ASSIGNMENT-4

1.

$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$+ \rightarrow$ addition

$\times \rightarrow$ multiplication.

a. $\forall a, b \in Z \; ((a+b = b+a) \wedge (a \times b = b \times a))$

b. $\forall a, b \in Z \; ((a+b \in Z) \wedge (a \times b \in Z))$

c. $\forall a, b, c \in Z \; (a \times (b+c) = (a \times b) + (a \times c))$

d. $\forall a, b, c \in Z \; (((a+b)+c = a+(b+c)) \wedge ((a \times b) \times c = a \times (b \times c)))$

e. $\forall a \in Z \; ((a+0 = a) \wedge (a \times 1 = a))$

2. $\alpha = \forall x \, (P(x) \vee Q(x))$

$\beta = \forall x \, P(x) \vee \forall x \, Q(x)$

Lets take $P(x) = x$

$Q(x) = \neg x$

and lets assume $P(x) = \text{true}$

$\Rightarrow Q(x) = \text{false}$

for α.

$\Rightarrow \alpha = \text{True}$

But since

$\forall x \, P(x)$ and $\forall x \, Q(x)$ are independent statements

and $\forall x \, P(x)$ can be false and $\forall x \, Q(x)$ can be false

In such a case $\beta = \text{False}$

$\therefore \alpha$ does not entail β.

3.

(a) Owns (Han Solo, Millennium Falcon)

(b) Unhappy (Princess Leia)   c) loves (Princess Leia, Han Solo)

(d) $\forall x$ ((owns (x, Millennium Falcon) $\vee$ Unhappy (x)) $\Rightarrow$ visits (x, Obi-wan Kenobi

(e) $\forall x$ (visits (x, obi-wan Kenobi) $\Rightarrow$ wise (x))

(f) $\forall x$ ((Owns (x, Millennium Falcon) $\wedge$ visits (x, Obi-wan Kenobi))
$\Rightarrow$ teaches to use lightsaber (Obi-wan Kenobi, x))

(g) $\forall x$ (((unhappy (x) $\vee$ owns (x, Millenium Falcon)) $\wedge$ teaches to use lightsaber (Obi-wan Kenobi, x))
$\Rightarrow$ joins (x, Rebel Alliance))

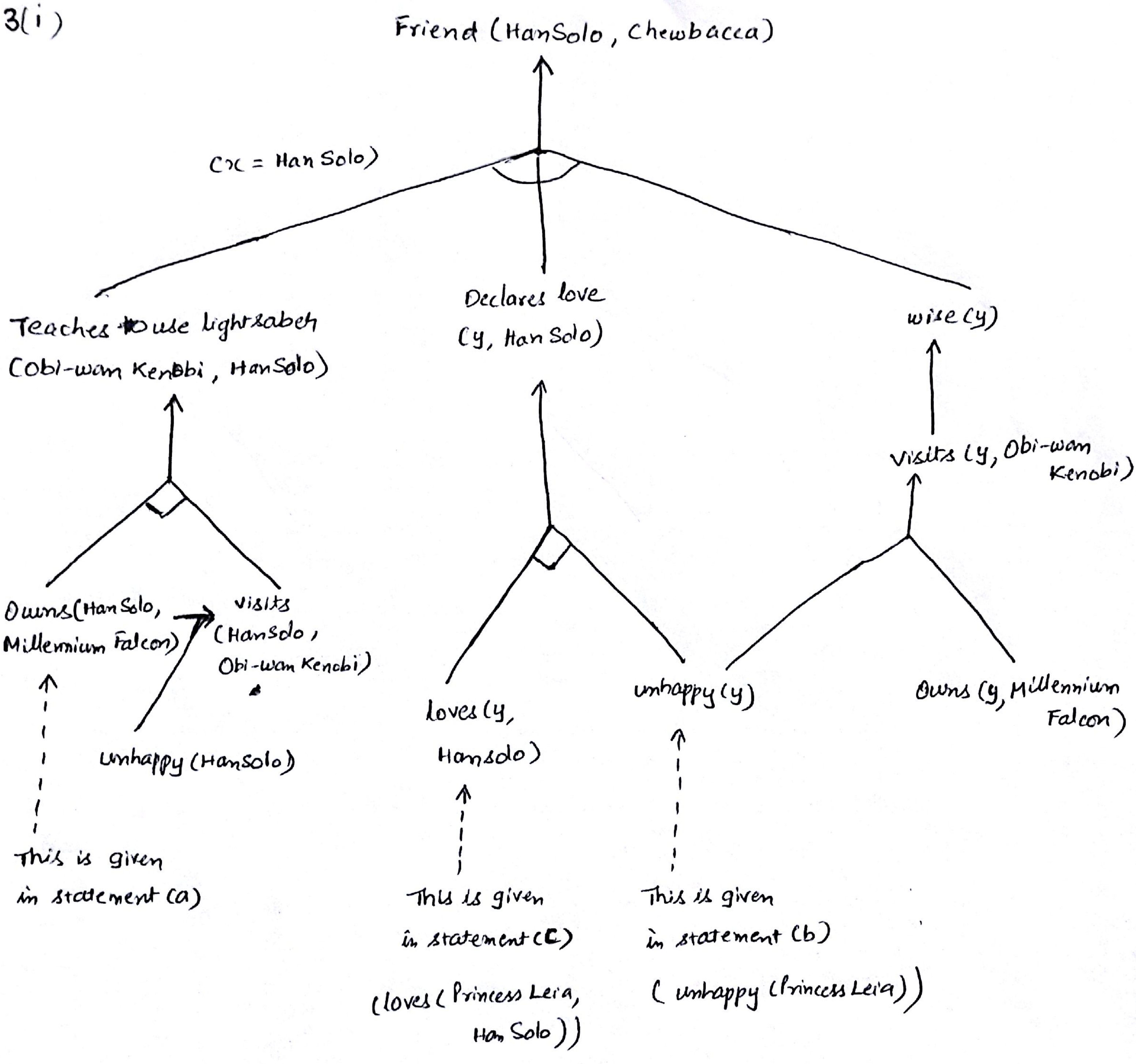
(h) $\forall x, y$ ((Unhappy (x) $\wedge$ loves (x, y)) $\Rightarrow$ declares love (x, y))

