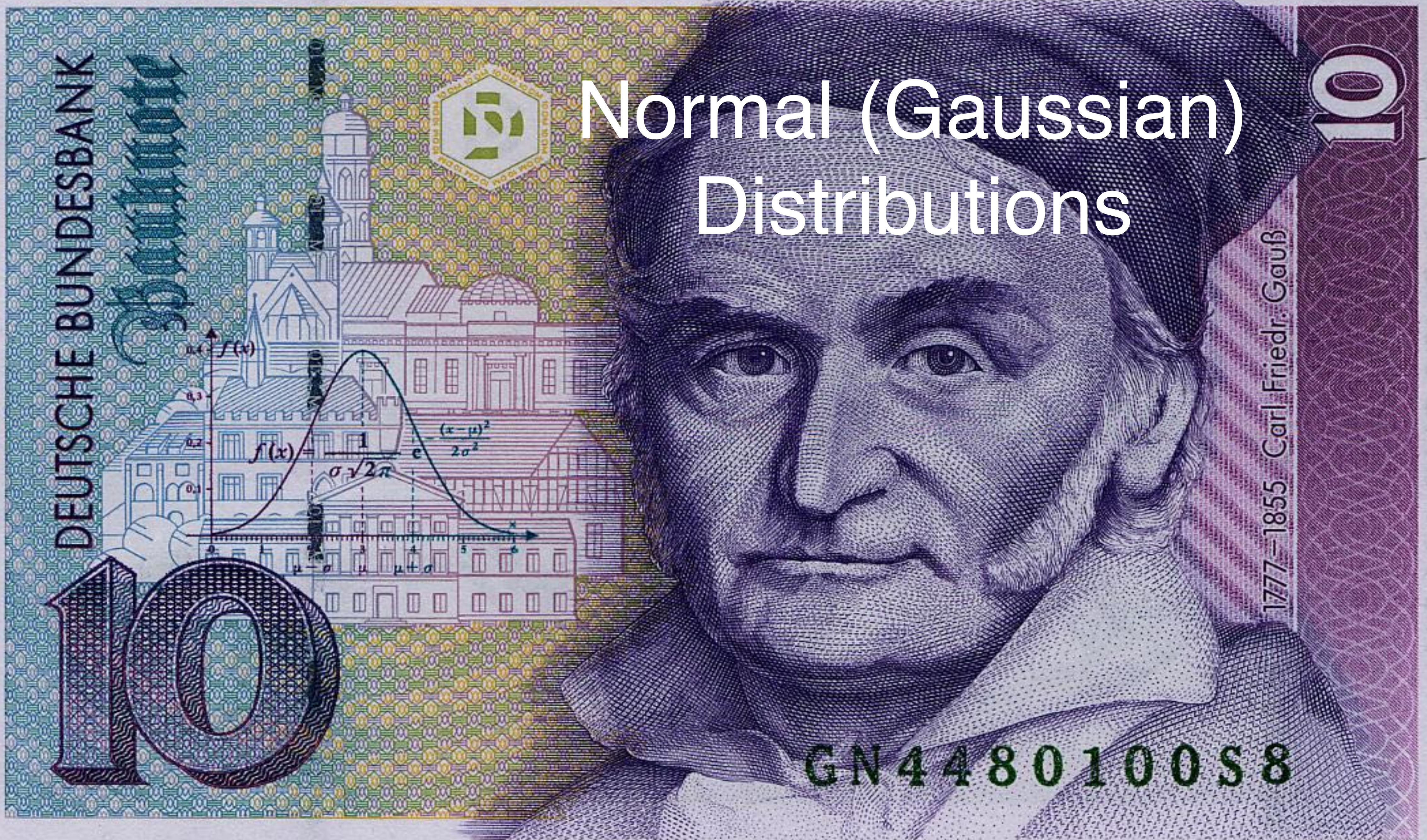


Normal (Gaussian) Distributions



Mean

Variance

Definition

$$X \sim N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

