Disjoint Unions

Addition Rule

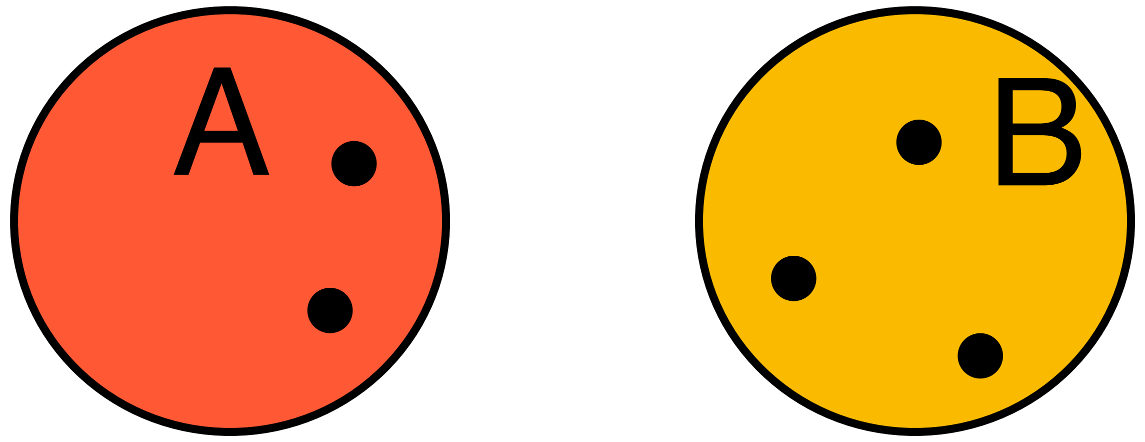
Subtraction Rule

Shaggy Problem

Hairy Problem

Disjoint Unions

$|A| = 2$ $|B| = 3$



$|A \cup B| = 2 + 3 = 5$

A union of disjoint sets is called a disjoint union

Use to emphasize a disjoint union

$\{0\} \cup \{1\} = \{0,1\}$

For disjoint sets,
the size of the union
is the sum of the sizes

$$\boxed{|A \cup B| = |A| + |B|}$$

Addition Rule

Numerous applications & implications

Reason $\cup \approx +$

Kids Play

Class has 2 boys and 3 girls

students = ?

∪

⊕

students = 2 + 3 = 5

Jar with Marbles

1 blue, 2 green, 3 red

marbles = ?

