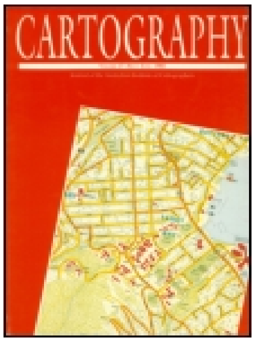

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Cartographic transformation of the thematic map base

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It is suggested that choropleth maps of population characteristics exhibit serious graphic distortions when the population concerned varies greatly in density. The topological cartogram is proposed as a viable alternative in such situations. The example of an urban cartogram provides a focus for discussion of the strengths and weaknesses of the proposal. It is concluded that the topological cartogram may provide an improvement in graphic communication by avoiding the terrestrial distortion inherent in the choropleth method.

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A recurrent topic in the literature on thematic cartography is the system of logic employed in the encoding procedure. One clearly definable segment of this discussion involves the problem arising from the lack of any clear and consistent relationship between, on the one hand, the terrestrial area of the data collection units which constitute the map base and, on the other, the size of the populations which exist within these units and of which a variety of characteristics may need to be mapped. Pannakoek (1968) made specific mention of encountering this problem in designing the National Atlas of the Netherlands.

A recent example of the significance of this problem is provided by a study of a federal election in Canada (Campbell & Knight, 1976). In a critical evaluation of the cartographic metholodogy used in that study, Coulson (1977) has argued that the highly irregular pattern of electoral districts resulted in maps which were "neither useful nor reliable". In judging the graphic image, he considered it inevitable that "the map pattern is controlled by relatively few large electoral districts" and extolled the virtues of dispersion histograms as a means of obtaining a "meaningful and valid" presentation of the electoral data. However, this latter method clearly precludes the opportunity to perceive the spatial relationships which were the original concern of Campbell and Knight. In these circumstances one may be tempted to suggest that a topological cartogram might have provided a satisfactory compromise, particularly when it is remembered that an 'isodemographic' map of Canada is available (Skoda & Robertson, 1972).

The use of the topological cartogram (or map) has varied across the continuum between facetiousness and sobriety, ranging from the whimsical portrayal of regional parochialism to exploratory research designs in spatial analysis.

Although none would claim that it provides a panacea for the problems faced by the thematic cartographer, it would be equally short-sighted and superficial to dismiss it as a technique of negligible significance. In fact, Tobler (1963) has argued convincingly that the topological cartogram represents an interesting and useful set of transformations, clearly deserving of discussion in any text on map projections.

Another matter of concern to the community of cartographers is the continuing trend for the increasing urbanization of mankind. Implicit within this trend is the certainty that future decision makers, whether public or private, are likely to manifest a growing demand for urban thematic maps. Some part of this interest is already evident in the form of the various SYMAP atlases which have appeared in recent years (e.g. Davis and Spearritt, 1974).

This paper will explore the potential value of the topological cartogram as a base map for the communication of the spatial variation of socio-economic data within urban areas.

The Topological Cartogram

Topological cartograms are essentially map projections which distort terrestrial areas in order to produce a constant density surface for some spatially distributed population. Some of the main characteristics of such transformations are illustrated in Figure 1. In this diagram, map A may be considered to represent some particular set of data collection units (DCU) in planimetric form. After deriving some data set, with quantities attributable to each of the DCU's, it is possible to adjust their boundaries so that the areas which they enclose become proportional to these quantities rather than to their areal extent. This may be done in an infinite number of ways, maps B and C representing two of that number.

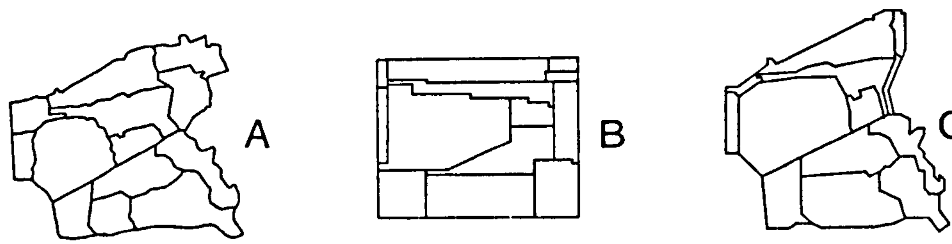

Figure 1 Planimetric Map (A) and Topological Transformations (B and C) of a Set of Data Collection Units.

Maps B and C provide evidence on the two basic principles of the topological cartogram. Firstly, that no such transformation has a unique solution, leaving the subjective choice in the hands of the individual map designer. Secondly, that the contiguity relationships between the constituent DCU's remain constant, so that maps A, B and C may be considered to be topologically identical.