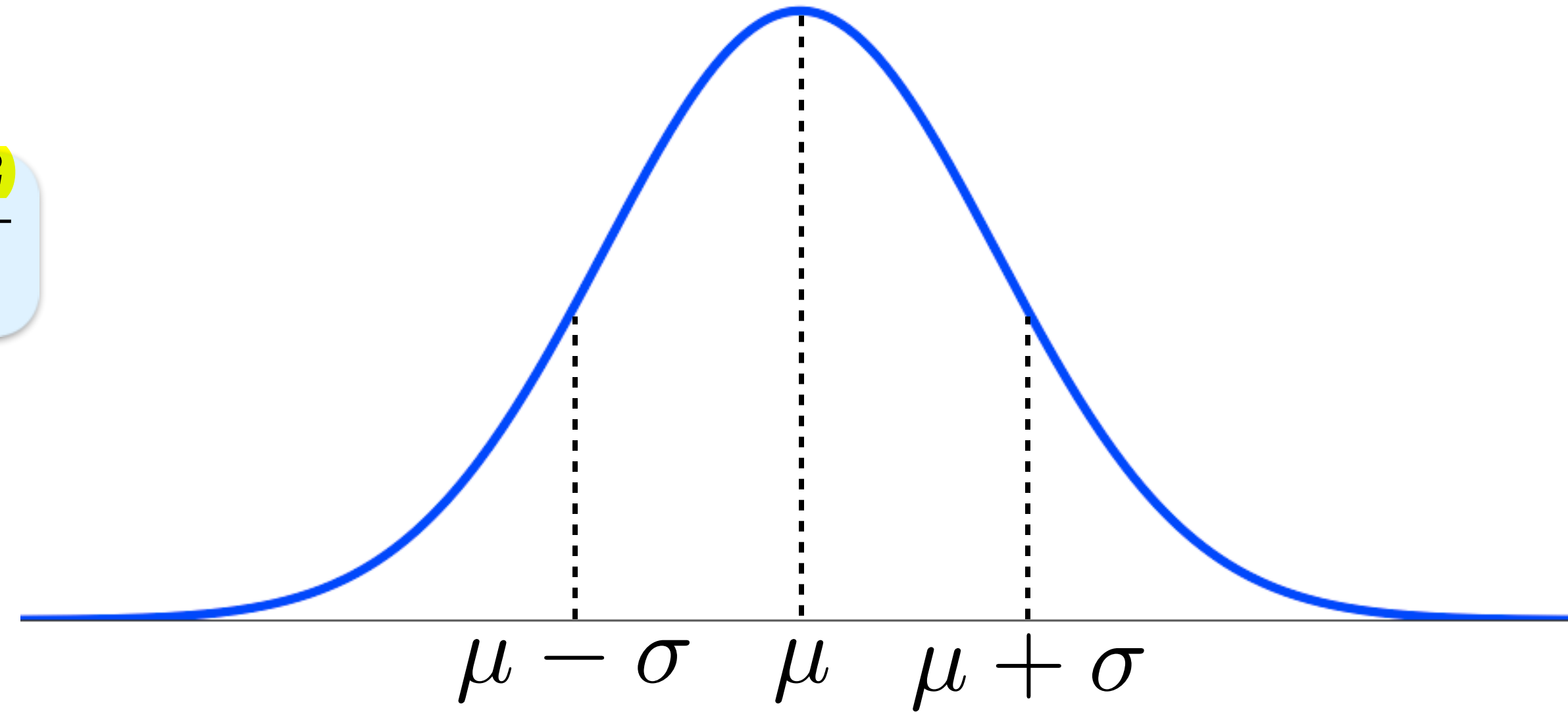
Bell Curve

Very common

Occurs whenever adding many independent factors

height, weight, rainfall, salaries,

