

ETC5510: Introduction to Data Analysis

Week 7, part B

Week of introduction

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May 2020

Recap

- Models as functions
- Linear models

Overview

- Correlation
- Model basics
- Let's look at R^2 again
- Using many models

Project deadline (Next Week)

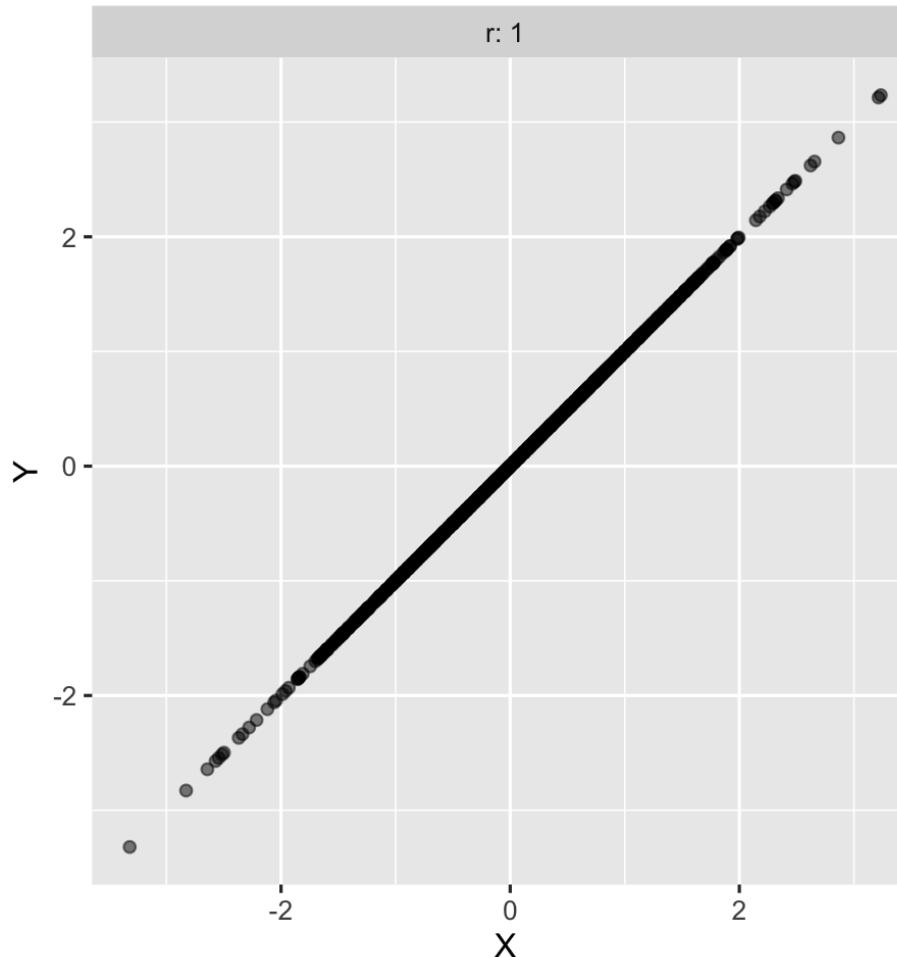
Find team members, and potential topics to study (ed quiz will be posted soon)

What is correlation?

- Linear association between two variables can be described by correlation
- Ranges from -1 to +1

