

# A Hexagon Tile Map Algorithm For Spatial Mapping To Create Alternative Australian Maps

by Stephanie Kobakian and Dianne Cook

**Abstract** This algorithm creates a tessellated hexagon display to represent a set of the spatial polygons. It allocates these hexagon in a manner that preserves the spatial relationship of the geographic units. The algorithm creates a display that showcases spatial distributions, by emphasising the small geographical regions that are often difficult to locate on geographic maps. Spatial distributions have been presented on alternative representations of geography for many years. In modern times, interactivity and animation have begun to play a larger role, as alternative representations have been popularised by online news sites, and atlas websites with a focus on public consumption. Applications are increasingly widespread, especially in the areas of disease mapping, and election results.

## Introduction

Many cancer atlases present geospatial cancer data on a choropleth map display. The Australian Cancer Atlas (?) is a recent addition to the many cancer atlas maps worldwide. The ground-breaking atlas for Australia presents a central map that shows the landmass overlaid with administrative boundaries. This choropleth display can highlight the geographic patterns in geospatially related cancer statistics (?).

In many countries, residents are increasingly gathering to live in urban areas around major cities (?). The creation of approximately equal population areas results in dramatically different square meterage of the geographic units. The geographic units are usually administrative, such as states or electorates. The units are filled with colour to represent the value of the statistic for each unit (?). This can lead to unequal emphasis on the statistical value of each of the geographic units. This allows choropleth map displays to misrepresent the spatial distributions of human related statistics due to area-size bias (?). Figure ?? is a choropleth map that uses colour to display the estimated Standardised Incidence Ratios (SIRs) of melanoma cancer for males for each of the Statistical Areas at Level 2 (SA2) used by the Australian Bureau of Statistics (?). The Australian choropleth map display draws attention to the expanse of light blue areas across the rural communities in all states. The SA2s around Brisbane stand out as more orange and red. To create this choropleth map the SA2 polygons for 2011 from the Australian Bureau of Statistics.

The solutions to this visualisation problem begin with the geography. Alternative maps shift the focus from land area and shape, to the value of the statistics in a group of areas (?). Different styles of cartograms apply transformations to the geographic areas, to highlight the values of the value of the statistic of interest. These displays result in a distortion of the map space to represent differences in the statistic across the areas (?) as the statistic of interest is used to determine the cartogram layout.

Alternative mapping methods allow increased understanding of the spatial distribution of a variable across the population, by fairly representing the population in each administrative area (?). This acknowledges that the number of residents can be different but recognises that each area, or person within it is equally important.

This paper contains a discussion of Existing Mapping Practices in Section 2. This is followed by details of the Algorithm in Section 3. Section 4 describes the implementation of the algorithm in the sugarbag package. Section 5 outlines the use and benefits of animation.

## Existing Mapping Practices

There are several established alternative visualisation methods. Tile maps, Rectangular cartograms (?) and Dorling cartograms (?) all use one simple shape to represent each geographic unit. They all minimise the emphasis on the size or shape of the geographic areas. These alternative map displays focus on the relationship between neighbours, attempting to preserve connections, and disregard the unique shapes of the administrative boundaries. Figure ?? shows a collection of alternative map displays, this includes a) a contiguous cartogram, b) a non-contiguous cartogram, c) a Dorling cartogram and d) a hexagon tile map. Each Statistical Area at Level 2 (SA2) (?) was designed to represent a community. These administrative areas are often used to aggregate census data and used to understand demographics of the Australian population.



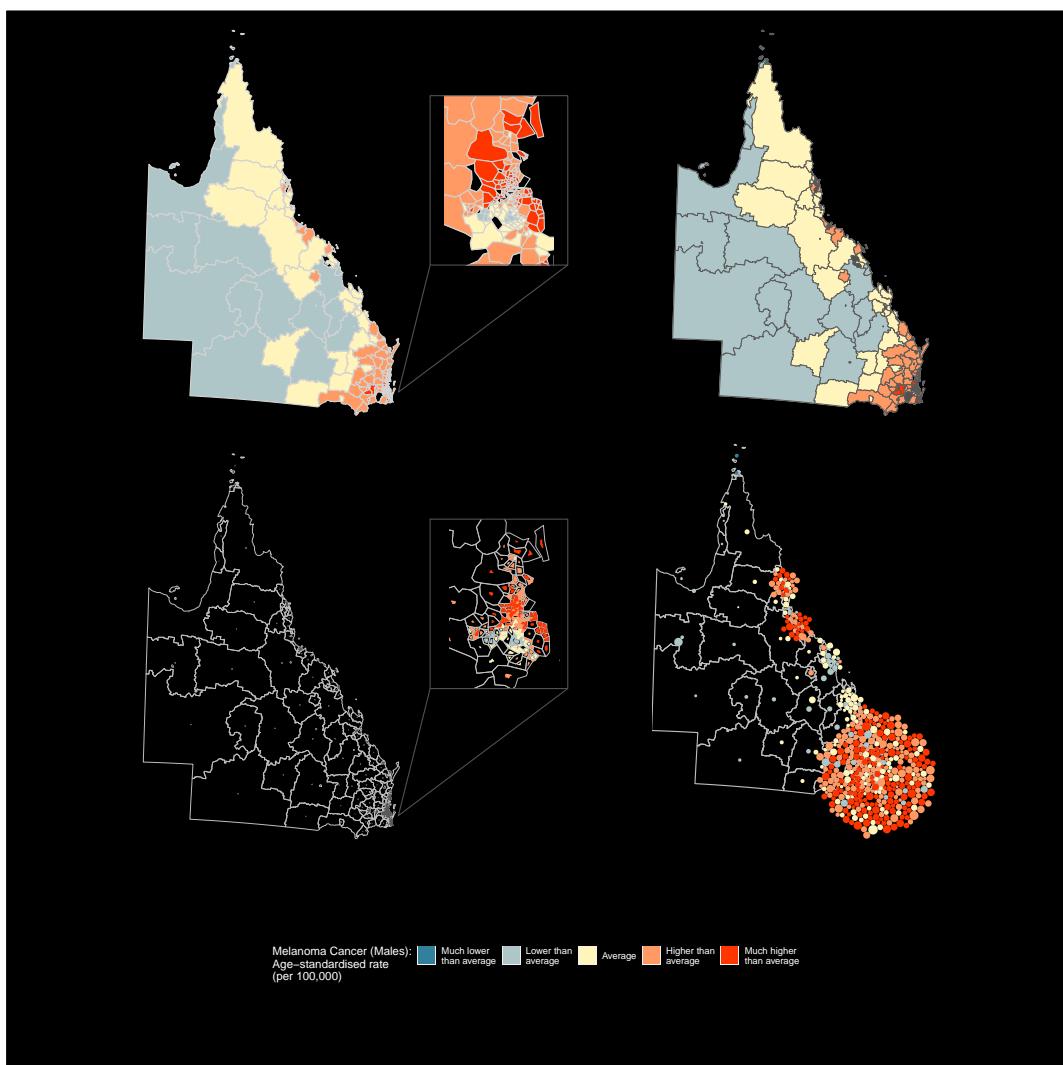
**Figure 1:** A choropleth map of the Statistical Areas of Australia at Level 2. The colours communicate the value of the estimated SIR of Melanoma for males, they range from much lower than average (blue) to much higher than average (red)