Approximates binomial distribution



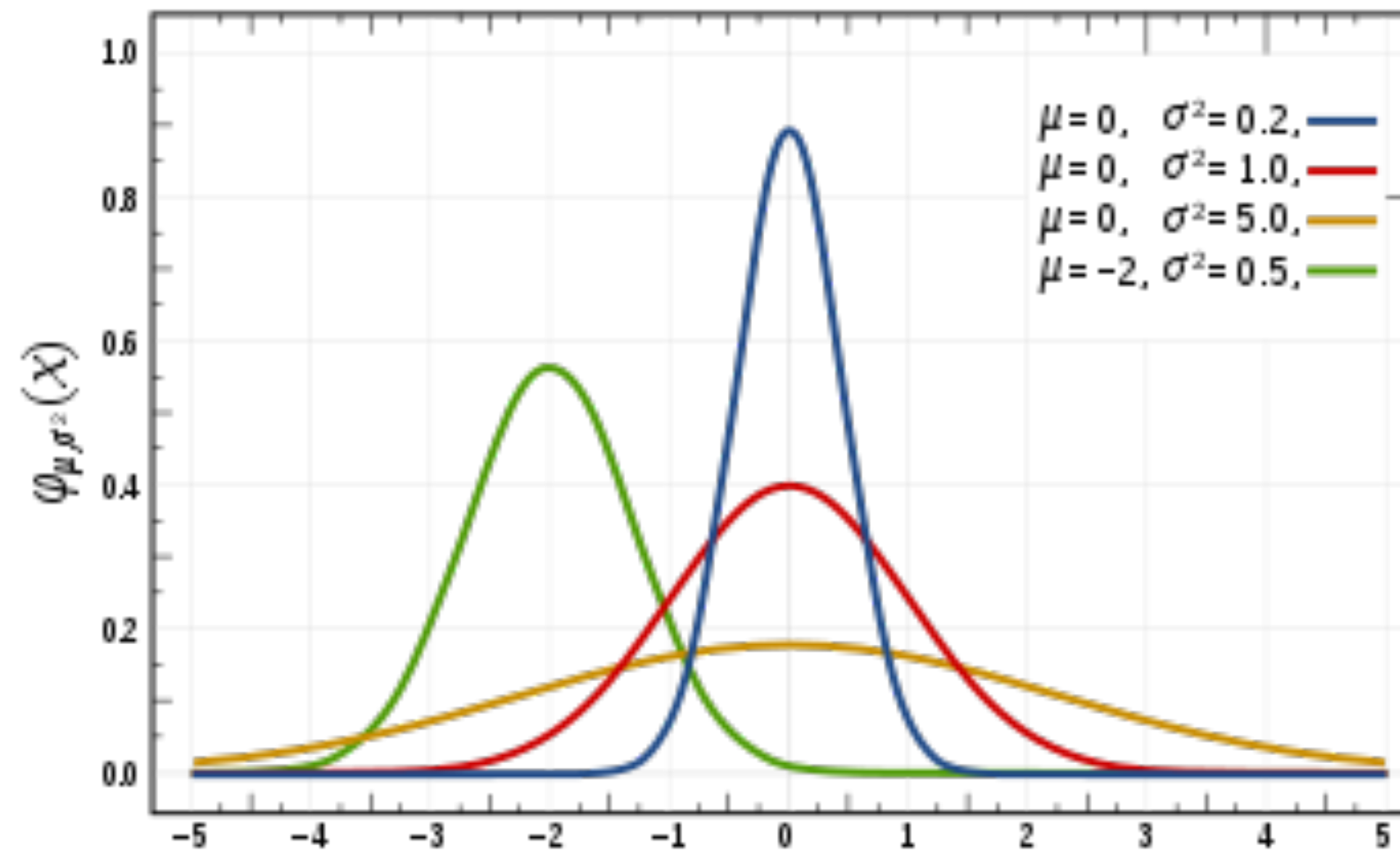
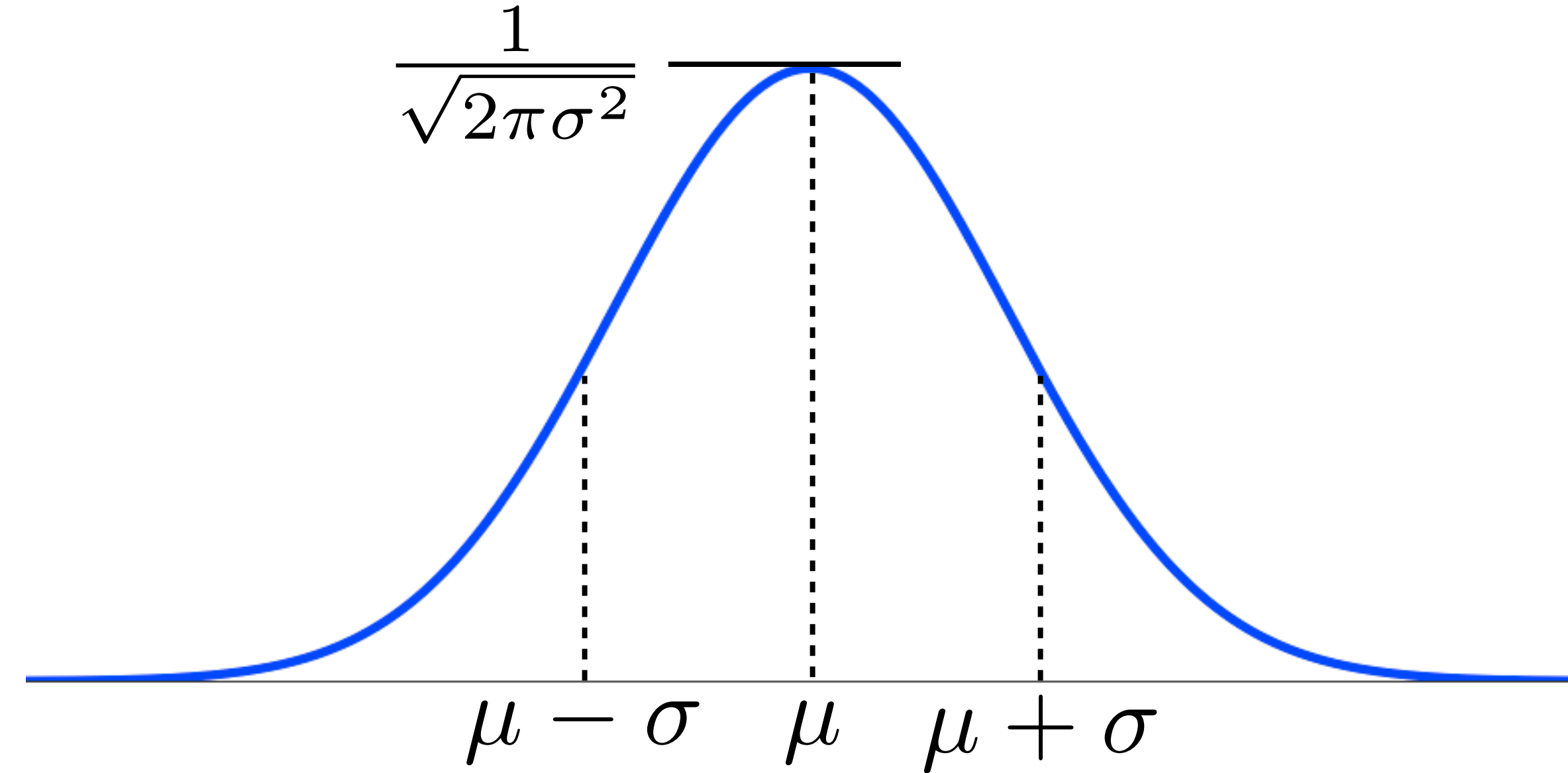
Observations

