In[1]:=

```
F = {{βA * N, βID * N, βID * N, βW * N}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

Out[1]= $\{\{N\,\beta A, N\,\beta ID, N\,\beta ID, N\,\beta W\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

In[2]:=

```
V = {{μ + σ, -γ, -γ, 0}, {-σ, μ + γ + α, 0, 0}, {0, -α, μ + γ + ξ, 0}, {-ρA, -ρID, -ρID, η}}
```

Out[2]= $\{\{\mu+\sigma, -\gamma, -\gamma, 0\}, \{-\sigma, \alpha+\gamma+\mu, 0, 0\}, \{0, -\alpha, \gamma+\mu+\xi, 0\}, \{-\rho A, -\rho ID, -\rho ID, \eta\}\}$

In[3]:=

```
Vinv = Inverse[V]
```

Out[3]=

$$\left\{\left\{\frac{(\alpha+\gamma+\mu)\ (\gamma+\mu+\xi)}{-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)}, \frac{\alpha\gamma\eta+\gamma^2\eta+\gamma\eta\mu+\gamma\eta\xi}{\eta\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)},\right.\right.$$

$$\left.\frac{\alpha\gamma\eta+\gamma^2\eta+\gamma\eta\mu}{\eta\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)}, 0\right\},$$

$$\left\{\frac{\gamma\eta\sigma+\eta\mu\sigma+\eta\xi\sigma}{\eta\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)}, \frac{(\gamma+\mu+\xi)\ (\mu+\sigma)}{-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)},\right.$$

$$\left.\frac{\gamma\sigma}{-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)}, 0\right\},$$

$$\left\{\frac{\alpha\sigma}{-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)}, \frac{\alpha\eta\mu+\alpha\eta\sigma}{\eta\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)},\right.$$

$$\left.\frac{\alpha\eta\mu+\gamma\eta\mu+\eta\mu^2+\alpha\eta\sigma+\eta\mu\sigma}{\eta\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)}, 0\right\},$$

$$\left\{\frac{\alpha\rho ID\,\sigma+(\gamma+\mu+\xi)\ (\alpha\rho A+\gamma\rho A+\mu\rho A+\rho ID\,\sigma)}{\eta\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)},\right.$$

$$\frac{\alpha\gamma\rho A+\gamma^2\rho A+\gamma\mu\rho A+\gamma\xi\rho A+\alpha\mu\rho ID+\gamma\mu\rho ID+\mu^2\rho ID+\mu\xi\rho ID+\alpha\rho ID\,\sigma+\gamma\rho ID\,\sigma+\mu\rho ID\,\sigma+\xi\rho ID\,\sigma}{\eta\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)},$$

$$\left.\frac{\alpha\gamma\rho A+\gamma^2\rho A+\gamma\mu\rho A+\alpha\mu\rho ID+\gamma\mu\rho ID+\mu^2\rho ID+\alpha\rho ID\,\sigma+\gamma\rho ID\,\sigma+\mu\rho ID\,\sigma}{\eta\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\ \left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)}, \frac{1}{\eta}\right\}\right\}$$

In[4]:=
```
NGM = F.Vinv
```