The familiar choropleth map display is often used to allow the user to orient themselves. However, the emphasis on the land mass diminishes the features of the distribution in densely populated communities due to the small size on the display (?). The unique shapes of boundaries can be helpful for orienting users but may not contribute to their understanding of the spatial disease distribution as many of the communities are not visible in a choropleth display (?). This presents an opportunity for explicit map transformations to improve communication of distributions (?). In figure ??a) the choropleth map is presented with the alternative maps to allow for comparisons to be made.

When communicating information that is relevant to the population, each member of the population can be given equal representation by transforming the map (?). The impact of a disease over connected communities can be seen by allowing the boundaries to maintain connection in the transformed display. The contiguous cartogram displayed in figure ??b) draws attention to smaller geographic units when they are rescaled according to the population (?). These new shapes can now be coloured to represent a second variable. This display can create twisted and unfamiliar shapes from the geographic units as the algorithms must satisfy the topology conditions, especially when there are communities located geographically far from their neighbours (?). This is shown in the northern SA2s of Queensland with an average (yellow) estimated SIR of Melanoma for males.

The non-contiguous cartogram in figure ??c) also uses the population to rescale the geographic units. Unlike the contiguous cartogram, the SA2 areas maintain their geographic shape, but they may not retain the connection with their neighbours. The population of the SA2 areas is used to scale the geographic units in the non-contiguous cartogram (?). The amount of white space can be meaningful in non-contiguous cartograms (?), in this example the disparity between large rural areas and small urban areas means that the city of Brisbane, shown in the inset map looks reasonable, but these units are not visible in the context of the Queensland state map. Depending on the difference between the population and geographic land size, the amount of white space can also prevent meaningful understanding of the distribution (?).

The Dorling cartogram presents each geographic unit as a circle, the size of the circle is scaled according to the population value of each area (?). Figure ??d) shows the Queensland SA2 areas as



**Figure 2:** A collection of alternative maps of the Statistical Areas of Australia at Level 2. The colours communicate the value of the estimated SIR of Melanoma for males, ranging from much lower than average (blue) to much higher than average (red)

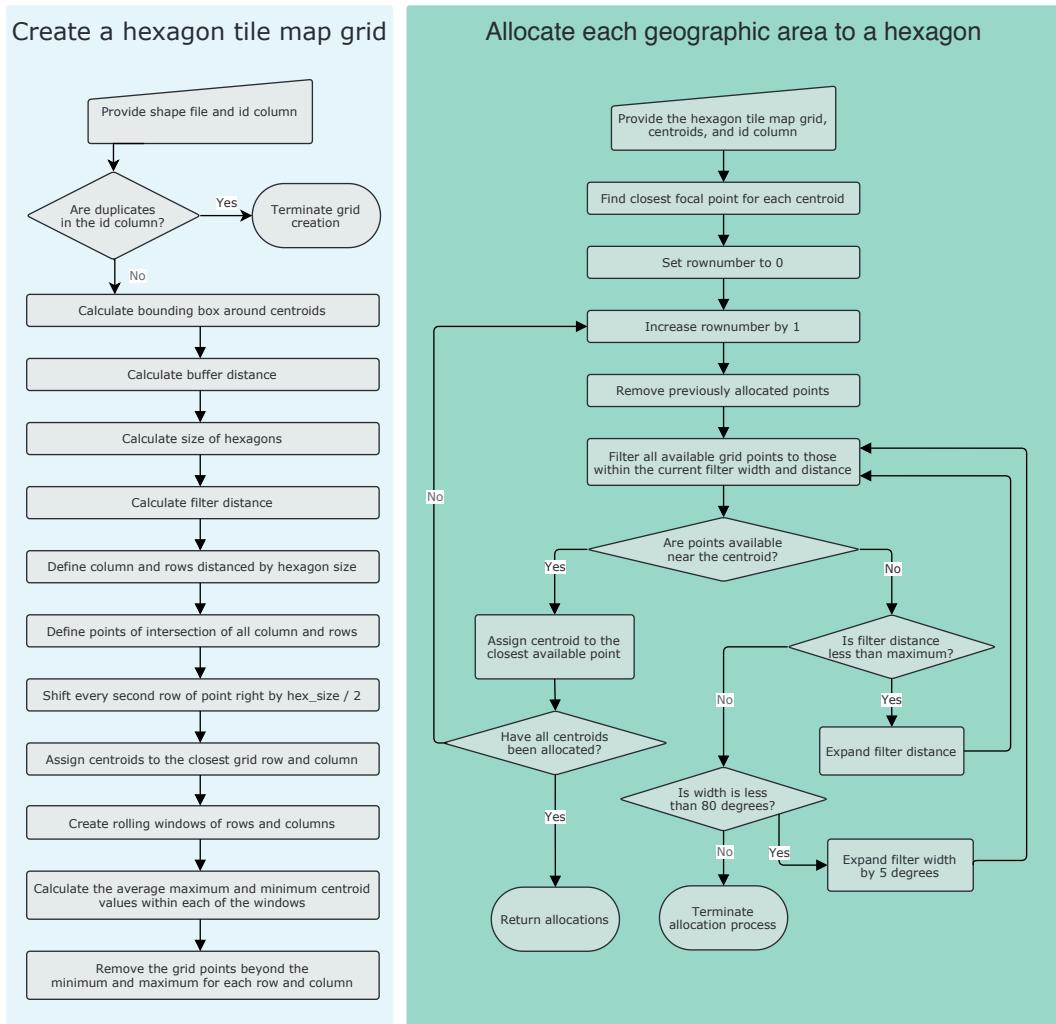
an individual circle located as close as possible to the geographic centroid location. This map draws attention to the collection of coastal cities in Queensland that were not apparent in figure ?? a), b) or c).