Symmetric around mean μ

μ most likely

$$f(\mu) = \frac{1}{\sqrt{2\pi\sigma^2}}$$

As σ grows, distribution gets more flat



Linear Transformations

Linear transformations of normal distributions are normal

$$X \sim N(\mu, \sigma)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$Y = aX + b$$

As for all r.v.

$$\mu_Y = a\mu_X + b$$

$$\sigma_Y = a\sigma_X$$

Show normal

Variable transformation

$$f_Y(y) = \frac{1}{(ax+b)'} f_X(x) \Big|_{y=ax+b} \longleftrightarrow x = \frac{y-b}{a}$$

$$= \frac{1}{a} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\frac{y-b}{a} - \mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\underbrace{\sqrt{2\pi(a\sigma)^2}}_{\sigma_Y}} e^{-\frac{(y - \underbrace{(a\mu+b)}_{\mu_Y})^2}{2\underbrace{(a\sigma)^2}_{\sigma_Y^2}}}$$

$$Y \sim N(a\mu + b, (a\sigma)^2)$$

Standard Normal Distribution

Without loss of generality consider $X \sim N(0,1)$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Helpful integral

$$\int x e^{-\frac{x^2}{2}} dx = -e^{-\frac{x^2}{2}}$$

Σ Will it ADD?