∪ of 3 sets

⊕

twice

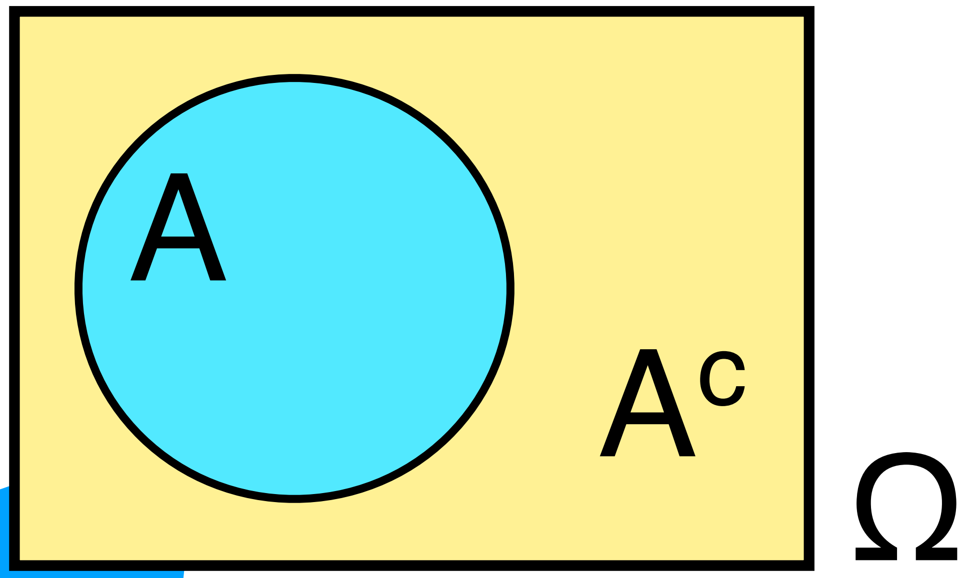
=1+2+3 = 6

Complements

Quintessential disjoint sets

A and A^c

$A \cup A^c = \Omega$



$|\,\Omega\,| = |\,A \cup A^c\,| = |A| + |A^c|$

Subtraction (or complement) Rule

$$\boxed{|A^c| = |\,\Omega\,| - |A|}$$

$D = \{\, i \in [6] : 3 \mid i \,\} = \{\, 3, 6 \,\}$ $|D|=2$

$D^c = \{\, i \in [6] : 3 \nmid i \,\} = \{1, 2, 4, 5\}$ $|D^c|=4$

$\Omega = [6] = \{1,..,6\}$

$|D^c| = 4 = 6 - 2 = |\Omega| - |D|$ ✅

Application of ⊕ rule

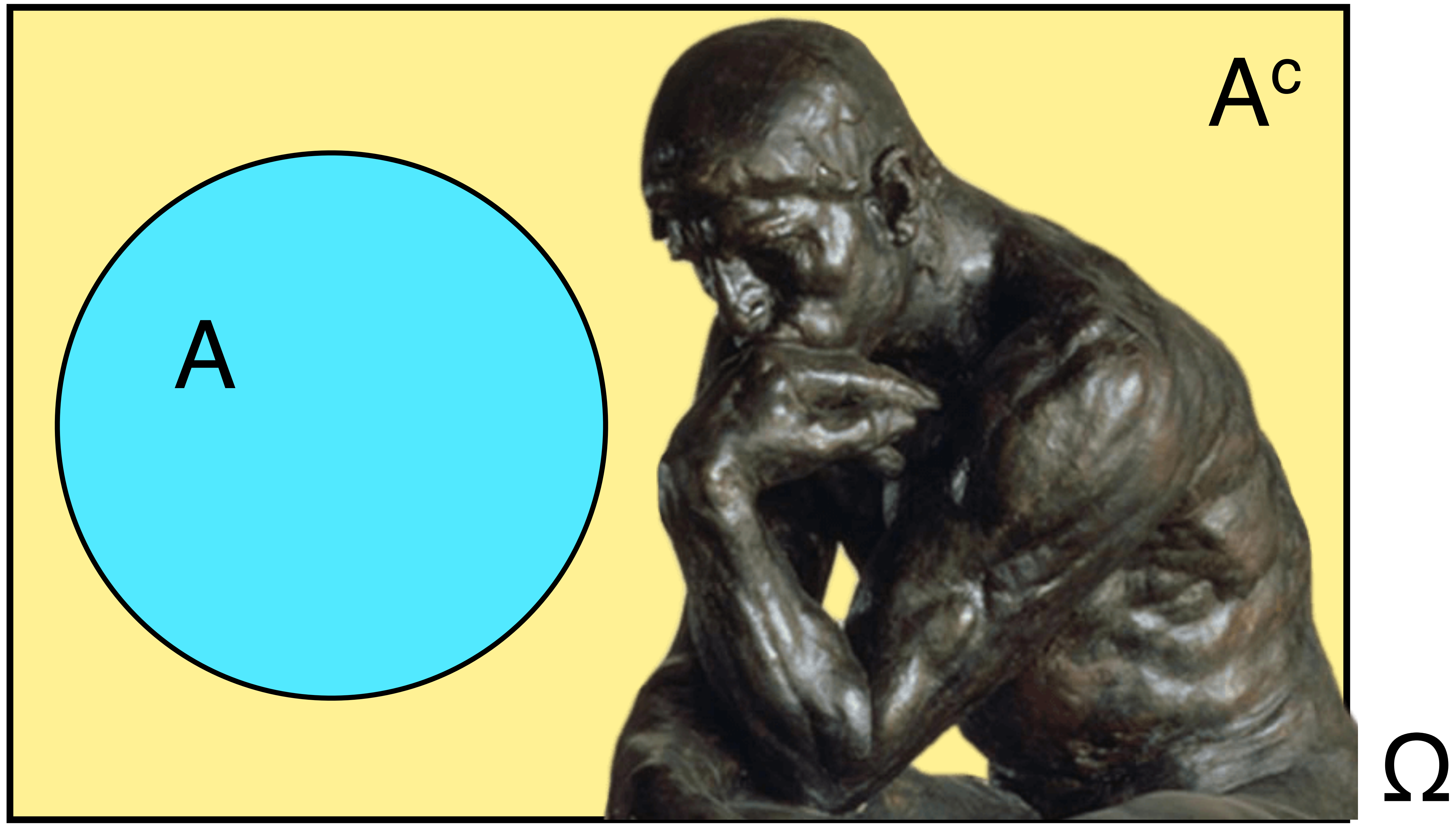
Reason set difference ≈ -

Think Outside the Circle

$|A^c| = |\Omega| - |A|$

$|A| = |\Omega| - |A^c|$

Handy for large or complex sets!



