
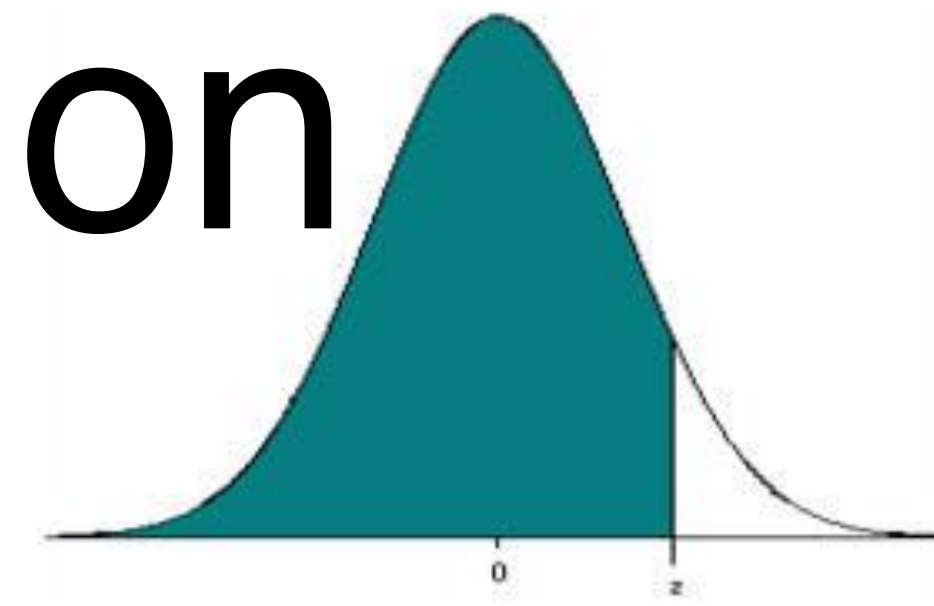


Normal



A normal distribution curve is shown with a horizontal axis. The area under the curve to the left of a point labeled z is shaded in teal. The mean of the distribution is marked at 0 on the axis.



Interval probabilities

Z table and scores

Standard deviation

Approximating Bernoulli

[illegible]

CDF

$$X \sim N(0, 1)$$

$$\Phi(x) \triangleq F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{y^2}{2}} dy$$

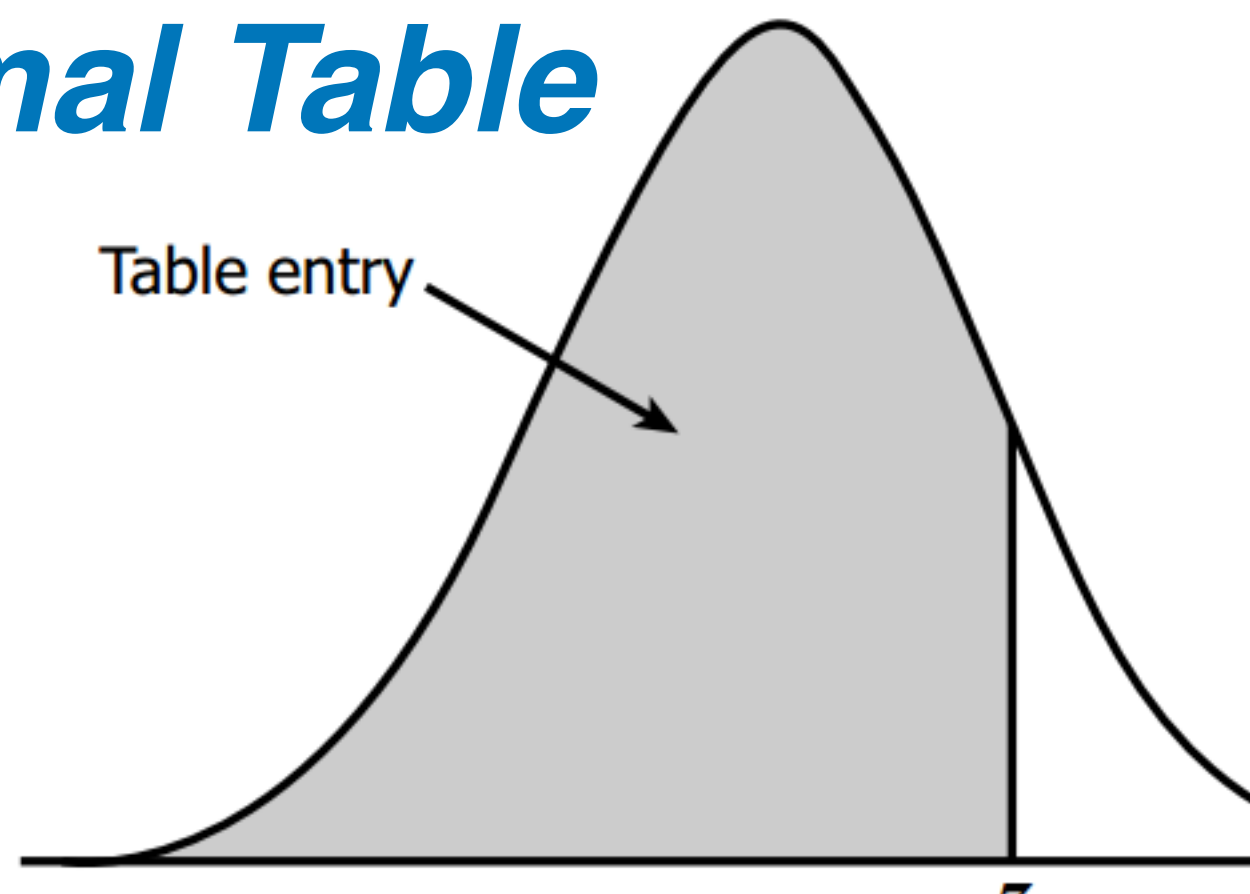
No known formula

Instead use table or computer

Table for each μ, σ ? 1 suffices!