$$I = \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$I^2 = \left(\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx \right) \left(\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}} dy \right)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{(x^2+y^2)}{2}} dx dy$$

$$= \int_0^{\infty} \int_0^{2\pi} e^{-\frac{r^2}{2}} r d\theta dr$$

$$= \int_0^{\infty} r e^{-\frac{r^2}{2}} \int_0^{2\pi} d\theta dr$$

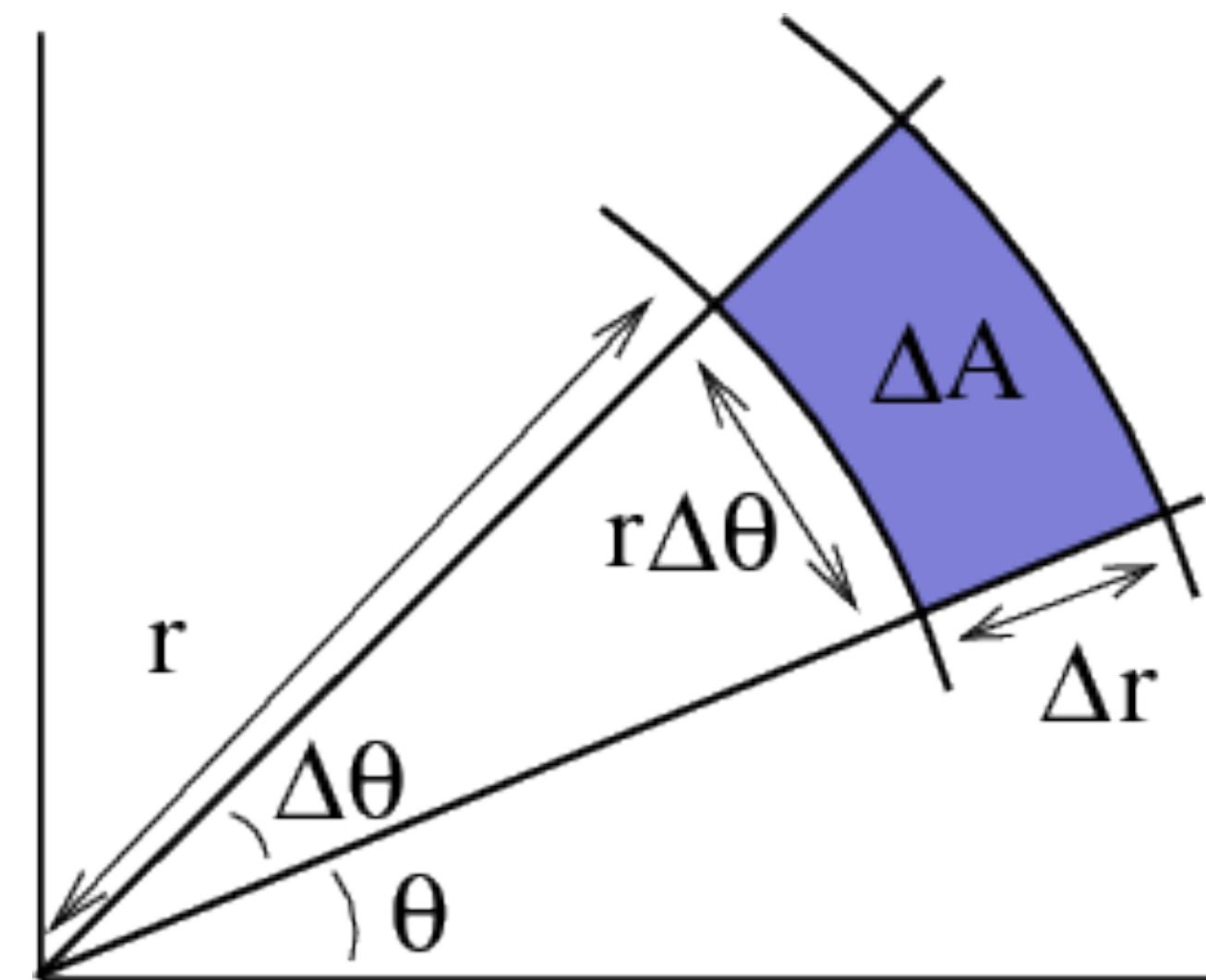
$$= 2\pi \int_0^{\infty} r e^{-\frac{r^2}{2}} dr$$

$$= -2\pi e^{-\frac{r^2}{2}} \Big|_0^{\infty} = 2\pi$$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$dx dy = r dr d\theta$$