Beyond these two fundamental principles there are a series of secondary properties which may be incorporated within the design of such topological cartograms. Two of these involve the concept of shape. The designer may wish the overall shape of the transformation to possess a strong likeness to that of the original planimetric map, as is the case for map C. Equally, there may be a desire to maintain as much as possible of the intricate detail of the DCU shapes, which is also a characteristic of map C. In contrast, the designer may wish to maximize the simplicity of the transformation, as exemplified in map B; a feature strongly evident in the rectangular statistical cartograms produced by Raisz (1934). Other properties such as the maintenance of directional accuracy or the attempt to influence and direct the map user's attention, may be combined with, or preferred to, those previously mentioned.

As noted above, an important difference between maps B and C in Figure 1 is that the latter, by its preservation of both internal and overall characteristics of shape, presents a 'more recognizable' image of the original. Tests involving relatively inexperienced map users have shown that this is an important factor for the acceptance of such transformations.

In 1974, a group of 112 first year students in the Department of Geography at the University of Adelaide were presented with a planimetric map (map A in Figure 1) and an array of seven topological cartograms, varying in the degree to which they retained the shape characteristics of the original and also in their linear complexity. They were asked to select the transformation which they considered the 'best', for the ease with which it could be related to the original. The least popular option, chosen by only 5% of the subjects, was that shown here as map B. In contrast, that presented as map C was the most popular transformation, selected by 34% of the subjects. It is assumed that the retention of internal and external shape characteristics was extremely influential in producing this result.

Subject preferences may also have been influenced by the higher degree of detail present in map C, which had exactly twice as many identifiable line segments as map B. However, this proposition was not supported by the remaining test results; the rank correlation between subject preferences and quantity of line detail proved to be positive, but non-significant at the .05 level of probability. In fact, the second most popular transformation was a heavily generalized version of map C, which ranked only fifth in its degree of linear complexity.

As an attempt to identify features of the topological cartogram which may be of communicative value for the map-user, these results may be considered encouraging. However, they must be treated with care. The author had been responsible for introducing the concept of topological transformations to the test subjects some ten weeks before testing, so that the problem of contamination of subject preferences cannot be ignored. Fortunately, these results are supported by the work of Dent (1972), who stresses the importance of the quality of shape and shows that linear complexity is increasingly superfluous beyond the point where 'sufficient' visual cues are provided.

The Urban Base Map

The rapid growth of the Australian state capitals in recent decades and the fragmented residential development of the rural-urban fringe have posed the continuing problem of accurately delimiting the metropolitan areas. One method of overcoming this problem when dealing with the study of temporal changes is to make comparative maps for the entire statistical division which encloses and is dominated by the urban area in question. However, this results in an 'overbounded' urban area (Haggett, Cliff & Frey, 1977), where the outer boundary includes substantial areas of rural land.

The Adelaide Statistical Division, as defined by the Australian Bureau of Statistics, provides a typical example of such an overbounded urban area. Figure 2a indicates that a considerable proportion of the division is not continuously developed, even though the urban influence may dominate in socio-economic terms. But, when any specific set of DCU's is considered in isolation, evidence of the extent of the continuously built-up area is lost. This is typified by the map of electoral subdivisions shown in Figure 2b. Consequently, although many of the peripheral DCU's may possess very small electoral populations, they may be expected to have a major impact upon the appearance of any thematic map of the whole division.

A map representing the net migration of electors (Figure 2c) exemplifies the problem of misinforming the map-user. The top quintile, represented by the heaviest shading, identifies the 20% of subdivisons with the highest migration rates. Yet the ten DCU's involved cover 40.7% of the total map area while accounting for only 13.8% of the total electoral population. Even greater confusion may be engendered by the differences that exist within this single class. The four units in the north-eastern block cover 10.2% of the map and represent 8.9% of the population, but the five units in the southern block cover 30.0% of the map area and account for only 4.5% of the population. Such variations, with a differential factor of the order of six, can hardly be considered as anything less than graphic distortions of an extremely serious