### Algorithm

The purpose of this algorithm is to create a map display that highlights the spatial distributions for populations. There has been an increasing need for displays that recognise the large number of people that live in dense urban environments. The algorithm intends to maintain the spatial relationships of a group of geographic units in two ways: between each unit and its neighbours; and between each unit and the closest focal point. The algorithm allocates geographic units to a representative hexagon, in order of their proximity to the closest focal point.

The algorithm is named for the *Trigona carbonaria* bee species. Native to northern and eastern Australia this stingless species builds flat layers of hexagonal brood cells, spiralling out from a central point (?). This hive design inspired the use of multiple focal points in the algorithm, individual spirals are constructed outward from various points on the geographic map base.

There are three key steps performed to create a tessellated hexagon tile map. First create the set of centroids from the polygons to allocate, then create the grid of hexagons locations. Each centroid can then be allocated to an available hexagon location. To make tessellated plots the points can be converted to hexagon shapes, with the hexagon size equal to the size used in the grid.



**Figure 3:** A flow diagram detailing the necessary steps to create a hexagon tile map.

**User choices** Only two inputs are necessary to begin using the algorithm, the shape file, and the id variable. The id column should uniquely identify each geographic unit in the shape file.

The centroids can be derived from the shape file. The amount of centroids is used to determine a hexagon size. The derived centroids are a necessary input for an appropriate grid to be constructed, in which each grid point is located a hexagon size distance from the next closest point in all six directions. The grid will initially cover the entire map space, encompassing all the centroid locations and will extend in all directions to the extent of the buffer distance. This buffer distance can be helpful to account for densely populated coastal areas, allowing the use of the sea for hexagon locations. The hexagon size will likely need adjustments depending on the density of the population.

The centroids derived from the shape file are necessary inputs when creating a grid. The creation of the grid and filtering out the points unlikely to be used in the hexagon map results in the hexagon tile map grid. At this point, the centroid set and the grid become the necessary inputs to the allocation process.

A set of reference locations can be provided as focal points, such as capital cities of states or countries. Using focal points, the algorithm will create the order of allocation in order of the closest centroid locations to the focal points. Alternatively, a user can specify the variable that should be used to determine the order for the allocation. When allocating representative hexagons, the width parameter can be used to determine the flexibility of positioning using the relationship with the nearest reference location. A larger width parameter will increase the amount of available hexagon locations nearer to the centroid of the geographic location. A smaller width will maintain the orientation from the focal point to the centroid when selecting the hexagon location, however this could mean it is placed further from the centre of the cluster.

## Implementation

Hexagon tile maps can be useful to understand a distribution across a collection of geographic areas. However, these maps are not easily drawn, especially as the number of areas increases. This algorithm was created to automate this process, and reduce the manual load involved in creating and implementing alternative displays. This allows map makers and data communicators to allocate their time to choosing the most effective display, rather than manually creating them.

The sugarbag package has been written for R software, it contains a set of functions that help R users to create a hexagon tile map. The algorithm presented in the sugarbag package operates on a set of simple feature geometry objects (?), this package allows R users to create sf objects by importing polygons stored in various formats. Users should provide a set of polygons that define geographic units by their administrative boundaries. The functions arrange the geographic units in order of proximity to a set of locations provided, such as the centre of capital cities. The centroid location of each geographic unit is used to measure the proximity. It emphasises the capital cities as population hubs, rather than emphasizing the size of large, rural geographic units.

A user can tweak the parameters of the hexagon map using the additional arguments to the create\_hexmap function, these options may affect the speed of the creation.

**Installation** The package can be installed from CRAN:

```
install.packages("sugarbag")
```

and the development version can be install from the GitHub repository:

```
devtools::install_github("srkobakian","sugarbag")
```

Load the library into your R session with:

```
library(sugarbag)
```

**Creating a hexagon tile map** The following code creates the hexagon tile map for all the Statistical Areas at Level 2 in Queensland. These steps can be executed by the main function, create\_hexmap, or can be implemented separately for more flexibility.

If a user would like to perform each of the steps of the algorithm themselves, the necessary inputs will change for each function.

Users may choose to only use the allocate function when they wish to use a set of centroids, rather than (?) polygons.

```

## Load data
qld_sa2 <- absmapsdata::sa22011 %>%
  filter(state_name_2011 == "Queensland")

## Create centroids set
centroids <- create_centroids(qld_sa2, "sa2_name_2011")

## Create hexagon grid
grid <- create_grid(centroids = centroids,
                     hex_size = 0.12,
                     buffer_dist = 1.2)

## Allocate polygon centroids to hexagon grid points
hex_allocated <- allocate(
  centroids = centroids,
  hex_grid = grid,
  sf_id = "sa2_name_2011",
  ## same column used in create_centroids
  hex_size = 0.12,
  ## same size used in create_grid
  hex_filter = 10,
  use_neighbours = qld_sa2,
  focal_points = capital_cities %>%
    filter(points == "Brisbane"),
  width = 35,
  verbose = FALSE
)
}

## Prepare to plot
fort_hex <- fortify_hexagon(data = hex_allocated,
                             sf_id = "sa2_name_2011",
                             hex_size = 0.12)

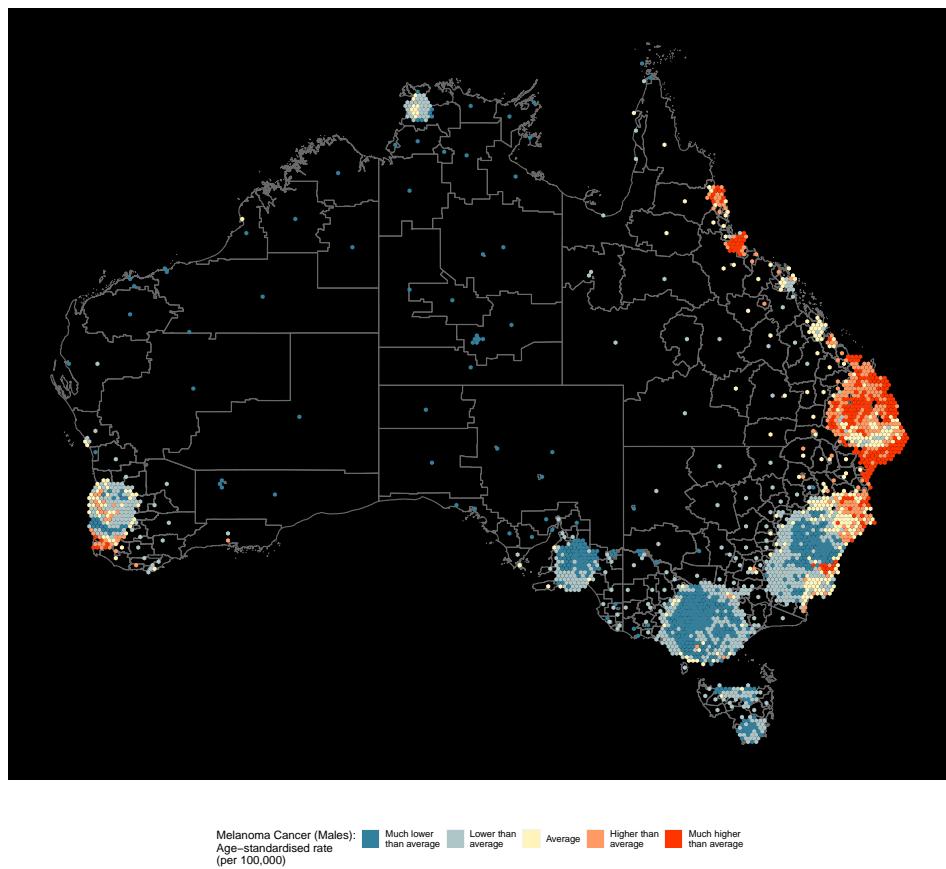
fort_qld <- qld_sa2 %>%
  fortify_sfc()

## Make a plot
library(ggplot2)
qld_hexmap <- ggplot() +
  geom_polygon(aes(
    x = long,
    y = lat,
    group = interaction(sa2_name_2011, polygon)
  ),
  fill = "white",
  colour = "lightgrey",
  data = fort_qld
) +
  geom_polygon(aes(
    x = long,
    y = lat,
    group = hex_id,
    fill = rownumber
  ),
  data = fort_hex) +
  scale_fill_distiller("", palette = "PRGn") +
  coord_equal() + theme_void()
qld_hexmap