(i) $\forall x, y$ ((teaches to use light saber (Obi-wan Kenobi, x) $\wedge$ declares love (y, x)
$\wedge$ wise (y)) $\Rightarrow$ friend (x, chewbacca))

(j) Backward Chaining :

Please Turn over.

3(i)

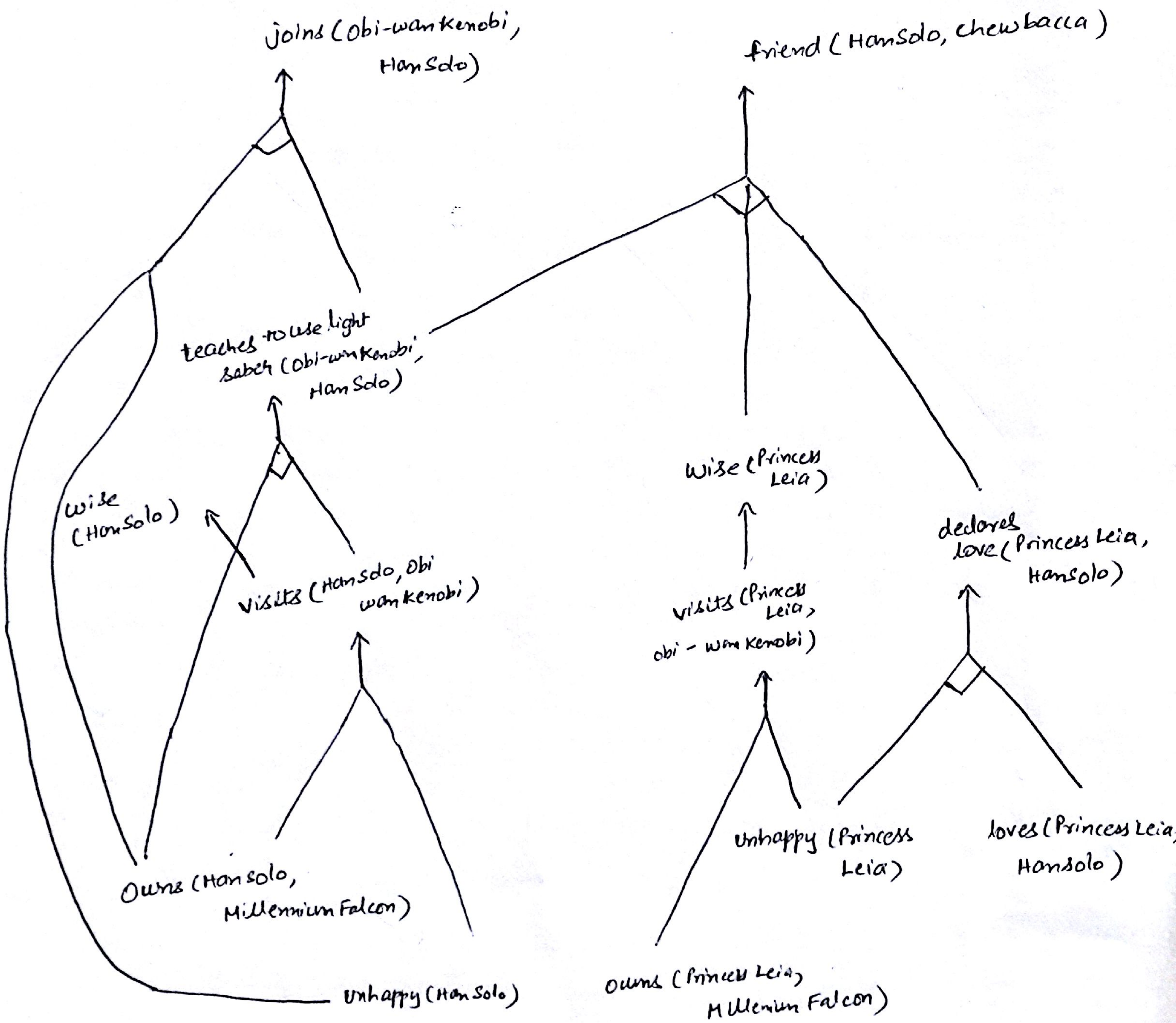


=> (y = Princess Leia)

Unhappy (HanSolo) & Owns (y, Millennium Falcon) are not given and cannot be evaluated.

3(ii) Forward Chaining:

- unhappy (Han Solo) and owns (Princess Leia, Millennium Falcon) are not given and have only taken for completing the diagram.



4:

