Posterior Predictive Distribution on Coin Toss

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After an experiment, posterior distribution for the prediction of y_2 depends on the knowledge of θ and the outcome of the first toss experiment

$$f(y_2|y_1) = \int f(y_2|\theta, y_1) f(\theta|y_1) d\theta$$

The experiment follows an IID, therefore y2 is not dependent on y1. The integral can be simplified to

$$f(y_2|y_1) = \int f(y_2|\theta) f(\theta|y_1) d\theta$$

Probability of y_2, given that we got a head in the first experiment:

$$\implies P(y_2|y_1 = 1) = \int_0^1 \theta^{y_2}(1-\theta)^{1-y_2} 2\theta d\theta$$

$$= \int_0^1 2\theta^{y_2+1}(1-\theta)^{(1-y_2)} d\theta$$

Probability of y_2=1 given y_1=1

$$P(y_2 = 1|y_1 = 1) = \int_0^1 2\theta^2 d\theta = \frac{2}{3}$$

Probability of y_2=0 (tails) given y_1=1 (heads)

$$P(y_2 = 0|y_1 = 1) = \frac{1}{3}$$