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NONCONTIGUOUS AREA CARTOGRAMS

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A CARTOGRAM is a map on which size is proportional to some variable other than actual earth size. If area is the size characteristic representing some distribution, such as population, we may refer to the cartogram as an area cartogram. If distance is the characteristic representing some phenomenon, such as travel time, we may refer to it as a distance cartogram *(1)*. Cartograms are usually visually striking and intellectually interesting, at least to those who are familiar with the ordinary map of the area. They are also an efficient mapping method since there is no loss of information through categorization of values represented. They provide, too, a method for depicting two logically related distributions on one map, with size representing one variable and superimposed patterns representing the second. Carolan, for example, mapped presidential victors by state on a base proportional to electoral votes and crime rates on a population base *(2)*.

Area cartograms may be of two types: contiguous and noncontiguous. On the contiguous version, shapes of units are distorted but observation units remain contiguous to the same units with which they share a border on an ordinary map *(3)*. On a noncontiguous cartogram, observation units stay in more or less their correct location and maintain correct shape, but at least some units are separated by gaps from their neighbors. An example of such a cartogram is shown in Figure 1; the areas of the units (states in this case) are proportional to the number of people sixty-five years of age and older *(4)*. The purpose of this paper is to present a coherent set of steps for constructing the noncontiguous area cartogram and to discuss some of the interesting properties, or characteristics, of this type of cartogram that may affect the decision to use it or not.

Mechanics of Construction. The construction of the noncontiguous cartogram involves little more than increasing or decreasing the relative sizes of observation units, the amounts depending on the original areas of the units and the values on the variable being mapped (such as population over 65). Since the terms "area," "unit," and "value" can be confusing in a discussion of cartograms, their definitions as used here are as follows: the term "area" refers to measured area such as square miles or square inches, whereas the term "unit" (or "observation unit") refers to an entity for which data are gathered. For example, Maryland is an observation unit with an area of 10,577 square miles and with approximately twice as much area as the observation unit Connecticut with an area of 5009 square miles. On the cartogram of population sixty-five years and over (Figure 1), Maryland has only slightly more area than Connecticut because it has only a slightly greater number of people who are sixty-five and over. The term "value" will refer to the value of an observation unit on the variable being represented; the value (persons sixty-five and over) for Maryland is 299,682 and for Connecticut is 288,908.

Projector Method. The raw data needed for a cartogram include: the areas and the values for the observation units (Table 1, columns 2 and 3). The desirable pieces of equipment are a Saltzman or similar projector and a calculator with a square root function and a capacity to store and recall a constant. An equal-area map of the region serves as a base map and is marked with the center of area of each observation unit (Figure 2). If the projector does not have a clearly marked scale indicating amount of enlargement or reduction, then a scale of arbitrary length can be included for scaling purposes. The worksheet on which compilation will be performed should have the outline of the entire area, the center points of units, and the scale line that is on the base map but extended to several times its length and divided into tenths (Figure 3).

The next step is to find the linear scaling numbers. These numbers are the relative linear scales to which the individual observation units must be adjusted relative

