theta} \qquad i,e. \quad \frac{(likelihood * prior)}{normalizing\ constant}$$

Coin toss follows a bernoulli distribution. Considering θ is the probability of getting a head

$$Likelihood, f(y|\theta) = \theta^1(1-\theta)^0$$

4 Experiment

Let's say we observe a head in the first toss.

Posterior probability of θ (after having seen the head)

$$f(\theta|y=1) = \frac{\theta^1(1-\theta)^0 * I_{[0<=\theta<=1]}}{\int_{-\infty}^{\infty} \theta^1(1-\theta)^0 * I_{[0<=\theta<=1]}d\theta}$$

$$= \frac{\theta I_{[0<=\theta<=1]}}{\int_0^1 \theta d\theta}$$

$$= 2\theta I_{[0<=\theta<=1]}$$

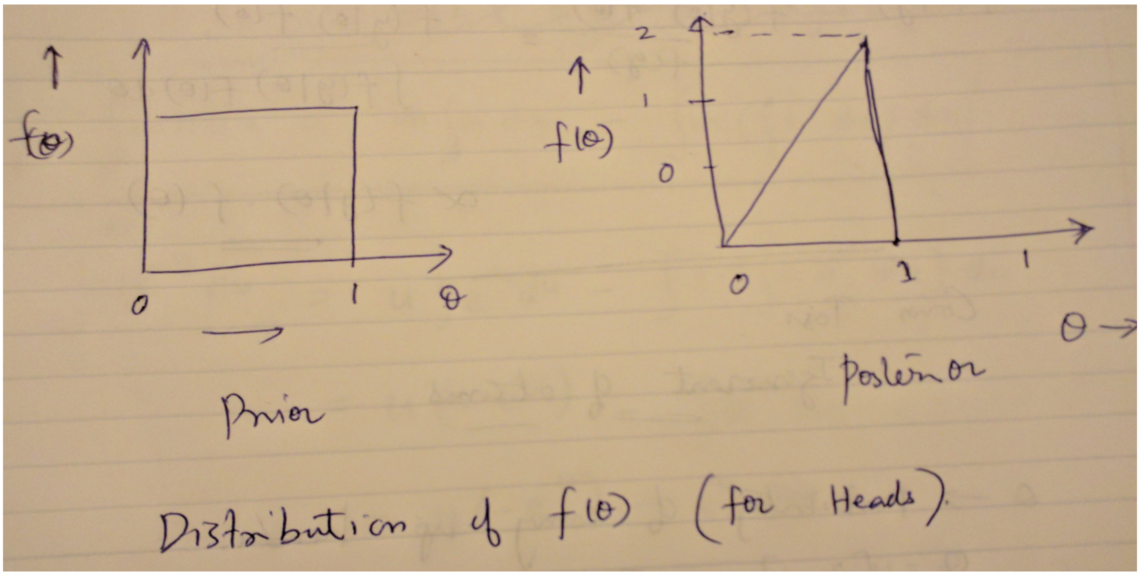
$$f(\theta|y) \propto f(y|\theta) * f(\theta) \propto \theta I_{[0<=\theta<=1]} = 2\theta I_{[0<=\theta<=1]}$$

also, Posterior probability of 1 - θ (landing a tail)

$$f(\theta|y=1) = 2(1-\theta)I_{[0<=\theta<=1]}$$

5 What do we do with this?

Prior and posterior (after observing a head in the first toss) probability distribution of θ before experiments



- From Prior Distribution, calculating the Probability Density Function:

$$P(.025 < \theta < 0.975) = \int_{.025}^{.975} d\theta = .95$$

$$P(\theta > 0.05) = \int_{.05}^{1} d\theta = .95$$

- From Posterior Distribution, calculating the Probability Density Function:

$$Here, f(\theta) = 2\theta I_{[0<=\theta<=1]}$$

$$Hence,\ \ P(\theta) = \theta^2$$

$$P(.025 < \theta < 0.975) = \int_{.025}^{.975} 2\theta d\theta = .95$$

No changes here even after including additional information.

However,

$$P(\theta > 0.05) = \int_{.05}^{1} 2\theta d\theta = .9975$$

i,e The PDF of the random variable θ for θ greater than 0.05 has shifted when we include the information from the experiment