EECE 5639- Homework 6: Homographies, Stereo and Motion

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Question 1

Homography is a 2D - 2D projective transformation between the pixel coordinates in two images that one viewing the same plane from different angles, taken from the camera. In homography, the camera is rotated about its centre of projection without any translation. In other words, planar homography relates the transformation between two planes.

$$\begin{bmatrix} x' \\ y' \\ 1' \end{bmatrix} = \mathrm{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{21} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

In stereo vision, which is the ability to infer information on the 3D structure and distance of a scene from two or more images taken from different viewpoints, we use Essential and Fundemental matrices to construct correspondence equations.

The essential matrix is a 3 x 3 matrix that encodes the epipolar geomtry of two views. Given a point in an image, multiplying by the Essential Matrix, will tell us the epipolar line in the second image where the corresponding point must be. If P_l and P_r are two two points from the left and right cameras of the stereo, we have:

$$P_r^T R.SP_l = 0$$

Where R is the rotation matrix and S is the matrix which depends on the baseline translation. Using the Longuet Higgins equation we also have

$$p_r^T R.Sp_l = 0$$

where p_l and p_r are the camera coordinates. Now adding instrinsic parameters to this equation, we have an equation of the form:

$$\overline{p_r}^T (M_r^{-T} E M_l^{-T}) \overline{p_l} = 0$$

or

$$\overline{p_r}^T F \overline{p_l} = 0$$

F, The fundemental matrix here is analogous to the Essential matrix, and tells how points in each image are related to epipolar lines in the other image. It depends on the extrinsic and intrinsic parameters.

Question 2

Epipoles in the canonical configuration will be at infinity.

Question 3

Planar Affine Camera Model

- $\begin{bmatrix} su \\ sv \\ s \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ 0 & 0 & p_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$

Since the scale doesn't matter,

$$\begin{bmatrix} u \\ v \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{21} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

- If the field of view is such that all points on the world plane have approximately same depth from the camera compared to the distance of the camera from plane.
- Since we have 6 degrees of freedom, we need 3 points.
- Planar affine transformation preserves the parallel property of parallel lines.

Note: Question 4 (scanned page) has been put below Question 5

Question 5

Normalizing the entries of the fundemental matrix:

Let the set of points be defined by

$$p_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}^T$$

Now,

$$\overline{x} = \frac{1}{n}\sum_i x_i \text{ } and \text{ } \overline{y} = \frac{1}{n}\sum_i y_i$$

where

$$\overline{d} = \frac{\sum_i \sqrt{(x_i - \overline{x})^2 + (y_i - \overline{y})^2}}{n\sqrt{2}}$$

$$H\mathbf{p} = \hat{p}_i$$

i.e,

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{21} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{x_i - \overline{x}}{d} \\ \frac{y_i - \overline{y}}{d} \\ 1 \end{bmatrix}$$

$$h_{11}x_i + h_{12}y_i + h_{13} = \frac{x_i - \overline{x}}{d} \tag{1}$$

$$h_{21}x_i + h_{22}y_i + h_{23} = \frac{y_i - \overline{y}}{d} \tag{2}$$

$$h_{31} + h_{32} + h_{33} = 1 \tag{3}$$

Equating with the co-efficients of x_i and y_i we have:

From eq (1) above $h_{11}x_i = \frac{x_i}{d}$, $h_{12} = 0$, $h_{13} = \frac{\overline{-x}}{d}$

From eq(2) $h_{21} = 0$, $h_{22} = \frac{1}{d}$, $h_{23} = \frac{\overline{y}}{d}$

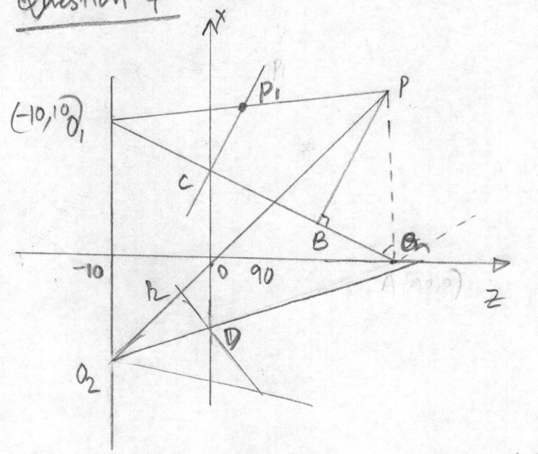
From (3) $h_{31} = 0$, $h_{32} = 0$, $h_{33} = 1$

Hence, the normalizing matrix is:

$$\begin{bmatrix} \frac{1}{d} & 0 & \frac{\overline{-x}}{d} \\ 0 & \frac{1}{d} & \frac{\overline{y}}{d} \\ 0 & 0 & 1 \end{bmatrix}$$

Question 4

Question 4



Line AO_1 from the points $A(90,0)$ and $O_1(^-10,10)$

$$X + .1Z - 9 = 0.$$

$$\therefore PB = \frac{(20 + 0.1 \times 40 - 9)}{\sqrt{1 + .1^2}} = \frac{15}{\sqrt{1.01}} = 14.9$$