a) able (zeus) ∧ willing to prevent evil (zeus) ⇒ prevent evil (zeus)

b) ¬able (zeus) ⇒ Impotent (zeus)

c) ¬ willing to prevent evil (zeus) ⇒ Malevolent (zeus)

d) ¬ preventevil (zeus)

e) exists (zeus) ⇒ ¬ Impotent (zeus) ∧ ¬ Malevolent (zeus)

1) CNF

a) ¬ (able (zeus) ∧ willing toprevent evil (zeus)) ∨ ~~p~~ preventevil (zeus)
⇒ ¬able (zeus) ∨ ¬ willingtopreventevil (zeus) ∨ prevent evil (zeus)

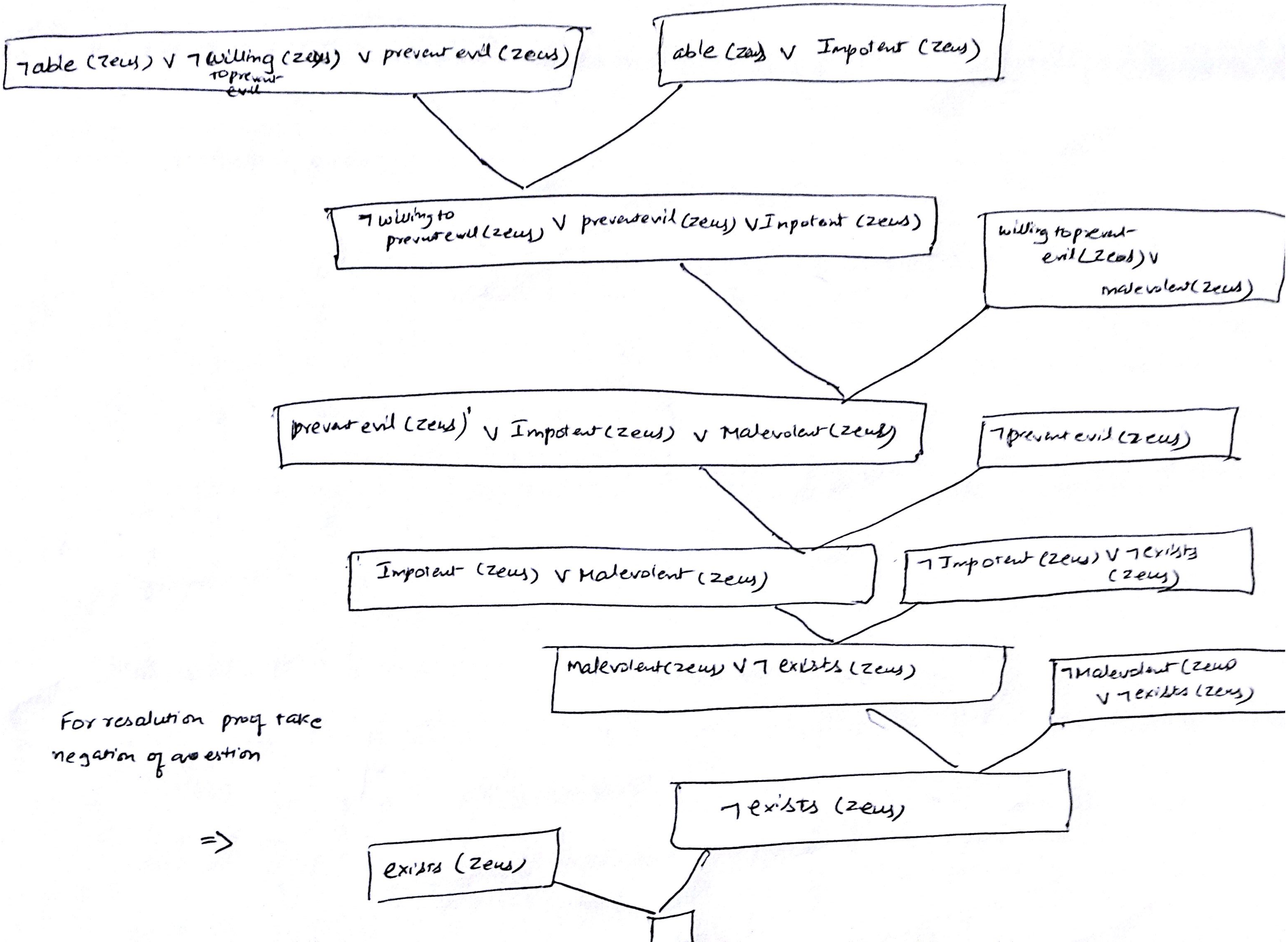
b) ¬ (¬able (zeus)) ∨ Impotent (zeus)
⇒ able (zeus) ∨ Impotent (zeus)

