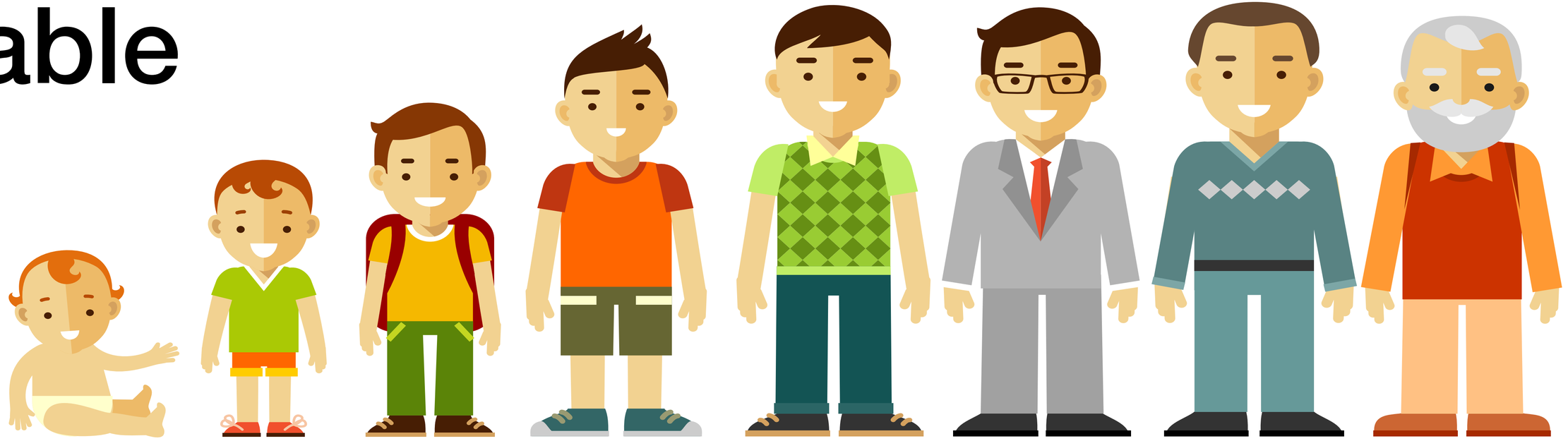
Variable



Modifications

(aka functions)

Functions of a Random Variable

Random variables X take values in $\mathbb{R}$

Often interested in related variable $Y = g(X)$ $g:\mathbb{R}\to\mathbb{R}$ is a fixed function

X Random salary in $

$10 raise $Y = X + 10$

10% raise $Y = 1.1X$

→ CEO $Y = X^2$

Deterministic Functions

$Y = g(X)$

g is a deterministic function over $\mathbb{R}$ (or Ω)