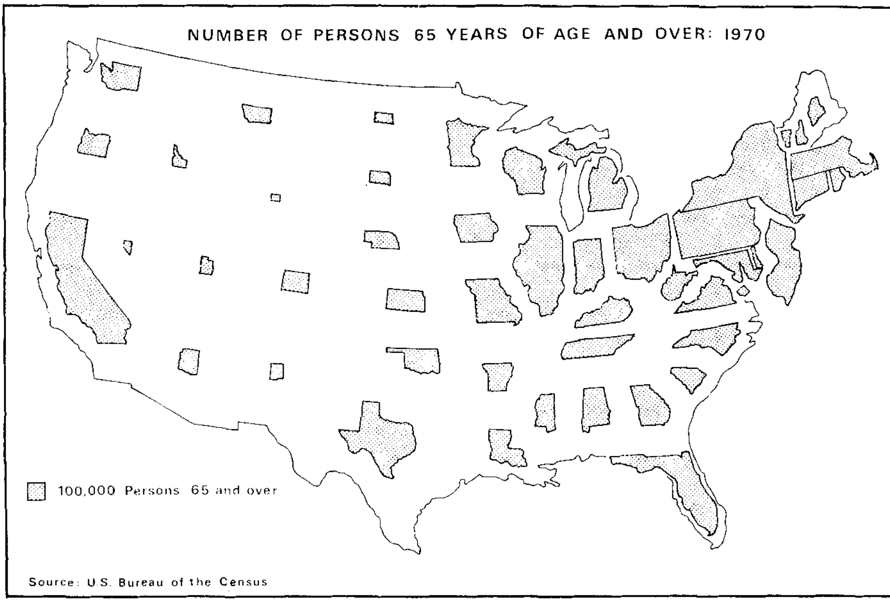


Figure 1. A noncontiguous area cartogram. The area of each state represents the number of persons sixty-five and older in that state. This cartogram was constructed with the use of a projector.

TABLE 1

THE ARRANGEMENT OF INFORMATION FOR CALCULATING THE LINEAR SCALING NUMBERS (LAST COLUMN)

(1) Unit	(2) Value	(3) Area	(4) $\sqrt{\text{Density}}$	(5) $k \cdot \sqrt{\text{density}}$
Alabama	325,961	51,609	2.513	.34
Arizona	161,474	113,909	1.191	.19
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
Washington, D.C.	70,803	67	32.508	5.17
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
New York	1,960,752	49,576	6.289	1.00
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

In this case, New York was chosen as the anchor unit; hence $k = 1/6.289 = .1590$ and the linear scaling number for New York is 1.00. Note that Washington, D.C. will be enlarged while Alabama and Arizona will be reduced.

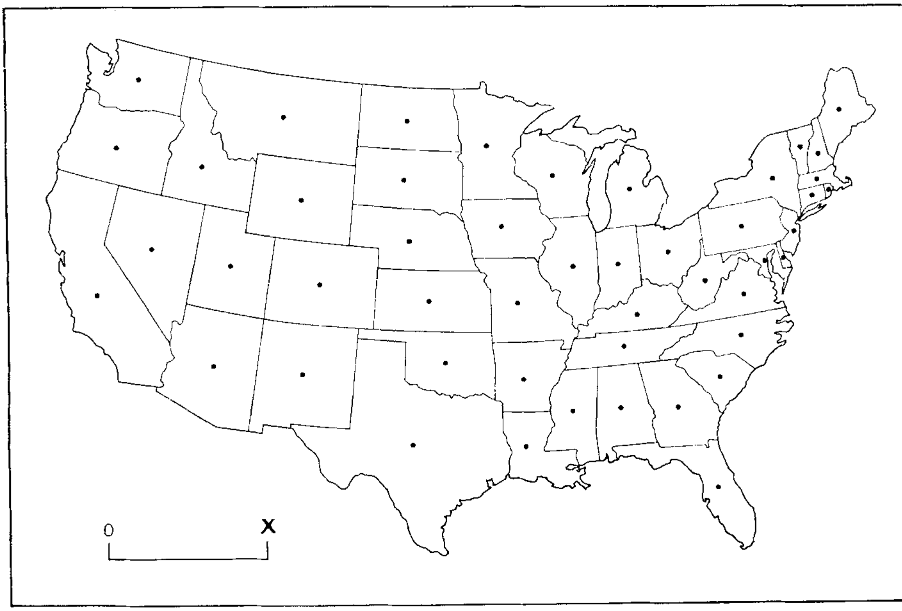


Figure 2. The base map (much reduced) used to construct the cartogram in Figure 1. The centers of states are marked and a scale of arbitrary length is included.

to the scale of the original map so that their new areas will be proportional to their values. In other words, if the linear scaling number for Maryland is .85, the projector will be set to that fraction of the original scale and its outline traced. If the projector is then set for Connecticut's linear scaling number of 1.21, Connecticut will be at its correct relative size and its area should be slightly smaller than Maryland because of its slightly lower population of persons sixty-five and over.

