EECE 5639- Homework 3

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Question 1

```
%Creating the image in the question
function image = get_image()
  image = zeros(8, 8);

  for x = 1:8
      for y = 1:8
          image(x,y) = abs(x-y);
      end
  end
end
```

Prewitt Mask

```
[PGmag, PGdir] = imgradient(image,'prewitt');
disp(PGmag);
disp(PGdir);

% Output - Disregarding boundaries

      % Gradient Magnitude

            0    5.6569    8.4853    8.4853    8.4853    8.4853
       5.6569         0    5.6569    8.4853    8.4853    8.4853
       8.4853    5.6569         0    5.6569    8.4853    8.4853
       8.4853    8.4853    5.6569         0    5.6569    8.4853
       8.4853    8.4853    8.4853    5.6569         0    5.6569
       8.4853    8.4853    8.4853    8.4853    5.6569         0

      % Gradient Orientation


               0   45.0000   45.0000   45.0000   45.0000   45.0000
       -135.0000         0   45.0000   45.0000   45.0000   45.0000
       -135.0000 -135.0000         0   45.0000   45.0000   45.0000
       -135.0000 -135.0000 -135.0000         0   45.0000   45.0000
       -135.0000 -135.0000 -135.0000 -135.0000         0   45.0000
       -135.0000 -135.0000 -135.0000 -135.0000 -135.0000         0
```

Sobel Mask

```
[SGmag, SGdir] = imgradientxy(image);
disp(SGmag);
disp(SGdir);

% Output - Disregarding boundaries

      % Gradient Magnitude

             0     6     8     8     8     8
            -6     0     6     8     8     8
            -8    -6     0     6     8     8
            -8    -8    -6     0     6     8
            -8    -8    -8    -6     0     6
            -8    -8    -8    -8    -6     0


      % Gradient Orientation

            0    -6    -8    -8    -8    -8
            6     0    -6    -8    -8    -8
            8     6     0    -6    -8    -8
            8     8     6     0    -6    -8
            8     8     8     6     0    -6
            8     8     8     8     6     0
```

Qn2 $\quad I(r,c) = pr + qc + h \quad , \; r, c = -1, 0, 1$

$\xrightarrow{\quad} c$, $\quad r \downarrow$

$r \backslash c$	-1	0	1
-1	$-p+q+h$	$-p+h$	$-p+q+h$
0	$-q+h$	h	$q+h$
1	$p-q+h$	$p+h$	$p+r+h$

(a) Convolve with $\boxed{-1 \mid 0 \mid 1}$ at $(0,0)$, we have

$$= (-1)(q+h) + (q+h)$$

$$= q - h + q + h = \underline{\underline{2q}}. \quad \text{(Vertical gradient)}$$

Now, convolving with $\begin{array}{|c|}\hline -1 \\ \hline 0 \\ \hline 1 \\ \hline\end{array}$ at $(0,0)$ we have

$$= -1(p+h) + (p+h) = p - h + p + h = \underline{\underline{2p}} \quad \text{(Horizontal gradient)}$$

(b) Applying the Prewitt mask, we have

$$I(0,0) = p+q-h + p-h + p-q-h + p-q+h + p+h + p+q+h$$

$$= \underline{\underline{6h}}$$

(c) Applyig the Laplacian mask, we have

$$-1(p+h) - 1(q+h) + 4h - 1(q+h) - 1(p+h)$$

$$= p-h + q-h + 4h - q-h - p-h = \underline{\underline{0}}$$

Hence, the laplacian mask has zero response

Question 3

```
function image = get_image_with_corners()
    image = zeros(21,20);
    for x = 11:21
        for y = 1:10
            image(x,y) = 40;
        end
    end

    for x = 1:10
        for y = 11:20
            image(x,y) = 40;
        end
    end
end

img2 = get_image_with_corners();

% Gradient in X - direction
maskx = [-1 -1 -1;0 0 0;1 1 1];
Gx = conv2(img2, maskx);

% Gradient in Y - direction
masky = [-1 0 1;-1 0 1;-1 0 1];
Gy = conv2(img2, masky);


Ex2  = Gx.*Gx;
Ey2  = Gy.*Gy;
ExEy = Gx.*Gy;

% Build the C Matrix from the Ex, Ey and ExEy matrices

function matrix = CreateCMatrix(Ex2, Ey2, ExEy, i, j)
    matrix = zeros(2,2);
    matrix(1,1) = Ex2(i-1, j-1) + Ex2(i-1, j) + Ex2(i-1, j+1) + ...
        + Ex2(i, j-1) + Ex2(i,j) + Ex2(i,j+1) + Ex2(i+1,j-1) + Ex2(i+1,j) + Ex2(i+1,j+1);
    matrix(1,2) = ExEy(i-1, j-1) + ExEy(i-1, j) + ExEy(i-1, j+1) + ...
        + ExEy(i, j-1) + ExEy(i,j) + ExEy(i,j+1) + ExEy(i+1,j-1) + ExEy(i+1,j) + ExEy(i+1,j+1);
    matrix(2,1) = matrix(1,2);
    matrix(2,2) = Ey2(i-1, j-1) + Ey2(i-1, j) + Ey2(i-1, j+1) + ...
        + Ey2(i, j-1) + Ey2(i,j) + Ey2(i,j+1) + Ey2(i+1,j-1) + Ey2(i+1,j) + Ey2(i+1,j+1);
end


CMatrix = zeros(2,2,21,20);


% Excluding borders while summing
for i=2:20
    for j=2:19
       CMatrix(:,:,i,j) = CreateCMatrix(Ex2, Ey2, ExEy, i, j);
    end
end
```