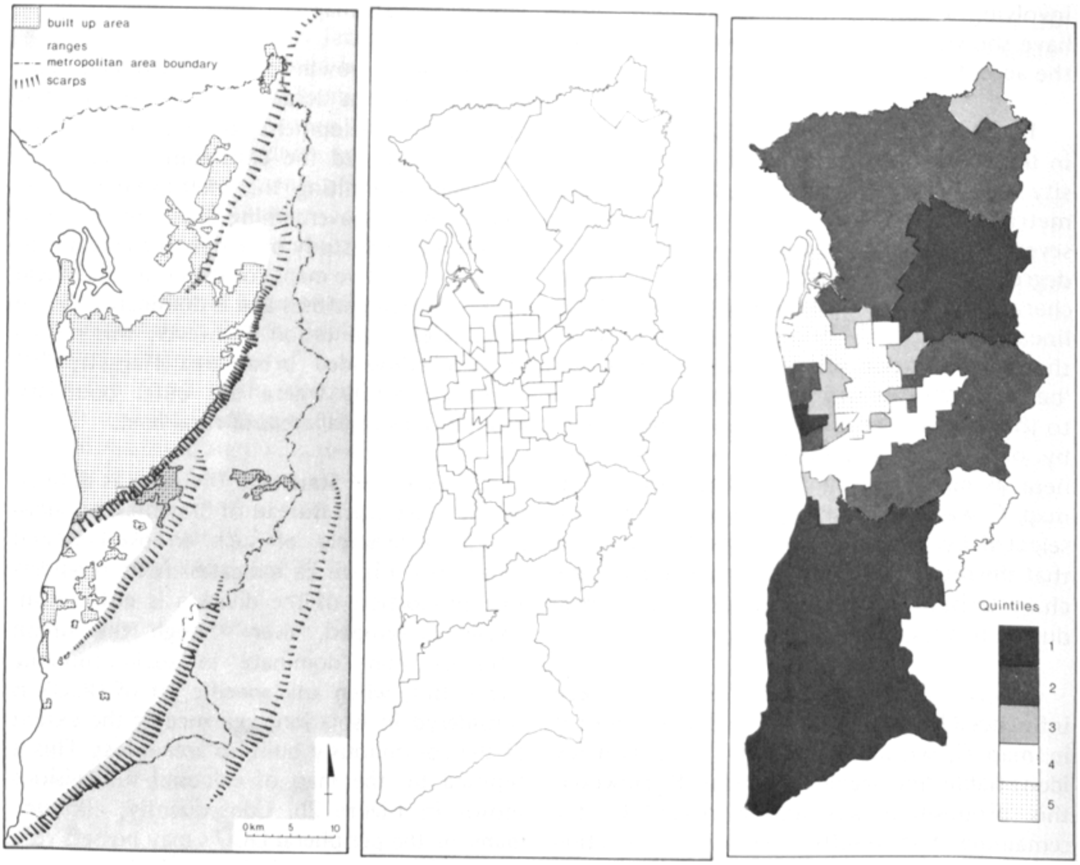

Figure 2 The Adelaide Statistical Division (a) General Location, (b) Electoral Subdivisions, (c) Net Migration of Electors, 1970-71 (data by permission, B.J. Ward, 1975).

nature. In fact, they are rather reminiscent of the areal distortions suffered on the Mercator projection when making comparison between regions in high and low latitudes!

At the most elementary level of comprehension, the map designer may fairly expect the map user to be able to identify the roughly concentric pattern of migration rates. But it would be excessively optimistic to expect even the knowledgeable map user to obtain a clear and unequivocal view of the relative importance of the migrational pattern displayed, whatever mental adjustments are attempted.

In situations such as this, which are far from rare, the planimetric distortions present in a topological cartogram may be no more, and possibly less, disturbing to the graphic message. As Monmonier (1977) has said: "map use cannot be effortless". This effort is now expended on the complex mental adjustments necessary to compensate for the radical variations in population density hidden behind the disarmingly simple facade of the conventional choropleth map, on the 'respectable' planimetric base. It might be more efficiently used in familiarization with the concept of the topological cartogram.

An Experimental Topological Cartogram

As a test of the feasibility of replacing the planimetric base map a topological cartogram was designed to represent the electoral population of the set of DCU's in Figure 2b. Bearing in mind the fundamental principles and the range of secondary properties noted previously, the following aims were set:

1) that the size of the electoral subdivisions should be made proportional to their respective electoral populations,
2) that the contiguity relationships between the DCU's should be maintained,
3) that true directions should be retained from the centre of the transformation,
4) that the main shape characteristics of the DCU's should be preserved, and
5) that the map should possess the overall shape of the built-up area.

Inevitably, some of these aims proved to be mutually supportive and others to be conflicting, but they are listed in the general order of importance conferred on them in the design process. The first two aims, considered as fundamental, were fulfilled precisely. The reasons for the last two have been discussed in some detail above. The third aim, implying the wish to produce an azimuthal transformation, requires some substantiation.

The anxiety to maintain the azimuthal quality of the original planimetric map stemmed from consideration of some of the standard generalizations about the form of the western city. Two of these features were influential in the design process. Firstly, that the central city area, with its Central Business District, is the economic and cultural node of the urban area. Thus, it provides the most suitable origin for any transformation. Secondly, that population density tends to decline with distance from the central city (Clark, 1951), although there is normally an area of low population density immediately adjacent to the central business district.