Standard Normal Table

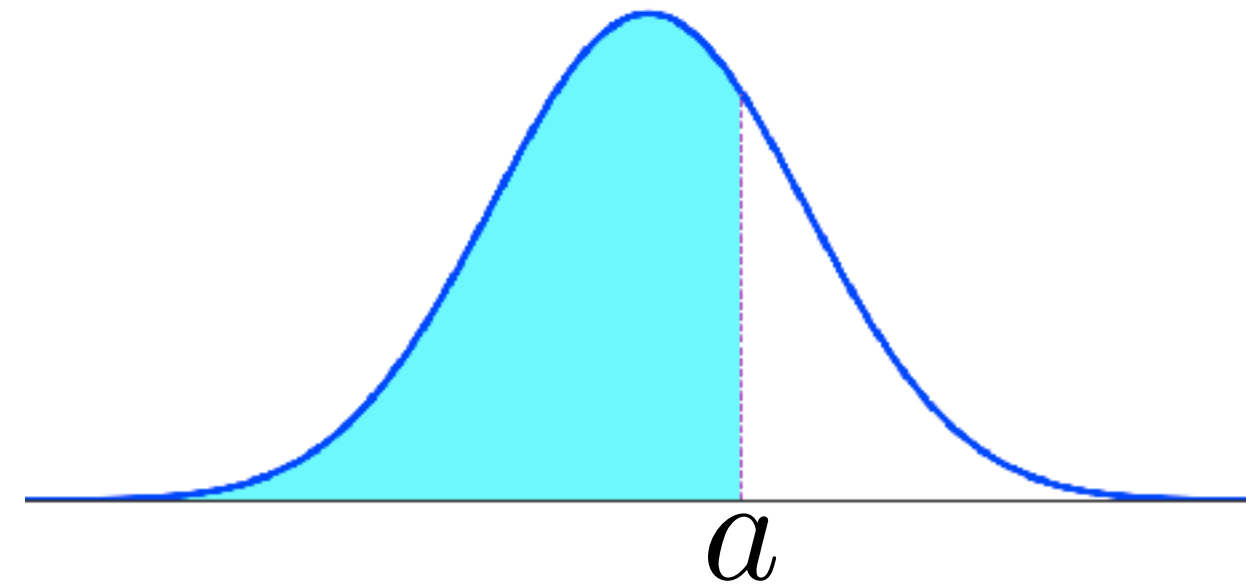
Z Table

[illegible]