Out[4]=
$$\left\{\left\{\frac{\mathrm{N}\,\beta\mathrm{A}\,(\alpha+\gamma+\mu)\,(\gamma+\mu+\xi)}{-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)}+\right.\right.$$
$$\frac{\mathrm{N}\,\alpha\,\beta\mathrm{ID}\,\sigma}{-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)}+\frac{\mathrm{N}\,\beta\mathrm{ID}\,(\gamma\eta\sigma+\eta\mu\sigma+\eta\xi\sigma)}{\eta\,\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)}+$$
$$\frac{\mathrm{N}\,\beta\mathrm{W}\,(\alpha\,\rho\mathrm{ID}\,\sigma+(\gamma+\mu+\xi)\,(\alpha\,\rho\mathrm{A}+\gamma\,\rho\mathrm{A}+\mu\,\rho\mathrm{A}+\rho\mathrm{ID}\,\sigma))}{\eta\,\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)},$$
$$\frac{\mathrm{N}\,\beta\mathrm{A}\,\left(\alpha\gamma\eta+\gamma^2\eta+\gamma\eta\mu+\gamma\eta\xi\right)}{\eta\,\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)}+\frac{\mathrm{N}\,\beta\mathrm{ID}\,(\gamma+\mu+\xi)\,(\mu+\sigma)}{-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)}+$$
$$\frac{\mathrm{N}\,\beta\mathrm{ID}\,(\alpha\eta\mu+\alpha\eta\sigma)}{\eta\,\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)}+$$
$$\left(\mathrm{N}\,\beta\mathrm{W}\,\left(\alpha\gamma\,\rho\mathrm{A}+\gamma^2\,\rho\mathrm{A}+\gamma\mu\,\rho\mathrm{A}+\gamma\xi\,\rho\mathrm{A}+\alpha\mu\,\rho\mathrm{ID}+\gamma\mu\,\rho\mathrm{ID}+\mu^2\,\rho\mathrm{ID}+\mu\xi\,\rho\mathrm{ID}+\alpha\,\rho\mathrm{ID}\,\sigma+\right.\right.$$
$$\left.\left.\gamma\,\rho\mathrm{ID}\,\sigma+\mu\,\rho\mathrm{ID}\,\sigma+\xi\,\rho\mathrm{ID}\,\sigma\right)\right)\Big/\left(\eta\,\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)\right),$$
$$\frac{\mathrm{N}\,\beta\mathrm{A}\,\left(\alpha\gamma\eta+\gamma^2\eta+\gamma\eta\mu\right)}{\eta\,\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)}+\frac{\mathrm{N}\,\beta\mathrm{ID}\,\gamma\sigma}{-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)}+$$
$$\frac{\mathrm{N}\,\beta\mathrm{ID}\,\left(\alpha\eta\mu+\gamma\eta\mu+\eta\mu^2+\alpha\eta\sigma+\eta\mu\sigma\right)}{\eta\,\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)}+$$
$$\left.\frac{\mathrm{N}\,\beta\mathrm{W}\,\left(\alpha\gamma\,\rho\mathrm{A}+\gamma^2\,\rho\mathrm{A}+\gamma\mu\,\rho\mathrm{A}+\alpha\mu\,\rho\mathrm{ID}+\gamma\mu\,\rho\mathrm{ID}+\mu^2\,\rho\mathrm{ID}+\alpha\,\rho\mathrm{ID}\,\sigma+\gamma\,\rho\mathrm{ID}\,\sigma+\mu\,\rho\mathrm{ID}\,\sigma\right)}{\eta\,\left(-\alpha\gamma\sigma+(\gamma+\mu+\xi)\,\left(\alpha\mu+\gamma\mu+\mu^2+\alpha\sigma+\mu\sigma\right)\right)},\ \frac{\mathrm{N}\,\beta\mathrm{W}}{\eta}\right\},$$