To find these linear scaling numbers, one observation unit from the region being mapped is usually chosen as the "anchor unit" and will remain the same size on the cartogram as it is on the base map. To avoid overlapping of areas on the cartogram, this observation unit must be chosen as the one with the highest density (population over sixty-five divided by area in the case of Figure 1). In practice, it is not always feasible to use the highest-density unit because it may cause many other units to become virtually invisible. Generally, however, a unit which is at least fairly high in density can be used. In any case, the relative linear scales for each observation unit will be $\sqrt{V_i/A_i} \cdot k$, where V_i is the value for the unit, A_i is the unit's area, and $k = 1/d$, where d is the square root of the density of the anchor unit. If the anchor unit has the highest density, then the scaling number for that observation unit is 1.00 and for all other observation units it is less than 1.00. Because it is desirable to find some suitable high-density unit to use as the anchor, the task of finding the scaling numbers may be broken down as follows (see Table 1):

(a) Find the square root of the density for each observation unit ($\sqrt{\text{value/area}}$).

(b) Examine the values to find several of the highest ones. If the highest densities occur in very small observation units, it is generally best to use the highest-density unit of reasonable original size as the anchor. The small observation units with higher densities will then be enlarged rather than reduced. On the map of population over sixty-five, several small observation units (Washington, D.C., New Jersey,

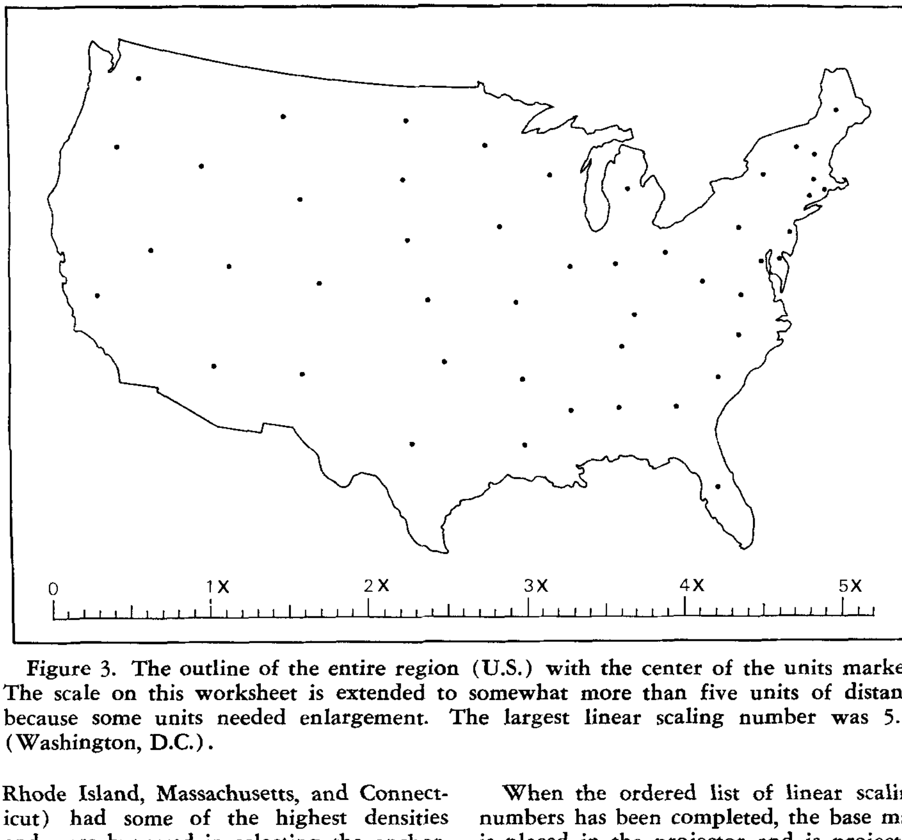


Figure 3. The outline of the entire region (U.S.) with the center of the units marked. The scale on this worksheet is extended to somewhat more than five units of distance because some units needed enlargement. The largest linear scaling number was 5.17 (Washington, D.C.).

Rhode Island, Massachusetts, and Connecticut) had some of the highest densities and were bypassed in selecting the anchor. The enlargement of the five bypassed areas did not create great problems because they were on the edge of the map. New York, with the highest density for a state with a large original area, was chosen as the anchor and remained the same size on the cartogram as on the base map.

(c) Once the anchor is chosen, calculate the constant $k = 1/d$, where d is the square root of the density of the anchor unit. Then multiply the numbers calculated in *a* above by this value k and the resulting values are the linear scaling numbers ($k \cdot \sqrt{\text{density}}$).

(d) Rearrange the listing in order of magnitude of the linear scaling numbers for convenience in setting the projector (Table 2). If at this stage the numbers are still not feasible, multiply each by a constant to produce a more suitable scale and, in effect, to change the anchor unit.

