CSCI E-106: Data Modeling

Assignment 0

Due: September, 9 2019 at 7:30pm EST

Instructions: Students should submit their reports on Canvas. The report needs to contain copies of the questions, step-by-step walk-through solutions, and final answers clearly indicated. For the questions 1 to 5, please solve it by hand, you can either submit a scanned version or (preferred) LaTeX or Rmd solution; for questions 6 to 8, please submit two files: (1) a R Markdown file (.Rmd extension) and (2) a PDF document generated using knitr.

There are three optional problems, but you should attempt to solve them. They serve as a diagnostic tool for you whether you should read additional recommended readings.

Please, note that homework assignments are due before the class.

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1. For the questions a to c, calculate the mean and variance for each of the following probability distributions

   a $f_X(x) = ax^{a-1},\ \ 0 < x < 1,\ a > 0$

   b $f_X(x) = \frac{1}{n}\ x = 1, \cdots, n,\quad n > 0\ an\ integer$

   c $f_X(x) = \frac{3}{2}(x-1)^2\ ,\ 0 < x < 2$

Questions for Joint Distribution:

2. Let X_1 and X_2 be independent standard normal distribution random variables $N(0,1)$. Find the PDF of $\frac{(X_1-X_2)^2}{2}$

3. [Optional] Let X_1 and X_2 be independent gamma distribution random variables with $gamma(\alpha_1,1)$ and $gamma(\alpha_2,1)$. Find the marginal distributions of $\frac{X_1}{X_1+X_2}$ and $\frac{X_2}{X_1+X_2}$

For the following questions, find the Maximum Likelihood Estimation (MLE).

4. [Optional] Let $X_1,\ldots, X_n$ be a random sample from a gamma(α,β) population. Find the MLE of β assuming α is known.

5. [Optional] Let X_1,... X_n be a random sample from the PDF. Find the MLE of θ:

$$f(x|\theta) = \theta X^{-2}\ ,\ 0 < \theta \leq x < \infty$$

R Programming:

For the following questions, let matrix $X = \begin{bmatrix} 10 & 1 & 9 \\ 3 & 8 & 7 \\ 5 & 2 & 4 \end{bmatrix}$ $Y = \begin{bmatrix} 2 & 8 & 3 \\ 5 & 1 & 12 \\ 13 & 4 & 7 \end{bmatrix}$

(Please note that A^T is a transpose of a matrix A, a matrix such that $[A^T]_{ij} = A_{ji}$)

6. For matrix *X and Y* above, calculate (*X* + *Y*)

7. For matrix X *and Y* above, calculate $(X^TX)^{-1}X^TY$

8. Write R code to draw 10,000 random samples from uniform distribution and calculate the 99% percentile.