$A = \{\, i \in [100] : 3 \nmid i \,\} = \{1,2,4,5,7,\ldots,100\,\}$

integers between 1 and 100 not divisible by 3

$\Omega = \{1,\ldots,100\}$

$A^c = \{i \in [100] : 3 \mid i\} = \{\,3,\, 6,\, 9,\ldots,\, 99\,\}$

$|A^c| = 33$

$|A| = |\Omega| - |A^c| = 100 - 33 = 67$

Days

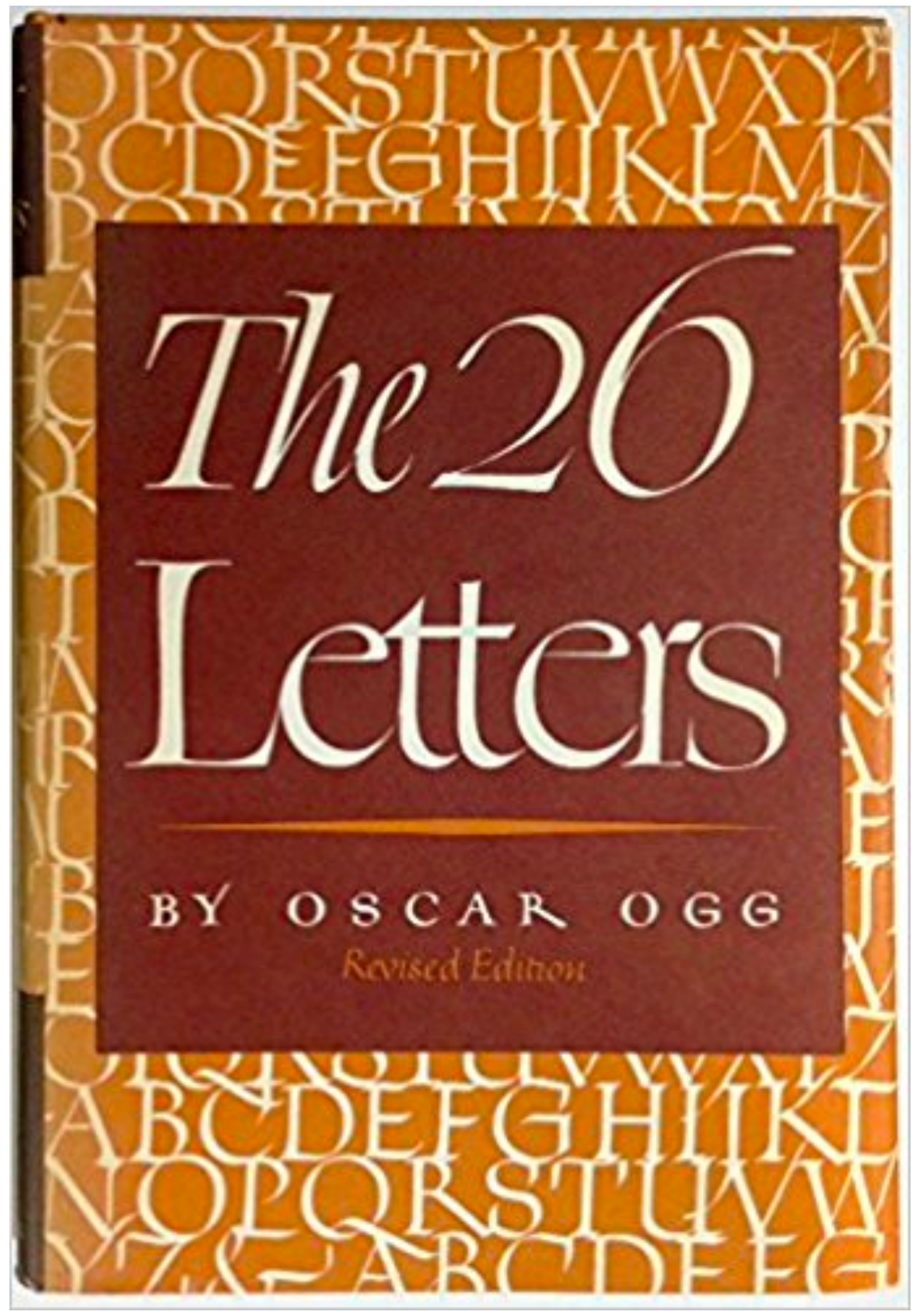
M Tu W Th F Sa Su

weekdays?

1, 2, 3, …, 5

7-2 = 5

Letters



Consonants

Vowels

consonants?

1, 2, 3, ….

26 - 5 = 21

Facetious question

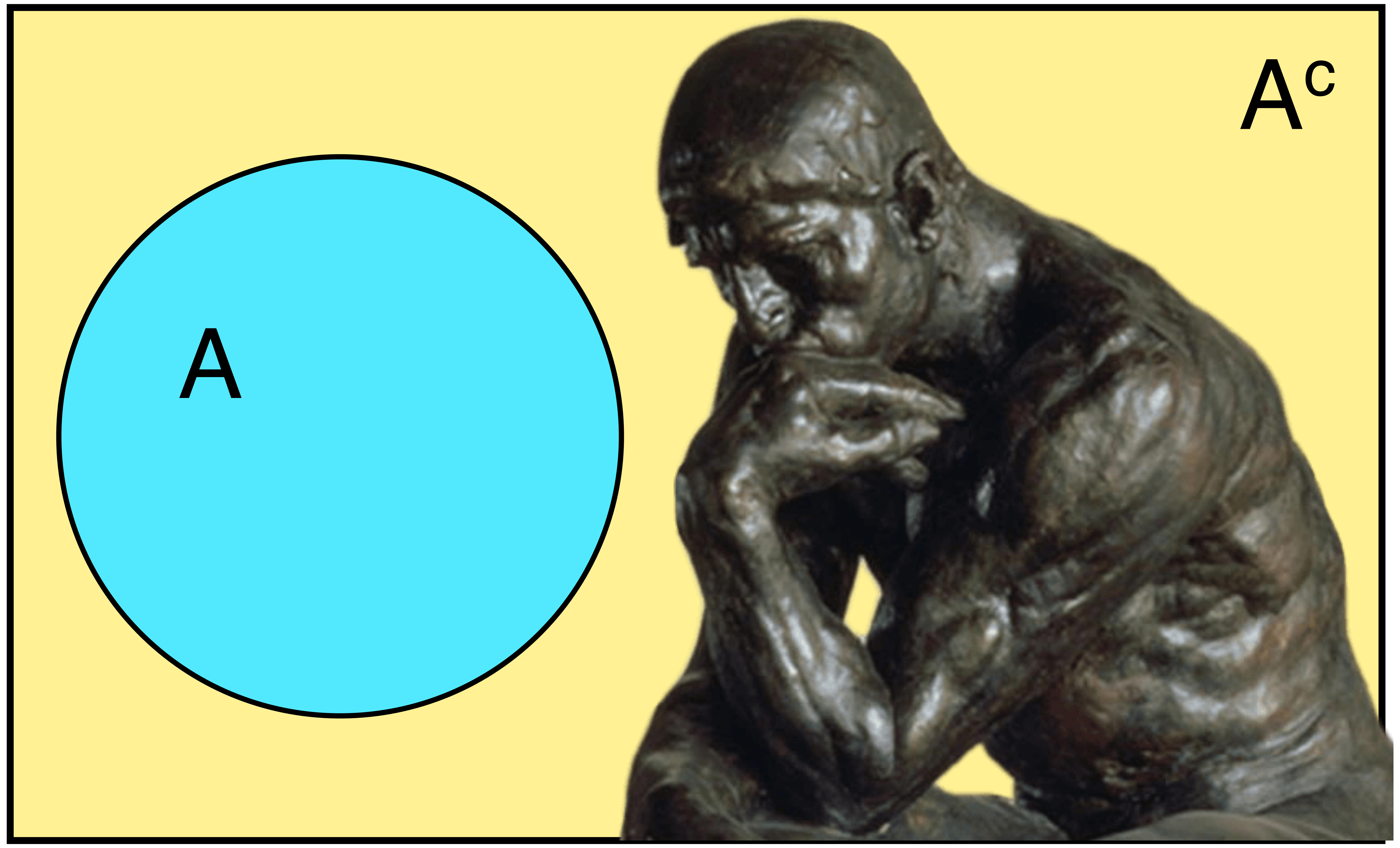
Word containing all 5 vowels, in order?