When the ordered list of linear scaling numbers has been completed, the base map is placed in the projector and is projected to the desired relative linear scale for each unit. Each unit's outline is traced in its proper position, displacing from the original center only those which must be enlarged. After finishing the projector work, the observation units may be retraced onto another sheet if the arrangement needs condensing or improvement. In the case of Figure 1, states were moved closer together and surrounded by a smaller-scale United States outline. The cartogram was then ready for final design touches and drafting.

The time involved in developing a cartogram depends upon the type of calculator and projector but is generally not much longer than that required for other commonly used thematic mapping methods. The method of construction is relatively easy to understand, and the linear-scaling formula can be explained by an example: assume that unit H, with area A_H and value V_H, is chosen as the anchor unit for the region being mapped. Unit J, with area A_J and value V_J, has four times as much geographic area but ¾ as many people sixty-five and over. The area of J then has to be reduced to one fourth (A_H/A_J) of its original size just to make it the same area as H, and then to ¾ of that size (V_J/V_H) to bring it to the correct proportion relative to number of persons sixty-five and over. Thus the overall reduction of J's area is

$$(A_H/A_J) \cdot (V_J/V_H) = 1/4 \cdot 3/4 = 3/16.$$

The square root of this value,

$$\sqrt{(A_H/A_J) \cdot (V_J/V_H)} = \sqrt{3/16} = .43,$$

is the linear scale to which the observation unit J must be reduced relative to H. Since A_H and V_H are constant for a given cartogram, this relative linear scale, or linear reduction number, can be rewritten $k \cdot \sqrt{V_J/A_J}$, where $k = \sqrt{A_H/V_H}$ or $1/d$ where d is the square root of the density of the anchor unit.

Computer Method. The same basic formula can be used to produce noncontiguous cartograms by computer and plotter. If a choropleth mapping program is available that plots units from digitized boundary points, some relatively minor modifications will produce a cartogram. The raw data required, in addition to the digitized boundaries of the units, are again the areas of the units and the values to be depicted by the transformed areas. Also, a computer cartogram requires a digitized reference point for each unit such as the center of the area or provision in the program for the calculation of such a point. In the program, the distances of the X-Y coordinates of the boundary points from this center point are adjusted to produce units varying in size according to the distribution being represented. The adjustment is proportional to the linear scaling number for each unit. For each boundary point, then, the following FORTRAN statements will properly transform the X and Y coordinates:

```
XNEW(I) =
   L(J)*(X(I)-CX(J)) + CX(J)
YNEW(I) =
   L(J)*(Y(I)-CY(J)) + CY(J)
```

TABLE 2
THE LINEAR SCALING NUMBERS IN RANK ORDER

Washington, D.C.	5.17
New Jersey	1.50
Rhode Island	1.47
Massachusetts	1.40
Connecticut	1.21
New York	1.00
Maryland	.85
Pennsylvania	.84
Ohio	.78
.	.
.	.
.	.

where $XNEW(I)$ and $YNEW(I)$ are the coordinates of the point i of unit j on the cartogram, $L(J)$ is the linear scaling number for unit j and $CX(J)$ and $CY(J)$ are the coordinates of the center point of unit j. To show that the resulting area is that desired, let us omit the subscript J and denote the coordinates of any boundary point on unit j as X_i, Y_i. The original area of each unit can be found using the formula

$$\text{Area} = \tfrac{1}{2} \cdot \sum_{i=1}^{n} X_i \cdot Y_{i+1} - X_{i+1} \cdot Y_i,$$

where n is the number of boundary points for the unit and X_{i+1} and Y_{i+1} are the coordinates of the starting point (X_1, Y_1) when $i = n$. The area of the adjusted unit would be

$$\begin{aligned}\text{Area} = \tfrac{1}{2} \cdot \sum_{i=1}^{n} & (L(X_i - CX) + CX) \\ & \cdot (L(Y_{i+1} - CY) + CY) \\ & - (L(X_{i+1} - CX) + CX) \\ & \cdot (L(Y_i - CY) + CY) \\ = \tfrac{1}{2} \cdot L^2 \cdot & \sum_{i+1}^{n} X_i \cdot Y_{i+1} - X_{i+1} \cdot Y_i,\end{aligned}$$

where L is the linear scaling number and CX and CY are the coordinates of the center point. Since L is expressed relative to linear scale, its square is equal to the correction for area and the resulting unit then has the proper relative area.