```

**Polygon set** The polygon set of Statistical Areas at Level 2 (SA2) (?) of Queensland in 2011 is provided with the sugarbag package as `tas_sa2`. A single column of the data set is used to identify the unique areas. In this case, the unique SA2 names for each SA2 have been used.

The longitude and latitude centre of the capital cities of Australia are used as focal points to allocate



**Figure 4:** A hexagon tile map of the Statistical Areas of Australia at Level 2. The colours communicate the value of the estimated SIR, they range from much lower than average (blue) to much higher than average (red)

each geographic area around the closest capital city. Brisbane will be the common focal point, as this example uses only the areas in the state of Queensland.

```
data(capital_cities)
```

The buffer distance, hexagon size, hexagon amount to filter and width of angle are parameters that will be determined within `create_hexmap` if they are not provided. They are created as they are needed throughout the following example.

**Step 1: Derive the set of centroid points** A set of centroids may be used directly. The set of polygons should be provided as an `sf` object, this is a data frame containing a `geometry` column. The `read_shape` function can assist in creating this object for use in R.

The centroids can be derived from the set of polygons using the `create_centroids` function:

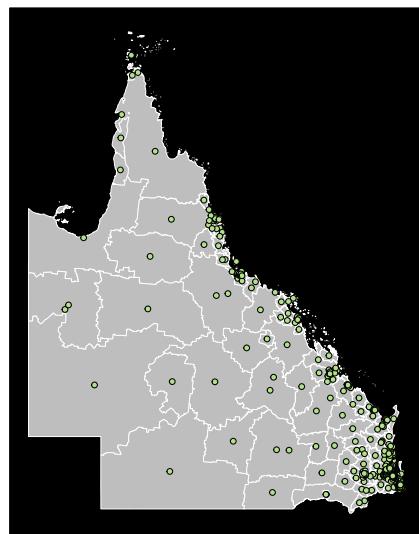
```
centroids <- create_centroids(shp_sf = tas_sa2, sf_id = "SA2_NAME16")
```

**Step 2: Create the hexagon grid points** A grid is created to ensure tessellation between the hexagons that represent the geographic units on a hexagon tile map.

The grid of possible hexagon locations is made using the `create_grid` function. It uses the centroids, the hexagon size and the buffer distance.

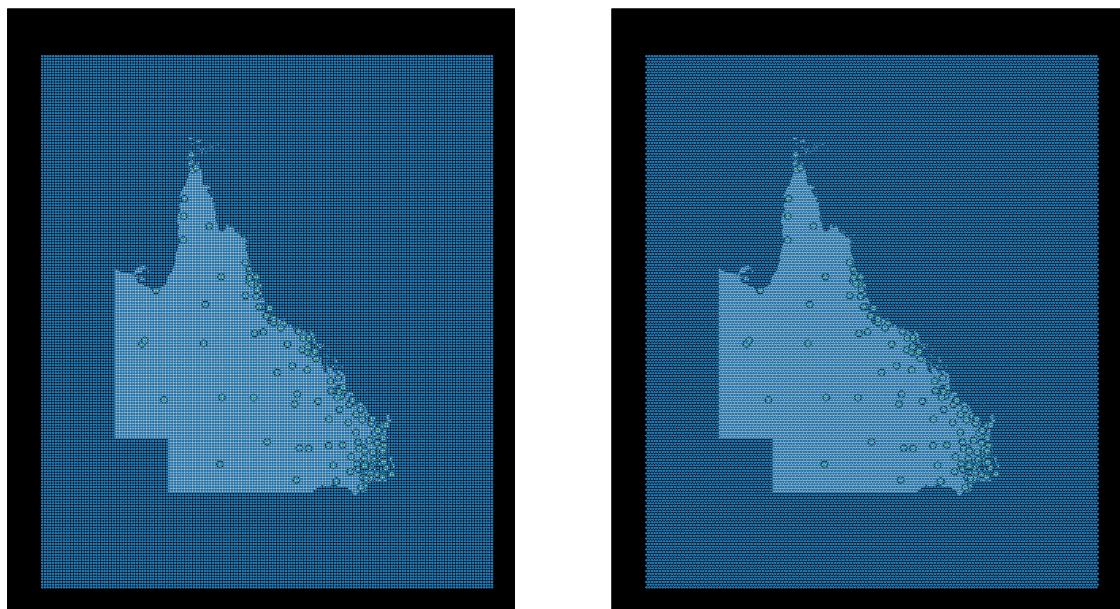
```
grid <- create_grid(centroids = centroids, hex_size = 0.2, buffer_dist = 1.2)
```

**Creating a tessellated grid** A set of longitude columns, and latitude rows are created to define the locations of the hexagons. The distance between each row and column is the size specified by



**Figure 5:** The geographic shapes of the Statistical Areas of Queensland at Level 2. The points show the locations of the centroids of the SA2 areas.