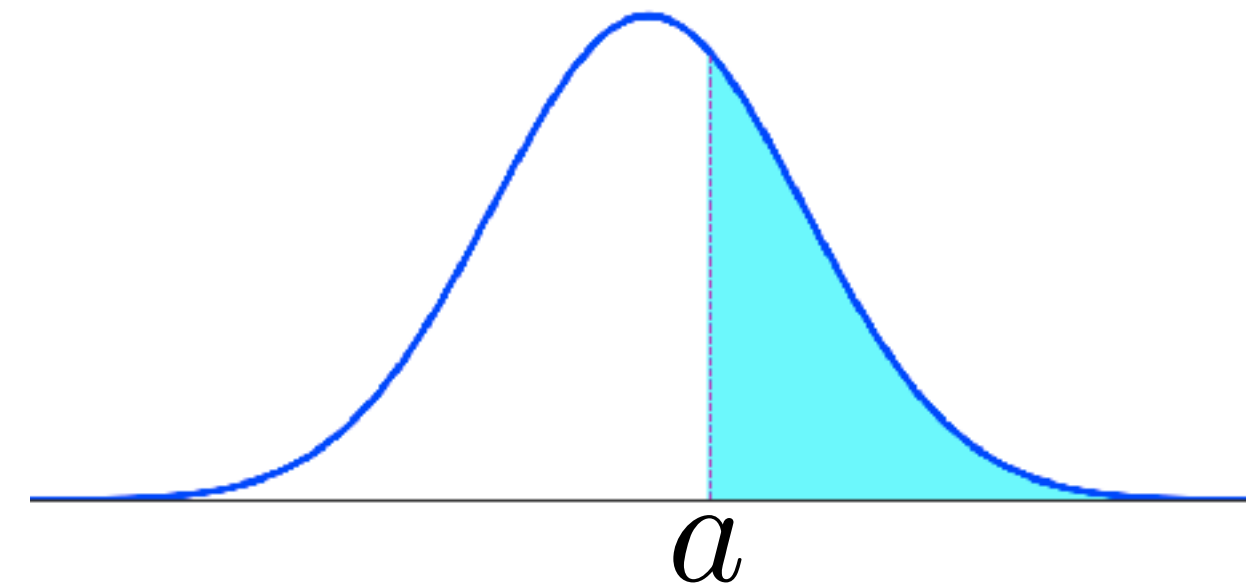
Using the Z Table

$$X \sim N(0, 1)$$

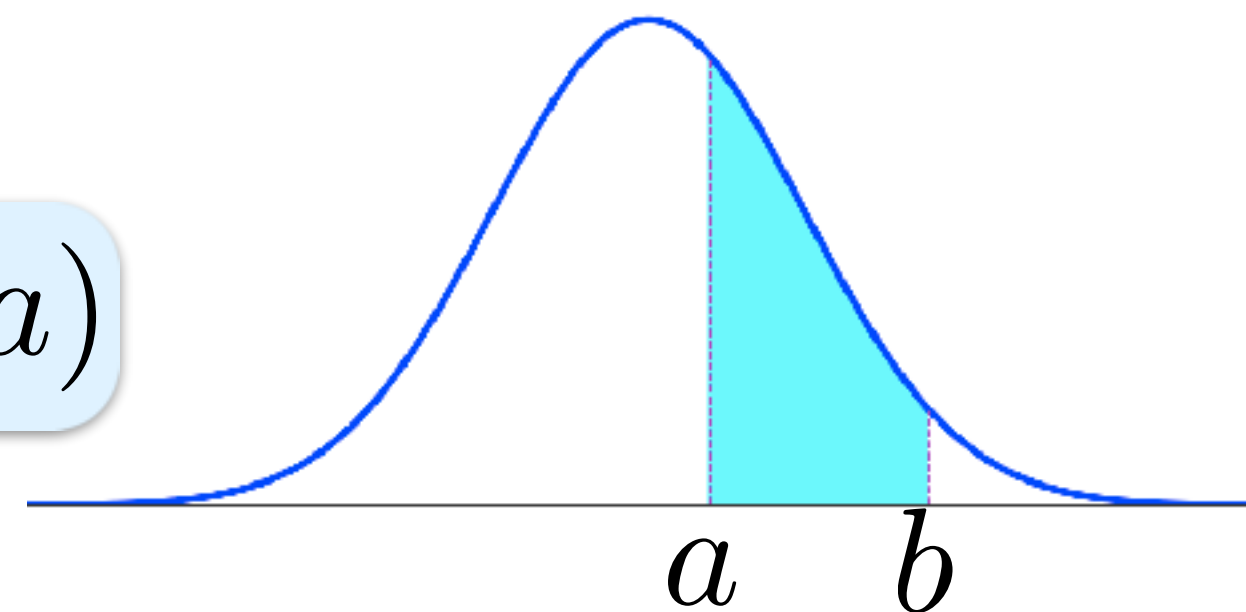
$$P(X \leq a) = \Phi(a)$$



$$P(X \geq a) = 1 - \Phi(a)$$



$$P(a \leq X \leq b) = \Phi(b) - \Phi(a)$$



