CS-E-106: Data Modeling

Assignment 0

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Solution 1:

(a).

$$f_X(x) = ax^{a-1}$$

$$\mathbb{E}[X] = \int_{-\inf}^{\inf} x.f_X(x)dx = \int_0^1 x.a.x^{a-1}dx = a\int_0^1 x^a dx = a\frac{1}{(a+1)}x^{a+1}|_0^1 = \frac{a}{a+1}$$

$$Var[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \int_0^1 x^2.a.x^{a-1}dx - (\mathbb{E}[X])^2$$

$$= a\int_0^1 x^{a+1}dx - (\mathbb{E}[X])^2 = a\frac{1}{(a+2)}x^{a+2}|_0^1 - (\frac{a}{a+1})^2$$

$$= \frac{a}{(a+2)(a+1)^2}$$

(b).

$$f_X(x) = \frac{1}{n}$$

$$\mathbb{E}[X] = \sum_{-\inf}^{\inf} xf_X(x) = \frac{1}{n}\sum_{x=1}^{n} x = \frac{1}{n}.\frac{n(n+1)}{2} = \frac{(n+1)}{2}$$

$$Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{1}{n}\sum_{x=1}^{n} x^2 - (\mathbb{E}[X])^2$$

$$= \frac{1}{n}.(\frac{n(n+1)(2n+1)}{6}) - [\frac{(n+1)}{2}]^2 = \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$

$$= \frac{(n+1)(4n+2-3n-3)}{12} = \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$$

(c).

$$f_X(x) = \frac{3}{2}(x-1)^2$$

$$\mathbb{E}[X] = \int_{-\inf}^{\inf} x.f_X(x)dx = \int_0^2 x.\frac{3}{2}(x-1)^2dx = \frac{3}{2}\int_0^2 (x^3 - 2x^2 + x)dx = \frac{3}{2}[\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2]|_0^2 = 1$$

$$Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \int_0^2 x^2.\frac{3}{2}(x-1)^2 dx - 1 = \frac{3}{2}[\frac{1}{5}x^5 - \frac{1}{2}x^4 + \frac{1}{3}x^3]|_0^2 - 1 = \frac{3}{5}$$

R Programming Questions

```r
X <- matrix(c(10,3,5,1,8,2,9,7,4), nrow = 3, ncol = 3)
print(X)
```

```
##      [,1] [,2] [,3]
## [1,]   10    1    9
## [2,]    3    8    7
## [3,]    5    2    4
```

```r
Y <- matrix(c(2,8,3,5,1,12,13,4,7), nrow = 3, ncol = 3)
print(Y)
```

```
##      [,1] [,2] [,3]
## [1,]    2    5   13
## [2,]    8    1    4
## [3,]    3   12    7
```

Solution 6:

```r
print(X+Y)
```

```
##      [,1] [,2] [,3]
## [1,]   12    6   22
## [2,]   11    9   11
## [3,]    8   14   11
```

Solution 7:

```r
solve((t(X)%*%X))%*%t(X)%*%Y
```

```
##            [,1]      [,2]       [,3]
## [1,]  0.4563107  6.563107  1.6019417
## [2,]  1.1941748  3.941748  0.2135922
## [3,] -0.4174757 -7.174757 -0.3592233
```

Solution 8:

```r
samples <- runif(10000)
percentile <- quantile(samples, probs = c(0.99))
print(percentile)
```

```
##       99%
## 0.9887164
```