c) ¬ (¬ willing to prevent evil (zeus)) ∨ malevolent (zeus)
⇒ willing to preventevil (zeus) ∨ malevolent (zeus)

d) ¬ Prevent evil (zeus)

e) ¬ exists (zeus) ∨ (¬Impotent (zeus) ∧ ¬malevolent (zeus))

⇒ (¬ exists (zeus) ∨ ¬ Impotent (zeus)) ∧
(¬ exists (zeus) ∨ ¬malevolent (zeus))



For resolution proof take
negation of question

=>

5.

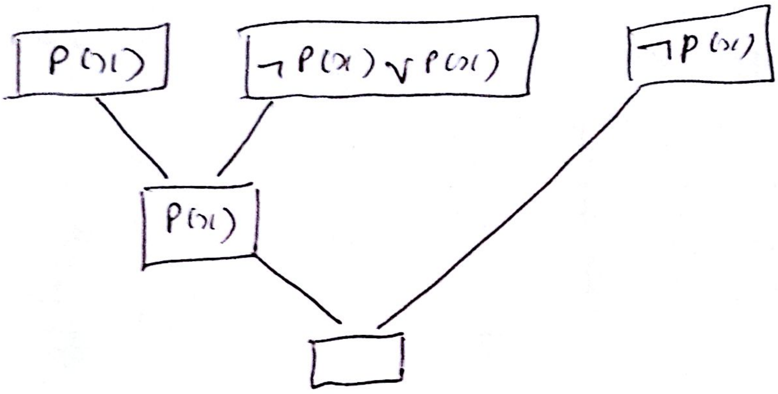
a) $= \forall x \, (P(x) \Rightarrow P(x))$

CNF $\equiv$

$\forall x \, (\neg P(x) \vee P(x))$

$\forall x \, \neg P(x) \vee \forall x \, P(x)$

$\neg P(x) \vee P(x)$ (Removing universal quantifier)



Empty set proves the statement

b) $(\neg \exists x \; P(x)) \Rightarrow (\forall x \, \neg P(x))$

Negation of conclusion $\Rightarrow$ $\neg (\forall x \, (\neg P(x)))$

$\exists x \, P(x)$

$P(f(x))$

Knowledge Base :

$(\neg \exists x \, P(x))$

$\forall x \, \neg P(x)$

$\neg P(x)$ (removing universal quantifiers)

CNF

$\neg \exists x\, P(x) \Rightarrow \forall x\, \neg P(x)$

$\neg(\neg \exists x\, P(x)) \vee \forall x\, \neg P(x)$

$\exists x\, \neg P(x) \vee \forall x\, \neg P(x)$

$\neg P(f(x)) \vee \neg P(x)$