`hex_size`. Equally spaced columns are created from the minimum longitude minus the buffer distance, up to the maximum longitude plus the buffer distance. Similarly, the rows are created from the latitude values and the buffer distance. A unique hexagon location is created from all intersections of the longitude columns and latitude rows. Figure ?? shows the original grid on the left, to allow for tessellating hexagons, every second latitude row on the grid is shifted right, by half of the hexagon size. The grid for tessellation is shown on the right.



**Figure 6:** Grid points to create a hexagon tile map.

**Rolling windows** Not all of the grid points will be used, especially if islands result in a large grid space. To filter the grid for appropriate hexagon locations for allocation, the `create_buffer` function is used by `create_grid`. It finds the grid points needed to best capture the set of centroids on a hexagon tile map.

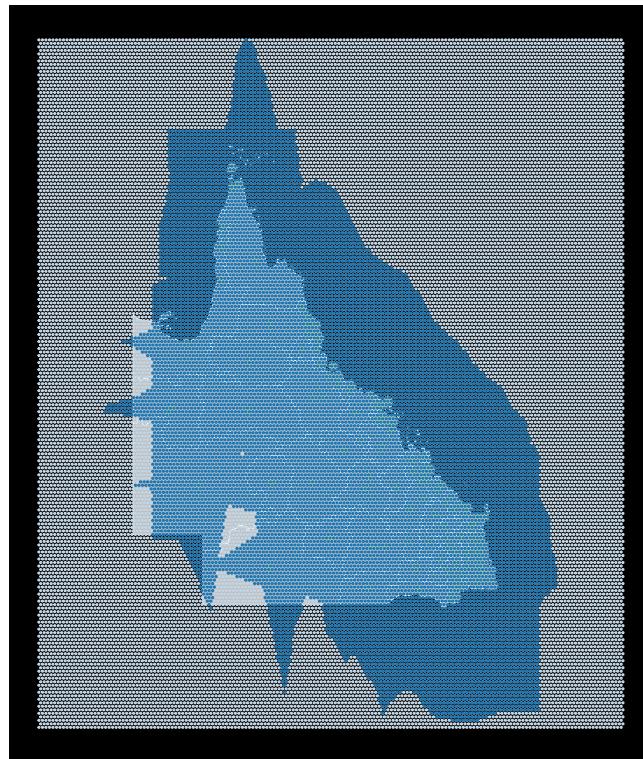
The closest latitude row and longitude column are found for each centroid location. Then rows and columns of centroids are divided into 20 groups. The amount of rows in each latitude group and the amount of columns in each longitude group are used as the width of rolling windows. The rolling windows can be seen on the bottom and right of the grid shown in Figure ???. This will tailor the available grid points to those most likely to be used. It also helps reduce the amount of time taken,

as it decreases the amount of points considered for each centroid allocation.

The first rolling window function finds the minimum and maximum centroid values for the sliding window groups of longitude columns and the groups of latitude rows.

The second rolling window function finds the average of the rolling minimum and maximum centroid values, for the longitude columns and latitude rows.

**Filtering the grid** The grid points are kept only if they fall between the rolling average of the minimum and maximum centroid values after accounting for the buffer distance, for each row and column of the grid. The sparsely populated South-West region of National Park has much fewer points available compared to the South-East region containing the city of Brisbane.



**Figure 7:** The remaining hexagon locations from all the possible points in the initial grid after the buffer is applied. The blue dots show the grid points left to choose from after the buffer step.

**Centroid to focal point distance** The distance between each centroid in the set, and each of the focal points provided is calculated. The name of the closest focal point, and the distance and angle from focal point to polygon centroid is joined to polygon data set. To minimise time taken for this step only one option is provided, Queensland's capital city Brisbane. The order for allocation is determined by the distance between the polygon centroid and its closest focal point. The points are arranged from the centroid closest to the focal point(s), to the furthest.

**Step 3: Allocate each centroid to a hexagon grid point** Allocation of all centroids takes place using the set of polygon centroids and the hexagon map grid. Centroid allocation begins with the closest centroid to a focal point. This will preserve spatial relationships with the focal point, as the inner city areas are allocated first, they will be placed closest to the capital, and the areas that are further will then be accommodated. The possible hexagon grid points reduces by one after each allocation, then only those that have not yet been allocated are considered.

The possible hexagon locations to consider for a centroid are determined by the `hex_filter`. This is the maximum amount of hexagons between the centroid and the furthest considered hexagon. It is used to subset possible grid points to only those surrounding the polygon centroid within an appropriate range. A smaller distance will increase speed, but can decrease accuracy when width of the angle increases.

```
hexmap_allocation <- allocate(
  centroids = centroids %>% select(SA2_NAME16, longitude, latitude),
```

```

sf_id = "SA2_NAME16",
hex_grid = grid,
hex_size = 0.2, ## same size used in create_grid
hex_filter = 10,
width = 35,
focal_points = capital_cities,
verbose = TRUE)

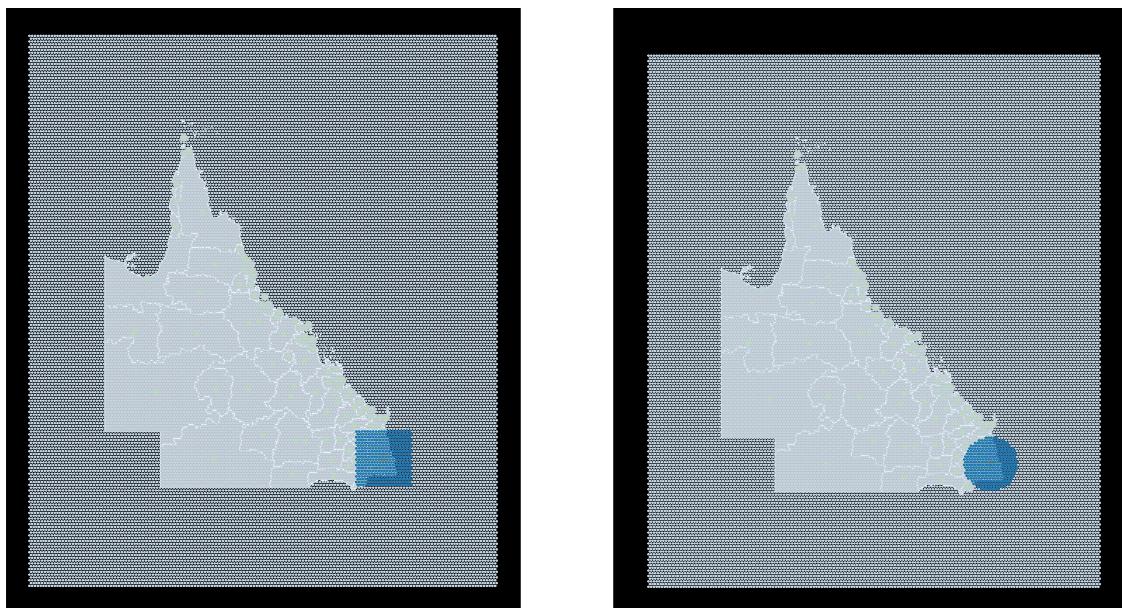
```

The following example considers the first of the Statistical Areas at Level 2. Within the algorithm, these steps are repeated for each polygon.