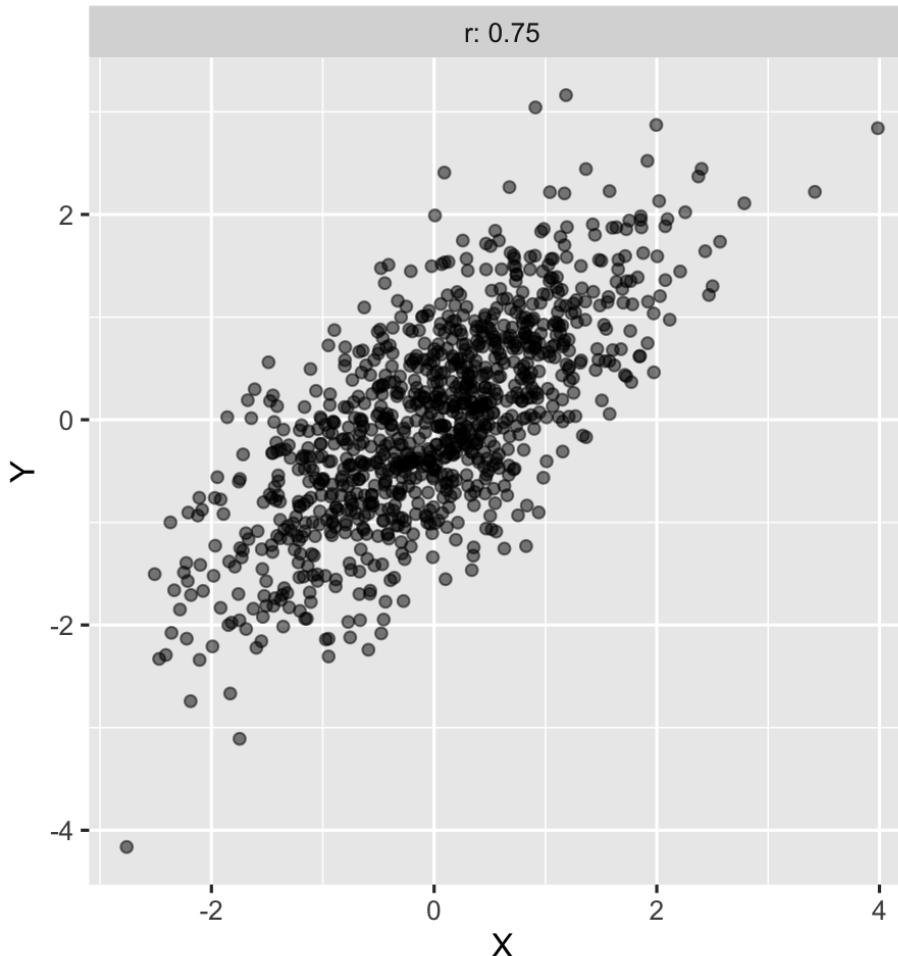
Strong Positive correlation

As one variable increases, so does another

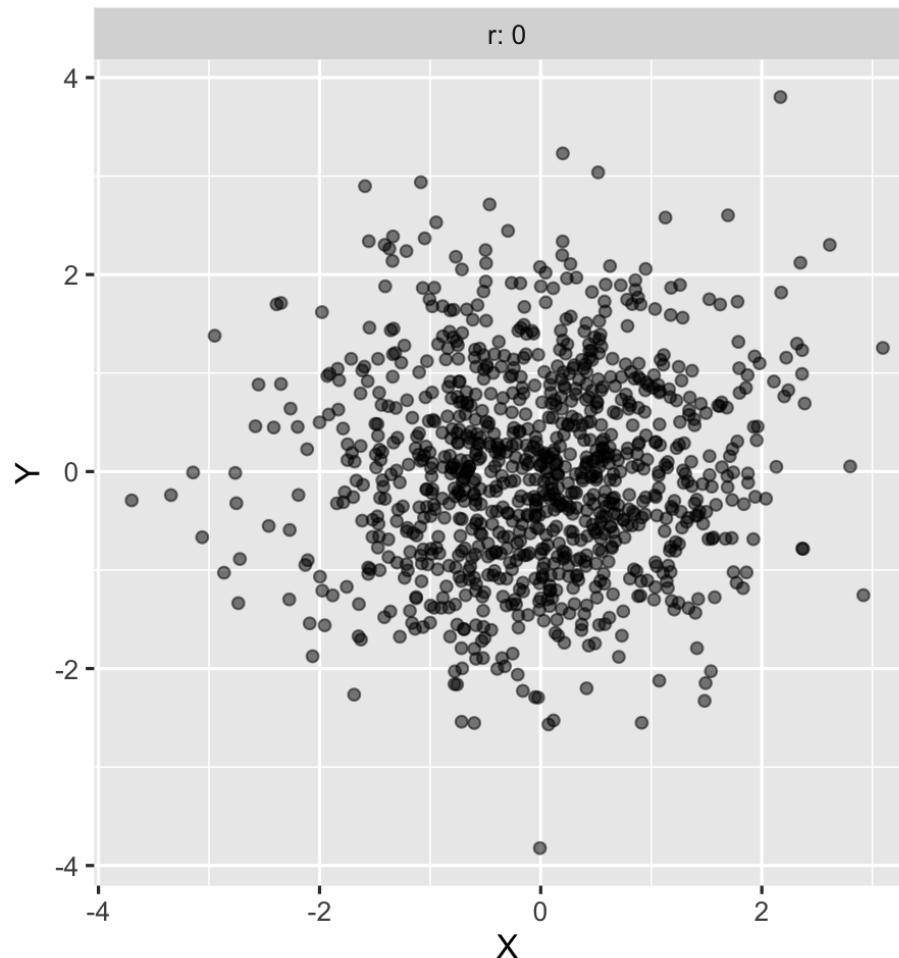


Strong Positive correlation

As one variable increases, so does another variable

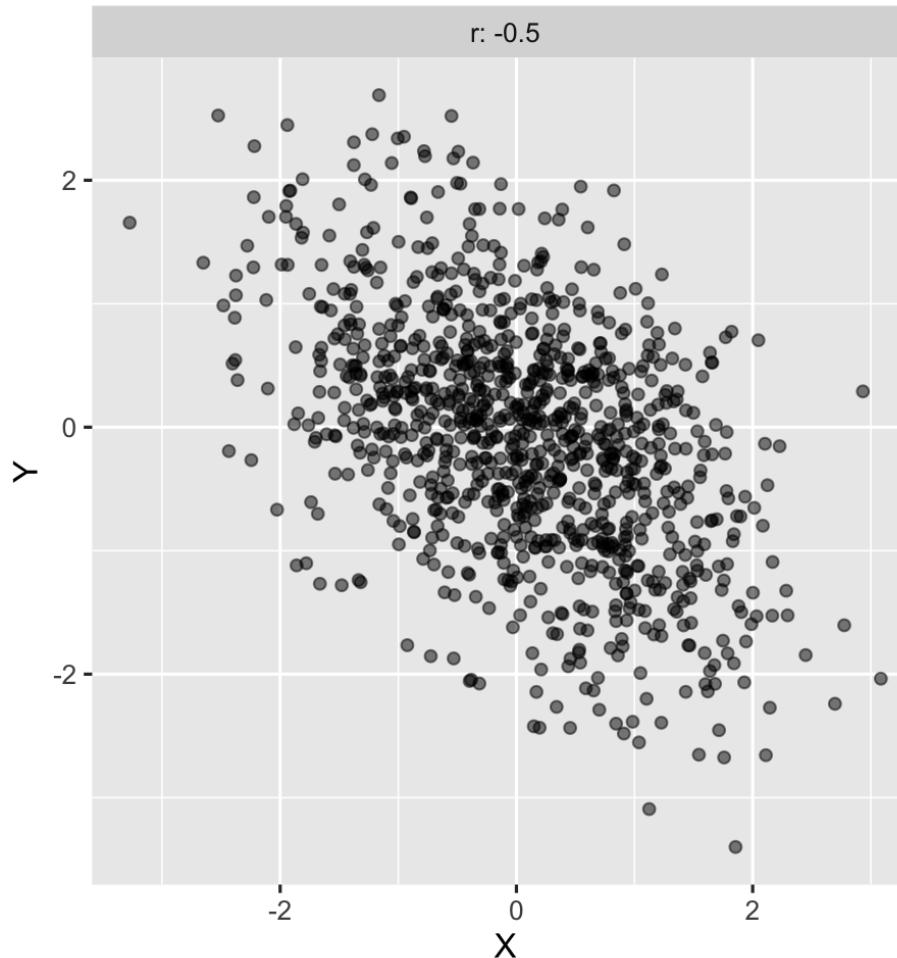


Zero correlation: neither variables are related



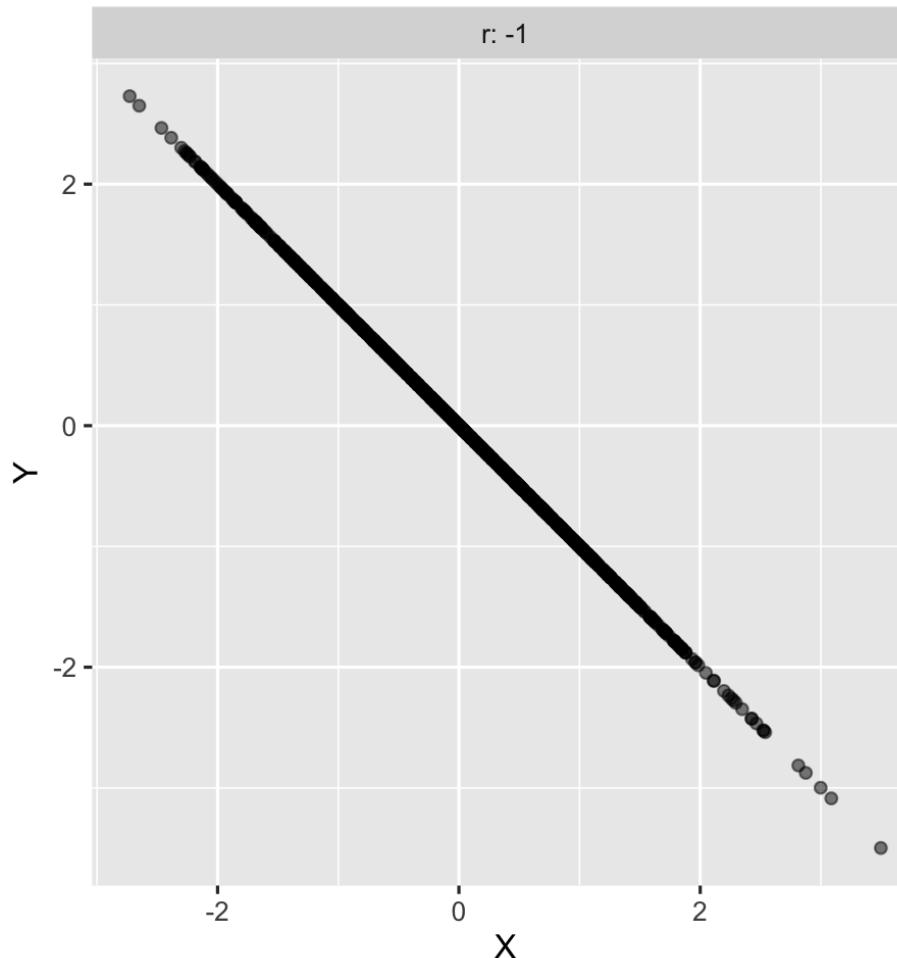
Strong negative correlation

As one variable increases, another decreases



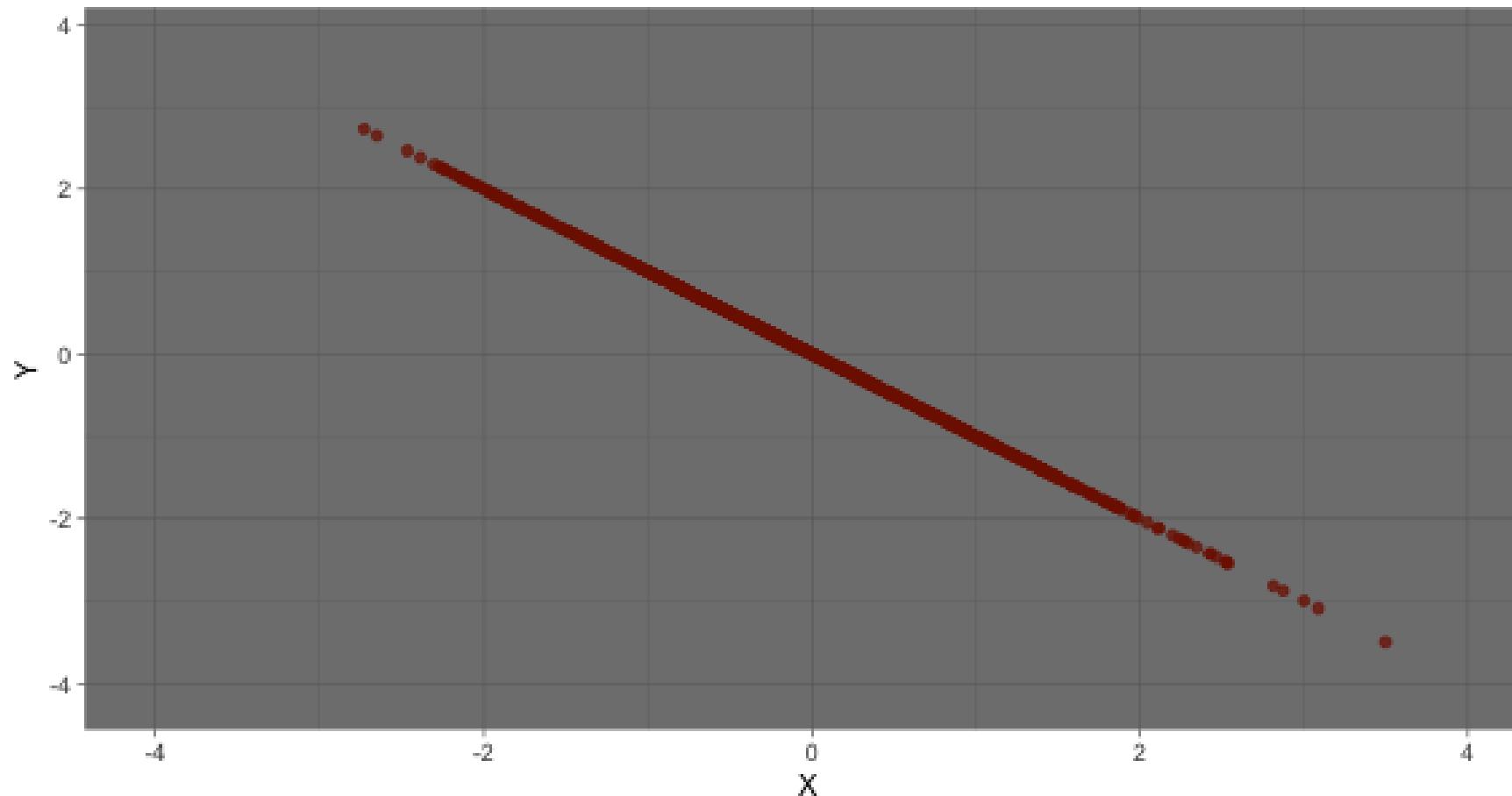
STRONG negative correlation

As one variable increases, another decreases



Correlation: The animation

Now showing $R = -1$



definition of correlation

For two variables X, Y , correlation is:

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} = \frac{cov(X, Y)}{s_x s_y}$$

Dance of correlation

Dancing statistics: explaining the statistical concept of correlation through dance



**Remember! Correlation
does not equal causation**

What is R^2 ?

- (model variance)/(total variance), the amount of variance in response explained by the model.
- Always ranges between 0 and 1, with 1 indicating a perfect fit.
- Adding more variables to the model will always increase R^2 , so what is important is how big an increase is gained. - Adjusted R^2 reduces this for every additional variable added.

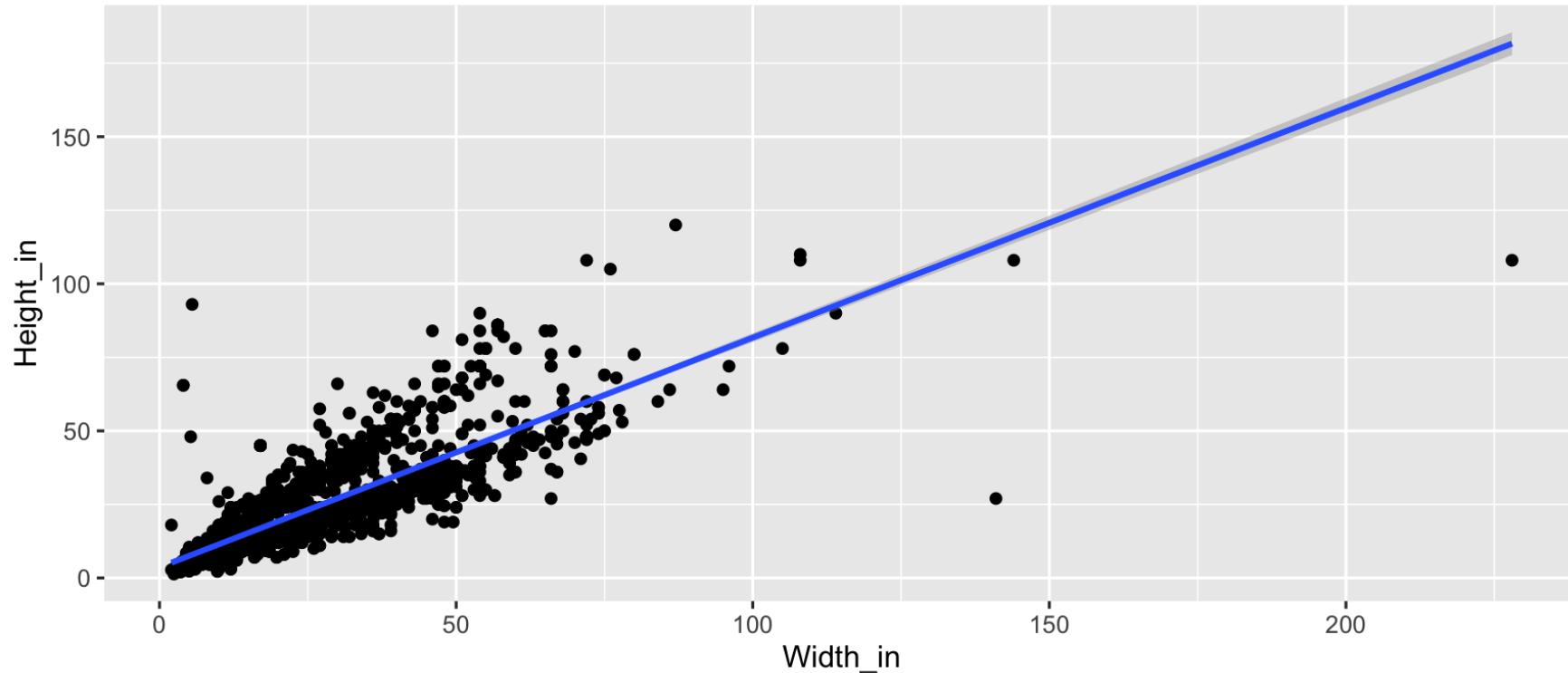
unpacking lm and model objects

```
(pp <- read_csv("data/paris-paintings.csv", na = c("n/a", "", "NA")))

## # A tibble: 3,393 x 61
##   name    sale    lot    position dealer    year origin_author origin_cat school_pntg
##   <chr>   <dbl>   <dbl>   <dbl>     <chr>   <dbl>   <chr>       <chr>      <chr>
## 1 L176... 0.0328 L     1764 F        0         F
## 2 L176... 0.0492 L     1764 I        0         I
## 3 L176... 0.0656 L     1764 X        0         D/FL
## 4 L176... 0.0820 L     1764 F        0         F
## 5 L176... 0.0820 L     1764 F        0         F
## 6 L176... 0.0984 L     1764 X        0         I
## 7 L176... 0.115  L     1764 F        0         F
## 8 L176... 0.115  L     1764 F        0         F
## 9 L176... 0.131  L     1764 X        0         I
## 10 L176... 0.148  L    1764 D/FL      0         D/FL
## # ... with 3,383 more rows, and 52 more variables: diff_origin <dbl>,
## #   price <dbl>, count <dbl>, subject <chr>, authorstandard <chr>, artistliving <db
## #   authorstyle <chr>, author <chr>, winningbidder <chr>, winningbiddertype <chr>,
## #   endbuyer <chr>, Interm <dbl>, type_intermed <chr>, Height_in <dbl>, Width_in <d
## #   Surface_Rect <dbl>, Diam_in <dbl>, Surface_Rnd <dbl>, Shape <chr>, Surface <dbl>
```