Both generalizations were appropriate in the Adelaide case, where the central business district is not far removed from the centre of the built-up area. Additionally, the population density of the 49 non-central DCU's proved to be inversely related to the logarithm of the distances between the central city and the DCU centroids. Although the scatter of values was quite wide, the Pearsonian correlation coefficient was a highly significant -0.82 (Figure 3). The negative exponential form in which this relationship is usually expressed has been avoided because the cartogram required a constant population density and because it possessed a lower correlation and involved two extremely skewed distributions.

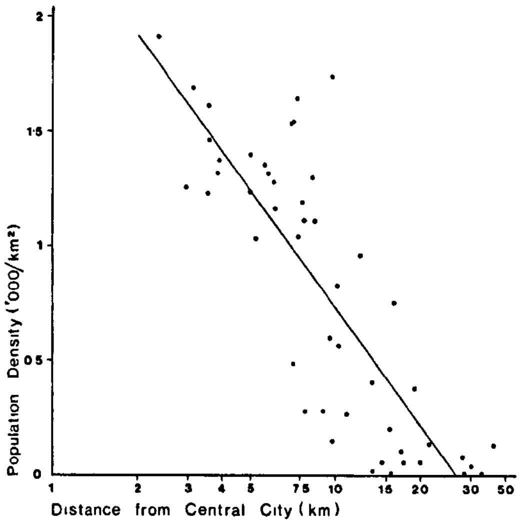

Figure 3 Relationship between Population Density and Distance from the City Centre, Adelaide, 1971.

The regression line shown in Figure 3 could have been used to identify the polar coordinates of an azimuthal projection. However, that would have resulted in the need to forfeit the first of the aims listed above, which was considered unjustifiable in this context.

The method employed in the construction of the topological cartogram was purely manual. An approach which has been commended by Tobler (1963) for cases involving sets of areal units, where an analytical solution may not be feasible. However, it should be noted that automation of this procedure would help to ensure a strict conformity to design aims and permit easy replication (Sen, 1976). But, although such an approach has much to commend it, the problems involved in the maintenance of the quality of shape are not easily overcome. Consequently, the method employed was similar to the graphic approach outlined in standard texts (Cole & King, 1968; Taylor, 1977), though care was taken to maximize the azimuthal and shape qualities. The cartogram, comprising fifty DCU's, took close to twenty hours to complete and is presented as Figure 4.

It fulfilled the first two aims exactly, but it proved impossible to achieve a similar level of precision with the other three aims.

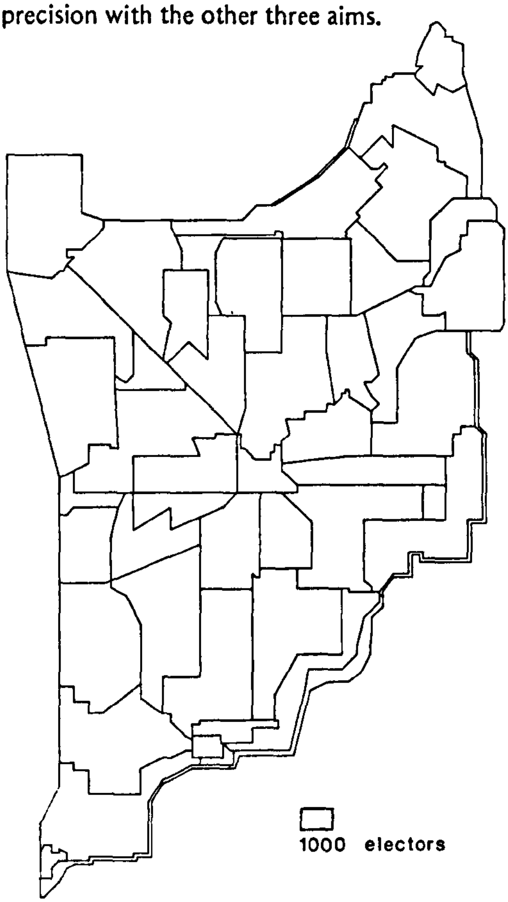

Figure 4 Topological Transformation of Adelaide's Electoral Subdivisions.

The internal shape characteristics which were retained were chosen intuitively. This involved the problem of subjective choice, but is an approach supported by Dent's (1972) research which indicated that test subjects showed a high degree of conformity in the selection of 'significant' identifiable boundary elements. Thus, a very large number of the easily recognizable line elements are reproduced in the transformation, although DCU's which suffered extreme diminution lost most of their shape character.

The general shape of the DCU's is also a matter of concern. Judgment of important changes in general shapes usually relies upon one or more of the simple shape indices which have been developed in recent years. As a means of measuring the changes in general DCU shape which had occurred during the transformation the Index of Compactness, proposed by Blair and Biss (1967), was employed. This index is derived by the application of infinitesimal calculus to the position of every element of area within the shape and is related to the radius of gyration in the field of mechanics. It has a value of 1.0 for a circle and would approach zero in the case of an area of infinite length and infinitesimal thickness.