In producing cartograms by computer it is particularly important to avoid overlap by choosing the highest density unit as the

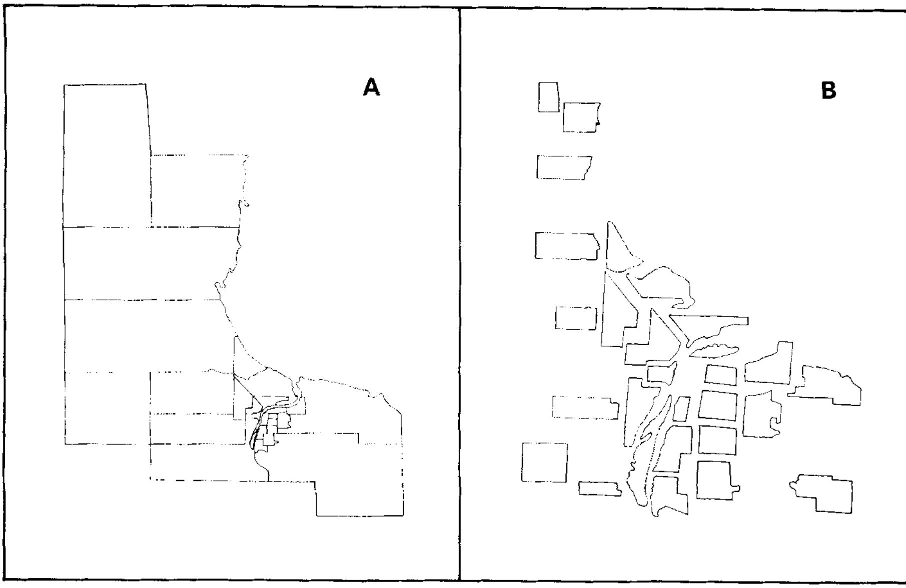


Figure 4. Computer-produced ordinary base (A) and population cartogram (B) of census tracts in Bay City, Michigan (traced from computer output and reduced).

anchor. If the sizes of units become miniscule, the program can simply be rerun with a larger scale factor. Again the units may be retraced for improvement of arrangement.

The map and cartogram illustrated in Figure 4 have been produced with the aid of a modified version of Tobler's program that produces choropleth maps without class intervals *(5)*. In Figure 4, A is a tracing of the choropleth map base of census tracts in Bay City, Michigan, and B is the population cartogram base, i.e., a base on which observation units (census tracts) are proportional to their populations. On the actual computer output a distribution represented by a series of line patterns was superimposed on each base; these superimposed patterns were omitted here for simplification. In tracing the cartogram, the spacing between units was altered to reduce the large gaps between them. The modifications made to the program to obtain cartograms such as that in B of Figure 4 included: a loop that finds the unit with the highest density for use as the anchor; the addition of the cards that transform the boundary point coordinates; provision for reading cartogram values in addition to the values of the superimposed distribution ordinarily required; provision for reading the center points, which had previously been calculated and stored for this particular data set; and an increase in storage space to accommmodate the changes *(6)*.