unpacking linear models

```
ggplot(data = pp, aes(x = Width_in, y = Height_in)) +  
  geom_point() +  
  geom_smooth(method = "lm") # lm for linear model
```



template for linear model

`lm(<FORMULA>, <DATA>)`

<FORMULA>

RESPONSE ~ EXPLANATORY VARIABLES

Fitting a linear model

```
m_ht_wt <- lm(Height_in ~ Width_in, data = pp)
```

```
m_ht_wt
```

```
##  
## Call:  
## lm(formula = Height_in ~ Width_in, data = pp)  
##  
## Coefficients:  
## (Intercept)    Width_in  
##           3.6214        0.7808
```

using tidy, augment, glance

tidy: return a tidy table of model information

`tidy(<MODEL OBJECT>)`

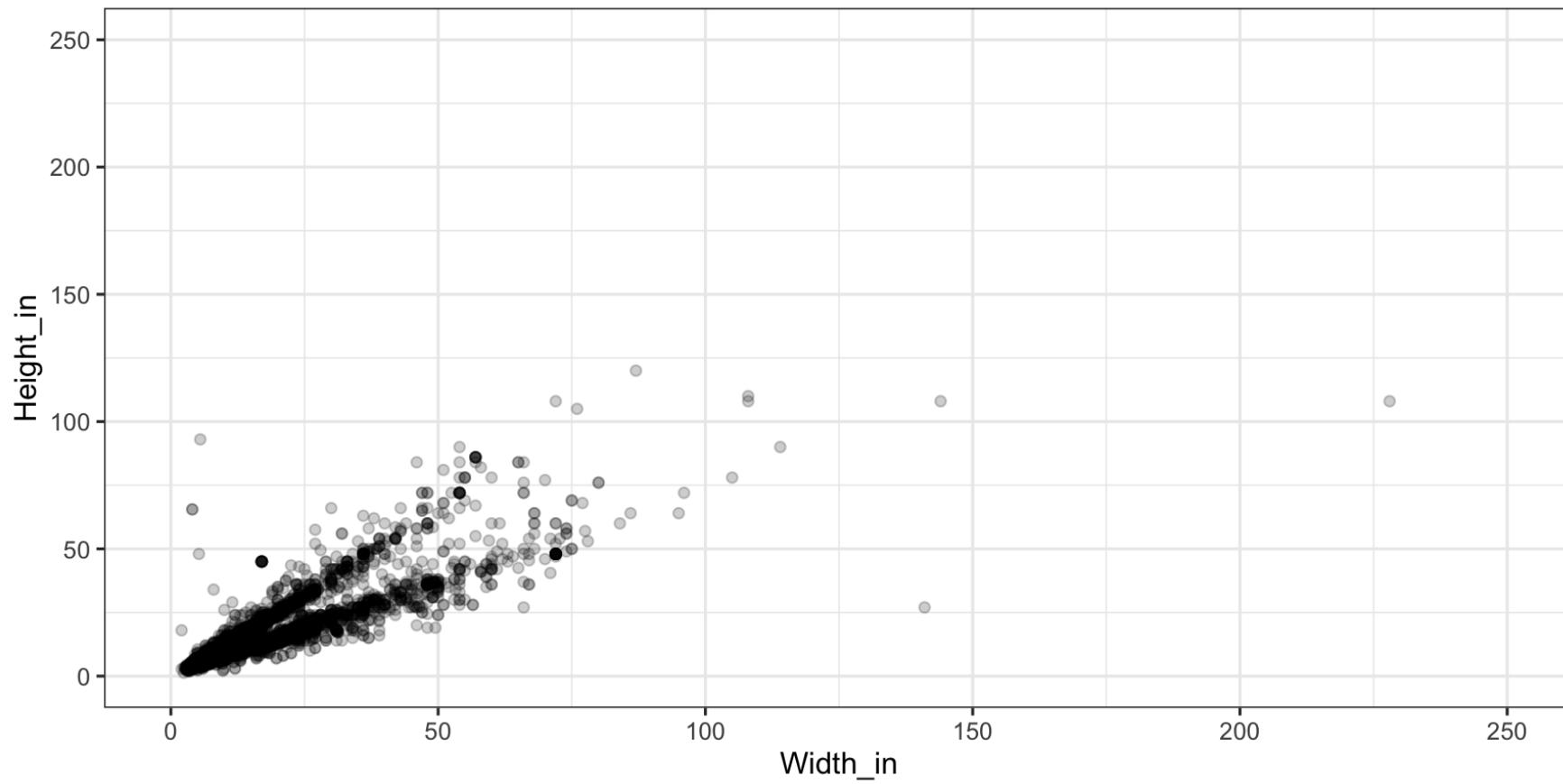
```
tidy(m_ht_wt)

## # A tibble: 2 x 5
##   term      estimate std.error statistic p.value
##   <chr>      <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)  3.62     0.254     14.3 8.82e-45
## 2 Width_in     0.781    0.00950    82.1  0.
```

