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Homework 5

EECE 5639 : Computer Vision

① From the equations of the stereo system,

$$Z = f \cdot \frac{T}{d}$$

where $d = X_r - X_L$

Taking partial derivatives of Z, wrt f, T and d we have:

$$\frac{\partial Z}{\partial f} = \frac{T}{d} \; ; \; \frac{\partial Z}{\partial T} = \frac{f}{d} \; ; \; \frac{\partial Z}{\partial d} = -f\frac{T}{d^2} \quad -①$$

The error in calculation of X_r and X_L propogates to the error 'd'.

As $\frac{\partial Z}{\partial f}$ and $\frac{\partial Z}{\partial T}$ are inversely proportional to d, error in d increase the error in depth estimation.

For, $\frac{\partial Z}{\partial d} = -f\frac{T}{d^2}$, the smaller the d we get because of error, the larger is the value of $|f \cdot \frac{T}{d^2}|$ and hence larger error in the estimation of depth Z.

(2) From the origin of the camera : (10, 1, 3)

(a) Translation : $\begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Rotate w.r.t z: $\begin{bmatrix} \cos 90 & -\sin 90 & 0 & 0 \\ \sin 90 & \cos 90 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Rotate wrt x : $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 & 0 \\ 0 & \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Putting it all together:-

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -3 \\ -1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Since the transformations are similar:-

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) Transform the 2 points into world co-ordinates

$$P_1 = T_1^{-1} p_1 = \begin{bmatrix} 0 & 0 & -1 & 10 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3/10 \\ 1/10 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ 13/10 \\ 31/10 \\ 1 \end{bmatrix}$$

$$P_2 = T_2^{-1} p_2 = \begin{bmatrix} 0 & 0 & -1 & 7 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3/7 \\ 2/7 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 10/7 \\ 16/7 \\ 1 \end{bmatrix}$$

Line passing through C_1 and P_1 is :-

$$C_1 + \lambda (P - C_1)$$

Line passing through C_2 and P_2 is

$$C_2 + \lambda (P_2 - C_2)$$

At Intersection :

$$C_1 + \lambda (P - C_1) = C_2 + \lambda (P_2 - C_2)$$

$$(10, 1, 3) + \lambda_1 \left(\begin{bmatrix} 9 \\ 13/10 \\ 31/10 \end{bmatrix} - \begin{bmatrix} 10 \\ 1 \\ 3 \end{bmatrix} \right) = (7, 1, 2) + \lambda_2 \left(\begin{bmatrix} 6 \\ 10/7 \\ 16/7 \end{bmatrix} - \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix} \right)$$

$$(10, 1, 3) + \lambda_1 \left(\begin{bmatrix} -1 \\ 3/10 \\ 1/10 \end{bmatrix} \right) = (7, 1, 2) + \lambda_2 \begin{bmatrix} -1 \\ 3/7 \\ 2/7 \end{bmatrix}$$

$$(3, 0, 1) = (-\lambda_2, 3/7\lambda_2, 2/7\lambda_2) - (-\lambda_1, 3/10\lambda_1\ 1/10\lambda_1)$$

So, $\lambda_1 = 10$ $\lambda_2 = 7$

When $\lambda_1 = 10$ and $\lambda_2 = 7$, $L_1 = C_1 + 10(P_1 - C_1)$

$P = (0, 4, 4)$

(3)
(a) Location of the object before motion:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 30 \\ 60 \\ 10 \end{bmatrix}$$

$$x = f\frac{X}{Z} = 1 \times \frac{30}{10}$$

$$= 3$$

$$y = \frac{fY}{Z} = \frac{1 \times 60}{10} = 6$$

$\therefore$ At time, $t = 0$ point is at $(3, 6)$

(b) Image Co-ordinates of focus of expansion:

$$P_0 = \begin{bmatrix} f\frac{T_x}{T_z} \\ f\frac{T_y}{T_z} \end{bmatrix} = \begin{bmatrix} u/w \\ v/w \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$\therefore P_0 = (5, 10)$

(c) Time of the object collision with observer:

$$t = \frac{10}{1} = \underline{\underline{10 \text{ seconds}}}$$

(4) Components of the acceleration :-

$80 = a_x t^2$

$a_x = 20$

In X direction $\Rightarrow (50-10) = \frac{1}{2} a_x t^2$

$$80 = a_x \cdot t^2$$

$$\underline{\underline{a_x = 20}}$$

In Y: $100 - 20 = \frac{1}{2} a_y t^2$

$$80 \times 2 = a_y \times 4$$

$$a_y = 40 \,//$$

In Z: $20 - 0 = \frac{1}{2} a_z t^2$

$$40 = a_z \cdot 4$$

$$\underline{\underline{a_z = 10}}$$

$\therefore$ Acceleration vector: $(20, 40, 10)$

Initial velocity: $(0, 0, 0)$

Since the motion has constant acceleration

Velocity at time t: $(20t, 40t, 10t)$
(t_{2sec} t_{1sec})

This motion is purely translation

$$\therefore p_0 = \begin{bmatrix} x_0 \\ y_0 \\ f \end{bmatrix} = \begin{bmatrix} f\frac{T_x}{T_z} \\ f\frac{T_y}{T_z} \\ f \end{bmatrix} = \begin{bmatrix} 2f \\ 4f \\ f \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

So, they will meet at $(2, 4)$