- Motivation, intuition
- Formulation
- Proof
- Example

Markov → Chebyshev

Markov

Probability that non-neg X is α times larger than its mean is $\leq 1/\alpha$

Chebyshev

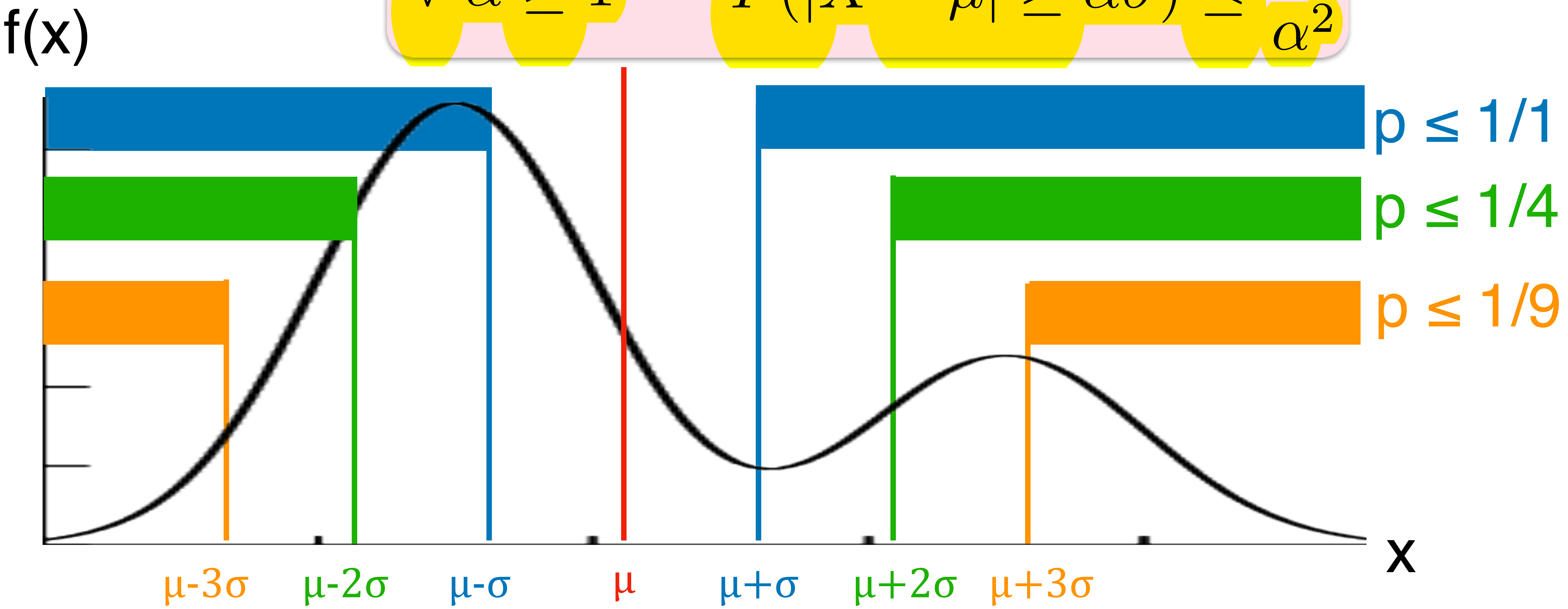
Probability that any X is more than $\alpha\sigma$ times further from μ is $\leq 1/\alpha^2$

Chebyshev's Inequality

Same two versions 1 Easier to illustrate, understand, remember

X is any discrete or continuous r.v. with finite mean μ and std σ

$$\forall\ \alpha \geq 1 \qquad P(|X-\mu| \geq \alpha\sigma) \leq \frac{1}{\alpha^2}$$



Two Formulations

X is any discrete or continuous r.v. with finite mean μ and std σ

1 Easier to visualize, understand, remember

$$\forall\ \alpha \geq 1 \qquad P(|X-\mu| \geq \alpha\sigma) \leq \frac{1}{\alpha^2}$$

2 Easier to prove, use

$a = \alpha\sigma$

$$\forall\ a \geq \sigma \qquad P(|X-\mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

Towards a Proof

Markov's Inequality

$$\forall\ a \geq \mu \qquad P(X \geq a) \leq \frac{\mu}{a}$$

Chebyshev's Inequality

$$P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

$$P((X - \mu)^2 \geq a^2) \leq \frac{\sigma^2}{a^2}$$

Need: nonnegative r.v. with mean σ^2

Proof

$$\boxed{P(|X-\mu_X| \ge a) \le \frac{\sigma_X^2}{a^2}}$$

Y
Soon

X - any random variable $\quad \mu_X = EX \quad \sigma_X^2 = V(X) = E(X-\mu_X)^2$

$$Y = (X-\mu_X)^2 \qquad Y \ge 0 \qquad \mu_Y = E(X-\mu_X)^2 = \sigma_X^2$$

$$P(|X-\mu_X| \ge a) = P((X-\mu_X)^2 \ge a^2)$$

$$= P(Y \ge a^2)$$

Markov

$$\le \frac{\mu_Y}{a^2} = \frac{\sigma_X^2}{a^2}$$

Citations

X - # paper citations

$\mu = 8$

Suppose $\sigma = 5$

$P(X \geq 28)$?