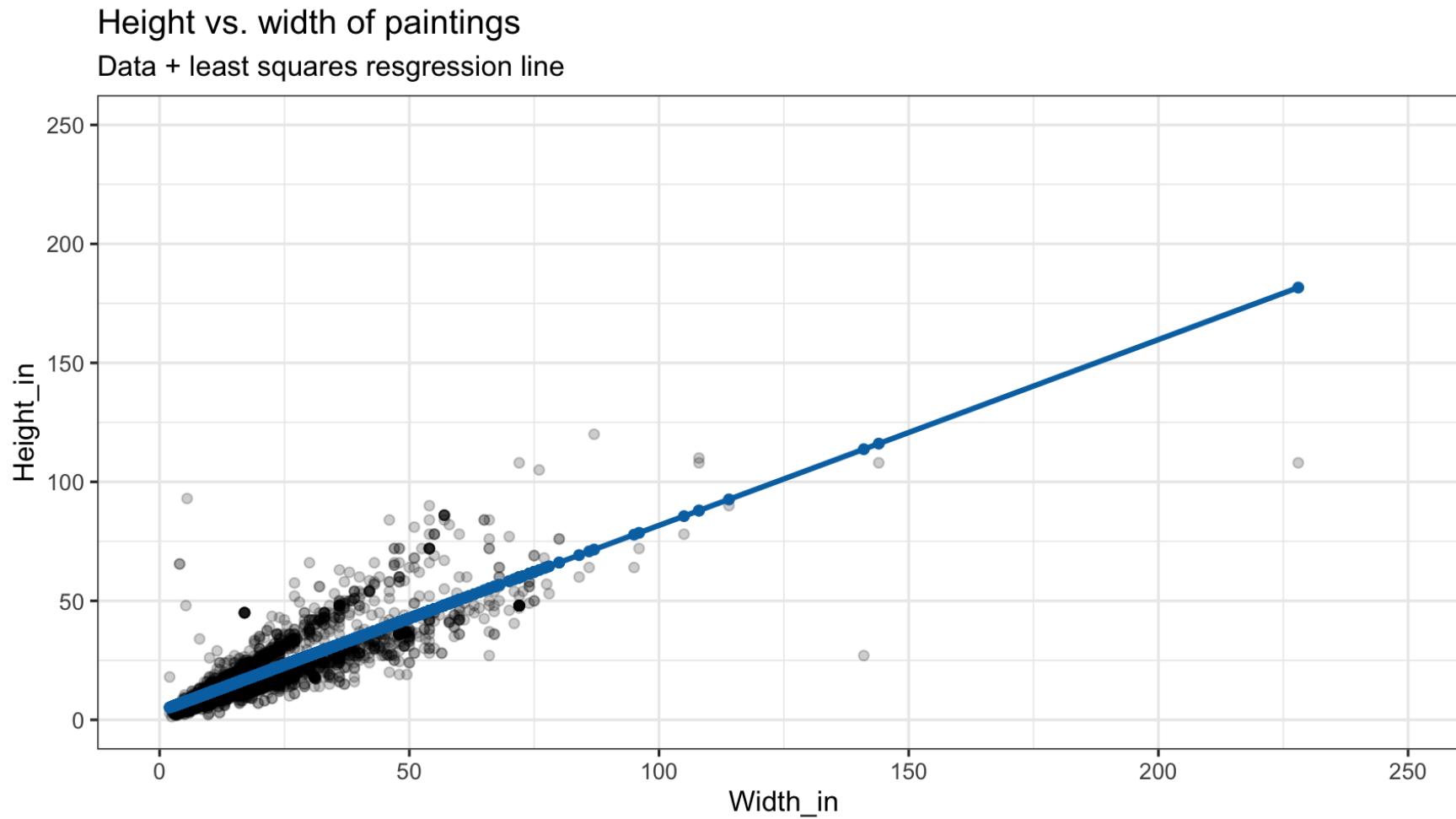
Visualizing residuals

Height vs. width of paintings

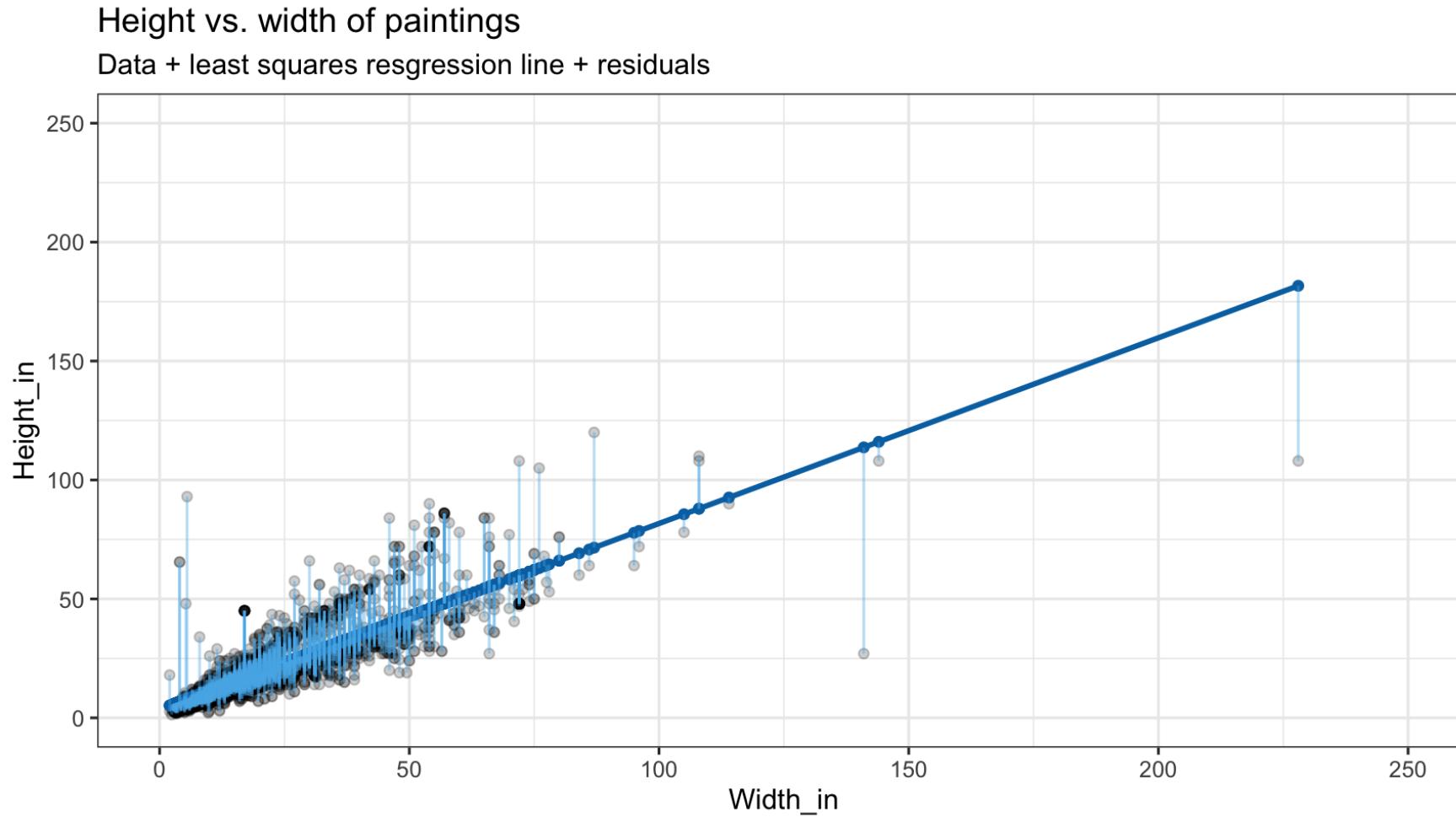
Just the data



Visualizing residuals (cont.)



Visualizing residuals (cont.)



glance: get a one-row summary out

`glance(<MODEL OBJECT>)`

```
glance(m_ht_wt)

## # A tibble: 1 x 11
##   r.squared adj.r.squared sigma statistic p.value    df logLik     AIC     BIC devia
##       <dbl>          <dbl>  <dbl>    <dbl>    <int>  <dbl>  <dbl>  <dbl>  <dbl>    <d
## 1     0.683          0.683  8.30    6749.      0      2 -11083. 22173. 22191. 2160
## # ... with 1 more variable: df.residual <int>
```

AIC, BIC, Deviance

- **AIC, BIC, and Deviance** are evidence to make a decision
- Deviance is the residual variation, how much variation in response that IS NOT explained by the model. The closer to 0 the better, but it is not on a standard scale. In comparing two models if one has substantially lower deviance, then it is a better model.
- Similarly BIC (Bayes Information Criterion) indicates how well the model fits, best used to compare two models. Lower is better.

augment: get the data

augment<MODEL>

or

augment(<MODEL> , <DATA>)

augment

```
augment(m_ht_wt)

## # A tibble: 3,135 x 10
##   .rownames Height_in Width_in .fitted .se.fit .resid     .hat .sigma .cooksdi .stdi
##   <chr>       <dbl>    <dbl>     <dbl>    <dbl>    <dbl>    <dbl>    <dbl>    <dbl>
## 1 1             37      29.5     26.7    0.166   10.3    0.000399  8.30   3.10e-4
## 2 2             18      14        14.6    0.165   3.45    0.000396  8.31   3.42e-5
## 3 3             13      16        16.1    0.158  -3.11    0.000361  8.31   2.54e-5
## 4 4             14      18        17.7    0.152  -3.68    0.000337  8.31   3.30e-5
## 5 5             14      18        17.7    0.152  -3.68    0.000337  8.31   3.30e-5
## 6 6              7      10        11.4    0.185  -4.43    0.000498  8.31   7.09e-5
## 7 7              6      13        13.8    0.170  -7.77    0.000418  8.30   1.83e-4
## 8 8              6      13        13.8    0.170  -7.77    0.000418  8.30   1.83e-4
## 9 9             15      15        15.3    0.161  -0.333   0.000377  8.31   3.04e-7
## 10 10            9       7        9.09    0.204  -0.0870   0.000601  8.31   3.30e-8
## # ... with 3,125 more rows
```

understanding residuals

- variation explained by the model
- residual variation: what's left over after fitting the model

**Your turn: go to studio and
start exercise 7B**

Going beyond a single model

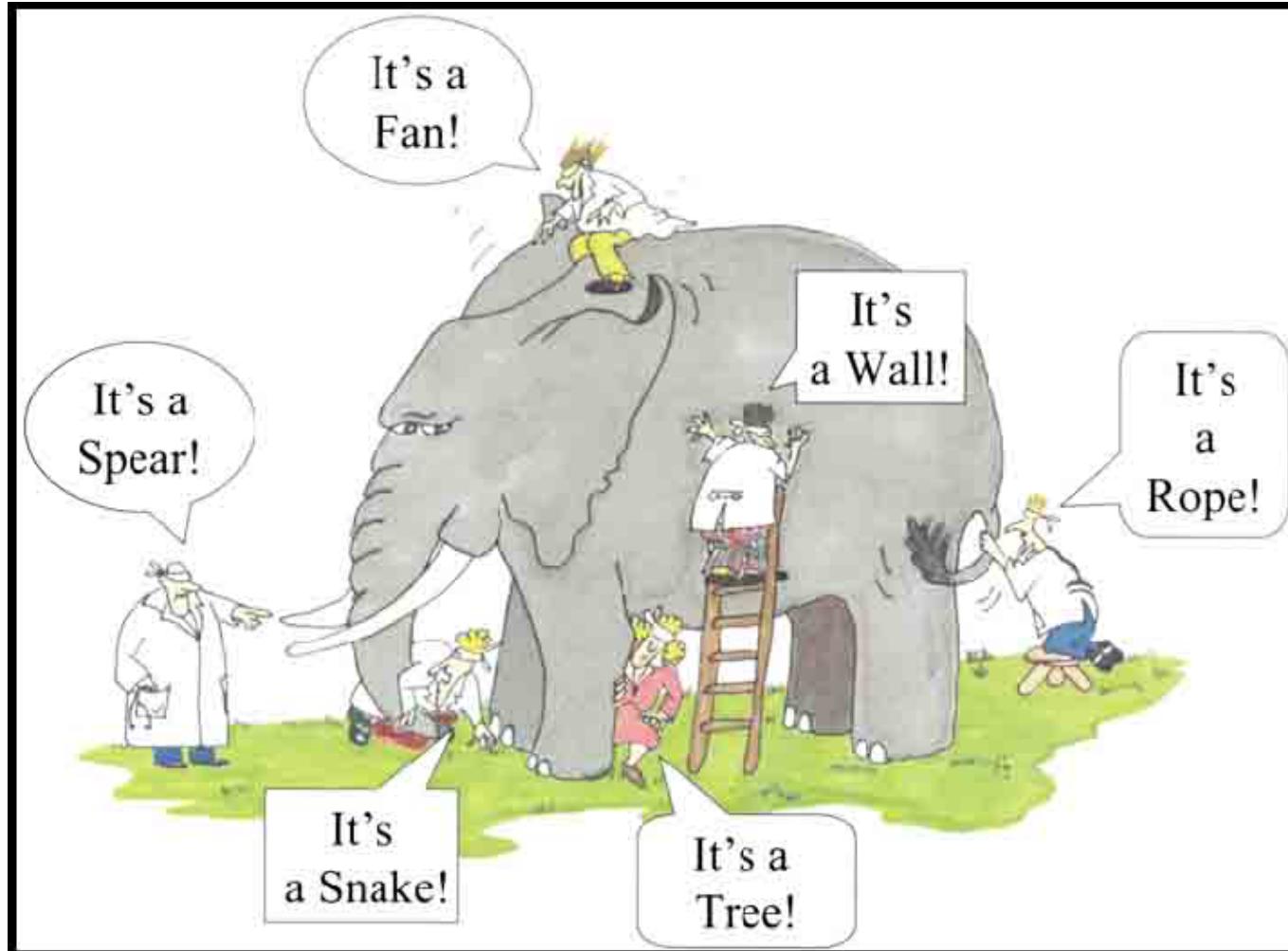


Image source: <https://balajiviswanathan.quora.com/Lessons-from-the-Blind-men-and-the-elephant>

Going beyond a single model

- Beyond a single model
- Fitting many models

Gapminder

- Hans Rosling was a Swedish doctor, academic and statistician, Professor of International Health at Karolinska Institute. Sadly he passed away in 2017.
- He developed a keen interest in health and wealth across the globe, and the relationship with other factors like agriculture, education, energy.
- You can play with the gapminder data using animations at <https://www.gapminder.org/tools/>.