Numbers

$|A^c| = |\Omega| - |A|$

$|A| = |\Omega| - |A^c|$

Handy for large or complex sets!



Ω

$A = \{\, i \in [100] : 3 \nmid i \,\} = \{1,2,4,5,7,\ldots,100\}$

integers between 1 and 100 not divisible by 3

$\Omega = \{1,\ldots,100\}$

$A^c = \{i \in [100] : 3 \mid i\} = \{\,3, 6, 9, \ldots, 99\,\}$

$|A^c| = 33$

$|A| = |\Omega| - |A^c| = 100 - 33 = 67$

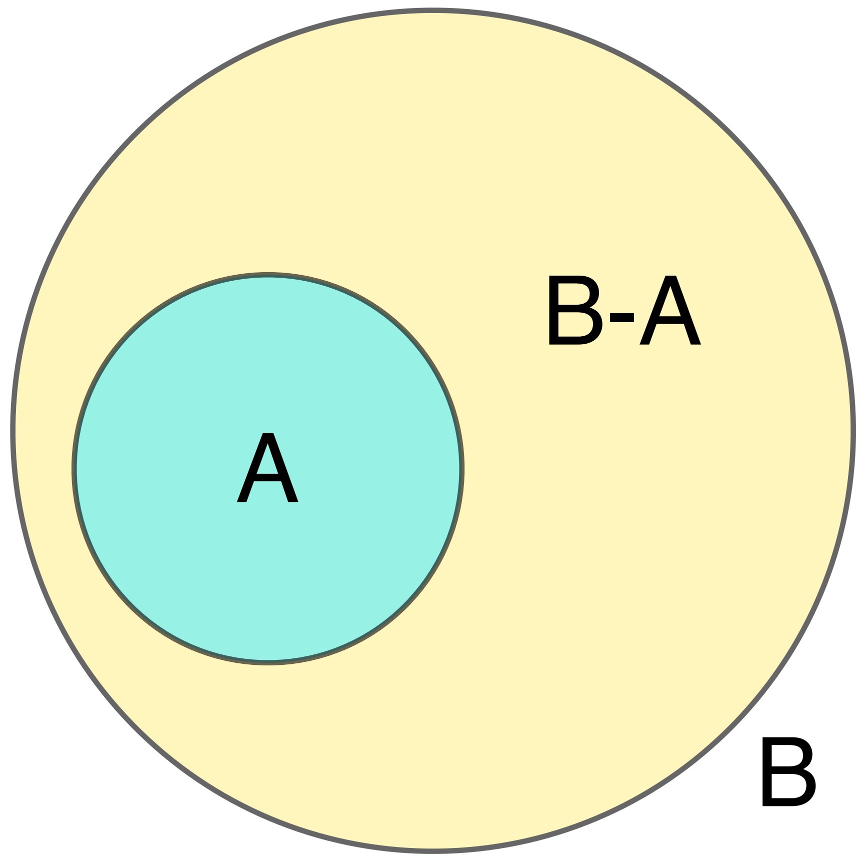
General Subtraction Rule

$|B-A| = |B| - |A|$

$A \subseteq B \rightarrow B = A \cup (B-A)$

$|B| = |A| + |B-A|$

$|B-A| = |B| - |A|$



Subtraction Rule

Disjoint Unions

Addition Rule

Subtraction Rule

Shaggy and Hairy

Two “real life” examples


How to weigh
your pet

Another Hairy Problem

A friend claims she can determine the size of any set instantly and exactly

She can determine the exact # of hairs on your head

Can you ask her some questions to be fairly certain if she tells the truth?

Hint

It doesn't just work for humans

See you next video

Shaggy or Hairy

Two “real life” examples