$Y = X + 3$

All randomness in Y derives from X

Deterministically modified by g

$X = 5$ $Y = 8$

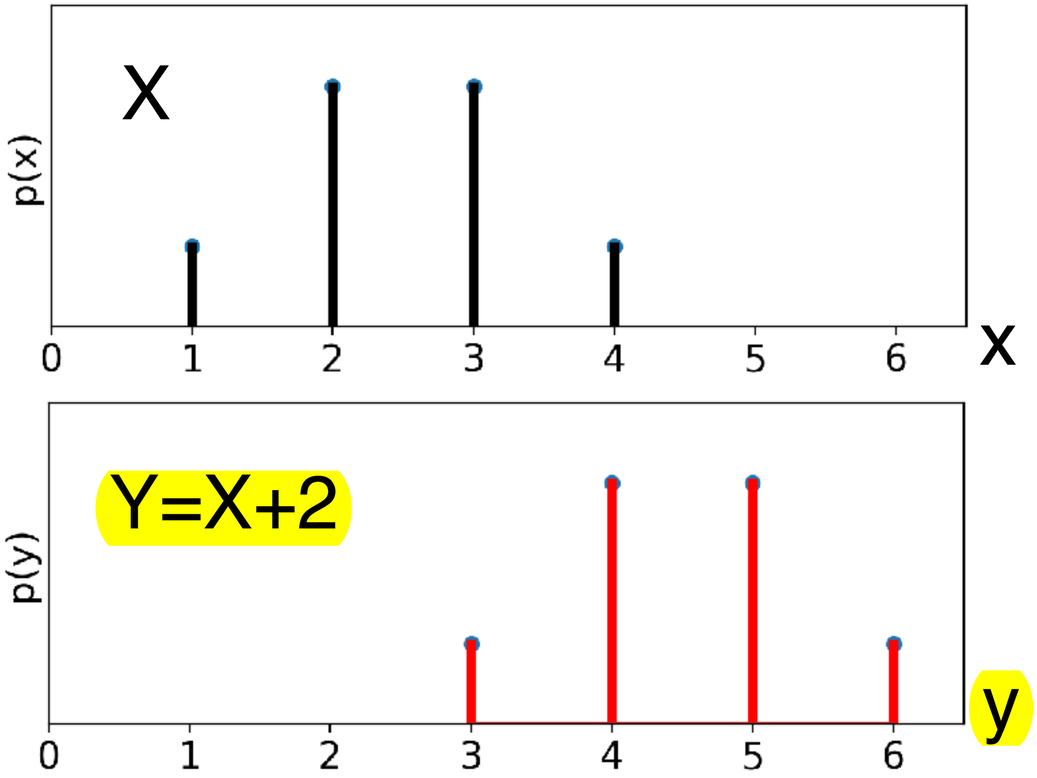
Translation

Add constant b to X

Translate X by b

$Y = X + b$

$P(Y=y) = P(X+b=y) = P(X=y-b)$



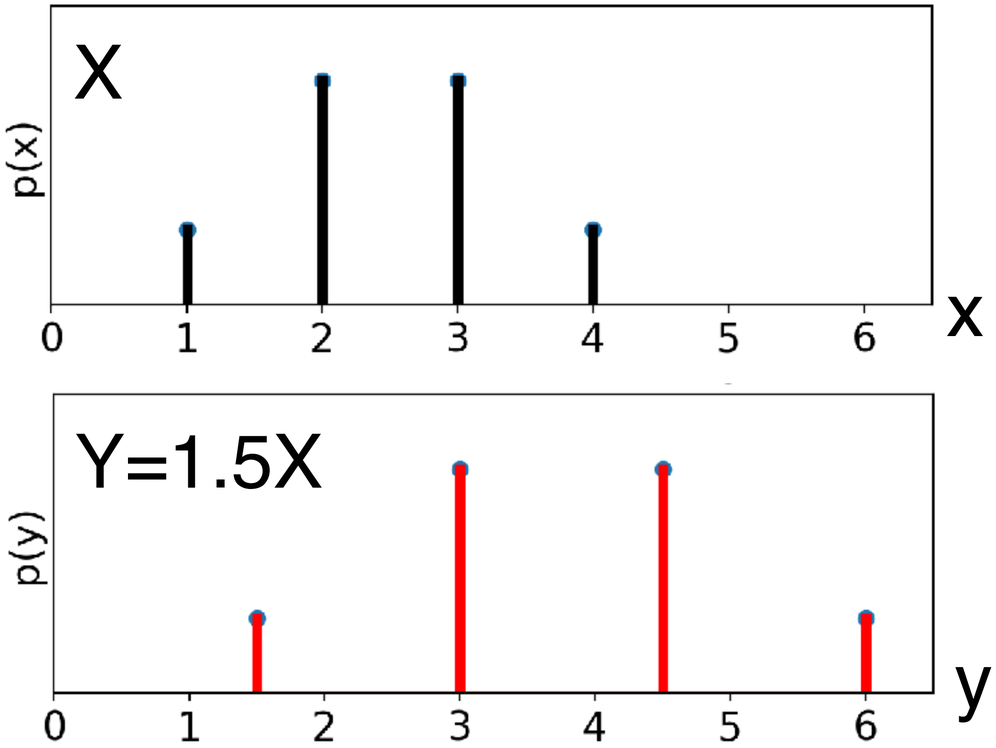
Scaling

Multiply X by a constant b

Scale X by a factor b

$Y = b \cdot X$

$P(Y=y) = P(bX=y) = P(X=y/b)$



Two Square Examples

