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Homework 5

EECE 5639 : Computer Vision

① From the equations of the stereo system,

$$z = f \cdot \frac{T}{d}$$

where $d = X_r - X_L$

Taking partial derivatives of z , wrt f , T and d we have.

$$\frac{\partial z}{\partial f} = \frac{T}{d} ; \frac{\partial z}{\partial T} = \frac{f}{d} ; \frac{\partial z}{\partial d} = -f \frac{T}{d^2} \quad \text{--- ①}$$

The error in calculation of X_r and X_L propagates to the error in 'd'.

'd' is disparity (Difference between two distances)

From $\frac{\partial z}{\partial f} = \frac{T}{d}$,

$$\frac{\partial z}{\partial T} = \frac{f}{d},$$

and from $\frac{\partial z}{\partial d} = -f \frac{T}{d^2}$, the smaller the d we get

because of error, the larger is the value of $|f \cdot \frac{T}{d^2}|$
and hence larger error in the estimation of depth z .

(2) From the origin of the Camera: $(10, 1, 3)$

(a) Translation:
$$\begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate w.r.t z:
$$\begin{bmatrix} \cos 90 & -\sin 90 & 0 & 0 \\ \sin 90 & \cos 90 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate w.r.t x:
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 & 0 \\ 0 & \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Putting it all together:-

$$\begin{aligned} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -3 \\ -1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

(b) Since the transformations are similar:-

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

© Transform the 2 points into world Co-ordinates

$$P_1 = T_1^{-1} p_1 = \begin{bmatrix} 0 & 0 & -1 & 10 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3/10 \\ 1/10 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ 13/10 \\ 31/10 \\ 1 \end{bmatrix}$$

$$P_2 = T_2^{-1} p_2 = \begin{bmatrix} 0 & 0 & -1 & 7 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3/7 \\ 2/7 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 10/7 \\ 16/7 \\ 1 \end{bmatrix}$$

Line passing through C_1 and P_1 is :-

$$C_1 + \lambda (P_1 - C_1)$$

Line passing through C_2 and P_2 is

$$C_2 + \lambda (P_2 - C_2)$$

At Intersection :

$$C_1 + \lambda (P_1 - C_1) = C_2 + \lambda (P_2 - C_2)$$

$$(10, 1, 3) + \lambda \left(\begin{bmatrix} 9 \\ 13/10 \\ 31/10 \end{bmatrix} - \begin{bmatrix} 10 \\ 1 \\ 3 \end{bmatrix} \right) = (7, 1, 2) + \lambda \left(\begin{bmatrix} 6 \\ 10/7 \\ 16/7 \end{bmatrix} - \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix} \right)$$

$$(10, 1, 3) + \lambda_1 \begin{bmatrix} -1 \\ 3/10 \\ 1/10 \end{bmatrix} = (7, 1, 2) + \lambda_2 \begin{bmatrix} -1 \\ 3/7 \\ 2/7 \end{bmatrix}$$

$$(3, 0, 1) = (-\lambda_2, 3/7 \lambda_2, 2/7 \lambda_2) - (-\lambda_1, 3/10 \lambda_1, 1/10 \lambda_1)$$

$$S_0, \lambda_1 = 10 \quad \lambda_2 = 7$$

$$\text{When } \lambda_1 = 10 \text{ and } \lambda_2 = 7, L_1 = C_1 + 10(P_1 - C_1)$$

$$P = (0, 4, 4)$$

③ Location of the object before motion:

④

$$x = f \frac{X}{Z} = 1 \times \frac{30}{10}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 30 \\ 60 \\ 10 \end{bmatrix}$$

$$= \underline{\underline{3}}$$

$$y = \frac{fY}{Z} = \frac{1 \times 60}{10} = \underline{\underline{6}}$$

\therefore At time, $t=0$ point is at $(3, 6)$

⑥ Image co-ordinates of focus of expansion:

$$P_0 = \begin{bmatrix} f \frac{T_x}{T_z} \\ f \frac{T_y}{T_z} \end{bmatrix} = \begin{bmatrix} u/w \\ v/w \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\therefore P_0 = (5, 10)$$

(c) Time of the object collision with observer.

$$t = \frac{10}{1} = \underline{\underline{10 \text{ seconds}}}$$

(4) Components of the acceleration :-

$$\text{In X direction} \Rightarrow (50 - 10) = \frac{1}{2} a_x t^2$$

$$80 = a_x \cdot t^2$$

$$a_x = \underline{\underline{20}}$$

$$\text{In Y: } 100 - 20 = \frac{1}{2} a_y t^2$$

$$80 \times 2 = a_y \times 4$$

$$a_y = 40 //$$

$$\text{In Z: } 20 - 0 = \frac{1}{2} a_z t^2$$

$$40 = a_z \cdot 4$$

$$\underline{\underline{a_z = 10}}$$

\therefore Acceleration vector : $(20, 40, 10)$

Initial velocity : $(0, 0, 0)$

Since the motion has constant acceleration

Velocity at time $t = (20t, 40t, 10t)$
 $(t_{2 \text{ sec}}, t_{1 \text{ sec}})$

$$80 = a_x t^2$$
$$a_x = 20$$

This motion is purely translation

$$\therefore P_0 = \begin{bmatrix} x_0 \\ y_0 \\ f \end{bmatrix}$$

$$= \begin{bmatrix} f \frac{T_x}{T_z} \\ f \frac{T_y}{T_z} \\ f \end{bmatrix} = \begin{bmatrix} 2f \\ 4f \\ f \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

So, they will meet at (2, 4)