Negative Values

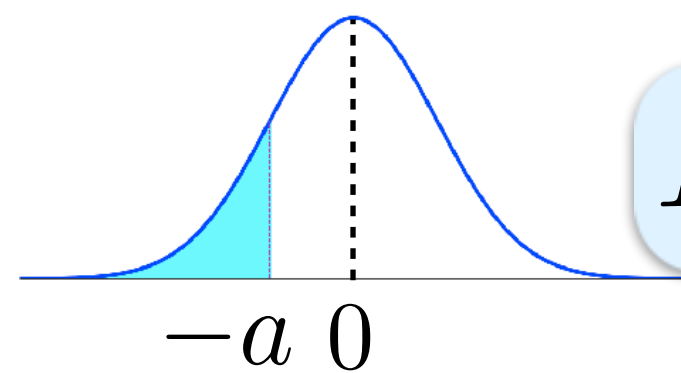
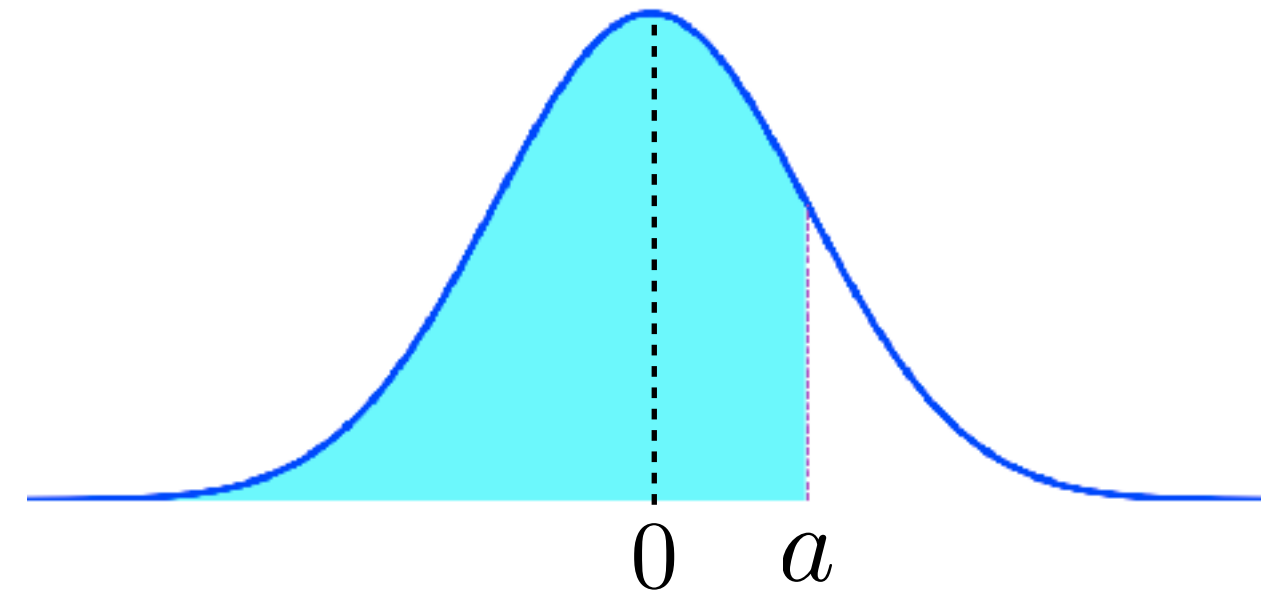
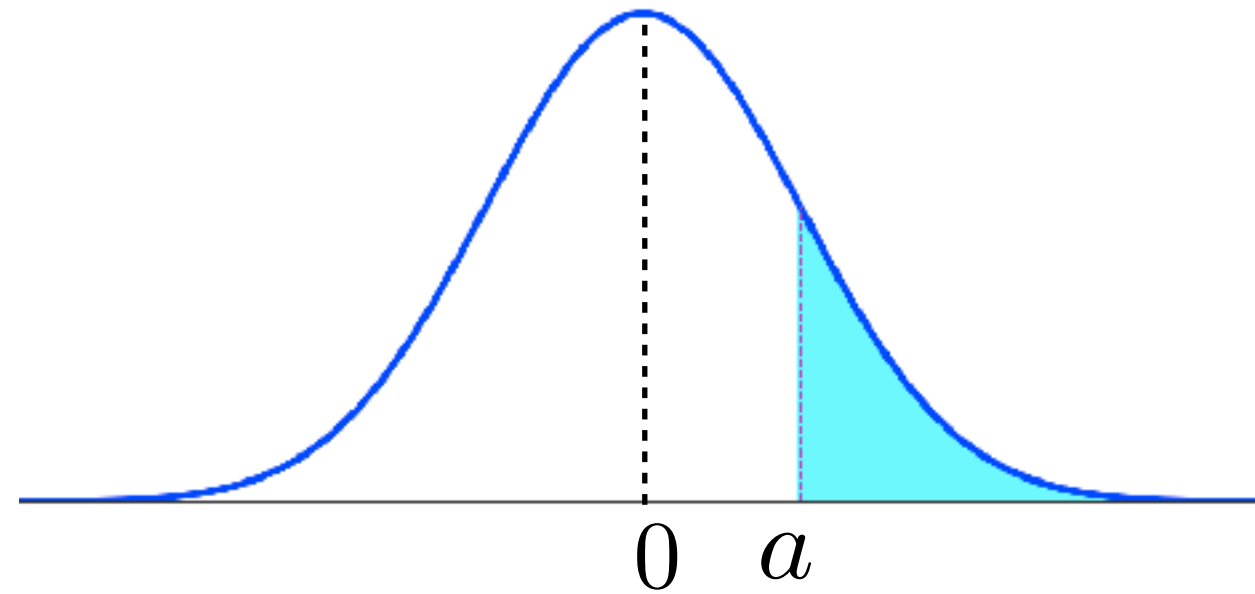
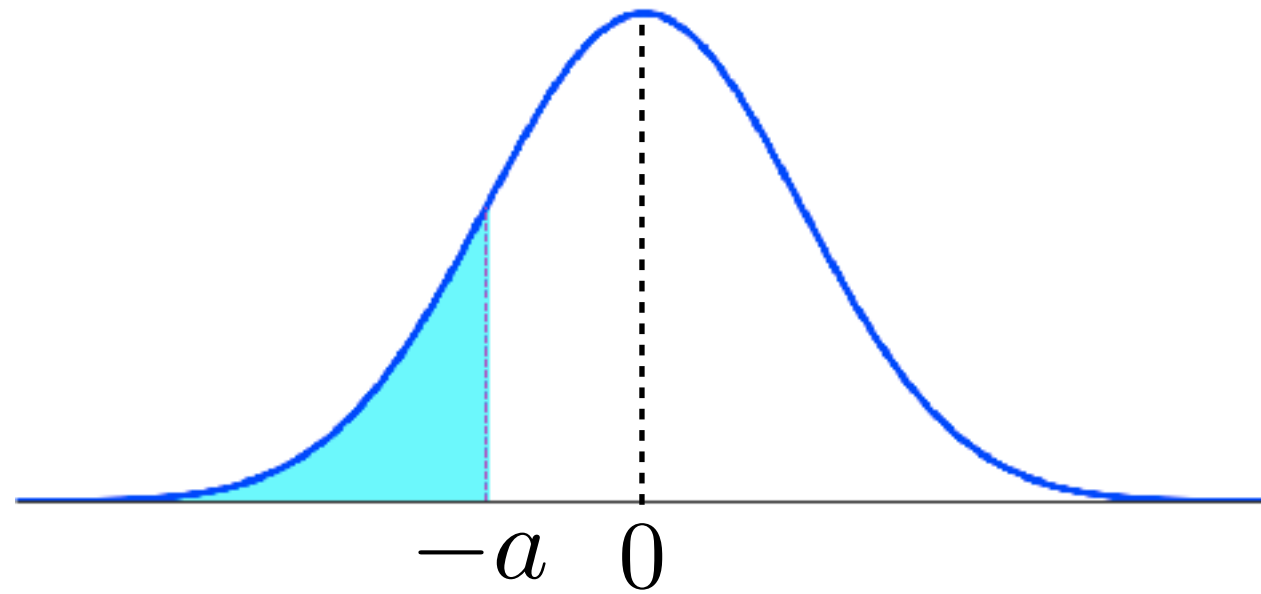
$$a > 0$$