The overall shape of the transformation was shown to be more compact than the original map, with the index rising from a value of 0.74 to one of 0.88. However, the constituent DCU's showed a general tendency to decline in compactness, with 34 lower and only 16 higher values. Whereas the range of values on the original was from 0.97 to 0.59, the range on the transformation was from 0.97 to 0.17. This may be attributed largely to the extreme elongation of some of the peripheral DCU's particularly those along the south-eastern edge. In fact, 70% of the DCU's altered their degree of compactness by less than 14% during the transformation process, which indicates a high level of preservation of general shape.

The retention of correct azimuth in the transformation is equally difficult to measure with any high degree of precision. The two characteristics which were measured for comparative purposes were the directions from the origin to the DCU centroids and the angles which the DCU's subtended at the origin.

In Figure 5, C represents the central DCU from an original planimetric map and C′ from a transformation, both positioned so that their centroids coincide. The region A represents another DCU on the original and A′ its image on the transformation. Following the calculation of the position of the centroids of A and A′, it is possible to establish the difference in direction from the combined centroid of C and C′. This angle (angle a) may then be used as a measure of the distortion in azimuth.

The largest error identified by this means,

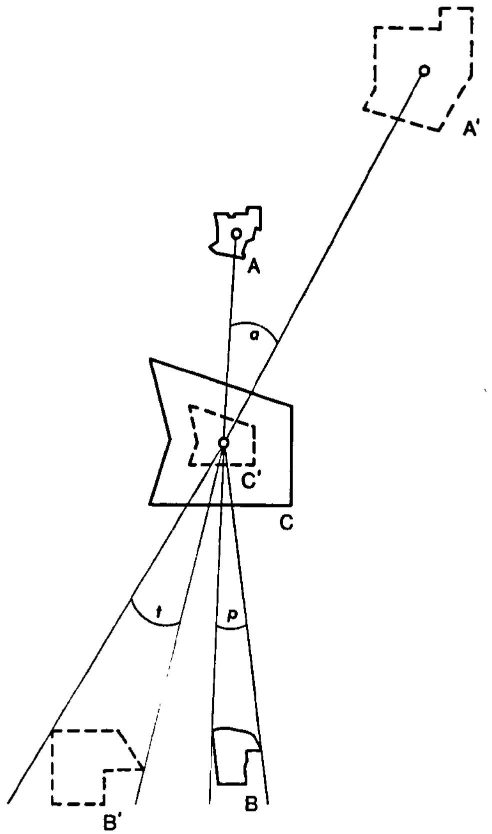

Figure 5 Schematic Portrayal of the Measures of Azimuthal Distortion in the Topological Transformation.

from one of the elongated peripheral DCU's, was 20°/30'. However, 43 of the 49 angles involved were of less than ten degrees, and the median error was 4° 33'.

The other measure of distortion is also exemplified in Figure 5, where regions B and B' are the original and image, respectively, of another DCU. Identification of the extremities of each, as viewed from the centroid of C and C', allows the measurement of the two angles (p and t) subtended at that centroid. The difference in these two angles may then be used as a measure of the distortion in azimuth.

The difference in the subtended angles reached a maximum of 38° 57' for one of the peripheral DCU's. However, only four of the differences exceeded fifteen degrees and the median difference was 6° 54'. Although these differences were relatively small in absolute terms, the proportional differences were somewhat greater: for 13 of the 49 original angles the changes exceeded thirty per cent.

Clearly, the retention of true azimuth from the origin of the transformation could not be controlled precisely within the limits of manual construction. This was partially due to the desire to retain the visual cues related to DCU shape, but also to the great changes in relative area that certain of the DCU's suffered. There were, for example, a number of cases where peripheral DCU's had to be elongated to maintain the contiguity relationships, but could not be stretched to the point where adjacent boundaries might fuse at reproduction scale.

The final user of any such topological cartogram may be interested not only in the directions between the constituent DCU's and the central city, but also in the spatial relationship between any of the DCU's. Although this transformation did not, and could not, try to maintain true direction throughout, it is interesting to view the amount of angular distortion which it possesses. A matrix of the angular errors between DCU centroids was calculated, taking into account the direction of difference. This has been collapsed into a histogram in Figure 6. The 1225 errors form a leptokurtic symmetrical distribution with a mean value of 3° 19' and a standard deviation of 11° 41'. This implies a slight clockwise rotation of the DCU set. Also, with one exception, all errors in excess of thirty degrees are related to peripheral DCU's with very small electoral populations.