**Filter the grid for unassigned hexagon points** Keeping only the available hexagon points prevents multiple geographic units from being allocated to the same hexagon.

**Filter the grid points for those closest to the centroid** A box of possible hexagon locations around the centroid allows only the closest points that are not yet assigned to be considered. The corners of the box may not appear square if the buffer step has already removed unnecessary points from over the ocean.

The algorithm then removes the outer corners of the square, creating a circle of points, by only keeping points within a certain radial distance around the original centroid location.



**Figure 8:** The remaining available hexagon locations after filtering for the grid points within a square distance, then circular around the centroid.

The width parameter is used to take a slice of the remaining points. The width is the amount of degrees used on either side of the angle from the focal point to centroid location. This uses the angle from the closest capital city, to the current centroid as seen in Figure ?? . This allows the spatial relationship to be preserved, even when it is allocated to a hexagon that is further from the focal point than the original centroid location.

If no available hexagon grid point is found within the original filter distance and angle, the distance is expanded, only when a maximum distance is reached will the angle expand to accommodate more possible grid points.

By default the angle filter to hexagon grid points that fall within the bounds of the angle from the focal point to the geographic centroid, plus and minus 30 degrees. This will increase if no points can be found within the hex\_filter distance. The default angle of 30 was chosen to allow the algorithm to choose hexagons that best maintained the spatial relationship between the focal point and geographic centroid.

A complete hexagon tile map of Queensland is created by applying the algorithm steps to each centroid. The hexagon tile map visualisation is used below to visualise the Australian Cancer Atlas data (?). Two views of the same data are produced by filling according to the Lung Cancer Standardised Incidence Rates (SIRs) downloaded from the Australian Cancer Atlas site. This small example in

Figure ?? shows the group of blue areas in the Brisbane CBD more prominently in the hexagon tile map (b). The small red areas visible in the choropleth map (a) along the north coast are much larger in the hexagon tile maps. The hexagon tile map shows less yellow, this no longer overwhelms the map space with the information regarding the rural areas.

**Neighbour relationships** It is possible to consider the neighbouring areas for each SA2, for stronger preservation of the spatial distribution.

An additional step can be included to allow the neighbours that have already been allocated to influence the placement of the current centroid. This requires specifying the `sf` object as the argument for the `use_neighbours` parameter. This calculates neighbours using intersections of their polygons. This occurs for all areas before any allocations begin.

During the allocation of each centroid, the list of neighbours is consulted. If any neighbour was already allocated, the hexagons surrounding the neighbours on the grid are prioritised. For multiple neighbours, the neighbouring hexagon grid points are aggregated and considered in order of distance from the original centroid.

### Hexagon tile map of Australia

This algorithm has been applied to the complete set of all Australian Statistical Areas at Level 2. This display highlights the density of Australian capital cities, as it draws attention to the many communities in Sydney, Melbourne and Brisbane. This display also highlights the disparity in the burden of Melanoma cancer for males in the communities of these three cities. There are several collections of red hexagons in Brisbane that draw attention and they represent the communities with much higher rates of diagnosis than the Australian average. The communities south of Brisbane show a gradient of colour as the values progress from higher than average, to much lower than average for the communities closer to the Sydney CBD. This pattern progresses into the communities with lower than average rates in Melbourne and Tasmania.

Compared to the choropleth map display, the low rates in the rural Australian communities are no longer dominating the display. The much higher than average rates in Sydney draw more attention in the hexagon tile map display unlike the red and orange areas in the city of Brisbane that draw attention in both displays.

### Animation

The `ganimate` (?) package can be used to make an animation. It requires connecting the polygons for each area in two displays, which can be done using the `sf_id` variable, such as the SA2 name. The animation<sup>1</sup> connecting these two displays will highlight the rapid growth of the inner-city areas that emphasises the density of the communities. The hexagons that move the furthest will move rapidly in the animation. The rapid decreases of the large rural areas also show how much greater the landmass of Statistical Areas can be.

### Conclusion

It is possible to use alternative maps to communicate spatial distributions. While a choropleth map display is the current practice spatial visualisation of geographical data. Current methods do not always work for Australia due to the large geographic space between the densely populated capital cities. The administrative boundaries may also distract from the statistics communicated using colour.

Alternative maps highlight the value of the statistics across the geographic units. Alternative mapping methods allow increased understanding of the spatial distribution of a variable across the population, by fairly representing each administrative area. This acknowledges that the amount of residents can be different but recognises that each population area is equally important. The solution to this visualisation problem has equally sized areas, with neighbourhood boundary connections. This map algorithm is implemented in the `sugarbag` (?) package written for R (?). The `sugarbag` package creates tessellated hexagon tile maps. The Australian application preserves the spatial relationships, emphasising capital cities. The hexagon tile map is a visualisation solution that highlights spatial distributions.

These hexagons equally represent each area. However, the tessellation does not allow the size of the hexagons to represent another variable, similar to the choropleth maps. The algorithm is

<sup>1</sup>This animation can be viewed at: <https://sugarbagjss.netlify.com/>

heavily dependent on the focal points used, as this determines the order of allocation. It works on the assumption that viewers can use directional relationships to identify their neighbourhoods but this can be aided by the animation. With careful consideration of the choropleth map, the small geographic inner city areas may have been noticed by viewers, but the hexagon tile map display emphasises them. The communities in northern Queensland and the Northern territory do not draw attention because of their size as in the choropleth, but their colour is still noticeably below average when contrasted with the hexagons further south.