$$\Phi(-a) = P(X \leq -a)$$

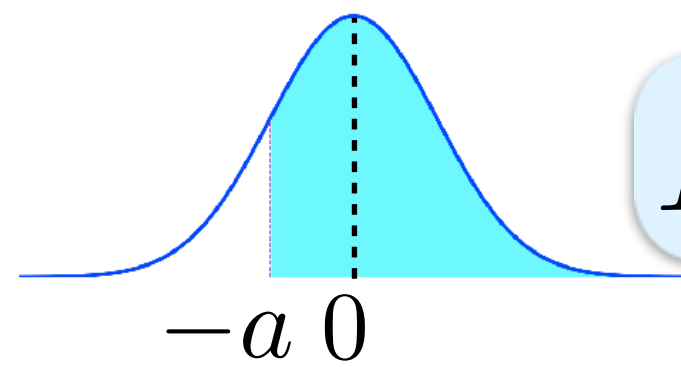
$$= P(X \geq a)$$

$$= 1 - P(X \leq a)$$

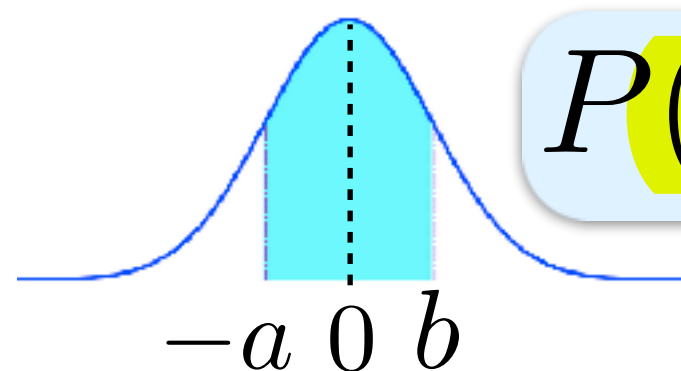
$$= 1 - \Phi(a)$$



$$P(X \leq -a) = \Phi(-a) = 1 - \Phi(a)$$



$$P(X \geq -a) = 1 - \Phi(-a) = \Phi(a)$$



$$P(-a \leq X \leq b) = \Phi(b) - \Phi(-a) = \Phi(b) - (1 - \Phi(a))$$

$$= \Phi(a) + \Phi(b) - 1$$

General Normal

$$X \sim N(\mu, \sigma^2)$$

Variable	E(X)	V(X)
X	μ	σ^2
$X - \mu$	0	σ^2
$\frac{X - \mu}{\sigma}$	0	1

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

Standardized Version of X

$$P(a \leq X \leq b)$$

$$= P\left(\frac{a - \mu}{\sigma} \leq \frac{X - \mu}{\sigma} \leq \frac{b - \mu}{\sigma}\right)$$

$$= P\left(\frac{a - \mu}{\sigma} \leq Z \leq \frac{b - \mu}{\sigma}\right)$$

Example

$$X \sim N(15, 4)$$

$$\mu = 15$$

$$\sigma = 2$$

Z score

$$Z = \frac{X - 15}{2}$$

$$P(11 \leq X \leq 17) = P\left(\frac{11-15}{2} \leq Z \leq \frac{17-15}{2}\right)$$

$$= P(-2 \leq Z \leq 1)$$

$$= \Phi(1) + \Phi(2) - 1$$

$$\approx 0.8413 + 0.9772 - 1$$

$$= 0.8185$$

<i>z</i>	.00	.01	.02	.03	.04
0.0	.5000	.5040	.5080	.5120	.5160
0.1	.5398	.5438	.5478	.5517	.5557
0.2	.5793	.5832	.5871	.5910	.5948
0.3	.6179	.6217	.6255	.6293	.6331
0.4	.6554	.6591	.6628	.6664	.6700
0.5	.6915	.6950	.6985	.7019	.7054
0.6	.7257	.7291	.7324	.7357	.7389
0.7	.7580	.7611	.7642	.7673	.7704
0.8	.7881	.7910	.7939	.7967	.7995
0.9	.8159	.8186	.8212	.8238	.8264
1.0	.8413	.8438	.8461	.8485	.8508
1.1	.8643	.8665	.8686	.8708	.8729
1.2	.8849	.8869	.8888	.8907	.8925
1.3	.9032	.9049	.9066	.9082	.9099
1.4	.9192	.9207	.9222	.9236	.9251
1.5	.9332	.9345	.9357	.9370	.9382
1.6	.9452	.9463	.9474	.9484	.9495
1.7	.9554	.9564	.9573	.9582	.9591
1.8	.9641	.9649	.9656	.9664	.9671
1.9	.9713	.9719	.9726	.9732	.9738
2.0	.9772	.9778	.9783	.9788	.9793
2.1	.9821	.9826	.9830	.9834	.9838
2.2	.9861	.9864	.9868	.9871	.9875
2.3	.9893	.9896	.9898	.9901	.9904
2.4	.9918	.9920	.9922	.9925	.9927
2.5	.9938	.9940	.9941	.9943	.9945
2.6	.9953	.9955	.9956	.9957	.9959