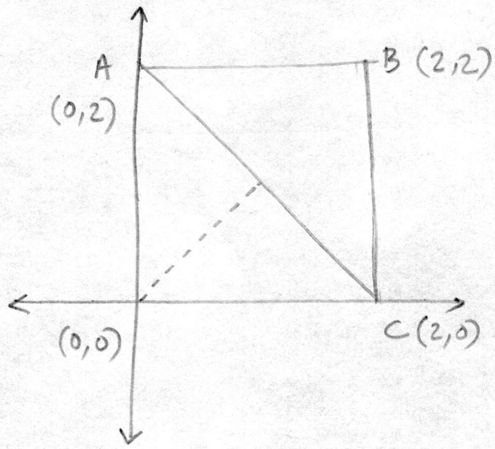
```
% Finding those values for which CMatrix has Sum(Ex^2) >> Threshold &&
% Sum(Ey^2) >> Threshold  && Sum(ExEy) =0 && Sum(EyEx) = 0
for i=1:19
    for j=1:18
        CM = CMatrix(:,:,i,j);
        if CM(2) == 0  && CM(3) == 0 && CM(1) > 10000 && CM(4) > 10000
            fprintf("(%d,%d)\n", i, j);
        end
    end
end


% Output

(10,11)
(10,12)
(11,10)
(11,11)
(11,12)
(11,13)
(12,10)
(12,11)
(12,12)
(12,13)
(13,11)
(13,12)
```

These are the indices of pixels where the corners are found.

(4) Vertices : A(2,0) B(2,2) C(0,2)



Hough transform of the lines are as follows:

	ρ	θ
AB	2	90°
BC	2	0°
AC	$\sqrt{2}$	45°

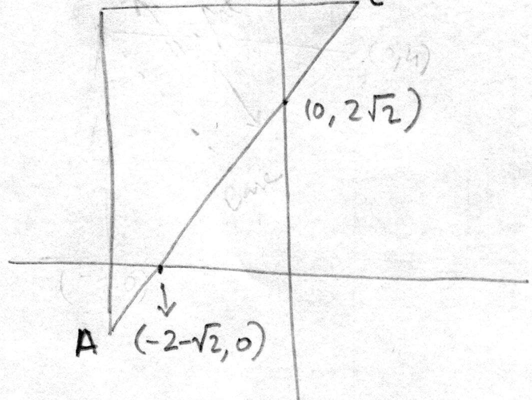
Modifying the object with ρ^2, $\theta + 90$, we have :-

	ρ^2	$\theta+90$
AB	-4	0°
BC	-4	90°
AC	-2	-45°

Area of the transformed object :-

Base $= 4 + (4 - 2\sqrt{2})$

Height $= 4 + (4 - 2\sqrt{2})$



$\therefore$ Area of transformed object

$= \frac{1}{2} (4 + 4 - 2\sqrt{2})(4 + 4 - 2\sqrt{2})$

$= 13.374 \text{ units}^2$

(5)

	ρ	θ
AB	3	90°
BC	3	0°
CD	$\sqrt{3}$	60°
DA	$-\sqrt{2}/2$	-45°

From the sides AB and BC, we have

$$\begin{aligned} X_B \cos 90 + Y_B \sin 90 &= 3 \\ X_B \cos 0 + Y_B \sin 0 &= 3 \end{aligned} \Rightarrow \begin{aligned} Y_B &= 3 \\ X_B &= 3 \end{aligned} \Rightarrow B:(3,3)$$

From sides BC and CD, we have

$$\begin{aligned} X_C \cos 0 + Y_C \sin 0 &= 3 \\ X_C \cos 60 + Y_C \sin 60 &= \sqrt{3} \end{aligned} \Rightarrow \begin{aligned} X_C &= 3 \\ \frac{X_C}{2} + \frac{\sqrt{3}}{2} Y_C &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} X_C &= 3 \\ X_C + \sqrt{3}\, Y_C &= 2\sqrt{3} \end{aligned} \Rightarrow C\left(3, \frac{2\sqrt{3}-3}{\sqrt{3}}\right)$$

$$C(3, 0.26)$$

From sides CD and DA, we have

$$\begin{aligned} X_D \cos 60 + Y_D \sin 60 &= \sqrt{3} \\ X_D \cos(-45) + Y_D \sin(-45) &= -\frac{\sqrt{2}}{2} \end{aligned} \Rightarrow \begin{aligned} \frac{X_D}{2} + \frac{\sqrt{3}}{2} Y_D &= \sqrt{3} \\ \frac{X_D}{\sqrt{2}} - \frac{Y_D}{\sqrt{2}} &= \frac{-1}{\sqrt{2}} \end{aligned}$$