$$\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\Big\}$$

In[5]:=
```
Eigenvalues[NGM]
```

Out[5]=
$$\{0,0,0,$$
$$\left(\mathrm{N}\,\left(\alpha\,\beta\mathrm{A}\,\gamma\,\eta+\beta\mathrm{A}\,\gamma^2\,\eta+\alpha\,\beta\mathrm{A}\,\eta\,\mu+2\,\beta\mathrm{A}\,\gamma\,\eta\,\mu+\beta\mathrm{A}\,\eta\,\mu^2+\alpha\,\beta\mathrm{A}\,\eta\,\xi+\beta\mathrm{A}\,\gamma\,\eta\,\xi+\beta\mathrm{A}\,\eta\,\mu\,\xi+\alpha\,\beta\mathrm{W}\,\gamma\,\rho\mathrm{A}+\beta\mathrm{W}\,\gamma^2\,\rho\mathrm{A}+\right.\right.$$
$$\alpha\,\beta\mathrm{W}\,\mu\,\rho\mathrm{A}+2\,\beta\mathrm{W}\,\gamma\,\mu\,\rho\mathrm{A}+\beta\mathrm{W}\,\mu^2\,\rho\mathrm{A}+\alpha\,\beta\mathrm{W}\,\xi\,\rho\mathrm{A}+\beta\mathrm{W}\,\gamma\,\xi\,\rho\mathrm{A}+\beta\mathrm{W}\,\mu\,\xi\,\rho\mathrm{A}+\alpha\,\beta\mathrm{ID}\,\eta\,\sigma+$$
$$\left.\left.\beta\mathrm{ID}\,\gamma\,\eta\,\sigma+\beta\mathrm{ID}\,\eta\,\mu\,\sigma+\beta\mathrm{ID}\,\eta\,\xi\,\sigma+\alpha\,\beta\mathrm{W}\,\rho\mathrm{ID}\,\sigma+\beta\mathrm{W}\,\gamma\,\rho\mathrm{ID}\,\sigma+\beta\mathrm{W}\,\mu\,\rho\mathrm{ID}\,\sigma+\beta\mathrm{W}\,\xi\,\rho\mathrm{ID}\,\sigma\right)\right)\Big/$$
$$\left.\left(\eta\,\left(\alpha\,\gamma\,\mu+\gamma^2\,\mu+\alpha\,\mu^2+2\,\gamma\,\mu^2+\mu^3+\alpha\,\mu\,\xi+\gamma\,\mu\,\xi+\mu^2\,\xi+\alpha\,\mu\,\sigma+\gamma\,\mu\,\sigma+\mu^2\,\sigma+\alpha\,\xi\,\sigma+\mu\,\xi\,\sigma\right)\right)\right\}$$

In[6]:=
```
R0 =
```
$$\left(\mathrm{N}\,\left(\alpha\,\beta\mathrm{A}\,\gamma\,\eta+\beta\mathrm{A}\,\gamma^2\,\eta+\alpha\,\beta\mathrm{A}\,\eta\,\mu+2\,\beta\mathrm{A}\,\gamma\,\eta\,\mu+\beta\mathrm{A}\,\eta\,\mu^2+\alpha\,\beta\mathrm{A}\,\eta\,\xi+\beta\mathrm{A}\,\gamma\,\eta\,\xi+\beta\mathrm{A}\,\eta\,\mu\,\xi+\alpha\,\beta\mathrm{W}\,\gamma\,\rho\mathrm{A}+\beta\mathrm{W}\,\gamma^2\,\rho\mathrm{A}+\right.\right.$$
$$\alpha\,\beta\mathrm{W}\,\mu\,\rho\mathrm{A}+2\,\beta\mathrm{W}\,\gamma\,\mu\,\rho\mathrm{A}+\beta\mathrm{W}\,\mu^2\,\rho\mathrm{A}+\alpha\,\beta\mathrm{W}\,\xi\,\rho\mathrm{A}+\beta\mathrm{W}\,\gamma\,\xi\,\rho\mathrm{A}+\beta\mathrm{W}\,\mu\,\xi\,\rho\mathrm{A}+\alpha\,\beta\mathrm{ID}\,\eta\,\sigma+$$
$$\left.\left.\beta\mathrm{ID}\,\gamma\,\eta\,\sigma+\beta\mathrm{ID}\,\eta\,\mu\,\sigma+\beta\mathrm{ID}\,\eta\,\xi\,\sigma+\alpha\,\beta\mathrm{W}\,\rho\mathrm{ID}\,\sigma+\beta\mathrm{W}\,\gamma\,\rho\mathrm{ID}\,\sigma+\beta\mathrm{W}\,\mu\,\rho\mathrm{ID}\,\sigma+\beta\mathrm{W}\,\xi\,\rho\mathrm{ID}\,\sigma\right)\right)\Big/$$
