EECE 5639- Homework 6: Homographies, Stereo and Motion

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Question 1

Homography is a 2D - 2D projective transformation between the pixel coordinates in two images that one viewing the same plane from different angles, taken from the camera. In homography, the camera is rotated about its centre of projection without any translation. In other words, planar homography relates the transformation between two planes.

$$\begin{bmatrix} x' \\ y' \\ 1' \end{bmatrix} = \mathrm{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{21} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

The essential matrix is a 3 x 3 matrix that encodes the epipolar geomtry of two views. Given a point in an image, multiplying by the Essential Matrix, will tell us the EPIPOLAR line in the second image where the corresponding point must be. If P_l and P_r are two two points from the left and right cameras of the stereo, we have:

$$P_r^T R.SP_l = 0$$

Where R is the rotation matrix and S is the matrix which depends on the baseline translation. Using the Longuet Higgins equation we also have

$$p_r^T R.Sp_l = 0$$

where p_l and p_r are the camera coordinates. Now adding instrinsic parameters to this equation, we have an equation of the form:

$$\overline{p_r}^T (M_r^{-T} E M_l^{-T}) \overline{p_l} = 0$$

or

$$\overline{p_r}^T F \overline{p_l} = 0$$

F, The fundemental matrix here is analogous to the Essential matrix, and tells how points in each image are related to epipolar lines in the other image. It depends on the extrinsic and intrinsic parameters.

Question 2

Epipoles in the canonical configuration will be at infinity.

Question 3

Planar Affine Camera Model

- $\begin{bmatrix} su \\ sv \\ s \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ 0 & 0 & p_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$

  Since the scale doesn't matter,

$$\begin{bmatrix} u \\ v \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{21} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

- If the field of view is such that all points on the world plane have approximately same depth from the camera compared to the distance of the camera from plane.
- Since we have 6 degrees of freedom, we need 3 points.
- Planar affine transformation preserves the parallel property of parallel lines.

Question 4

Question 5

Normalizing the entries of the fundemental matrix:

Let the set of points be defined by

$$p_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}^T$$

Now,

$$\overline{x} = \frac{1}{n}\sum_i x_i \ and \ \overline{y} = \frac{1}{n}\sum_i y_i$$

where

$$\overline{d} = \frac{\sum_i \sqrt{(x_i - \overline{x})^2 + (y_i - \overline{y})^2}}{n\sqrt{2}}$$

$$H\mathbf{p} = \hat{p_i}$$

i.e,

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{21} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{x_i - \overline{x}}{d} \\ \frac{y_i - \overline{y}}{d} \\ 1 \end{bmatrix}$$

$$h_{11}x_i + h_{12}y_i + h_{13} = \frac{x_i - \overline{x}}{d} \tag{1}$$

$$h_{21}x_i + h_{22}y_i + h_{23} = \frac{y_i - \overline{y}}{d} \tag{2}$$

$$h_{31} + h_{32} + h_{33} = 1 \tag{3}$$

Equating with the co-efficients of x_i and y_i we have:

From eq (1) above $h_{11}x_i = \frac{x_i}{d}$, $h_{12} = 0$, $h_{13} = \frac{\overline{-x}}{d}$

From eq(2) $h_{21} = 0$, $h_{22} = \frac{1}{d}$, $h_{23} = \frac{\overline{y}}{d}$

From (3) $h_{31} = 0$, $h_{32} = 0$, $h_{33} = 1$

Hence, the normalizing matrix is:

$$\begin{bmatrix} \frac{1}{d} & 0 & \frac{\overline{-x}}{d} \\ 0 & \frac{1}{d} & \frac{\overline{y}}{d} \\ 0 & 0 & 1 \end{bmatrix}$$