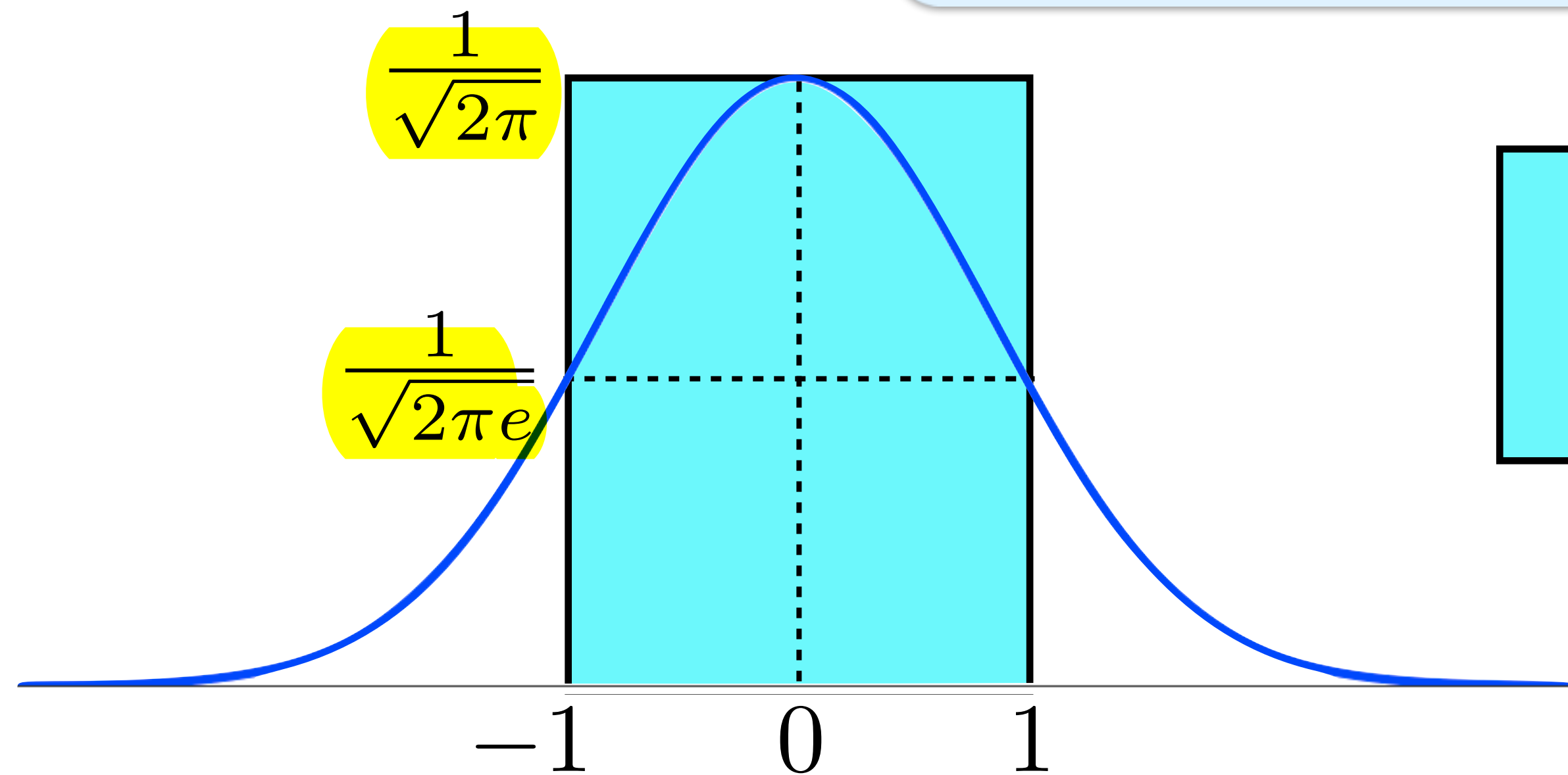
Standard Deviation and Probability

Probability of within σ

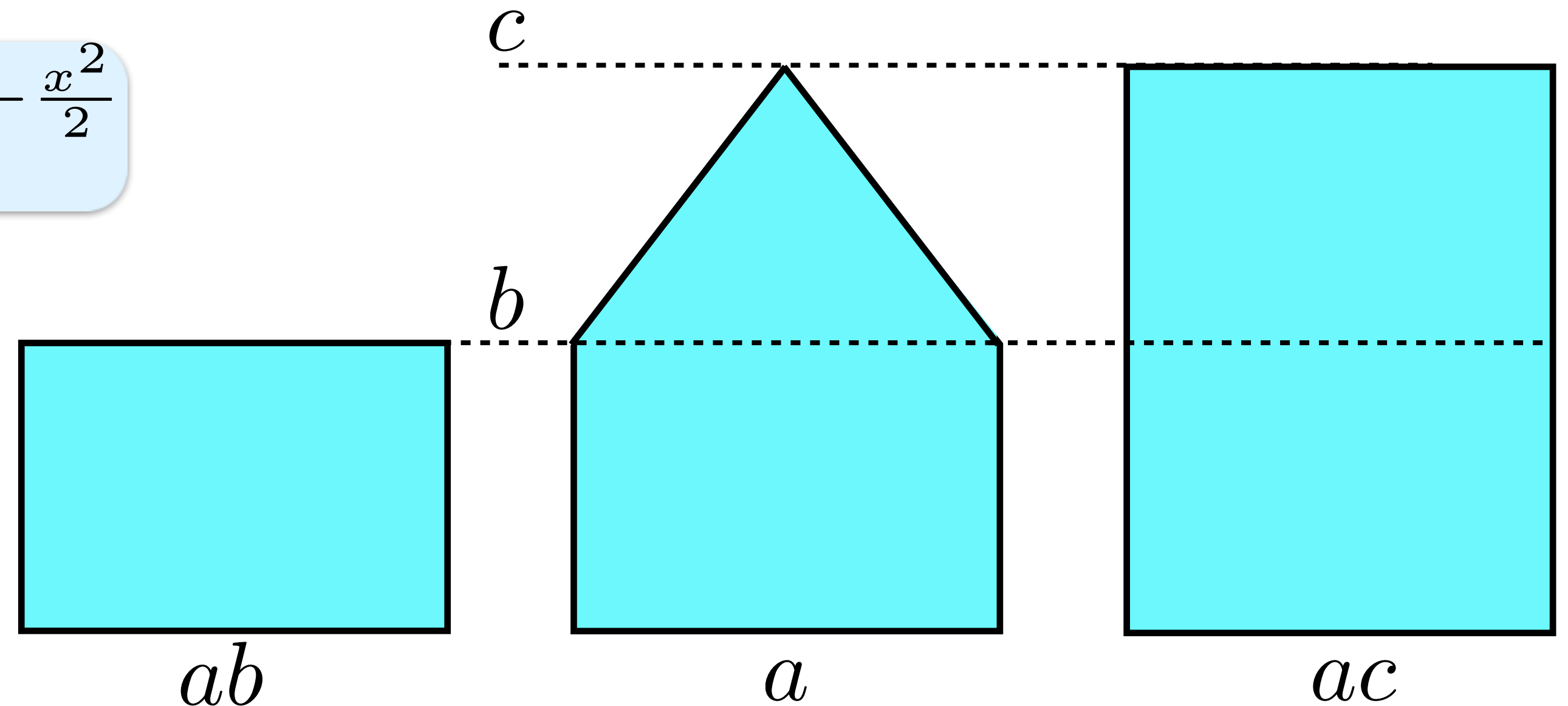
$$P(|X - \mu| \leq \sigma)?$$

Standard normal

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



How big's your house?



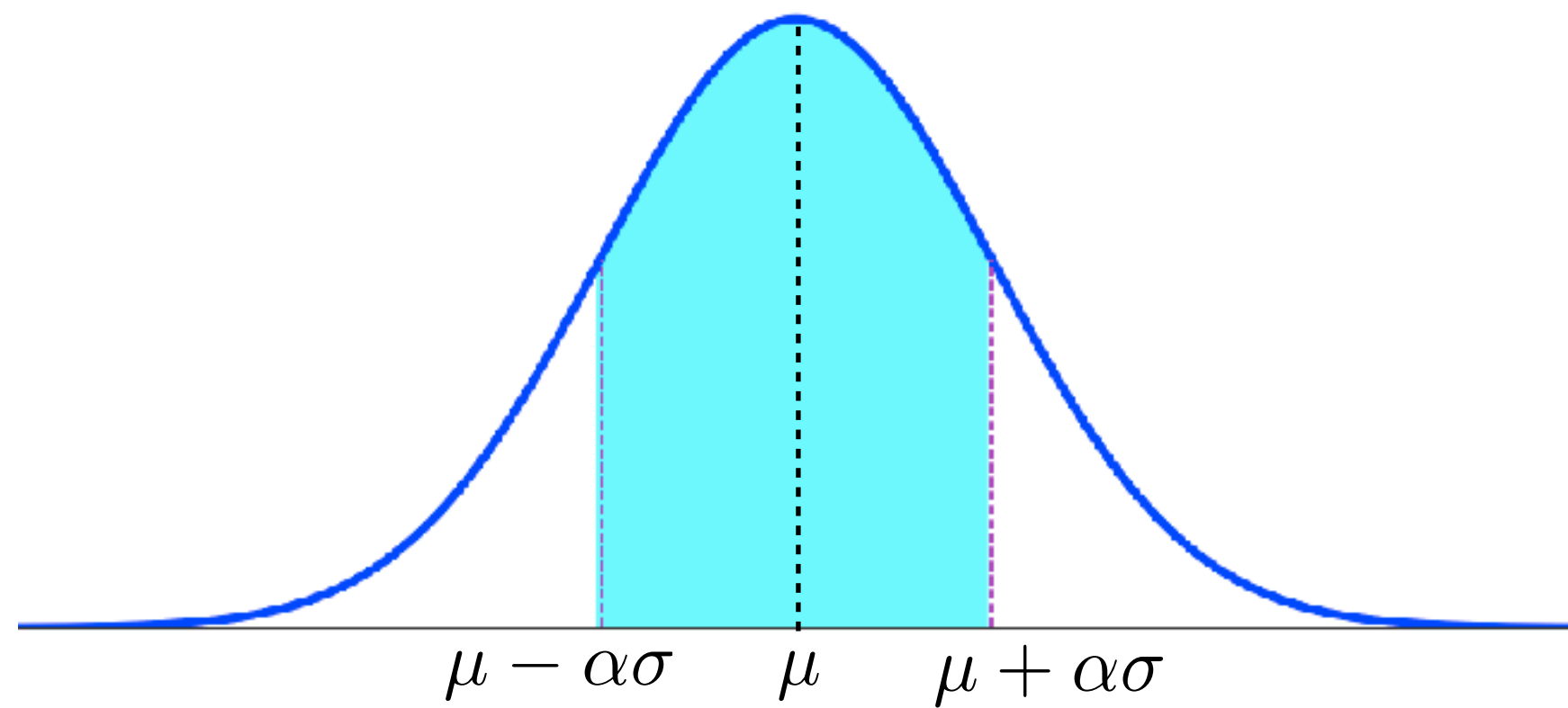
$$ab + \frac{a(c-b)}{2} = \frac{ab+ac}{2}$$

$$0.48 \approx \sqrt{\frac{2}{\pi e}} \leq P(|X - \mu| \leq \sigma) \leq \sqrt{\frac{2}{\pi}} \approx 0.8$$