Hans Rosling's 200 Countries, 200 Years, 4 Minutes - The Joy of Stats - BBC Four



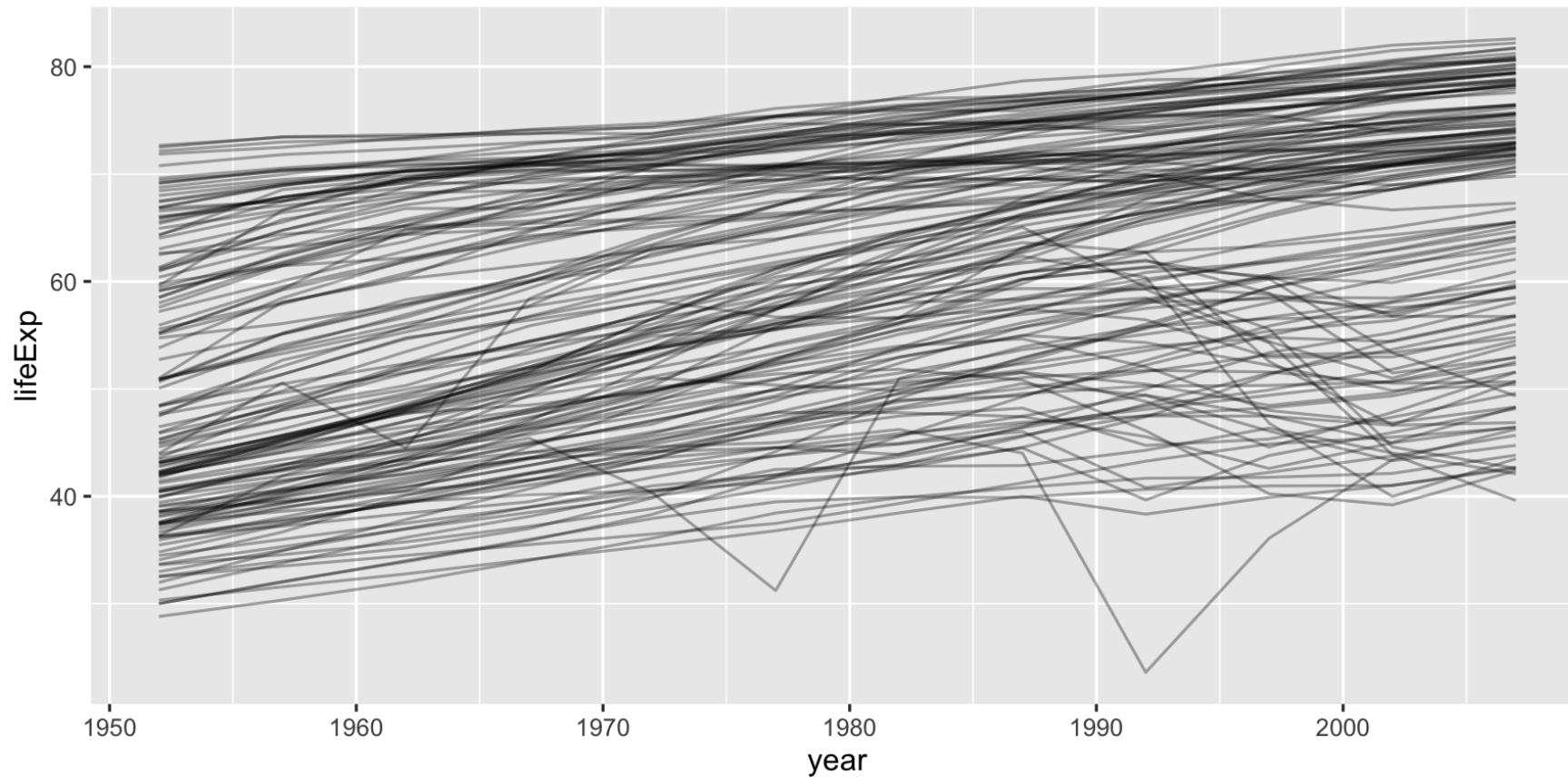
R package: gapminder

Contains subset of the data on five year intervals from 1952 to 2007.

```
library(gapminder)
glimpse(gapminder)

## #> #> Rows: 1,704
## #> #> Columns: 6
## #> #> $ country <fct> Afghanistan, Afghanistan, Afghanistan, Afghanistan, Afghanistan,
## #> #> $ continent <fct> Asia, Asia, Asia, Asia, Asia, Asia, Asia, Asia, Asia, Asia,
## #> #> $ year <int> 1952, 1957, 1962, 1967, 1972, 1977, 1982, 1987, 1992, 1997, 2002,
## #> #> $ lifeExp <dbl> 28.801, 30.332, 31.997, 34.020, 36.088, 38.438, 39.854, 40.822, 4
## #> #> $ pop <int> 8425333, 9240934, 10267083, 11537966, 13079460, 14880372, 1288181
## #> #> $ gdpPercap <dbl> 779.4453, 820.8530, 853.1007, 836.1971, 739.9811, 786.1134, 978.0
```

"Change in life expectancy in countries over time?"



"Change in life expectancy in countries over time?"

- There generally appears to be an increase in life expectancy
- A number of countries have big dips from the 70s through 90s
- a cluster of countries starts off with low life expectancy but ends up close to the highest by the end of the period.

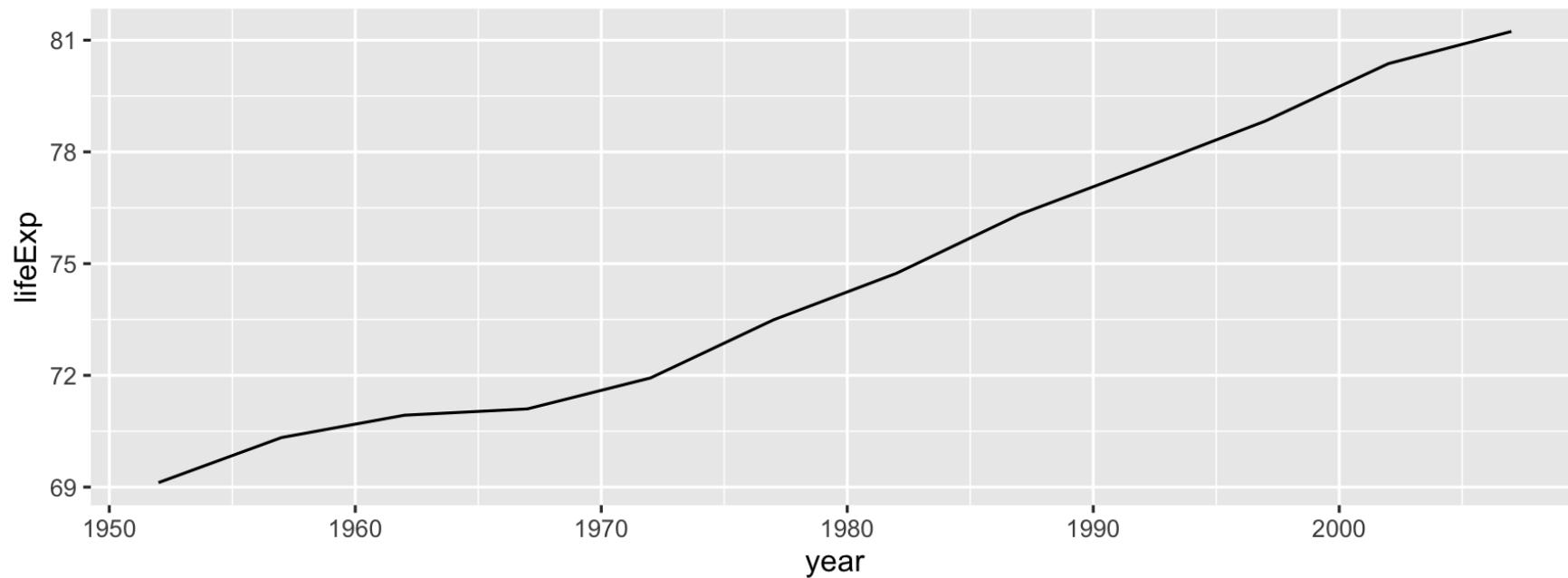
Gapminder: Australia

Australia was already had one of the top life expectancies in the 1950s.

```
oz <- gapminder %>% filter(country == "Australia")  
  
oz  
  
## # A tibble: 12 x 6  
##   country continent year  lifeExp      pop gdpPercap  
##   <fct>     <fct>   <int>   <dbl>    <int>     <dbl>  
## 1 Australia Oceania    1952    69.1  8691212    10040.  
## 2 Australia Oceania    1957    70.3  9712569    10950.  
## 3 Australia Oceania    1962    70.9 10794968    12217.  
## 4 Australia Oceania    1967    71.1 11872264    14526.  
## 5 Australia Oceania    1972    71.9 13177000    16789.  
## 6 Australia Oceania    1977    73.5 14074100    18334.  
## 7 Australia Oceania    1982    74.7 15184200    19477.  
## 8 Australia Oceania    1987    76.3 16257249    21889.  
## 9 Australia Oceania    1992    77.6 17481977    23425.  
## 10 Australia Oceania   1997    78.8 18565243    26998.
```

Gapminder: Australia

```
ggplot(data = oz,  
       aes(x = year,  
            y = lifeExp)) +  
  geom_line()
```



Gapminder: Australia

```
oz_lm <- lm(lifeExp ~ year, data = oz)

oz_lm

## 
## Call:
## lm(formula = lifeExp ~ year, data = oz)
## 
## Coefficients:
## (Intercept)      year
## -376.1163       0.2277
```

Tidy Gapminder Australia

```
tidy(oz_lm)

## # A tibble: 2 x 5
##   term      estimate std.error statistic p.value
##   <chr>        <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept) -376.      20.5     -18.3 5.09e- 9
## 2 year         0.228     0.0104    21.9 8.67e-10
```

$$\widehat{lifeExp} = -376.1163 - 0.2277 \text{ year}$$

Center year

- Let us treat 1950 is the first year
- so for model fitting we are going to shift year to begin in 1950
- This improved interpretability.

```
gap <- gapminder %>% mutate(year1950 = year - 1950)
oz <- gap %>% filter(country == "Australia")
```

Model for centered year

```
oz_lm <- lm(lifeExp ~ year1950, data = oz)

oz_lm

## 
## Call:
## lm(formula = lifeExp ~ year1950, data = oz)
## 
## Coefficients:
## (Intercept)    year1950
##       67.9451      0.2277
```

Tidy the model

```
tidy(oz_lm)

## # A tibble: 2 x 5
##   term      estimate std.error statistic p.value
##   <chr>        <dbl>     <dbl>      <dbl>    <dbl>
## 1 (Intercept)  67.9      0.355     192.  3.70e-19
## 2 year1950     0.228     0.0104     21.9  8.67e-10
```

$$\widehat{lifeExp} = 67.9 + 0.2277 \text{ year}$$

Augment

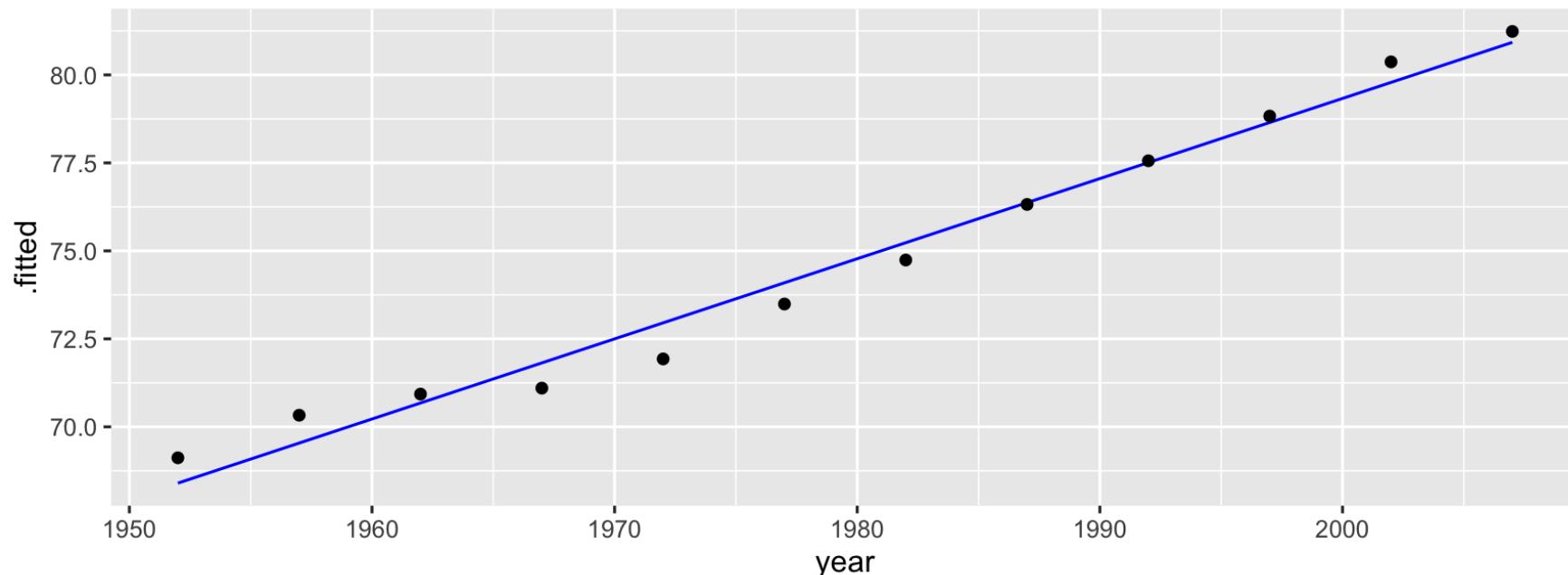
```
oz_aug <- augment(oz_lm, oz)
```

```
oz_aug
```

```
## # A tibble: 12 x 14
##   country continent year lifeExp     pop gdpPercap year1950 .fitted .se.fit .resid
##   <fct>    <fct>    <int>   <dbl>   <int>      <dbl>      <dbl>      <dbl>      <dbl>      <dbl>
## 1 Austra... Oceania    1952    69.1 8.69e6    10040.        2     68.4     0.337     0.719
## 2 Austra... Oceania    1957    70.3 9.71e6    10950.        7     69.5     0.294     0.791
## 3 Austra... Oceania    1962    70.9 1.08e7    12217.       12     70.7     0.255     0.252
## 4 Austra... Oceania    1967    71.1 1.19e7    14526.       17     71.8     0.221    -0.716
## 5 Austra... Oceania    1972    71.9 1.32e7    16789.       22     73.0     0.195    -1.02
## 6 Austra... Oceania    1977    73.5 1.41e7    18334.       27     74.1     0.181    -0.604
## 7 Austra... Oceania    1982    74.7 1.52e7    19477.       32     75.2     0.181    -0.492
## 8 Austra... Oceania    1987    76.3 1.63e7    21889.       37     76.4     0.195    -0.050
## 9 Austra... Oceania    1992    77.6 1.75e7    23425.       42     77.5     0.221     0.050
## 10 Austra... Oceania   1997    78.8 1.86e7    26998.       47     78.6     0.255     0.182
## 11 Austra... Oceania   2002    80.4 1.95e7    30688.       52     79.8     0.294     0.583
## 12 Austra... Oceania   2007    81.2 2.04e7    34435.       57     80.9     0.337     0.310
## # ... with 4 more variables: .hat <dbl>, .sigma <dbl>, .cooksD <dbl>, .std.resid <dbl>
```