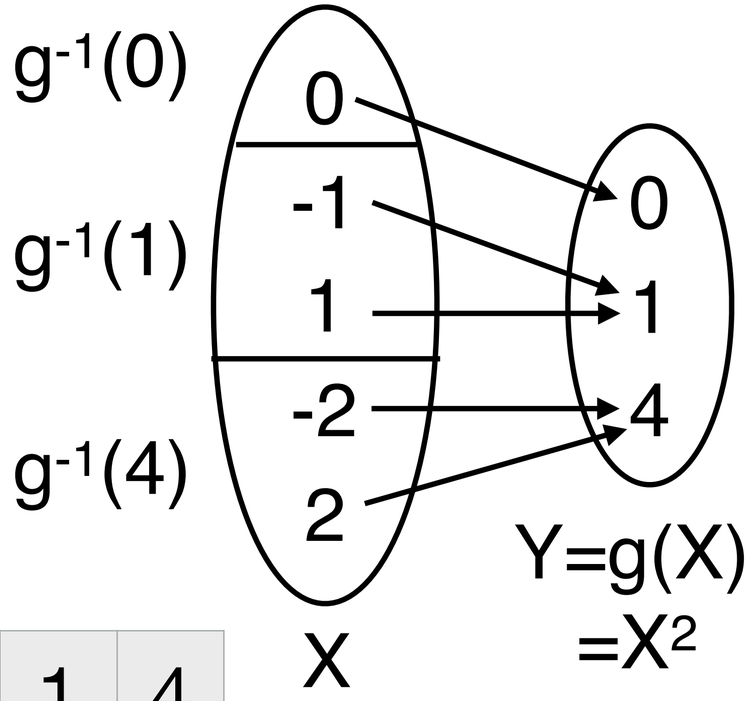
Square is 1-1

X

x	0	1	2
p(X = x)	½	⅓	⅙

$Y = X^2$

y	0	1	4
p(Y = y)	½	⅓	⅙



Square is many to 1

X

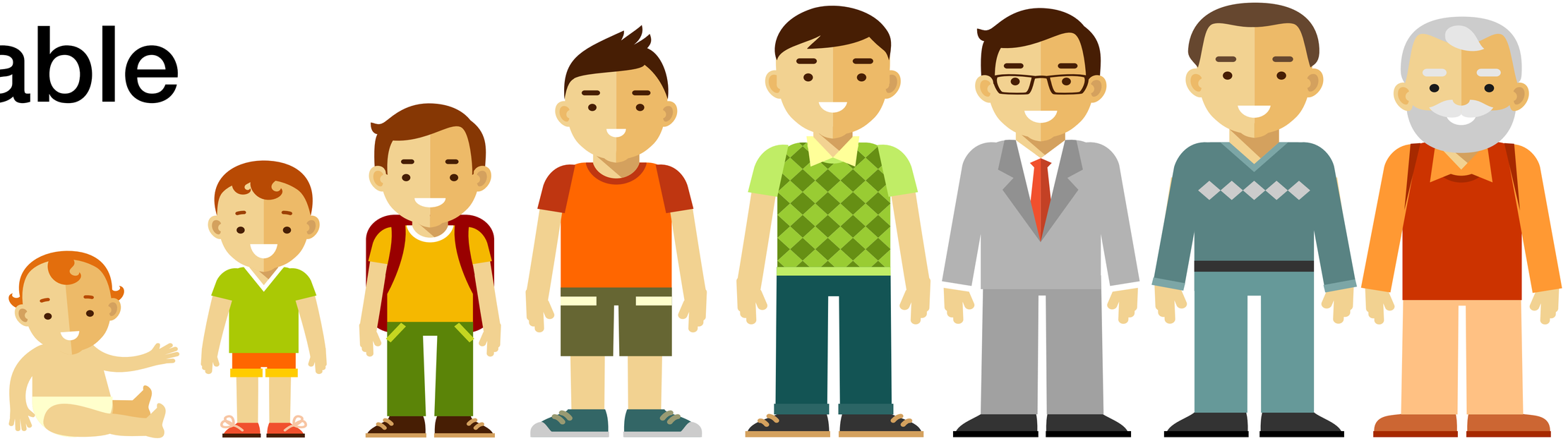
x	-2	-1	0	1	2
p(X = x)	⅕	⅕	⅕	⅕	⅕

$Y = X^2$

y	0	1	4
p(Y = y)	⅕	⅖	⅖

$$P(Y=y) = P(g(X)=y) = P(X \in g^{-1}(y)) = \sum_{x \in g^{-1}(y)} P(X=x)$$

Variable



Modifications

(aka functions)

Expectation of modification