6. Convert the decimal numbers to binary

a) 111

Mod 2							
Quotients	111	55	27	13	6	3	1
Remainders	1	1	1	1	0	1	1

$(111)_{10} = (1101111)_2$

b) 2137

Mod2												
Quotients	2137	1068	534	267	133	66	33	16	8	4	2	1
Remainders	1	0	0	1	1	0	1	0	0	0	0	1

$(2137)_{10} = (100001011001)_2$

c) 32333

Mod2										
Quotient	32333	16166	8083	4041	2020	1010	505	252	126	63
Remainder	1	0	1	1	0	0	1	0	0	1

Quotient	31	15	7	3	1
Remainder	1	1	1	1	1

$(32333)_{10} = (111111001001101)_2$

d) 93

Mod2							
Quotient	93	46	23	11	5	2	1
Remainder	1	0	1	1	1	0	1

$(93)_{10} = (1011101)_2$

7. Perform subtraction using complements

a) 111 - 31

$$
\begin{array}{r}
1 \quad 1 \; 1 \; 1 \\
+ \; 9 \; 6 \; 8 \\
+ \quad\quad 1 \\
\hline
0 \; 8 \; 0 \\
\hline
\end{array}
$$

discard carry of 1

9's complement of 031

$$
\begin{array}{r}
999 \\
-031 \\
\hline
968 \\
\hline
\end{array}
$$

The result is 80

b) 1021 - 55

$$
\begin{array}{r}
1 \quad 1021 \\
+ \; 9944 \\
+ \quad\quad 1 \\
\hline
0966 \\
\hline
\end{array}
$$

discard carry of 1

9's complement of 0055

$$
\begin{array}{r}
9999 \\
-0055 \\
\hline
9944 \\
\hline
\end{array}
$$

The result is 966

c) 156 - 99

$$
\begin{array}{r}
1 \quad 156 \\
+ \; 900 \\
+ \quad\; 1 \\
\hline
057 \\
\hline
\end{array}
$$

discard carry of 1

9's complement of 099

$$
\begin{array}{r}
999 \\
-099 \\
\hline
900 \\
\hline
\end{array}
$$

The result is 57

d) 2122 - 122

9's complement of 0122

$$\begin{array}{r} 9999 \\ -\ 0122 \\ \hline 9877 \\ \hline \end{array}$$

$$\begin{array}{r} 2122 \\ +\ 9877 \\ \hline 1\,1999 \\ +\ \quad 1 \\ \hline 2000 \end{array}$$

discard carry 4 1

The result is 2000

8) Decimal numbers to 8-bit 2's complement

a) -111

$(111)_{10} = (01101111)_2$

$(-111)_{10} =$

1s complement of 111 $= (10010000)_2$

2's complement of 111 $= (10010001)_2$

$\therefore (-111)_{10} = (1001\ 0001)_2$

b) -31

$(31)_{10} = (00011111)_2$

$(-31)_{10} =$

1's complement of 31 $= (11100000)_2$

2's complement of 31 $= (11100001)_2$

$\therefore (-31)_{10} = (11100001)_2$

c) -124

$(124)_{10} = (01111100)_2$

$(-124)_{10} =$ 1's complement of 124 $= (10000011)_2$

2's complement of 124 $= (10000100)_2$

$\therefore (-124)_{10} = (1000\,0100)_2$

d) 125

Since the number is not -ve, the binary representation is the only possible representation

	125	1
	62	0
	31	1
	15	1
	7	1
	3	1
	1	
	0	

$(125)_{10} = (01111101)_2$

e) 63

Since the number is not a negative number, the binary representation is the only possible representation. This is also equal to $-(-(63))$

2	63	1
2	31	1
2	15	1
2	7	1
2	3	1
	1	

$(63)_{10} = (00111111)_2$

Prove by Induction that for radix-r the largest number that can be represented with N digits is $r^N - 1$

For $N = 1$

The maximum number possible is $r-1$

which is true.

Let us assume that

for $N = n-1$

the max number is $= r^{n-1} - 1$

For $N = n$

the max number can be all $(r-1)$ at each digit

$$\Rightarrow (r-1)\, r^{n-1} + (r-1)\, r^{n-2} + \cdots (r+1)\, r^{0}$$

$$\Rightarrow r^{n} - \cancel{r^{n-1}} + \cancel{r^{n-1}} - \cancel{r^{n-2}} + \cdots + \cancel{r} - 1$$

$$\Rightarrow r^{n} - 1$$

Hence Proved.

For radix-r addition, the carry bits are 0 or 1

The max digit in a r-radix system is $r-1$

So, if we sum 2 $r-1$ values, the sum is

$2r-2$ which is less than $2r$

Since the value is less than $2r$, the carry bits are

always 0 or 1.