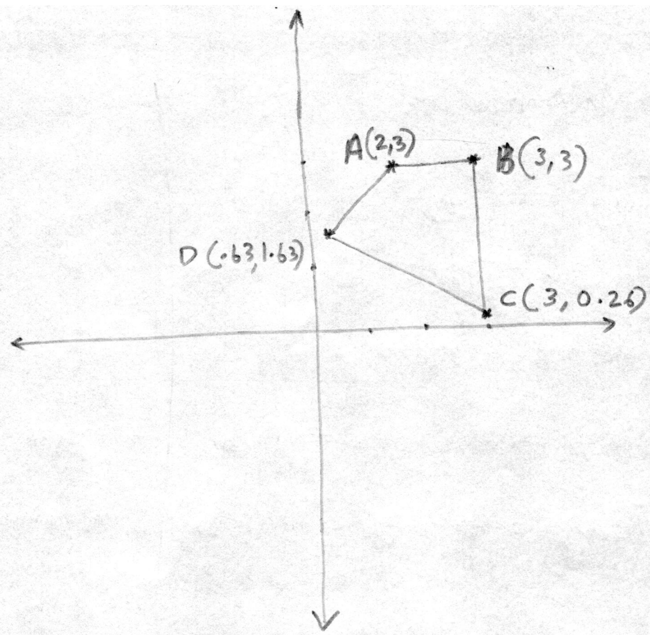
$$\begin{aligned} X_D + \sqrt{3}\, Y_D &= 2\sqrt{3} \\ X_D - Y_D &= -1 \end{aligned} = D\left(\frac{\sqrt{3}}{1+\sqrt{3}}, \frac{2\sqrt{3}+1}{\sqrt{3}+1}\right) := D(0.63, 1.63)$$

From sides DA and AB, we have

$$X_A \cos(-45) + Y_A \sin(-45) = -\frac{\sqrt{2}}{2} \Rightarrow \frac{X_A}{\sqrt{2}} - \frac{Y_A}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$X_A \cos 90 + Y_A \sin 90 = 3 \Rightarrow Y_A = 3$$

$$\begin{aligned} X_A - Y_A &= -1 \\ Y_A &= 3 \end{aligned} \Rightarrow A(2, 3)$$



(b) Hough transform after 30° rotation counter clockwise

	ρ	θ
AB	-3	-60°
BC	3	30°
CD	$\sqrt{3}$	90°
DA	$-\frac{\sqrt{2}}{2}$	-15°

⑥ If this is a region, then

$$H_0 = I(x,y) = Ax + By + C + N(0, \delta) \quad \text{and}$$

$Ax\ By + C$ is a constant

If this is not a region

$$I_1(x,y) = Ax + By + C + N(0, \delta_1) \quad m_1 \text{ points}$$

$$I_2(x,y) = Ax + By + C + N(0, \delta_2) \quad m_2 \text{ points}$$

$$L_0 = \frac{1}{(\sqrt{2\pi})^{m_1+m_2}\, \delta^{m_1+m_2}} e^{-\frac{1}{2}(m_1+m_2)}$$

$$L_1 = \frac{1}{\sqrt{2\pi}^{m_1+m_2}\, \delta^{m_1} \delta^{m_2}} e^{-m_1/2} e^{-m_2/2}$$

if $\frac{L_0}{L_1} = \frac{\delta^{m_1+m_2}}{\delta^{m_1}\delta^{m_2}} > 1$ then we need to split,

and if $\frac{L_0}{L_1} < 1$, then we need to merge.

(7) Histogram

Gray Value	1	2	3	4	5	6
Count	1	2	3	2	1	1

ⓐ $R = 4,\ X_0 = 2$

$$\mu = \frac{(2\cdot 2) + (3\cdot 3) + (4\cdot 2) + (5\cdot 1) + (6\cdot 1)}{2+3+2+1+1}$$

$$= \underline{\underline{3.55}}$$

$R = 2,\ X_1 = 3.55$

$$\mu = \frac{4+9+8+5}{2+3+2+1}$$

$$= \frac{26}{8} = 3.25$$

The window still remains, the same

$$\mu = \frac{4+9+8+5}{2+3+2+1} = \underline{\underline{3.25}}$$

ⓑ Fitting Error (Modeled as sum of square differences)

$$\sigma = \sum_{i=1}^{n} \frac{(\mu - x_i)^2}{N} =$$

$$= \frac{(3.25-1)^2 + 2(3.25-2)^2 + 3(3.25-3)^2 + 2(3.25-4)^2 + (3.25-5)^2 + (3.25-6)^2}{10}$$

$$= \frac{5.0625 + 3.125 + .1875 + 1.125 + 3.0625 + 7.56}{10}$$

$$= \frac{20.1225}{10} = 2.01225$$

(c) When $S =$ fitting error $= 2.012$ and this is a region

then

$$L_0 = \left(\frac{1}{\sqrt{2\pi} \cdot (2.012)}\right)^{10} e^{-1/2 \times \sum_{i=1}^{10} \left(\frac{x_i - 3.25}{2.01}\right)^{1/2}}$$