DATA MODELING: CSCI E-106

TA SESSION 3

THE DATA MODELING IS A GRADUATE-LEVEL COURSE!

- Read the syllabus, announcements and piazza; they contain useful info
- Homework should be done on Rstudio.cloud (we cannot accept excuses about home or work computers)
- TA sessions are not review sessions only
- Concept of minimal reproducible example

STATISTICAL SOFTWARE

- Various programs and packages have various methods (and, thus, results)
- Always check with known results
- Read manuals! Use google or stackoverflow.com
- Run examples!

LIKELIHOOD FUNCTION AND ESTIMATION

- From wikipedia: “… likelihood function (often simply the likelihood) is a function of the parameters of a statistical model, given specific observed data. ”

$$\prod_{i=1}^{n} \mathcal{L}(data|parameters) = \prod_{i=1}^{n} P(data|parameters)$$

- Different types of estimation (least squares, objective function minimization, moment matching, … and log-likelihood function minimization)
- Usually we use log-likelihood because taking a derivative of a sum is more convenient

LIKELIHOOD FUNCTION AND ESTIMATION (EXAMPLE)

2.53. (Calculus needed.)

a. Obtain the likelihood function for the sample observations $Y_1, \ldots, Y_n$ given $X_1, \ldots, X_n$, if the conditions on page 83 apply.

b. Obtain the maximum likelihood estimators of β_0, β_1, and σ^2. Are the estimators of β_0 and β_1 the same as those in (1.27) when the X_i are fixed?

LIKELIHOOD FUNCTION AND ESTIMATION (EXAMPLE)

- Can we still use regression model (2.1) if Y_1 and Y_2 are not bivariate normal? It can be shown that all results on estimation, testing, and prediction obtained from regression model (2.1) apply if $Y_1 = Y$ and $Y_2 = X$ are random variables, and if the following conditions hold:

  1. The conditional distributions of the Y_i, given X_i, are normal and independent, with conditional means $\beta_0 + \beta_1 X_i$ and conditional variance σ^2.
  2. The X_i are independent random variables whose probability distribution $g(X_i)$ does not involve the parameters β_0, β_1, σ^2.

$$(Y \,|\, X)\ N(\beta_0 + \beta_1 X, \sigma^2)$$

$$g(X_i \,|\, \beta_0, \beta_1, \sigma^2) = g(X_i)$$

LIKELIHOOD FUNCTION AND ESTIMATION (EXAMPLE)

- Obtain likelihood function

$$\prod_{i=1}^{n} \mathcal{L}(data|parameters) = \prod_{i=1}^{n} P(data|parameters)$$

$$P(Y = Y_i, X = X_i|\beta_0, \beta_1, \sigma^2)$$

LIKELIHOOD FUNCTION AND ESTIMATION (EXAMPLE)

$$P(Y = Y_i, X = X_i | \beta_0, \beta_1, \sigma^2)$$

$$P(Y = Y_i \cap X = X_i | \beta_0, \beta_1, \sigma^2)$$

- Using the chain rule of probability

$$P(A \cap B) = P(A|B) \cdot P(B)$$

LIKELIHOOD FUNCTION AND ESTIMATION (EXAMPLE)

$$P(Y = Y_i, X = X_i | \beta_0, \beta_1, \sigma^2)$$

$$P(Y = Y_i \cap X = X_i | \beta_0, \beta_1, \sigma^2)$$

- Using the chain rule of probability $P(A \cap B) = P(A|B) \cdot P(B)$

$$P(Y = Y_i \cap X = X_i | \beta_0, \beta_1, \sigma^2) = P(Y = Y_i | X = X_i, \beta_0, \beta_1, \sigma^2) \cdot P(X = X_i | \beta_0, \beta_1, \sigma^2)$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(Y_i - \beta_0 - \beta_1 X_i)^2\right] g(X_i)$$