A clear graphic picture of the deformation arising from the transformation may be obtained by relating both the planimetric and topological maps to a simple square graticule. The distortion suffered by these graticules in the transformation process is clearly exhibited in Figure 7.

Concluding Discussion

The experimental topological cartogram shown here certainly possesses its measure of faults. Even in the context of the stated aims it could not be considered to provide the 'best'

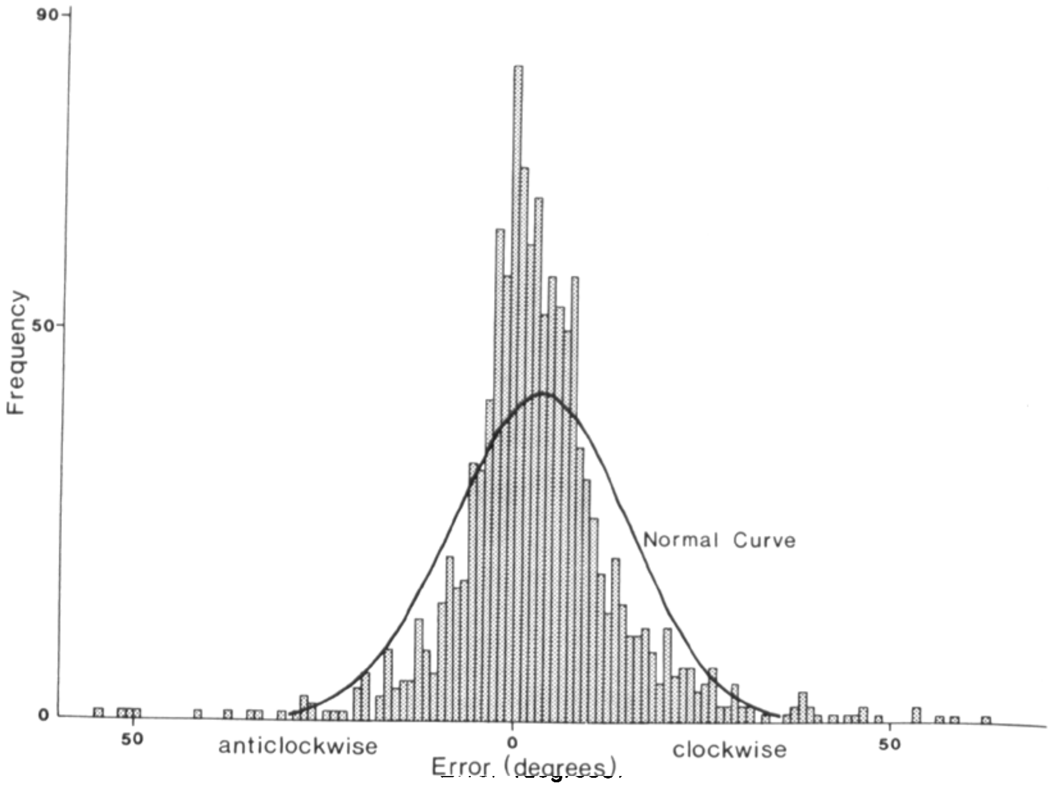

Figure 6 Histogram of the Angular Errors between all the Data Collection Unit Centroids on the Topological Map.

solution. The programming of a mathematical solution would, of course, have been more time-consuming. However, automation of the procedure does promise to solve many of the constructional problems inherent in the manual method (Kadmon & Shlomi, 1978). The danger will be that the novelty of an automated approach may lead to intemperate haste in its utilization, whereby both the merits and weaknesses of topological transformations may be submerged in the deluge of products.

A number of limitations related to graphic validity and to communicative ability may be isolated (Griffin, 1975). These may be defined as:

1) The repugnance that some map-users may feel toward the use of patently 'inaccurate' base maps in the encoding process; a valid concern arising from the concept of scientific integrity (Wright, 1942).
2) The danger that the map-user may be confused by the fundamental logic employed in the construction of any specific transformation unless the selected cartogrammetric properties are clearly stipulated.
3) The inevitable loss of the shape characteristics of some of the DCU components, which may nullify the attempts of the map-user to identify specific locations.
4) The frustration which the map-user may suffer in trying to relate the topological base and the information which it carries to the personal cognitive map of the area under consideration.
5) The danger that the first four limitations may eradicate the potential value of this graphic form as a communicative device.