The Dorling cartogram was the most influential in the design of the hexagon tile map. The use of a hexagon provides a simple shape that shifts the focus of the visualisation from the shape of SA2s to the distribution effects of a disease on communities. not impeded by the irregular and unusual shapes created by a contiguous cartogram and the immense white space created on the non-contiguous cartogram. Hexagons can emphasise the relationship between neighbouring communities by allowing their sides to touch. The tessellation created by adjoining hexagons for densely populated areas is helpful as it shows that most communities are not islands onto themselves, but are intrinsically connected to those around them. As seen in the Dorling cartogram in figure ??c), the tessellation that allows boundary connections was not imposed for rural communities, where centroid of the geographic unit was located far from others, a feature that allows distant but dense populations of the coastal towns of Queensland to be recognised.

The choropleth map that uses the geographic shapes of areas can be seen in figure ???. Figure ?? displays the Australian areas with equally sized hexagons. This display allows the small and densely populated inner city areas to be easily contrasted with their neighbouring areas.

Future work will include refining the algorithm. It would be possible to take a logarithmic function rather than a direct angle to help choose a closer hexagon to the original centroid location, before increasing the width of the angle used to filter the hexagons.

This algorithm has only been tested using single countries, and does not consider definite borders of countries. While the buffer allows extension beyond the furthest centroids, there is no mechanism to protect the borders and ensure centroids are placed within the geographic borders of a country.

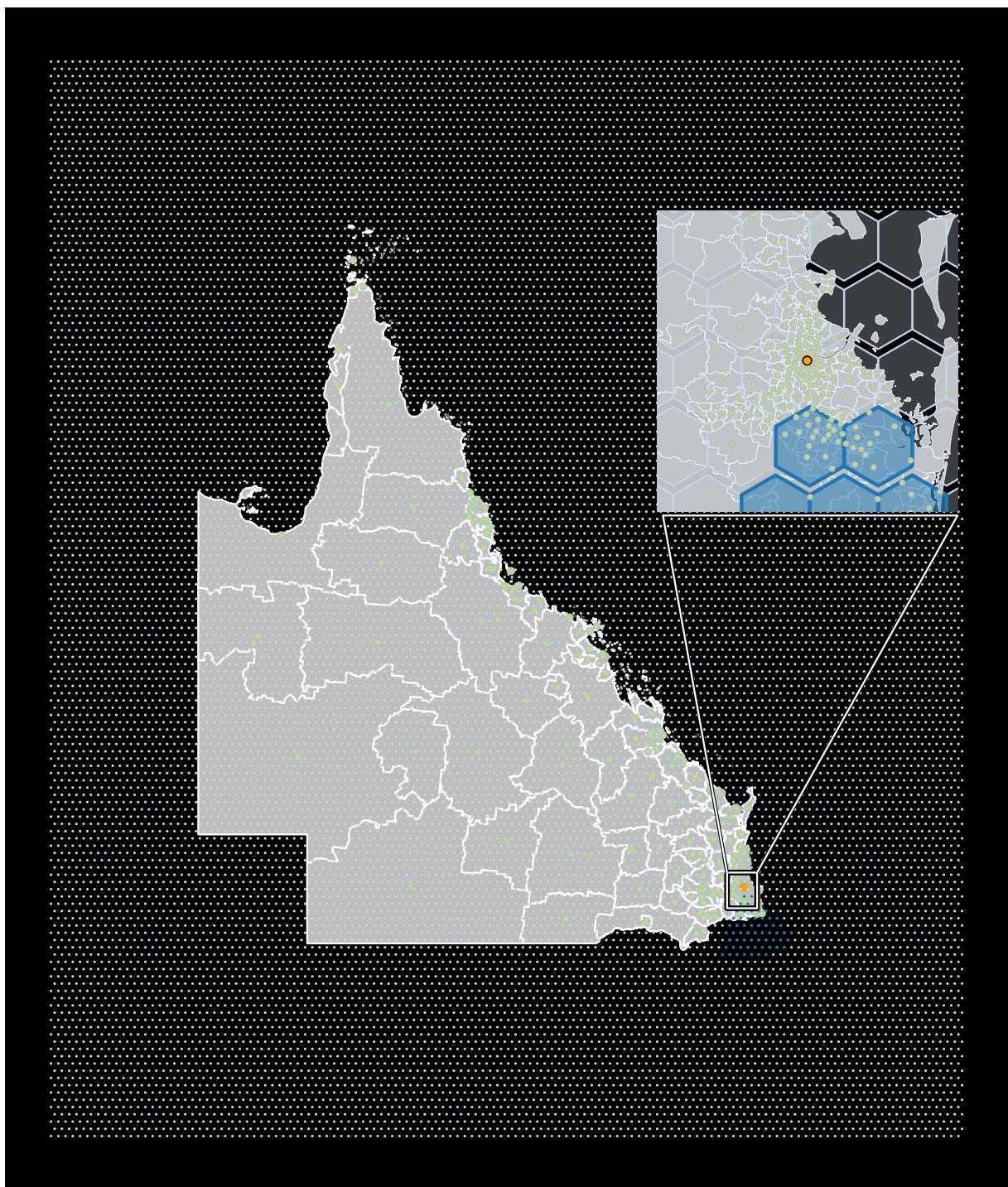
This algorithm is an effective start to creating hexagon tile maps for many geographic units.

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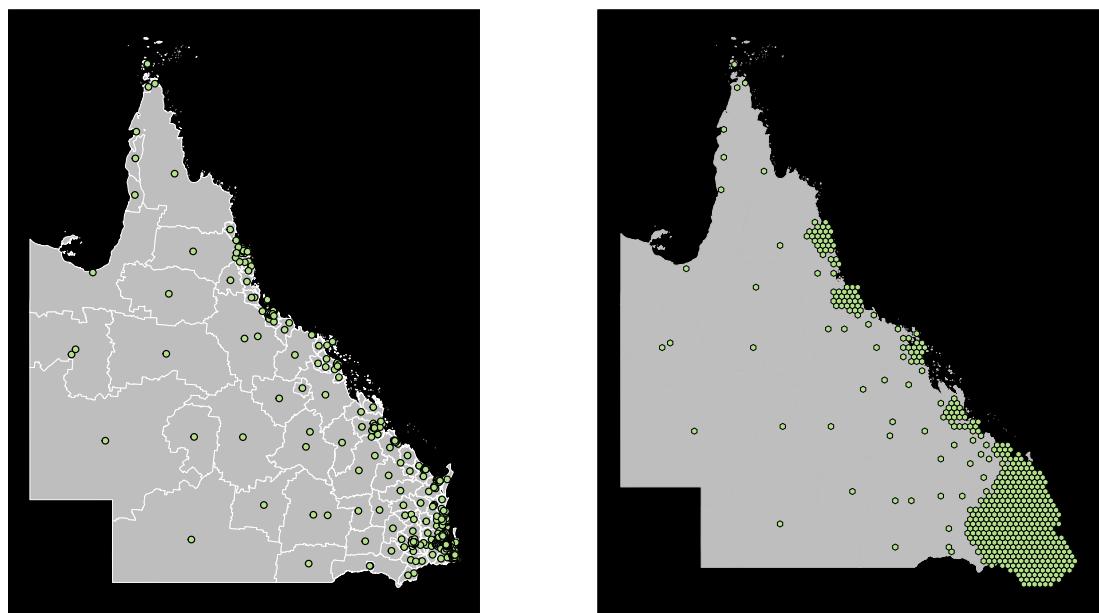
[stephanie.kobakian@monash.edu](mailto:stephanie.kobakian@monash.edu)

