Prior Predictive on Binomial Distribution

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A prior is a cumulative distribution function for the parameter.

P($\theta \leq$ c) for all c in $\mathbb{R}$

Generally if one has enough data, the information in the data will overwhelm the invasion of prior. And so it, prior is not particularly important in terms of what you get for the posterior. Any reasonable choice of prior will lead to approximately the same posterior.

Cases of Exception:

Setting $\theta = \frac{1}{2}$

f(θ | y) $\propto$ f(y | θ)f(θ) $\propto$ f(θ)

the data doesn't depend on θ anymore - something like a dirac delta function.

Predictive Intervals - With a certain confidence in the value of θ we can say that **y**, the data should fall in a certain range with a certain amount of confidence.

$$f(y) = \int f(y|\theta)f(\theta)d\theta = \int f(y, \theta)d\theta$$

1 Prior Predictive

Let the sum of the number of heads in a sequence of 10 coin flips be X.
Hence:

$$X = \sum_{i=1}^{10} Y_i$$

Assuming that the θ (the probability of getting a head) is equally distributed between the interval [0, 1], we have

$$f(\theta) = I_{\{0 \leq \theta \leq 1\}}$$

$$f(X) = \int f(X|\theta)f(\theta)d\theta$$

f(X|θ) follows a binomial density

$$f(X) = \int_0^1 \frac{10!}{X!(10-X)!}\theta^X(1-\theta)^{10-X}(1)d\theta$$

Now,

$$n! = \Gamma(n+1)$$

and

$$z = Beta(\alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)}.z^{\alpha-1}(1-z)^{\beta-1}$$

Rearraging f(x) in the form of a beta distribution

$$f(X) = \int_0^1 \frac{\Gamma(11)}{\Gamma(X+1)\Gamma(11-X)}.\theta^{(X+1)-1}(1-\theta)^{(1-X)-1}d\theta$$

$$= \frac{\Gamma(11)}{\Gamma(12)} \int_0^1 \frac{\Gamma(12)}{\Gamma(X+1)\Gamma(11-X)}.\theta^{(X+1)-1}(1-\theta)^{(1-X)-1}d\theta$$

Beta Density is a PDF, it integrates to 1

$$= \frac{\Gamma(10)}{\Gamma(11)}$$

$$= \frac{1}{11}\forall 0, 1, 2...10$$

i,e all X outcomes are equally likely