Careful design may surmount these obstacles, as exemplified by the topological network display of the London Underground system. However, the inexperienced map-user is bound to face

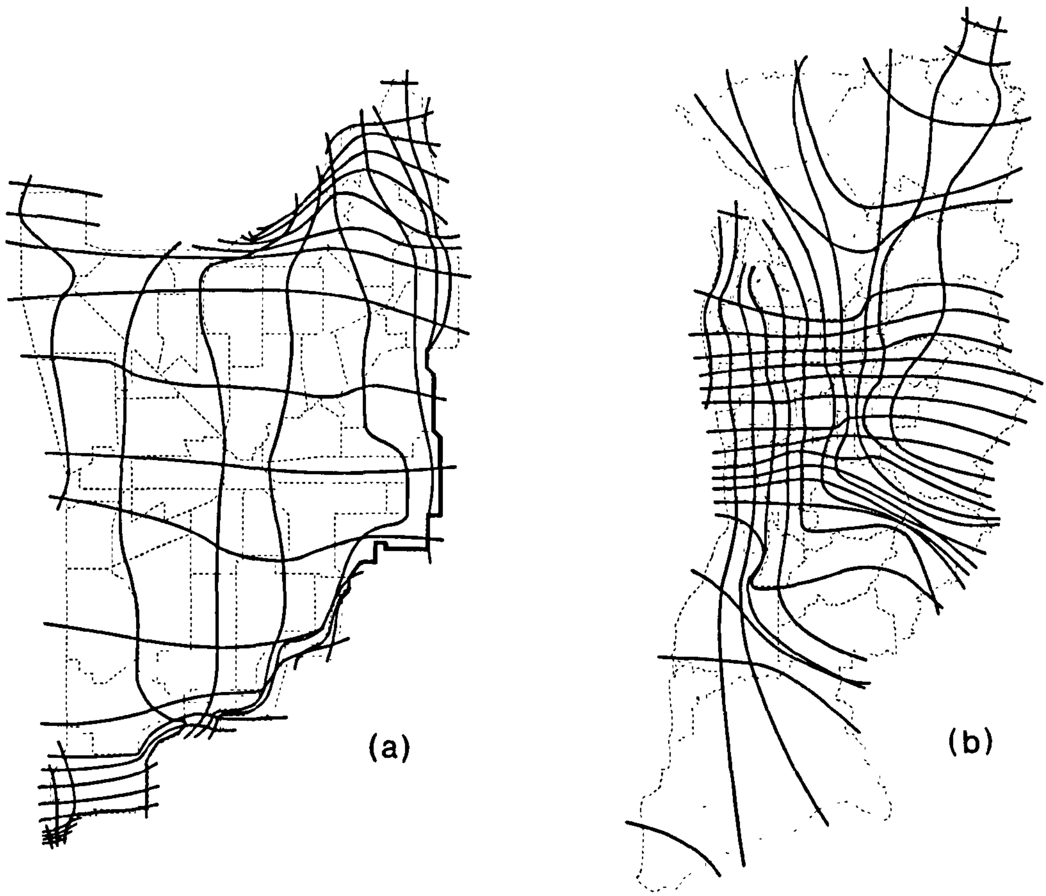

Figure 7 Deformation of a Regular Graticule by the Transformation Process: (a) Planimetric to Topological, (b) Topological to Planimetric.

some difficulties in comprehension. Some will be repulsed and others attracted by a map form which is innovative, unconventional in style and heavily generalized. Dent (1975) used the semantic differential technique to test undergraduate attitudes to a cartogram and found that its appearance gained a generally positive response, though doubt was thrown on its readability. The precise testing of functional utility awaits further research.

On the positive side, opportunities exist for the topological cartogram to be used to advantage. These may be considered in three categories:

1) As a device to shock the map-user by the graphic display of some unsuspected spatial peculiarities.
2) To promote clarity in a map which might otherwise be cluttered with detail of minor significance.
3) In the portrayal of statistical surfaces where interpretation would be obscured by the pervasive influence of terrestrial area.

It is the third category which concerns us here.

Map-users have realized that there is no perfect projection of the spherical Earth on to a plane surface and have learned to accept the areal distortion of the Mercator and the shape deformation of the Mollweide. It should be possible, therefore, for the map-user to become accustomed to the primary and selected secondary properties of a topological transformation. In the case of the experimental cartogram discussed above, we may consider the nature of the transformation by way of the standard projection characteristics of equivalence, conformality and azimuth.

Any map on an equivalent projection will retain the same relationship between its constitutent areas as exists for those areas on the surface of the sphere from which it was derived. The equivalence on a topological cartogram maintains such areal relationships as exist in the statistical population which it represents. Thus, this form of equivalence is one of the two fundamental principles of this type of map.

On the normal map projection the scale requirements for conformality and for equivalence are contradictory (Robinson, Sale & Morrison, 1978). For the topological map the same is invariably true. Total conformality is impossible, but the preservation of shape characteristics is a commendable aim, for the reasons mentioned above.