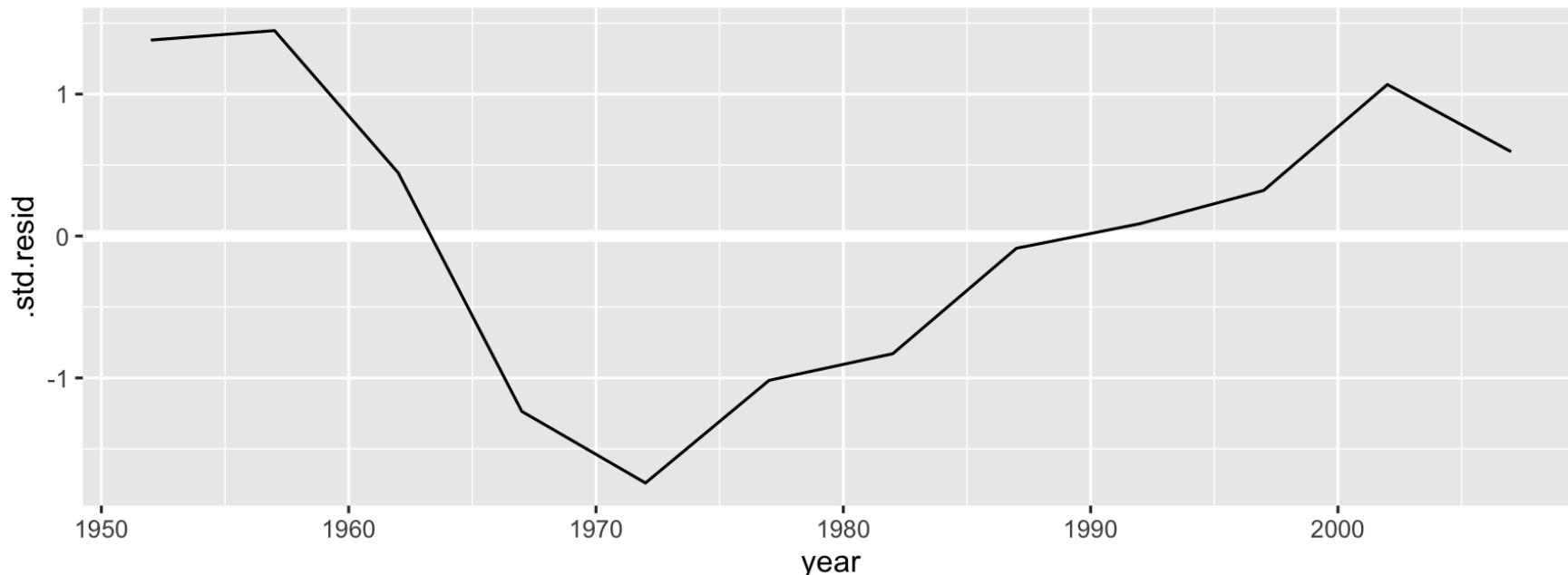
Plot fitted against values

```
ggplot(data = oz_aug,  
       aes(x = year,  
           y = .fitted)) +  
  geom_line(colour = "blue") +  
  geom_point(aes(x = year,  
                 y = lifeExp))
```



Plot studentised residuals against year

```
ggplot(data = oz_aug,  
       aes(x = year,  
            y = .std.resid)) +  
  geom_hline(yintercept = 0,  
             colour = "white",  
             size = 2) +  
  geom_line()
```



Making inferences from this

- Life expectancy has increased 2.3 years every decade, on average.
- There was a slow period from 1960 through to 1972, probably related to mortality during the Vietnam war.

Can we fit for New Zealand?

```
nz <- gap %>% filter(country == "New Zealand")
nz_lm <- lm(lifeExp ~ year1950, data = nz)
nz_lm

## 
## Call:
## lm(formula = lifeExp ~ year1950, data = nz)
## 
## Coefficients:
## (Intercept)    year1950
##       68.3013      0.1928
```

Can we fit for Japan?

```
japan <- gap %>% filter(country == "Japan")
japan_lm <- lm(lifeExp ~ year1950, data = japan)
japan_lm

## 
## Call:
## lm(formula = lifeExp ~ year1950, data = japan)
## 
## Coefficients:
## (Intercept)    year1950
##       64.4162      0.3529
```

Can we fit for Italy?

```
italy <- gap %>% filter(country == "Italy")
italy_lm <- lm(lifeExp ~ year1950, data = italy)
italy_lm

## 
## Call:
## lm(formula = lifeExp ~ year1950, data = italy)
## 
## Coefficients:
## (Intercept)    year1950
##       66.0574      0.2697
```

Is there a better way?

Like, what if we wanted to fit a model for ALL countries?
Let's tinker with the data.

nest() country level data (one row = one country)

```
by_country <- gap %>%
  select(country, year1950, lifeExp, continent) %>%
  group_by(country, continent) %>%
  nest()
```

by_country

```
## # A tibble: 142 x 3
## # Groups:   country, continent [710]
##   country     continent data
##   <fct>       <fct>    <list>
## 1 Afghanistan Asia     <tibble [12 x 2]>
## 2 Albania      Europe   <tibble [12 x 2]>
## 3 Algeria      Africa   <tibble [12 x 2]>
## 4 Angola       Africa   <tibble [12 x 2]>
## 5 Argentina    Americas <tibble [12 x 2]>
## 6 Australia    Oceania  <tibble [12 x 2]>
## 7 Austria      Europe   <tibble [12 x 2]>
## 8 Bahrain      Asia     <tibble [12 x 2]>
## 9 Bangladesh   Asia     <tibble [12 x 2]>
```

What is in data?

```
by_country$data[[1]]  
  
## # A tibble: 12 x 2  
##   year1950 lifeExp  
##       <dbl>    <dbl>  
## 1        2     28.8  
## 2        7     30.3  
## 3       12     32.0  
## 4       17     34.0  
## 5       22     36.1  
## 6       27     38.4  
## 7       32     39.9  
## 8       37     40.8  
## 9       42     41.7  
## 10      47     41.8  
## 11      52     42.1  
## 12      57     43.8
```

It's a list!

fit a linear model to each one?

```
lm_afghanistan <- lm(lifeExp ~ year1950, data = by_country$data[[1]])  
lm_albania <- lm(lifeExp ~ year1950, data = by_country$data[[2]])  
lm_algeria <- lm(lifeExp ~ year1950, data = by_country$data[[3]])
```

But we are copying and pasting this code **more than twice**...is there a better way?

A case for our friend, map ... ???

`map(<data object>, <function>)`

A case for map ???

```
mapped_lm <- map(.x = by_country$data,
                  .f = function(x){
                    lm(lifeExp ~ year1950, data = x)
                  })
```

```
mapped_lm
```

```
## [[1]]
##
## Call:
## lm(formula = lifeExp ~ year1950, data = x)
##
## Coefficients:
## (Intercept)    year1950
##       29.3566      0.2753
##
## 
## [[2]]
##
## Call:
```

Map inside the data?

```
country_model <- by_country %>%
  mutate(model = map(.x = data,
                     .f = function(x){
                       lm(lifeExp ~ year1950, data = x)
                     }))

country_model

## # A tibble: 142 x 4
## # Groups:   country, continent [710]
##   country     continent data          model
##   <fct>       <fct>    <list>        <list>
## 1 Afghanistan Asia     <tibble [12 x 2]> <lm>
## 2 Albania      Europe   <tibble [12 x 2]> <lm>
## 3 Algeria      Africa   <tibble [12 x 2]> <lm>
## 4 Angola       Africa   <tibble [12 x 2]> <lm>
## 5 Argentina    Americas <tibble [12 x 2]> <lm>
## 6 Australia    Oceania  <tibble [12 x 2]> <lm>
## 7 Austria      Europe   <tibble [12 x 2]> <lm>
## 8 Bahrain      Asia     <tibble [12 x 2]> <lm>
```

A case for map (shorthand function)

```
country_model <- by_country %>%
  mutate(model = map(.x = data,
                     .f = ~lm(lifeExp ~ year1950, data = .)))
```

```
country_model
```

```
## # A tibble: 142 x 4
## # Groups:   country, continent [710]
##   country     continent    data          model
##   <fct>       <fct>      <list>        <list>
## 1 Afghanistan Asia      <tibble [12 x 2]> <lm>
## 2 Albania      Europe    <tibble [12 x 2]> <lm>
## 3 Algeria      Africa    <tibble [12 x 2]> <lm>
## 4 Angola       Africa    <tibble [12 x 2]> <lm>
## 5 Argentina    Americas  <tibble [12 x 2]> <lm>
## 6 Australia    Oceania   <tibble [12 x 2]> <lm>
## 7 Austria      Europe    <tibble [12 x 2]> <lm>
## 8 Bahrain      Asia      <tibble [12 x 2]> <lm>
## 9 Bangladesh   Asia      <tibble [12 x 2]> <lm>
## 10 Belgium     Europe   <tibble [12 x 2]> <lm>
```

Where's the model?

```
country_model$model[[1]]  
  
##  
## Call:  
## lm(formula = lifeExp ~ year1950, data = .)  
##  
## Coefficients:  
## (Intercept)    year1950  
##      29.3566      0.2753
```

We need to summarise this content

```
tidy(country_model$model[[1]])  
  
## # A tibble: 2 x 5  
##   term      estimate std.error statistic p.value  
##   <chr>        <dbl>     <dbl>      <dbl>    <dbl>  
## 1 (Intercept)  29.4      0.699      42.0  1.40e-12  
## 2 year1950     0.275     0.0205     13.5  9.84e- 8
```

So should we repeat it for each one?

```
tidy(country_model$model[[1]])
```

```
## # A tibble: 2 x 5
##   term      estimate std.error statistic p.value
##   <chr>      <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept) 29.4      0.699     42.0  1.40e-12
## 2 year1950    0.275     0.0205    13.5  9.84e- 8
```

```
tidy(country_model$model[[2]])
```

```
## # A tibble: 2 x 5
##   term      estimate std.error statistic p.value
##   <chr>      <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept) 58.6      1.13      51.7  1.79e-13
## 2 year1950    0.335     0.0332    10.1  1.46e- 6
```

```
tidy(country_model$model[[3]])
```

```
## # A tibble: 2 x 5
##   term      estimate std.error statistic p.value
##   <chr>      <dbl>     <dbl>     <dbl>    <dbl>
```