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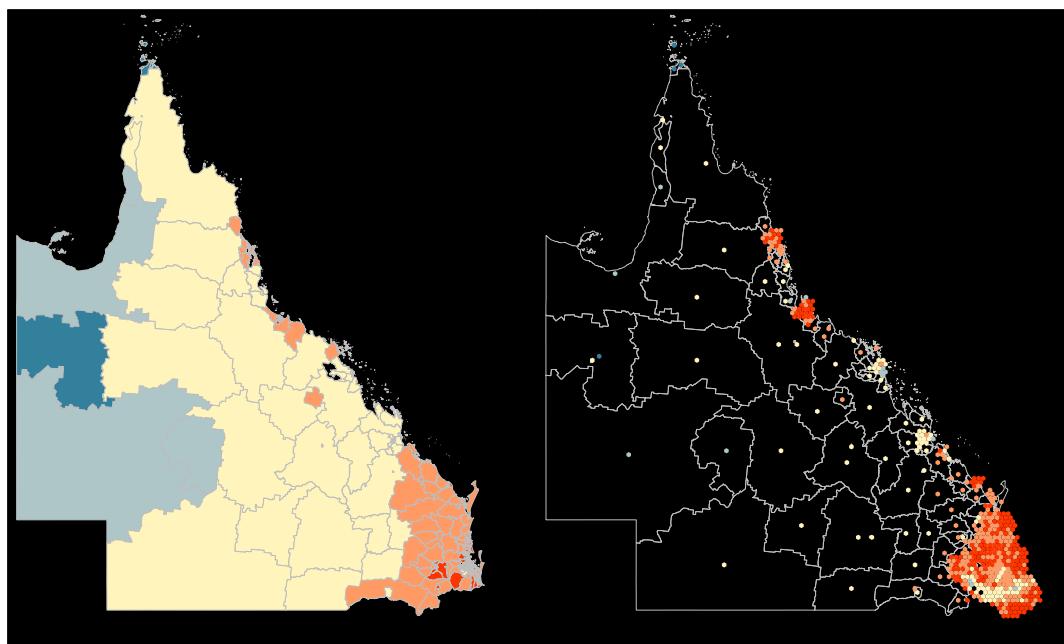
[dcook@monash.edu](mailto:dcook@monash.edu)



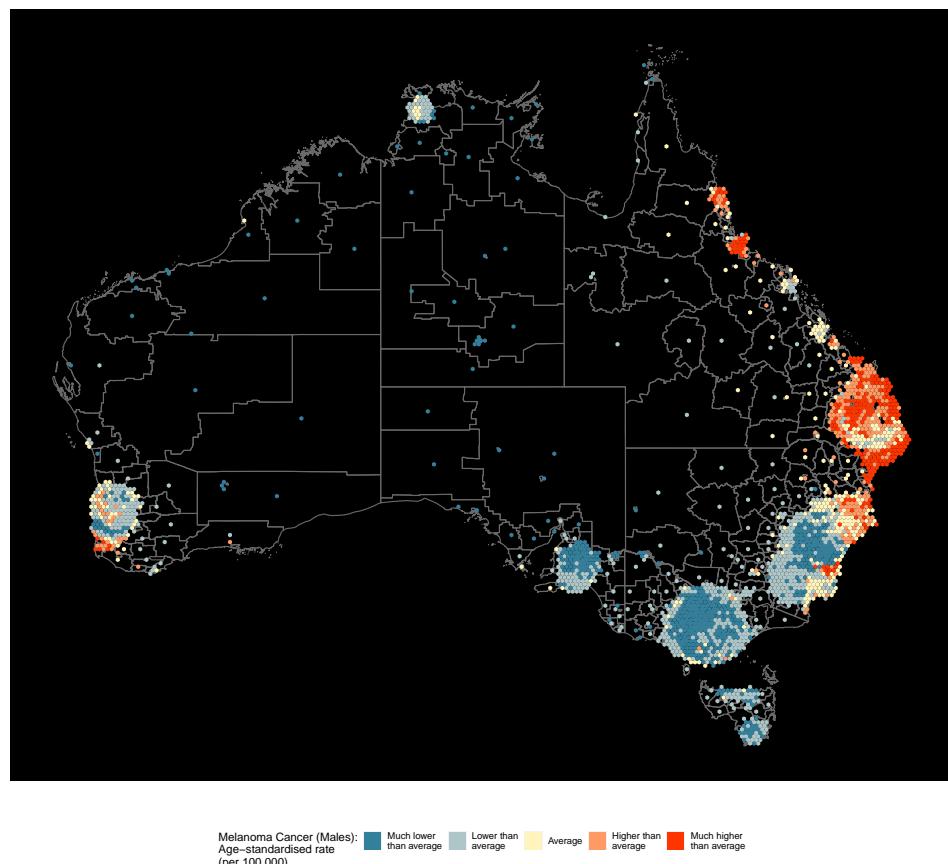
**Figure 9:** The remaining available hexagon locations after filtering for grid points within the angle from the focal point to the centroid.



**Figure 10:** A complete hexagon tile map of Queensland.



**Figure 11:** The Australian Cancer Atlas data displayed on a choropleth and hexagon tile map. The colour of each Statistical Area of Australia at Level 2 coloured by SIR. A choropleth map (a) of SIR is paired with a hexagon tile map (b) to contrast the colours that are made obvious when every SA2 is equally represented.



**Figure 12:** A hexagon tile map of the Statistical Areas of Australia at Level 2. The colours communicate the value of the estimated SIR of Melanoma for males, they range from much lower than average (blue) to much higher than average (red).