$$\int r e^{-\frac{r^2}{2}} dr = -e^{-\frac{r^2}{2}}$$



$$I = \sqrt{2\pi}$$

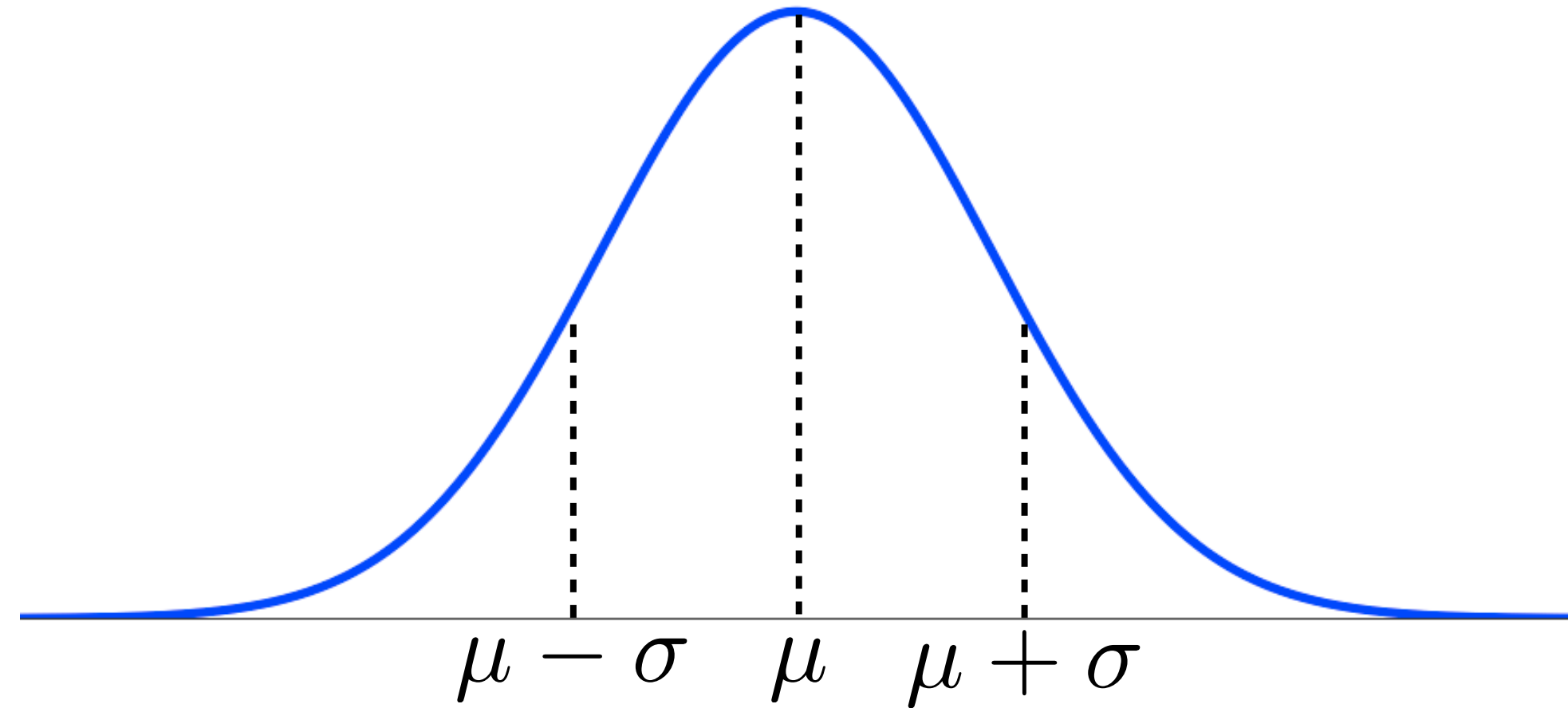
$$\int \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1$$

**YES IT
ADDS!**

Expectation

Symmetry

$$E(X) = 0$$



Calculation

$$E(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{x^2}{2}} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[-e^{-\frac{x^2}{2}} \right]_{-\infty}^{\infty} = 0$$

Variance

$$E(X^2) = \frac{1}{\sqrt{2\pi}} \int x^2 e^{-\frac{x^2}{2}} dx$$

$$u = x$$

$$dv = x e^{-\frac{x^2}{2}} dx$$

$$\int u dv = uv - \int v du$$

$$du = dx$$

$$v = -e^{-\frac{x^2}{2}}$$

$$= \frac{-1}{\sqrt{2\pi}} x e^{-\frac{x^2}{2}} \Big|_{-\infty}^{\infty} + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx = 0 + 1 = 1$$

$$V(X) = E(X^2) - (EX)^2 = 1 - 0 = 1$$

$$N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

$$EX = \mu$$

$$V = \sigma^2$$

$$\sigma = \sigma$$

Very common in nature

Normal (Gaussian) Distributions

Next Probabilities