Use map

```
country_model %>%  
  mutate(tidy = map(model, tidy))  
  
## # A tibble: 142 x 5  
## # Groups:   country, continent [710]  
##   country     continent data           model    tidy  
##   <fct>       <fct>    <list>        <list>   <list>  
## 1 Afghanistan Asia     <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## 2 Albania      Europe   <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## 3 Algeria      Africa   <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## 4 Angola       Africa   <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## 5 Argentina    Americas <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## 6 Australia    Oceania  <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## 7 Austria      Europe   <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## 8 Bahrain      Asia     <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## 9 Bangladesh   Asia     <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## 10 Belgium     Europe   <tibble [12 × 2]> <lm>    <tibble [2 × 5]>  
## # ... with 132 more rows
```

unnest

```
country_coefs <- country_model %>%  
  mutate(tidy = map(model, tidy)) %>%  
  unnest(tidy) %>%  
  select(country, continent, term, estimate)
```

```
country_coefs
```

```
## # A tibble: 284 x 4  
## # Groups:   country, continent [710]  
##       country     continent   term      estimate  
##       <fct>       <fct>     <chr>      <dbl>  
## 1 Afghanistan Asia (Intercept) 29.4  
## 2 Afghanistan Asia year1950  0.275  
## 3 Albania      Europe (Intercept) 58.6  
## 4 Albania      Europe year1950  0.335  
## 5 Algeria      Africa (Intercept) 42.2  
## 6 Algeria      Africa year1950  0.569  
## 7 Angola       Africa (Intercept) 31.7  
## 8 Angola       Africa year1950  0.209  
## 9 Argentina    Americas (Intercept) 62.2
```

Pivot the term

```
tidy_country_coefs <- country_coefs %>%
  pivot_wider(id_cols = c(term, country, continent),
              names_from = term,
              values_from = estimate) %>%
  rename(intercept = `Intercept`)
```

```
tidy_country_coefs
```

```
## # A tibble: 142 x 4
## # Groups:   country, continent [710]
##   country     continent intercept year1950
##   <fct>       <fct>        <dbl>      <dbl>
## 1 Afghanistan Asia         29.4      0.275
## 2 Albania      Europe      58.6      0.335
## 3 Algeria      Africa      42.2      0.569
## 4 Angola       Africa      31.7      0.209
## 5 Argentina    Americas    62.2      0.232
## 6 Australia    Oceania     67.9      0.228
## 7 Austria      Europe      66.0      0.242
## 8 Bahrain      Asia        51.8      0.468
```

Filter to only Australia

```
tidy_country_coefs %>%  
  filter(country == "Australia")  
  
## # A tibble: 1 x 4  
## # Groups:   country, continent [710]  
##   country   continent intercept year1950  
##   <fct>     <fct>        <dbl>      <dbl>  
## 1 Australia  Oceania       67.9      0.228
```

Your turn: Five minute challenge

- Fit the models to all countries
- Pick your favourite country (not Australia), print the coefficients, and make a hand sketch of the model fit.

Plot all the models

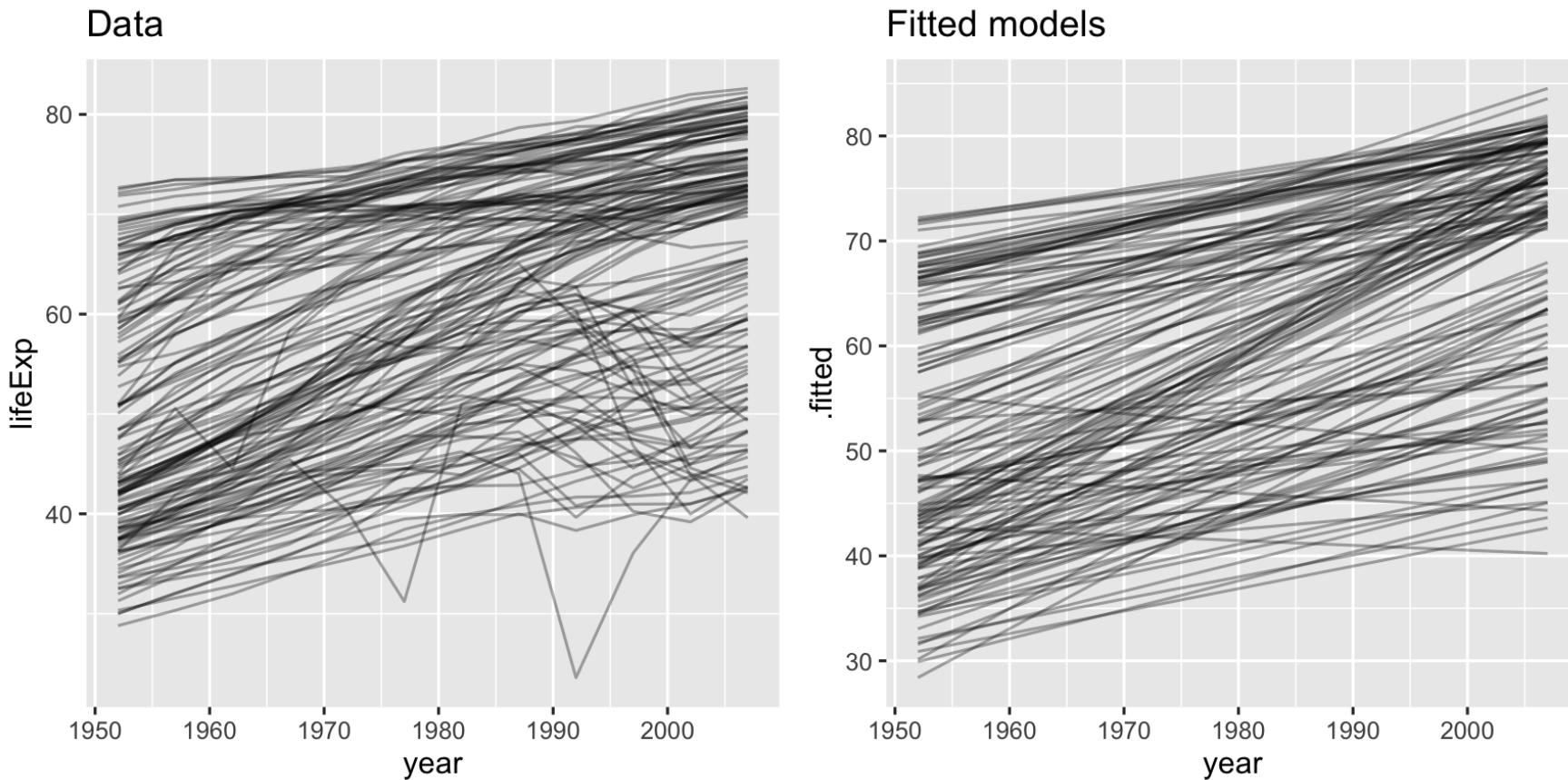
```
country_aug <- country_model %>%  
  mutate(augmented = map(model, augment)) %>%  
  unnest(augmented)  
  
country_aug  
  
## # A tibble: 1,704 x 13  
## # Groups: country, continent [710]  
##   country continent data model lifeExp year1950 .fitted .se.fit .resid .hat .s  
##   <fct>    <fct>  <lis> <lis>    <dbl>    <dbl>    <dbl>    <dbl>    <dbl>    <dbl>    <  
## 1 Afghan... Asia    <tib... <lm>    28.8      2     29.9    0.664 -1.11    0.295  
## 2 Afghan... Asia    <tib... <lm>    30.3      7     31.3    0.580 -0.952    0.225  
## 3 Afghan... Asia    <tib... <lm>    32.0     12     32.7    0.503 -0.664    0.169  
## 4 Afghan... Asia    <tib... <lm>    34.0     17     34.0    0.436 -0.0172   0.127  
## 5 Afghan... Asia    <tib... <lm>    36.1     22     35.4    0.385  0.674    0.0991  
## 6 Afghan... Asia    <tib... <lm>    38.4     27     36.8    0.357  1.65     0.0851  
## 7 Afghan... Asia    <tib... <lm>    39.9     32     38.2    0.357  1.69     0.0851  
## 8 Afghan... Asia    <tib... <lm>    40.8     37     39.5    0.385  1.28     0.0991  
## 9 Afghan... Asia    <tib... <lm>    41.7     42     40.9    0.436  0.754    0.127  
## 10 Afghan... Asia   <tib... <lm>    41.8     47     42.3    0.503 -0.534    0.169
```

Plot all the models

```
p1 <- gapminder %>%
  ggplot(aes(year, lifeExp, group = country)) +
  geom_line(alpha = 1/3) + labs(title = "Data")

p2 <- ggplot(country_aug) +
  geom_line(aes(x = year1950 + 1950,
                y = .fitted,
                group = country),
            alpha = 1/3) +
  labs(title = "Fitted models",
       x = "year")
```

Plot all the models

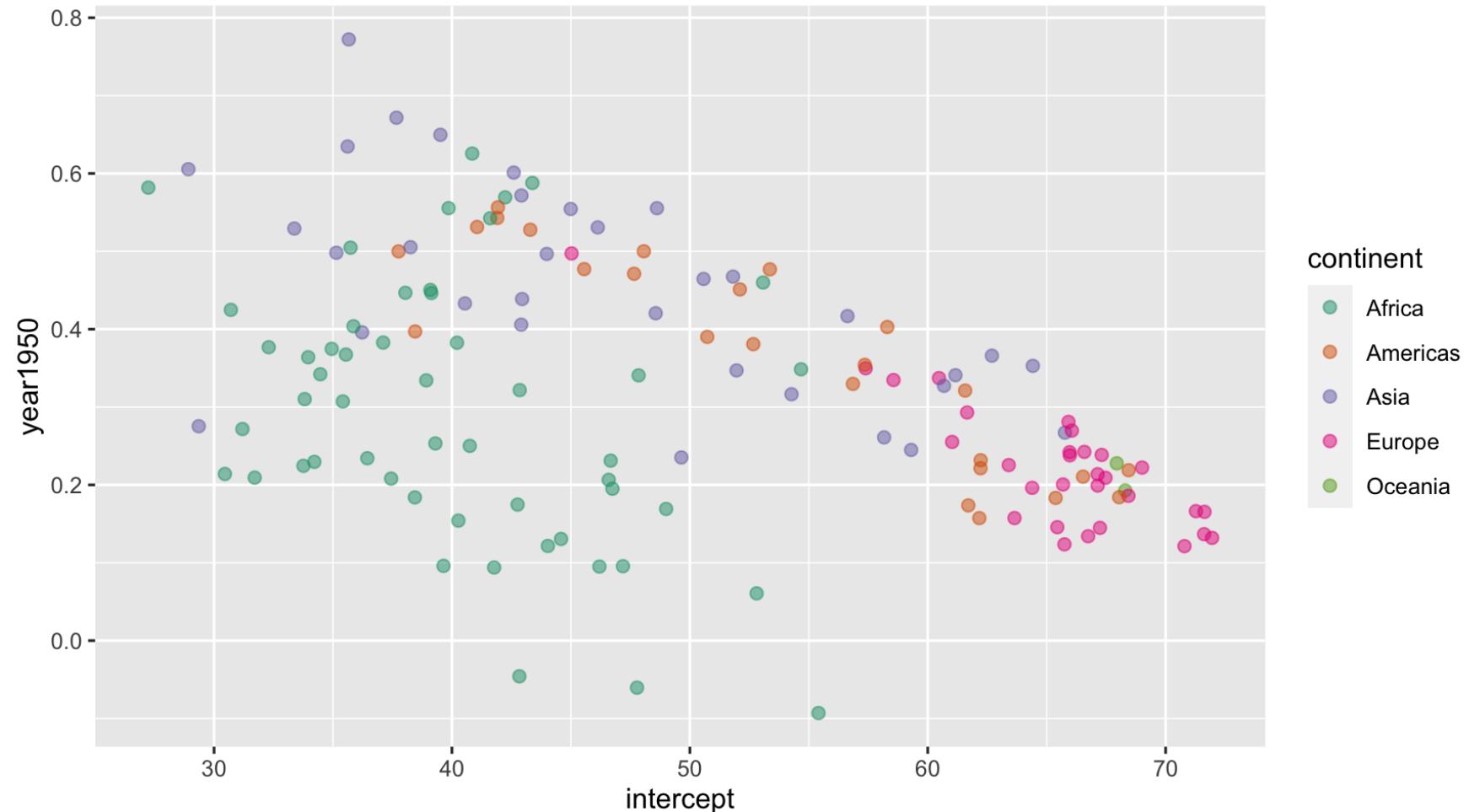


Plot all the model coefficients

```
p <- ggplot(tidy_country_coefs,  
             aes(x = intercept,  
                  y = year1950,  
                  colour = continent,  
                  label = country)) +  
  geom_point(alpha = 0.5,  
             size = 2) +  
  scale_color_brewer(palette = "Dark2")
```

Plot all the model coefficients

p



Make it interactive!

```
library(plotly)  
ggplotly(p)
```

Let's summarise the information learned from the model coefficients.