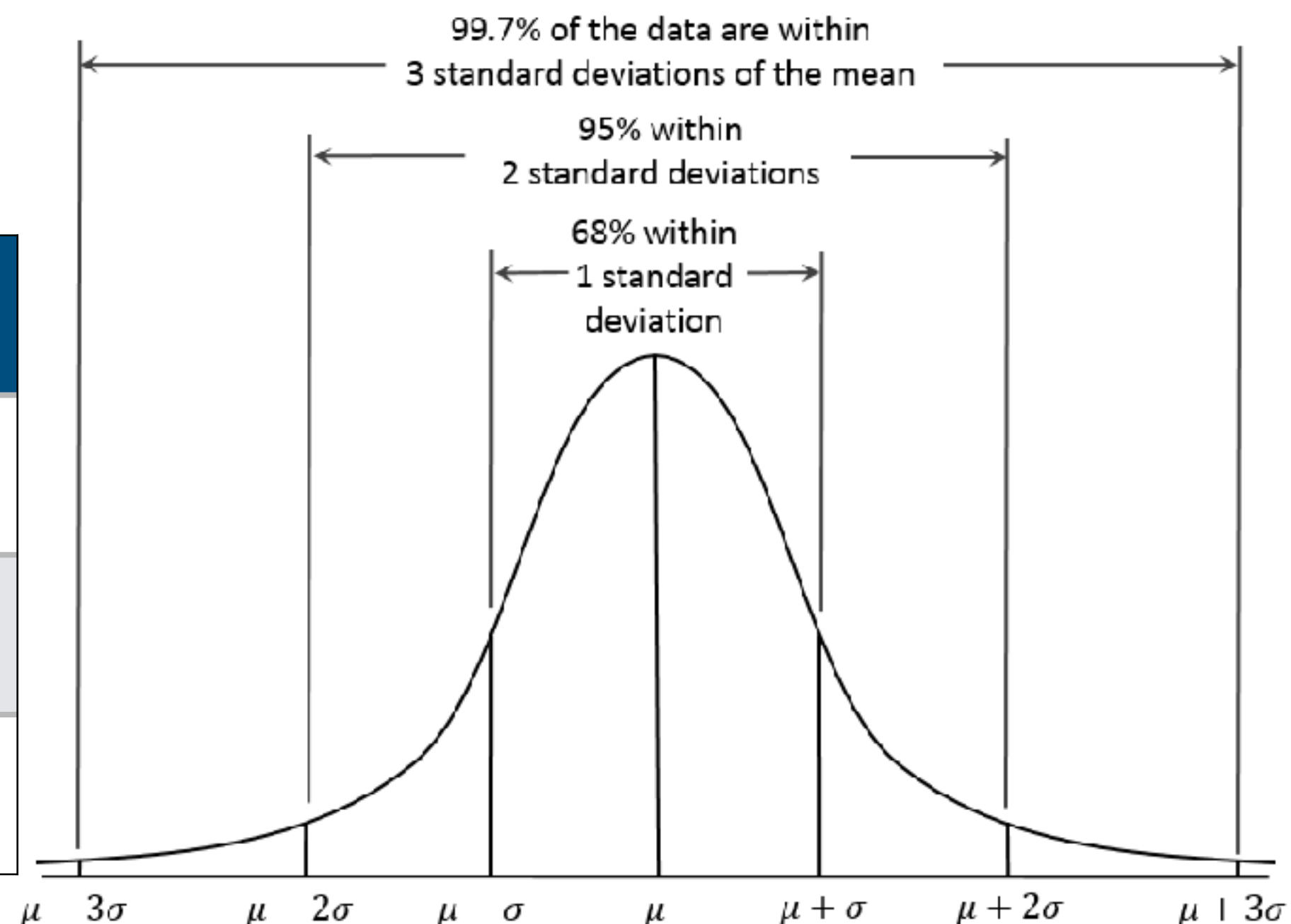
$$\frac{\sqrt{\frac{2}{\pi e}} + \sqrt{\frac{2}{\pi}}}{2} \approx 0.64$$

68 - 95 - 99.7 Rule

$$P(\mu - \alpha\sigma \leq X \leq \mu + \alpha\sigma) = P(-\alpha \leq Z \leq \alpha) = 2\Phi(\alpha) - 1$$



α	$P(X - \mu \leq \alpha\sigma)$
1	$2(0.8413) - 1 = 0.682$
2	$2(0.9772) - 1 = 0.9544$
3	$2(0.9987) - 1 = 0.9974$



Rare Events

α	$P(X - \mu \leq \alpha\sigma)$	$P(X - \mu > \alpha\sigma)$	Daily event ~
1	68.2%	31.8%	3 days
2	95.4%	4.6%	3 weeks
3	99.7%	0.3%	year
4	99.99%	0.01%	43 years
5	99.9999%	10^{-6}	4776 years
6	99.9999998%	$2 \cdot 10^{-9}$	1.3 millions years

6 σ philosophy

Life is normal

ab-

Normal Approximation of Binomial Distribution

Normal Approximation of Binomial

$$X \sim B_{n,p}$$

$$\mu = np$$

$$\sigma = \sqrt{npq}$$

$$Y \sim N(np, npq)$$

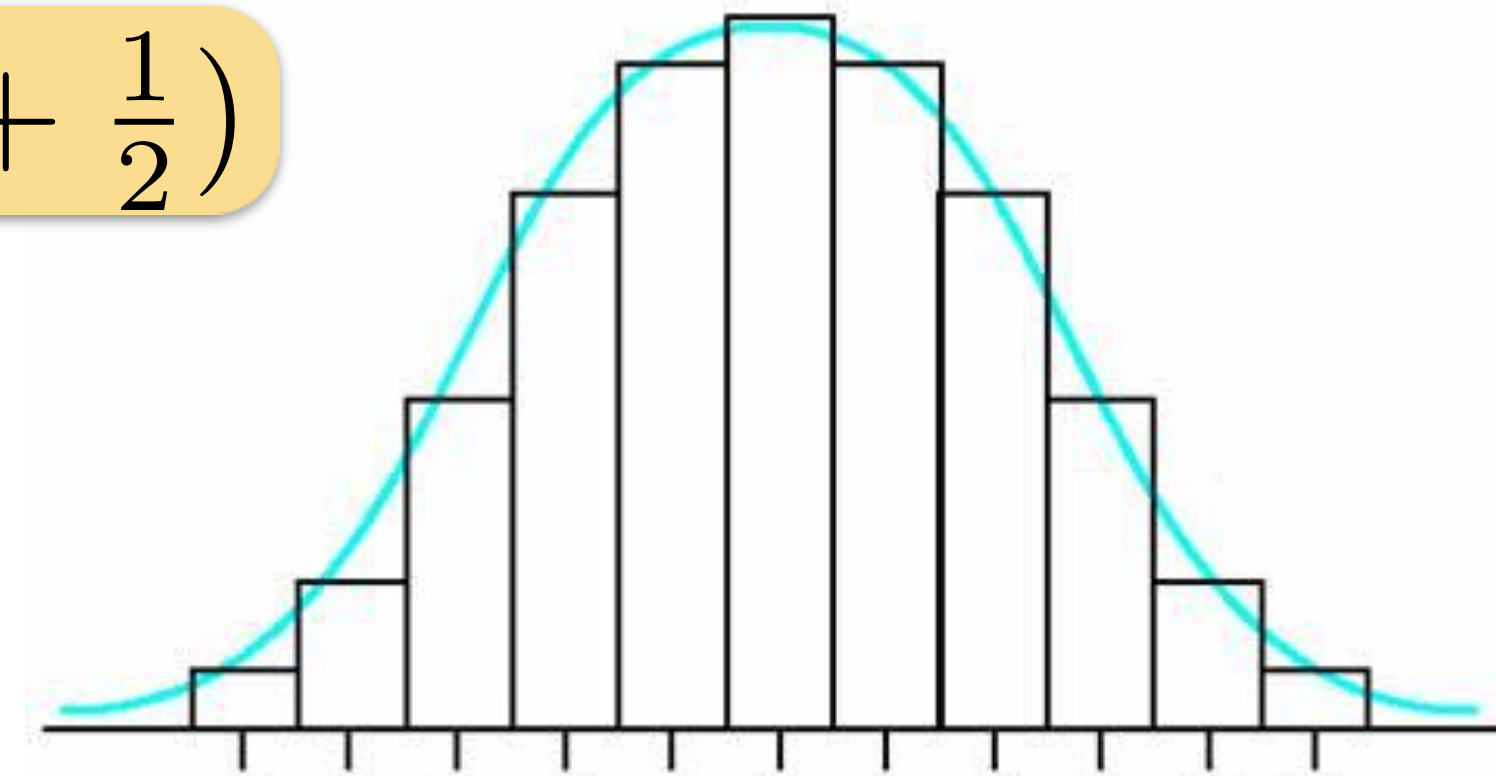
$$P(X = k) \approx P(k - \frac{1}{2} \leq Y \leq k + \frac{1}{2})$$