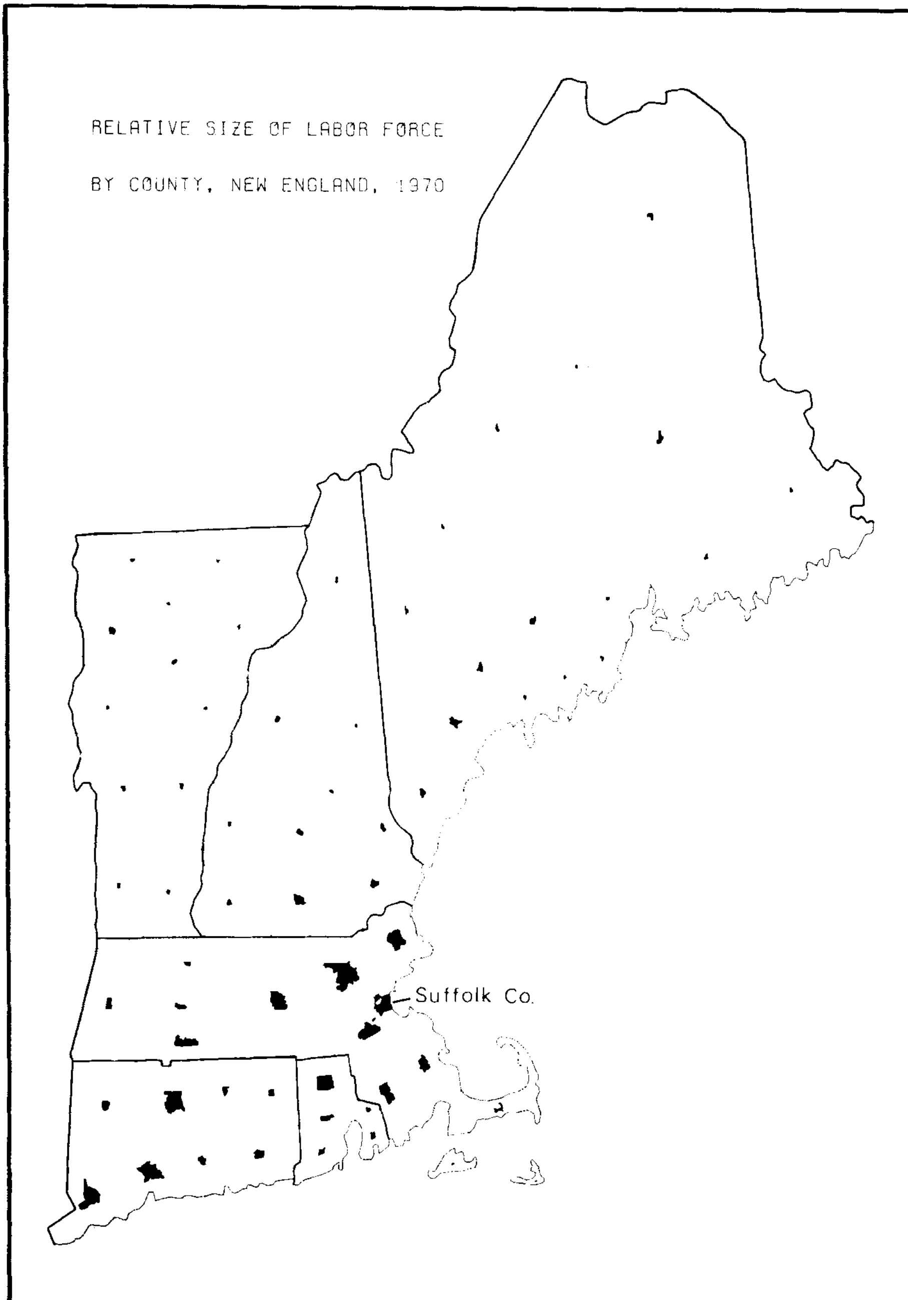
The resulting cartogram eliminates the visual dominance of the large outer tracts and should give a better overall impression of distributions that are related to population totals rather than to land area. Because of the lack of familiarity with Bay City on the part of most of us, such a cartogram is probably meaningful only when seen in conjunction with a map showing the ordinary base.

Variations. The steps outlined here for constructing a cartogram can be varied in many ways to alleviate various problems that may arise. Projector cartograms produced by Carolan, for example, use the unit of median density as the anchor unit so that extreme reductions and enlargements are avoided. When this variation is employed, the initial positioning of units cannot be based on unit centers since half the units are enlarged and many would overlap. Instead, positioning is at the discretion of the maker throughout the process. This may involve fitting problems but it does eliminate the need for constructing the work map with the original centers of units, searching for the appropriate center while using the projector, and then later retracing to form a more compact representation. It also results in a more predictable overall size of the resulting map. Carolan also maintains certain critical lines such as the northern boundary of the United States. This results in a pattern of units that looks reasonably similar to the ordinary map and preserves some very basic visual clues to the identity of the region even when the actual outline does not surround the units. He maintains an approximately uniform and minimum distance between units which helps the cartogram to resemble more closely the original arrangement and allows the maximum size of units within a given format *(7)*.