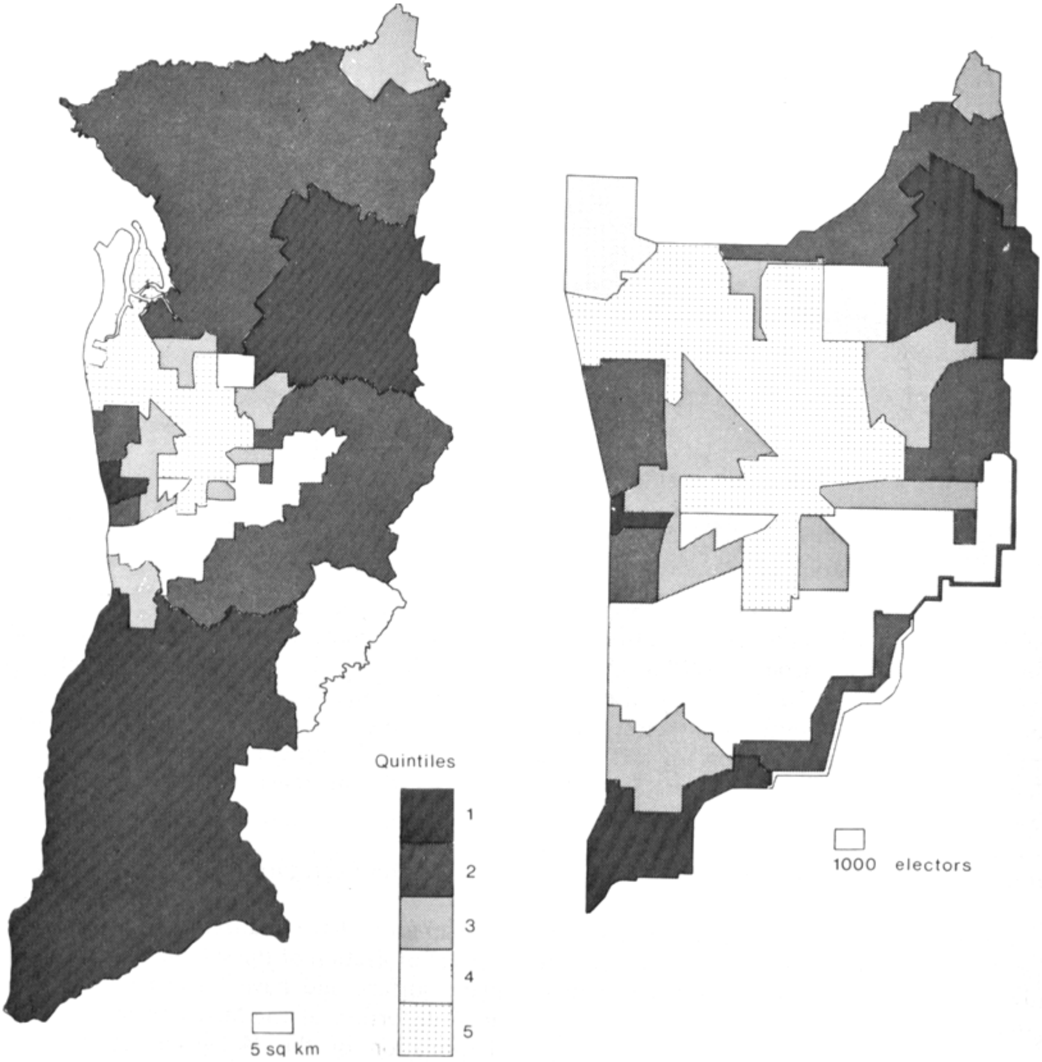

Figure 8 Planimetric and Topological Representation of the Net Migration of Electors in Adelaide, 1970-71 (data by permission, B.J. Ward, 1975).

True directions are normally maintained from only one point on an azimuthal projection. In the case of this topological cartogram the central city was chosen as that point and the design aimed at maintaining accurate directions to the remaining DCU's.

It may seem that one is straining the bounds of credibility by considering the properties of equivalence, conformality and azimuthality in the context of the topological cartogram. But there is value in doing so if it concentrates the cartographer's attention on the concise definition of cartogram properties and results in a better understanding of its progenitors.

The topological cartogram, once a rarity, has come into increasing use over the last two decades (Dent, 1975). The suggestion made here is that it may come into common use in thematic cartography for purposes other than its initial shock value. The azimuthal form shown here, if a relatively high degree of conformality can be maintained, may provide a very useful base map for the representation of data relating to urban populations. The communicative value of such maps must remain suspect until further research is undertaken, but the weakness in the logical basis of the planimetric alternative indicates that it should be considered. Figure 8 compares the planimetric and topological versions of net migration for Adelaide's electoral subdivisions. It might well be argued that the latter provides a clearer statement of the relative importance of the variations in the statistical surface, devoid of the graphic distortion imposed by terrestrial area.

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