Line $AO_1 = X + .1Z - 9 = 0$

$\therefore$ we have $C(9,0)$ So $O_1C = \sqrt{10^2+1}$

$= \sqrt{101} = \underline{\underline{10.05}}$

$$O_1P = \sqrt{10^2 + 50^2} = \sqrt{2600} = 50.99$$

$\therefore$ $PB = 14.9$

$$O_1B = \sqrt{50.99^2 - 14.9^2} = 48.76$$

From similar $\triangle$les properties,

$$\frac{P_1C}{PB} = \frac{O_1C}{O_1B} \Rightarrow \frac{P_1C}{14.9} = \frac{10.05}{48.76}$$

$P_1C = 3.07$

$O_2(-10,-10) \quad A(90,0)$

$O_2A \Rightarrow -x + .1y - 9 = 0$

$$PQ = \frac{-70 + .1 \times 40 - 9}{\sqrt{1 + .1^2}} = \frac{25}{\sqrt{1.01}} = 24.87$$

$O_1C = O_2D = 10.05$

$PQ = 24.87$

$O_2Q = \sqrt{7400 - 24.87} = 52.75$

So $\dfrac{P_2D}{PQ} = \dfrac{O_2P}{O_2Q} \Rightarrow \dfrac{P_2D}{24.87} = \dfrac{10.05}{52.75}$

$P_2D = 4.73$

$\therefore$ Disparity $= 4.73 - 3.071 = \underline{\underline{1.66}}$

Question 6

⑥ Suppose $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$$(x_i', y_i')^T = A(x_i, y_i)^T$$

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = A(x_i, y_i)^T$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Since we have 6 unknowns, therefore we need to solve 6 equations

i.e $x_i' = a_{11} x_i + a_{12} y_i + a_{13} \cdots$ ①

$y_i' = a_{21} x_i + a_{22} y_i + a_{23} \cdots$ ②

For ①: $\begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{bmatrix} = B \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix}$; $B = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix}$

For ②: $\begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_n' \end{bmatrix} = B \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix}$; $B: \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix}$

This way, the problem could be decomposed into two smaller sets.

Question 7

(7) (a) $C_1(0,0,0)$

$C_2'(1,2,1)$

C_1 and C_2' forms a line

So from the vector equation of lines,

$$x = \frac{f(0+\lambda(1-0))}{0+\lambda(1-0)} = \underline{\underline{1}}$$

$$y = \frac{f(0+\lambda(2-0))}{0+\lambda(1-0)} = 2$$

$$z = \underline{\underline{1}}$$

Co-ordinates of E_1 expressed in camera_1 cordinates $= (1,2,1)$

Similarly co-ordinates of E_2 expressed in camera_2 coordinates $= (-1,-2,-1)$

(b) $C_1\ (2,2,4)$

$C_2\ (4,3,3)$

$\downarrow$

$C_1 - C_2 = (2,1,1)$ Difference in translation between the camera in world co-ordinates

Adding the correct signs for coordinates we need 3 points to define the plane.

In C_2s co-ordinates

$C_2(0,0,0)$

Camera 1's origin : $C_1(-1,2,-1)$

Co-ordinates of the fly in terms of C_2's coordinates

$F_1^2 = (0,2,1) - (1,2,1)$

$= \underline{(-1,0,0)}$

Equation of the plane

$ax + by + cz + d = 0$

$d = 0$ — From C_2

$a = 0$ — From F_1^2

$-a - 2b - c = 0$

$b = -c/2$

Setting value of $c = 2$, we have $b = -1$

$y + 2z = 0$.

(c) In camera 1 co-ordinates $(-3,-2,1)$ becomes

$(-2,-3,1)$

Focus of expansion : $x' = f\frac{u}{w} = 1 \cdot \frac{-2}{1} = -2$

$y' = f\frac{v}{w} = 1 \cdot \frac{-3}{1} = -3$

$(-2,-3)$ is the camera coordinates of the FOE //

Question 8

(8) Hankel Matrix

$$H = \begin{bmatrix} 1 & 2 & 5 & 12 & 29 \\ 2 & 5 & 12 & x & 70 \\ 5 & 12 & 29 & 70 & y \end{bmatrix}$$

(a) Complexity of the dynamics $= \text{Rank}(H) = \text{Rank}\left(\begin{bmatrix} 1 & 2 & 5 \\ 2 & 5 & 12 \\ 5 & 12 & 29 \end{bmatrix}\right)$

$$= \underline{\underline{2}}$$

(b) Regressor

for $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 5 & 12 \\ 5 & 12 & 29 \end{bmatrix}$ we have

$$5 = 4a_1 + 2a_2$$
$$12 = 5a_1 + 2a_2$$

$a_1 = 2$, $a_2 = 1$

for $\begin{bmatrix} 1 & 2 & 5 & 12 \\ 2 & 5 & 12 & 29 \\ 5 & 12 & 29 & 70 \end{bmatrix} \Rightarrow 12 = 5a_1 + 2a_2 + a_3$

$$a_3 = 0.$$

for $\begin{bmatrix} 1 & 2 & 5 & 12 & 29 \\ 2 & 5 & 12 & 29 & 70 \\ 5 & 12 & 29 & 70 & y \end{bmatrix}$

$$29 = 12a_1 + 5a_2 + 2a_3 + a_4$$
$$= 24 + 5 + 0 + a_4$$

$$a_4 = 0.$$

(c) From the property of H, $x = 29$

Now, $y = 70a_1 + 29a_2 + 12a_3 + 5a_4 = 140 + 29 = \underline{\underline{169}}$