- Generally the relationship is negative: this means that if a country started with a high intercept tends to have lower rate of increase.
- There is a difference across the continents: Countries in Europe and Oceania tended to start with higher life expectancy and increased; countries in Asia and America tended to start lower but have high rates of improvement; Africa tends to start lower and have a huge range in rate of change.
- Three countries had negative growth in life expectancy: Rwanda, Zimbabwe, Zambia

Model diagnostics by country

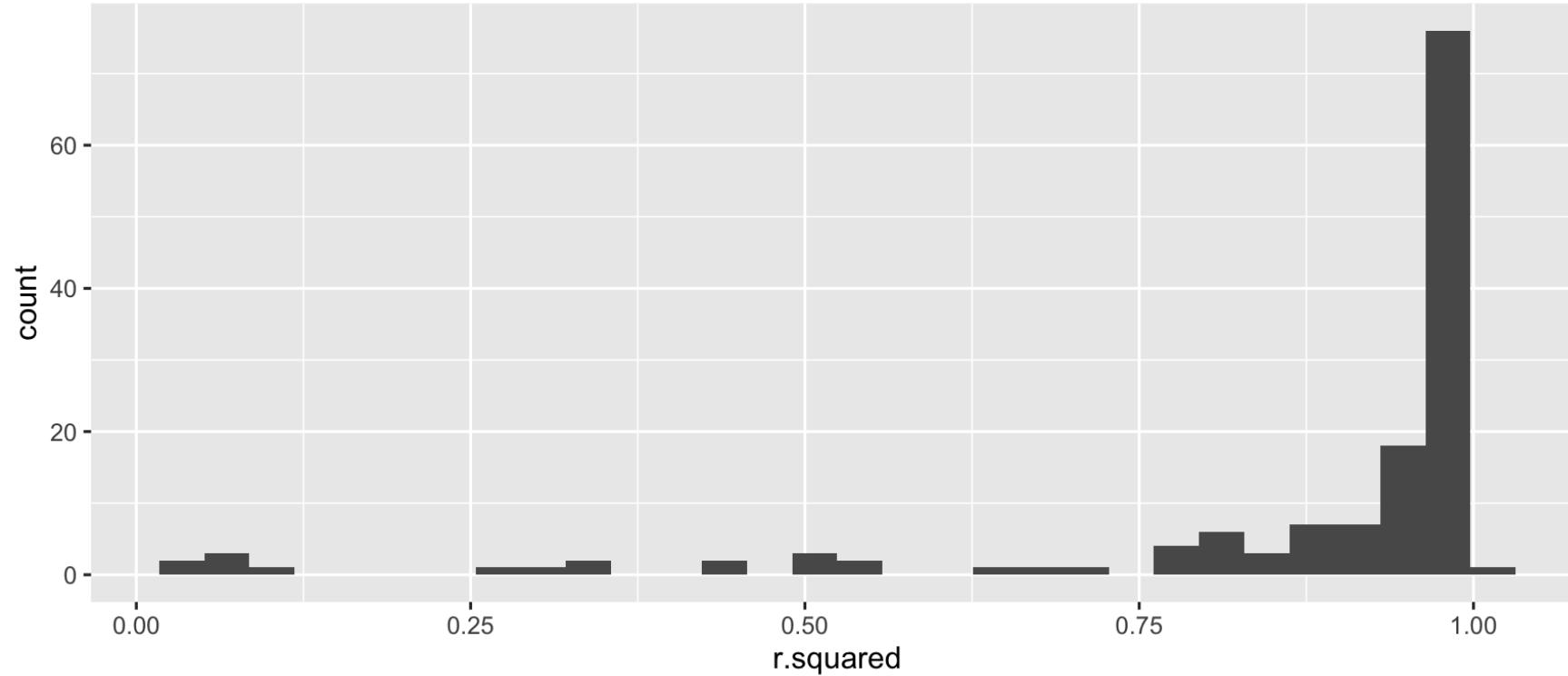
```
country_glance <- country_model %>%
  mutate(glance = map(model, glance)) %>%
  unnest(glance)

country_glance

## # A tibble: 142 x 15
## # Groups:   country, continent [710]
##   country continent data model r.squared adj.r.squared sigma statistic p.value
##   <fct>    <fct>   <lis> <lis>     <dbl>        <dbl> <dbl> <dbl>    <dbl> <
## 1 Afghan... Asia     <tib... <lm>    0.948       0.942 1.22 181.  9.84e- 8
## 2 Albania Europe   <tib... <lm>    0.911       0.902 1.98 102.  1.46e- 6
## 3 Algeria Africa   <tib... <lm>    0.985       0.984 1.32 662.  1.81e-10
## 4 Angola Africa    <tib... <lm>    0.888       0.877 1.41 79.1  4.59e- 6
## 5 Argent... Americas <tib... <lm>    0.996       0.995 0.292 2246. 4.22e-13
## 6 Australia Oceania <tib... <lm>    0.980       0.978 0.621 481.  8.67e-10
## 7 Austria Europe    <tib... <lm>    0.992       0.991 0.407 1261. 7.44e-12
## 8 Bahrain Asia      <tib... <lm>    0.967       0.963 1.64 291.  1.02e- 8
## 9 Bangla... Asia     <tib... <lm>    0.989       0.988 0.977 930.  3.37e-11
## 10 Belgium Europe   <tib... <lm>    0.995       0.994 0.293 1822. 1.20e-12
```

Plot the R^2 values as a histogram.

```
ggplot(country_glance,  
       aes(x = r.squared)) +  
  geom_histogram()
```



Countries with worst fit

Examine the countries with the worst fit, countries with $R^2 < 0.45$, by making scatterplots of the data, with the linear model overlaid.

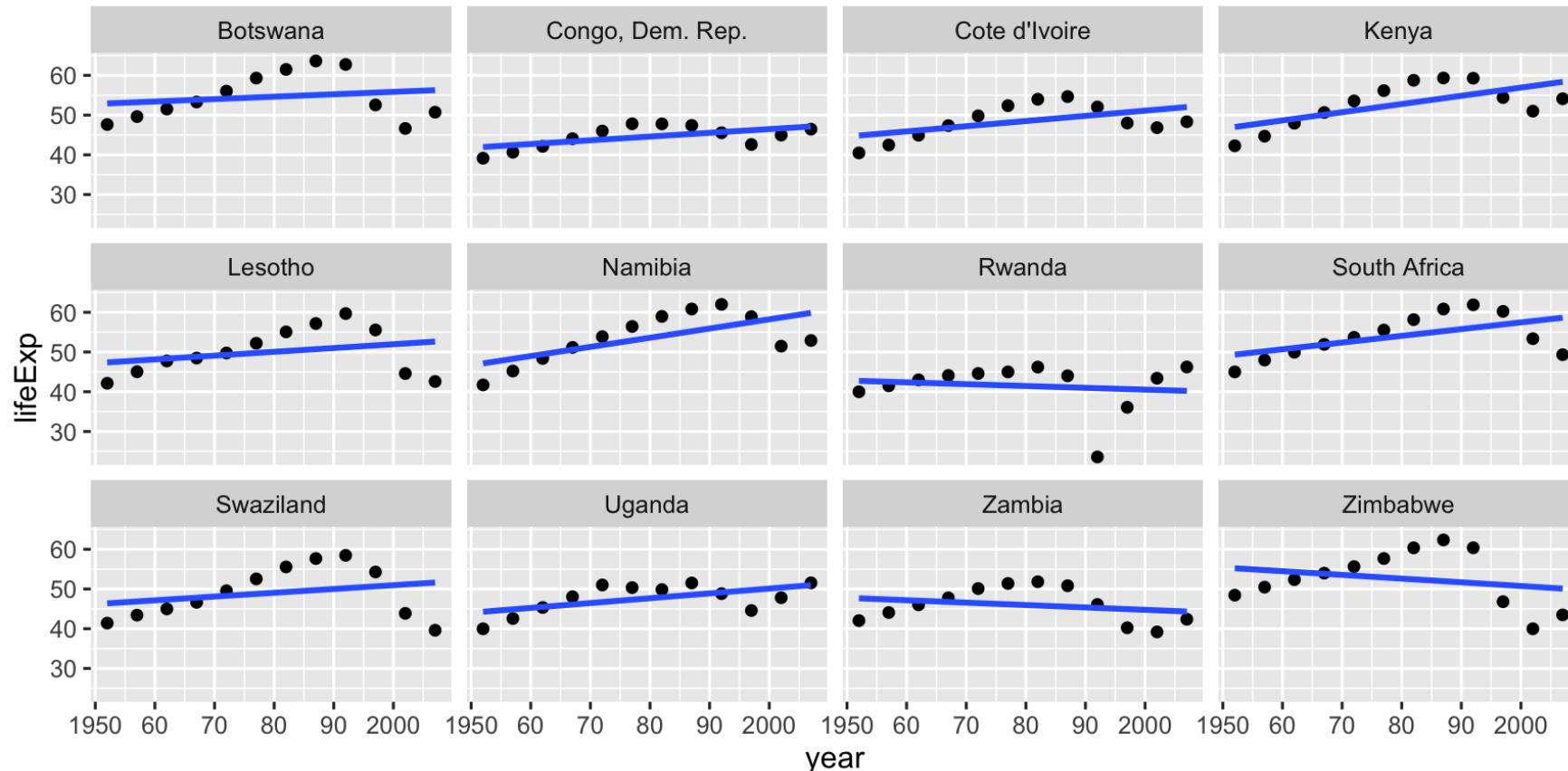
```
badfit <- country_glance %>% filter(r.squared <= 0.45)

gap_bad <- gap %>% filter(country %in% badfit$country)

gg_bad_fit <-
  ggplot(data = gap_bad,
         aes(x = year,
              y = lifeExp)) +
    geom_point() +
    facet_wrap(~country) +
    scale_x_continuous(breaks = seq(1950, 2000, 10),
                       labels = c("1950", "60", "70", "80", "90", "2000")) +
    geom_smooth(method = "lm",
                se = FALSE)
```

Countries with worst fit

Each of these countries had been moving on a nice trajectory of increasing life expectancy, and then suffered a big dip during the time period.



Your Turn:

- Use google to explain these dips using world history and current affairs information.
- finish the lab exercise (with new data)
- remember the project deadline: **Find team members, and potential topics to study (List of groups will be posted here)**