$$\left.\left(\eta\,\left(\alpha\,\gamma\,\mu+\gamma^2\,\mu+\alpha\,\mu^2+2\,\gamma\,\mu^2+\mu^3+\alpha\,\mu\,\xi+\gamma\,\mu\,\xi+\mu^2\,\xi+\alpha\,\mu\,\sigma+\gamma\,\mu\,\sigma+\mu^2\,\sigma+\alpha\,\xi\,\sigma+\mu\,\xi\,\sigma\right)\right)\right.$$

Out[6]=
$$\left(\mathrm{N}\,\left(\alpha\,\beta\mathrm{A}\,\gamma\,\eta+\beta\mathrm{A}\,\gamma^2\,\eta+\alpha\,\beta\mathrm{A}\,\eta\,\mu+2\,\beta\mathrm{A}\,\gamma\,\eta\,\mu+\beta\mathrm{A}\,\eta\,\mu^2+\alpha\,\beta\mathrm{A}\,\eta\,\xi+\beta\mathrm{A}\,\gamma\,\eta\,\xi+\beta\mathrm{A}\,\eta\,\mu\,\xi+\alpha\,\beta\mathrm{W}\,\gamma\,\rho\mathrm{A}+\right.\right.$$
$$\beta\mathrm{W}\,\gamma^2\,\rho\mathrm{A}+\alpha\,\beta\mathrm{W}\,\mu\,\rho\mathrm{A}+2\,\beta\mathrm{W}\,\gamma\,\mu\,\rho\mathrm{A}+\beta\mathrm{W}\,\mu^2\,\rho\mathrm{A}+\alpha\,\beta\mathrm{W}\,\xi\,\rho\mathrm{A}+\beta\mathrm{W}\,\gamma\,\xi\,\rho\mathrm{A}+\beta\mathrm{W}\,\mu\,\xi\,\rho\mathrm{A}+\alpha\,\beta\mathrm{ID}\,\eta\,\sigma+$$
$$\left.\left.\beta\mathrm{ID}\,\gamma\,\eta\,\sigma+\beta\mathrm{ID}\,\eta\,\mu\,\sigma+\beta\mathrm{ID}\,\eta\,\xi\,\sigma+\alpha\,\beta\mathrm{W}\,\rho\mathrm{ID}\,\sigma+\beta\mathrm{W}\,\gamma\,\rho\mathrm{ID}\,\sigma+\beta\mathrm{W}\,\mu\,\rho\mathrm{ID}\,\sigma+\beta\mathrm{W}\,\xi\,\rho\mathrm{ID}\,\sigma\right)\right)\Big/$$
$$\left.\left(\eta\,\left(\alpha\,\gamma\,\mu+\gamma^2\,\mu+\alpha\,\mu^2+2\,\gamma\,\mu^2+\mu^3+\alpha\,\mu\,\xi+\gamma\,\mu\,\xi+\mu^2\,\xi+\alpha\,\mu\,\sigma+\gamma\,\mu\,\sigma+\mu^2\,\sigma+\alpha\,\xi\,\sigma+\mu\,\xi\,\sigma\right)\right)\right.$$

In[7]:=
```
Simplify[R0]
```

Out[7]=
$$\left(\mathrm{N}\,\left(\left(\gamma+\mu+\xi\right)\,\left(\beta\mathrm{A}\,\eta\,\left(\gamma+\mu\right)+\beta\mathrm{ID}\,\eta\,\sigma+\beta\mathrm{W}\,\left(\gamma\,\rho\mathrm{A}+\mu\,\rho\mathrm{A}+\rho\mathrm{ID}\,\sigma\right)\right)+\right.\right.$$
$$\left.\left.\alpha\,\left(\beta\mathrm{A}\,\eta\,\left(\gamma+\mu+\xi\right)+\beta\mathrm{ID}\,\eta\,\sigma+\beta\mathrm{W}\,\left(\gamma\,\rho\mathrm{A}+\mu\,\rho\mathrm{A}+\xi\,\rho\mathrm{A}+\rho\mathrm{ID}\,\sigma\right)\right)\right)\right)\Big/$$
$$\left(\eta\,\left(\alpha\,\gamma\,\mu+\alpha\,\left(\mu+\xi\right)\,\left(\mu+\sigma\right)+\mu\,\left(\gamma+\mu+\xi\right)\,\left(\gamma+\mu+\sigma\right)\right)\right)$$