$$X \sim B_{100,0.5}$$

$$np = 50$$

$$\sqrt{npq} = 5$$

$$Y \sim N(50, 25)$$



$$Z = \frac{Y - 50}{5}$$

$$P(X = 60) \approx P(59.5 \leq Y \leq 60.5)$$

$$= P(1.9 = \frac{59.5 - 50}{5} \leq Z \leq \frac{60.5 - 50}{5} = 2.1)$$

$$= \Phi(2.1) - \Phi(1.9) \approx 0.9821 - 0.9713 = 0.0108$$

$$P(X = 60) = \binom{100}{60} 0.5^{100} \approx 0.0108$$

Interval Probabilities

$$X \sim B_{100,0.5}$$

$$np = 50$$

$$\sqrt{npq} = 5$$

$$Y \sim N(50, 25)$$

$$P(42 \leq X \leq 53) = P(41.5 \leq Y \leq 53.5)$$

$$= P\left(-1.7 = \frac{41.5-50}{5} \leq Z \leq \frac{53.5-50}{5} = 0.7\right)$$

$$= \Phi(0.7) + \Phi(1.7) - 1$$

$$\approx 0.7580 + 0.9554 - 1 = 0.7134$$

$$\text{Actual} = \sum_{k=42}^{53} \binom{100}{k} \left(\frac{1}{2}\right)^{100} \approx 0.7136$$



Normal Distribution Probabilities

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007
-3.0	0.0010	0.0010	0.0010	0.0011	0.0011	0.0011	0.0011	0.0010	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0044	0.0043	0.0042	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035
-2.5	0.0054	0.0053	0.0052	0.0051	0.0050	0.0049	0.0048	0.0047	0.0046	0.0045
-2.4	0.0064	0.0063	0.0062	0.0061	0.0060	0.0059	0.0058	0.0057	0.0056	0.0055
-2.3	0.0077	0.0075	0.0074	0.0073	0.0072	0.0071	0.0070	0.0069	0.0068	0.0067
-2.2	0.0090	0.0088	0.0087	0.0086	0.0085	0.0084	0.0083	0.0082	0.0081	0.0080
-2.1	0.0109	0.0107	0.0106	0.0105	0.0104	0.0103	0.0102	0.0101	0.0100	0.0099
-2.0	0.0139	0.0136	0.0134	0.0133	0.0132	0.0131	0.0129	0.0128	0.0127	0.0126
-1.9	0.0179	0.0174	0.0172	0.0171	0.0169	0.0168	0.0167	0.0166	0.0164	0.0163
-1.8	0.0228	0.0222	0.0219	0.0217	0.0215	0.0214	0.0212	0.0211	0.0209	0.0208
-1.7	0.0287	0.0280	0.0276	0.0273	0.0271	0.0269	0.0267	0.0266	0.0264	0.0263
-1.6	0.0359	0.0351	0.0347	0.0344	0.0341	0.0339	0.0337	0.0335	0.0334	0.0332
-1.5	0.0448	0.0438	0.0434	0.0431	0.0428	0.0426	0.0424	0.0423	0.0421	0.0420
-1.4	0.0539	0.0528	0.0523	0.0519	0.0516	0.0513	0.0511	0.0509	0.0508	0.0506
-1.3	0.0636	0.0623	0.0617	0.0613	0.0609	0.0606	0.0603	0.0601	0.0599	0.0598
-1.2	0.0749	0.0734	0.0728	0.0723	0.0719	0.0716	0.0713	0.0711	0.0709	0.0707
-1.1	0.0879	0.0861	0.0854	0.0849	0.0844	0.0841	0.0838	0.0836	0.0834	0.0832
-1.0	0.1044	0.1025	0.1018	0.1013	0.1009	0.1005	0.1002	0.1000	0.0998	0.0996
-0.9	0.1255	0.1233	0.1225	0.1219	0.1214	0.1210	0.1207	0.1205	0.1203	0.1201
-0.8	0.1493	0.1469	0.1461	0.1454	0.1448	0.1443	0.1438	0.1434	0.1430	0.1427
-0.7	0.1772	0.1742	0.1734	0.1726	0.1719	0.1714	0.1709	0.1705	0.1701	0.1698
-0.6	0.2147	0.2107	0.2099	0.2090	0.2083	0.2076	0.2071	0.2066	0.2062	0.2058
-0.5	0.2643	0.2599	0.2590	0.2581	0.2573	0.2566	0.2559	0.2553	0.2548	0.2543
-0.4	0.3085	0.3039	0.3030	0.3021	0.3013	0.3006	0.2999	0.2993	0.2988	0.2983
-0.3	0.3745	0.3699	0.3689	0.3679	0.3670	0.3662	0.3654	0.3647	0.3641	0.3636
-0.2	0.4362	0.4314	0.4304	0.4294	0.4285	0.4277	0.4269	0.4262	0.4256	0.4251
-0.1	0.4602	0.4552	0.4542	0.4532	0.4522	0.4514	0.4506	0.4499	0.4493	0.4487
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Interval probabilities

Z table and scores

Standard deviation

Approximating Bernoulli

Next

Inequalities and limits