USEFUL CHARACTERISTICS OF NONCONTIGUOUS CARTOGRAMS. The steps for constructing noncontiguous cartograms are relatively straightforward and as such are one of the advantages of this type of mapping method. Simplicity of construction should not be the sole criterion for using or not using a particular method, however, and there are at least two other sets of criteria by which we might judge relative utility. The first is the nature or inherent properties of the device itself, and the second is the ease and accuracy with which map users are able to extract information from it. A thorough treatment of the latter would necessarily involve the testing of human subjects and such testing is not included here. Rather, discussion is limited to the former set of criteria, the characteristics of the device itself.

Probably one of the most interesting aspects of the noncontiguous cartogram is that the empty area between units is meaningful. If the highest-density unit is used as the anchor, then the empty areas reflect the degree of discrepancy between the density in the most-crowded unit and the density in other units. The effect can be quite dramatic. Figure 5, for example, shows a cartogram representation of the labor force in New England superimposed on an ordinary base map. Only relatively small (but densely populated) Suffolk County, Massachusetts, remains at its original size, and the degree of relative shrinkage of most other counties is extreme. Although the fact that each cartogram unit represents an even density within is also an interesting property, it is the degree of difference from the original map that is the real message of a cartogram, and generally a major reason for constructing it. Such a representation as that in Figure 5 does not rely solely on the reader's mental recollection of the original map for comparison but includes the important empty area as well. A cartogram using the unit with the mean or median density as the anchor would also show relative discrepancies, but it is more difficult to produce by computer because many units increase in size and overlap.

Another inherent property of such a map is that the representation is of the discrete units for which information is available. Thus, like a choropleth map, nothing about the distribution within these observation units is implied and any manipulation of values, such as aggregation to *n* equal-valued subregions of the overall area, would be approximated by aggregating discrete neighboring units. Contiguous cartograms have been treated as devices of known density within as well as between units when procedures such as superimposition of latitude and longitude or division into *n* equal-valued areas with boundaries passing through observation units are carried out *(8)*. Such procedures are not strictly valid; whereas the generalizing assumption involved is not much different from that underlying the interpolation of isarithmic maps, strictly speaking the only meaningful enclosing lines on any cartogram (and the only lines which can be accurately relocated on the original base) are the boundaries of the observation units from which the cartogram was constructed. This property would be very minor if the observation units were very small relative to the total area of the map (and each contained a very small portion of the total distribution being represented in cartogram form), but generally a cartogram is constructed using units that are


RELATIVE SIZE OF LABOR FORCE
BY COUNTY, NEW ENGLAND, 1970
Suffolk Co.

fairly large and generally recognized, such as states on a United States map or counties over a limited area. The success of the visual representation (contiguous or noncontiguous) depends, in fact, on the readers' recognition of the units shown and such recognizable units are generally fairly large for the area represented. As a result, manipulations which make assumptions about distributions within units are of questionable validity and the maintenance of the discreteness of units on the noncontiguous cartogram discourages such assumptions.

The discreteness of units might also be of pragmatic importance in aggregating into equal-valued subareas in that sometimes the division of the existing units would be undesirable. A planner setting up aggregations of area containing a certain total population for administration of a government program, for example, may well wish to avoid the division of existing administrative units, such as counties. Again, the inherent discreteness of the noncontiguous cartogram may be conceptually advantageous.

The idea of recognition of observation units brings up another point about cartograms. In comparing the nature of contiguous and noncontiguous versions, one of the important differences is that although both are manipulations of scale within the map, the contiguous cartogram necessarily involves distortion of the shapes of observation units while noncontiguous versions do not. This is of utmost importance if it is agreed that recognition of the observation units by the map user is important. It is well known that constant shapes are easily recognized regardless of scale; we would be quite surprised, for instance, to find someone who could recognize the states on a wall map but could not recognize them on a page-size map. Distortion of shape on the contiguous cartogram presents an additional hurdle to visual recognition and this hurdle is not only eliminated on the noncontiguous cartogram but is replaced by the meaningful empty-space property.

It will be noted that in discussing the properties of the noncontiguous cartogram, the device has been compared in each instance, implicitly or explicitly, with the contiguous cartogram. The noncontiguous version does not, of course, possess the mathematical elegance of the contiguous variety and does not possess its inherent compactness of form. There are undoubtedly appropriate and inappropriate uses of each, the contiguous most important perhaps for its stimulation of thought about the general nature of scale changes on maps. From a pragmatic point of view, and with visual communication as the *raison d'etre*, we will probably do at least as well to consider the noncontiguous variety.