Markov

$P(X \geq 28) \leq 8/28 \approx 29\%$

$$P(X \geq a) \leq \frac{\mu}{a}$$

Chebyshev

$P(X \geq 28) = P(X - \mu \geq 20)$

$\leq P(|X - \mu| \geq 20)$

$\leq \left(\frac{\sigma}{20}\right)^2 = \left(\frac{5}{20}\right)^2 = \frac{1}{16} \approx 6.3\%$

$$P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

Markov: $\leq 0.02\%$

$P(X \geq 40{,}000) = P(X - \mu \geq 39{,}992)$

$\leq P(|X - \mu| \geq 39{,}992)$

$\leq \left(\frac{\sigma}{39{,}992}\right)^2 = \left(\frac{5}{39{,}992}\right)^2 = 1.6 \times 10^{-6}\ \%$

Survey Responses

Survey expected to result in μ = 1M responses with σ = 50K

Bound P(0.8M < # responses < 1.2M)

$0.8M = \mu - 4\sigma$ $\quad$ $1.2M = \mu + 4\sigma$

$P(\mu - 4\sigma < X < \mu + 4\sigma) = P(|X - \mu| < 4\sigma)$

$= 1 - P(|X - \mu| \geq 4\sigma)$

$\geq 1 - 1/16$

$= 15/16$

Mark x Che

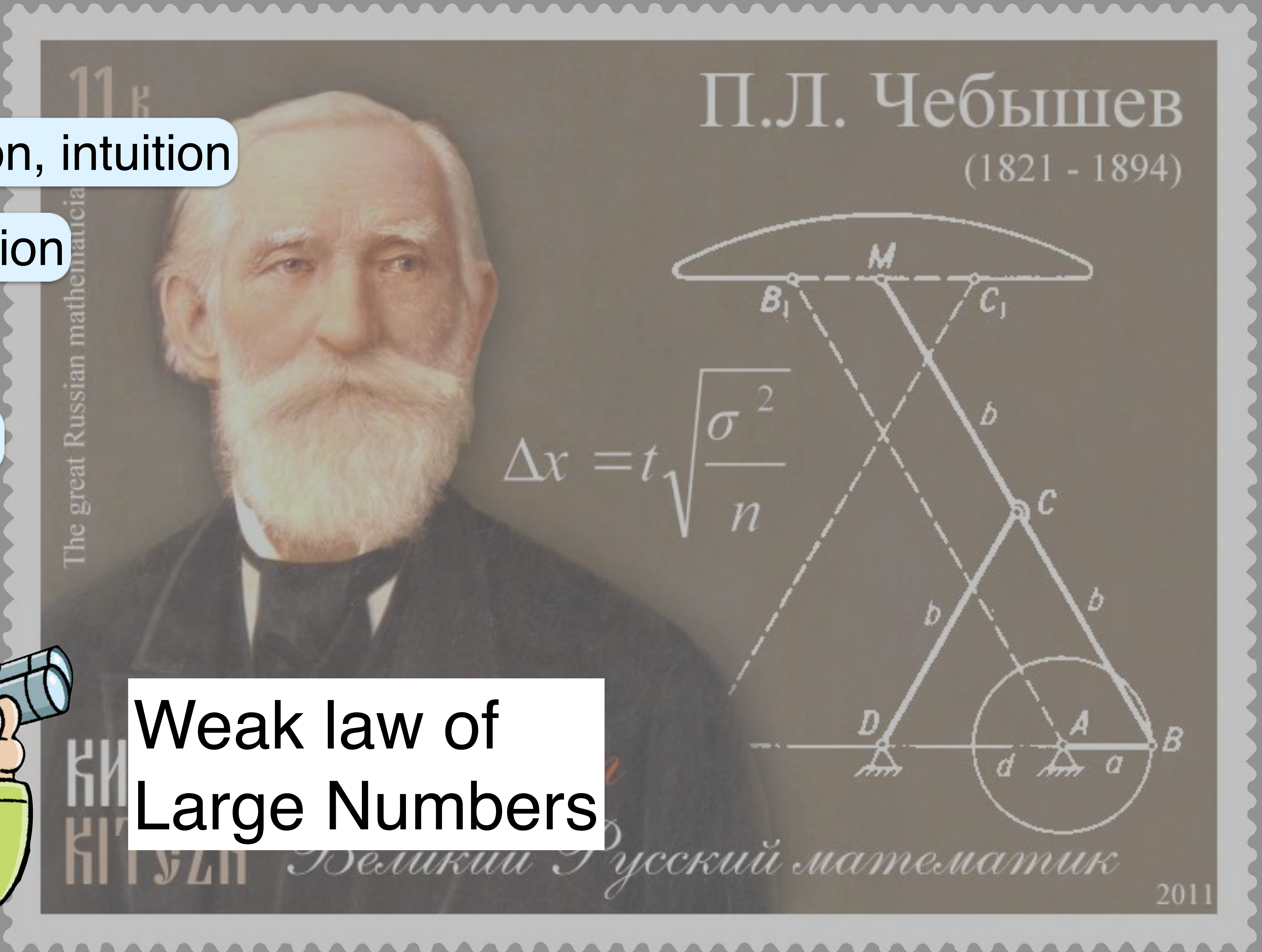
	Formula	Applies	Input	Range	Decreases
Markov	$P(X \geq a) \leq \frac{\mu}{a}$	X≥0	μ	a ≥ μ	Linearly
Chebyshev	$P(\lvert X - \mu \rvert \geq a) \leq \frac{\sigma^2}{a^2}$	Any X	μ and σ	a ≥ σ	Quadratically