FINAL COMMENTS. The noncontiguous area cartogram is relatively simple to construct and the instructions here provide a systematic approach to the steps involved. Cartograms produced by projector or by computer use a linear scaling value to adjust the scale of each unit. In the projector method, the machine is set at the linear scaling number and the unit is traced. In the computer method, the scaling number is applied to the distances of boundary points from the center of the unit before the map is plotted.

The inherent properties of the noncontiguous cartogram which make it a potentially useful device include: (a) that the empty areas, or gaps, between observation units are meaningful representations of discrepancies of values, these discrepancies generally being a major reason for constructing a cartogram; (b) the representation and manipulations involve only the discrete units for which information is available and only the lines which can be accurately relocated on the original map appear on the noncontiguous cartogram; (c) because only sizes of units change, not their shapes, recognition of the units represented is relatively uncomplicated for the reader.

A thorough evaluation of the usefulness to the reader of noncontiguous cartograms

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Figure 5. Cartogram representation of the size of labor force, superimposed on the ordinary base map of New England. This figure was plotted using the same computer program as used for Figure 4; center points were calculated within the program.

has not been evaluated. Such an evaluation, however, demands that such products be easily obtainable. It is hoped that the description of the projector and computer methods of construction and the discussion of the properties of the device will stimulate further study of the noncontiguous cartogram as a mapping technique.

* * *

(1) Examples of both area and distance cartograms can be found in Arthur Lockwood, *Diagrams* (New York: Watson-Guptill, 1969).

(2) A number of cartograms have been constructed by William Carolan (formerly of Clark University). Some of them have appeared in publications including: "Population," *Encyclopedia Americana Yearbook*, 1971, p. 549; Population Reference Bureau, *Annual Report 1968*, pp. 8–9; Gwyn Rowley, "Landslide by Cartogram," *Geographical Magazine*, Vol. 45 (1973), p. 344.

(3) See for example: John M. Hunter and Johnathan C. Young, "A Technique for the Construction of Quantitative Cartograms by Physical Accretion Models," *The Professional Geographer*, Vol. 20 (1968), pp. 402–407. Computer programs to produce contiguous cartograms are listed and discussed briefly in: University of Michigan Cartographic Laboratory, "Cartogram Programs," Report No. 3, Department of Geography, University of Michigan, Ann Arbor (no date).

(4) Carolan's cartograms, reference 2 above, are of the contiguous type.

(5) Waldo R. Tobler, "Choropleth Maps without Class Intervals?" *Geographical Analysis*, Vol. 3 (1973), pp. 262–265.

(6) The example shown in Figure 4 was produced at the U.S. Bureau of the Census. My thanks to Ed Adami of the Geography Division for supplying the computer output.

(7) William Carolan, personal correspondence.

(8) Waldo R. Tobler, "A Continuous Transformation Useful for Districting," *Annals*, New York Academy of Sciences, Vol. 219 (1973), pp. 215–220.

The University of Alberta's Department of Geography has issued a monograph about changing land use and physical change in Edmonton, Alberta, which could be most useful to geographers interested in the evolution of urban landscapes. Entitled *Neighbourhoods in Transition: Processes of Land Use and Physical Change in Edmonton's Residential Areas*, the author, L. D. McCann, attempts to answer such questions as, what is happening to the housing stock of the city? What is the spatial pattern of conversion and redevelopment? What is the evolutionary sequence of neighborhood change? The book may be purchased by writing to the Department of Geography, University of Alberta, Edmonton, Alberta, Canada. Its cost is $2.50.

A symposium was held at Villanova University in March, 1976 on the interaction of science, citizens and government in the resolution of regional problems. The Department of Geography, sponsor of the meeting, has issued a monograph in which three of the papers are published: "Science of, by and for the Urban Region," by James P. Latham, "Inter-Governmental Decision-Making in Urban Metropolitan Regions," by Arthur F. Loeben, and "The Citizen's Role: Is It Needed? Where Does It Fit In? How Can It Be Most Effective?" by Richard A. Walter. The monograph, entitled *Metropolitan Regionalism*, may be obtained from the Department of Geography, Villanova University, Villanova, PA 19085.