Motivation, intuition

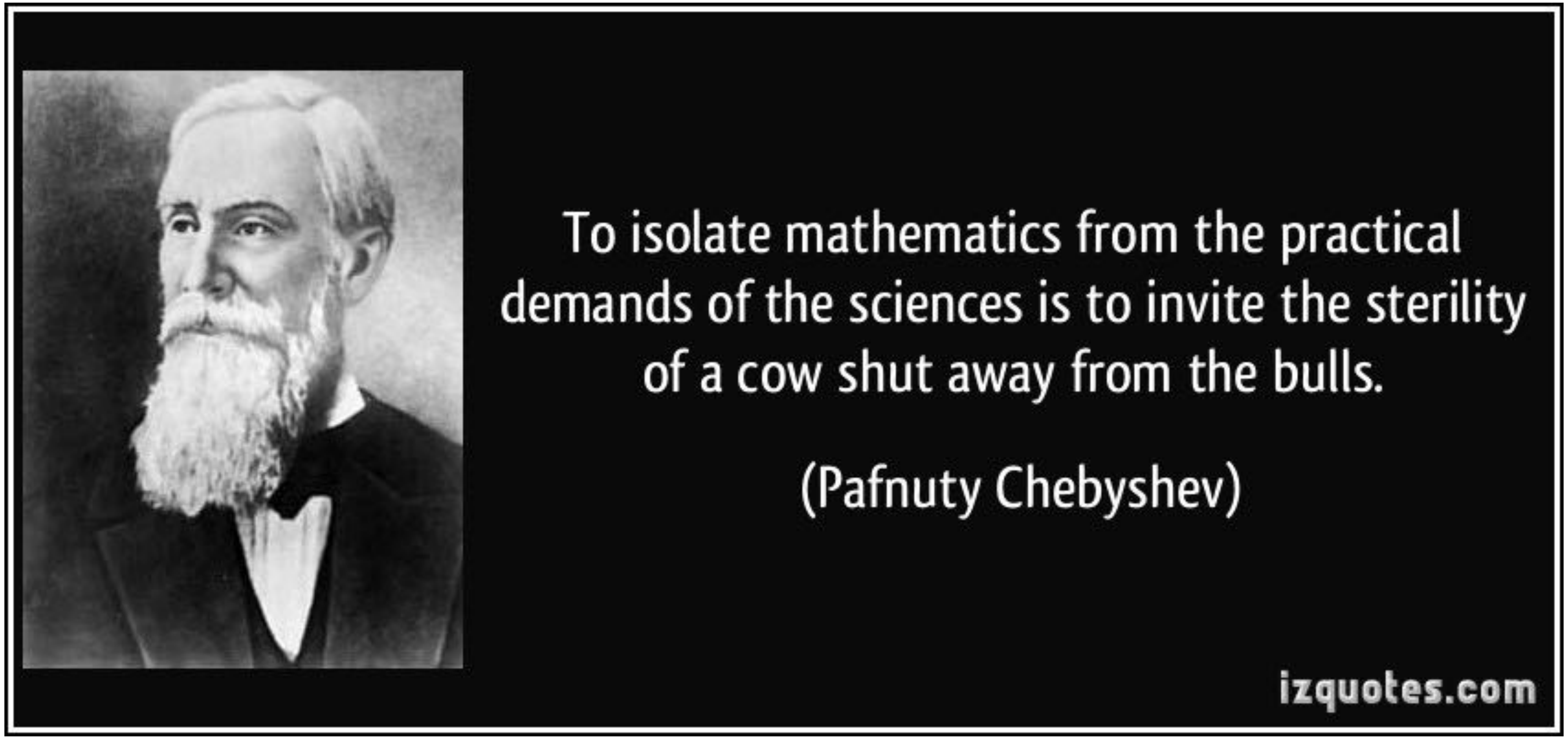
Formulation

Proof

Example



Weak law of Large Numbers



Alternative spellings: *Chebychev*, *Chebysheff*, Chebychov, *Chebyshov*;*Tchebychev*, *Tchebycheff, Tschebyschev*, *Tschebyschef*, *Tschebyscheff*