STAC58 - Statistical Inference

January 7, 2019 11:15 AM

Evaluations - Midterm 40%
- final 60%

Textbook: Probability and statistics - Evans & Rosenthal, Chapter 5-9

Measuring Statistical Evidence using relative belief
- M. Evans

website: http://www.utstat.utoronto.ca/mikevans/stac58/stac58.html

Statistical Inference

January 6, 2019 9:08 PM

Basics: Introduction

- Statistical inference is not so much about the methods of statistics but the "why".
- what is statistics as a subject all about?
- statistical methods are used in:
  - Finance
  - Machine learning
  - medicine
  - quantum physics

    ⋮

    more!
- Furthermore, "statistical reasoning" is becoming more and more important!
- It is being used as a tool to reason about reality.
- Note: significant decisions are made based on statistical analysis.
- So we want the rules of statistical reasoning to be ____ = logical, free of contradictions, ____, etc ... so we feel confident that whatever the conclusion / inference we draw makes sense.

- Current state of statistics
  - Many different points of view about what the the correct statistical reasoning is.
  - This makes learning the subject hard.

- Purpose of this course (STAC58 - Statistical Inference)

  1.) Survey the various approaches

  2) present the outline of a logical way to develop a theory of statistical reasoning.

- Some phenomenon / context in the real world that we have questions about
- Questions like: 1) what is the value of some quantity of interest?

  eg. mean half life length of a neutron

  Answer: An estimate of assessment of its error

  2) Does a certain quantity take a particular value?

  Answer: hypothesis assessment - evidence for or against and a measure of strength.
- when can statistical inference play a role?

Theory tells Jon how accurate estimate is.

Statistical Problems

- The first thing we need to do is be very clear about what a statistical problem is.
- It is all based on "measuring" and counting.
- we have a population Ω = a finite set of objects of interests.

Eg. Ω = set of all students enrolled at UofT on Jan 7, 2019

- $\#(\Omega) < \infty$

cardinality / # of items in the set

- we have a measurement(s) defined on Ω

$X : \Omega \to \mathcal{X}$ $\quad \omega \in \Omega \quad X(\omega)$

- for $\omega \in \Omega$ = set of students at UofT.

Define

$X_1(\omega)$ = height of ω in cm (interval)

$X_2(\omega)$ = weight of ω in kg (interval)

$X_3(\omega)$ = gender of ω (categorical)

$$X(\omega) = \begin{pmatrix} X_1(\omega) \\ X_2(\omega) \\ X_3(\omega) \end{pmatrix}$$

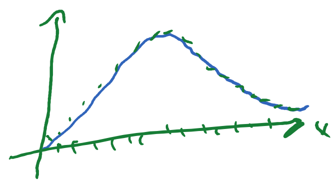
$X(\omega)$ is 3-d measurements

$$X: \Omega \to \underbrace{R^1 \times R^1 \times \{M, F\}}_{\mathcal{X}}$$

- $X = (X_1, X_2, X_3): \Omega \to R \times R \times \{M, F\}$
- Ω and X define relative frequency distribution over $\mathcal{X}$.

$$f_X(x) = \frac{\#\{\omega : X(\omega) = x\}}{\#(\Omega)}$$

= proportion of individuals in Ω whose X measurements is $x \in \mathcal{X}$.



simplify by introducing continuous approximation

discrete is too hard for approx it.

- note: i) $0 \le f_X(x) \le 1$ $\forall x \in \mathcal{X}$

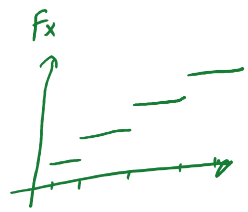
ii) $\sum_{x \in \mathcal{X}} f_X(x) = 1$

and only finitely many $x \in \mathcal{X}$ have $f_X(x) > 0$.

- when $\mathcal{X} = R$ (or an interval)

$$F_X(x) = \frac{\#\{\omega : X(\omega) \le x\}}{\#(\Omega)}$$ = Cumulative distribution function of X (CDF of X)

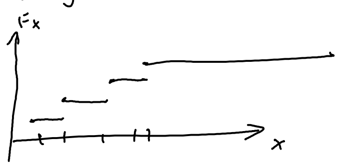
$$= \sum_{z \le x} f_X(z)$$



} Step function

$$f_X(x) = F_X(x) - F_X(x - \underbrace{\epsilon}_{\text{epsilon}}), \text{ where } F_X(x-\epsilon) = \lim_{z \uparrow x} F_X(z)$$

- So F_X and f_X are two equivalent ways of presenting a frequency distribution.

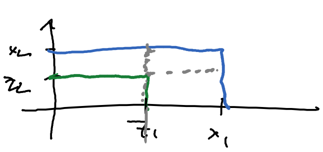


- when $\mathcal{X} = \mathbb{R}^2$

$$F_X(x_1, x_2) = \frac{\#\{\omega : X_1(\omega) \le x_1, X_2(\omega) \le x_2\}}{\#(\Omega)}$$

$$= \sum_{\substack{z_1 \le x_1 \\ z_2 \le x_2}} f_X(z_1, z_2)$$

$$f_X(x_1, x_2) = \lim_{z_1 \uparrow x_1} \left[F_X(x_1, x_2) - F_X(x_1, z_2) - F_X(z_1, x_2) + F_X(z_1, z_2) \right]$$



So, $F_X \leftrightarrow f_X$

- The whole point of any statistical analysis is to learn something about F_X.
- how do we do this?
- If possible we do a census, namely compute $X(\omega)$ $\forall \omega \in \Omega$ of the form f_X.
- Typically count (return to this in a moment)
- why do we want to know F_X?

eg relationships among variables.

- Suppose (x, y), where $x: \Omega \to x$, $y: \Omega \to y$ and we want to know if there is a relationship between x & y on Ω.

- form the conditional relative frequency distribution.

$$f_{y|x}(y|x) = \frac{\# \{\omega \mid x(\omega) = x,\ y_2(\omega) = y\}}{\# \{\omega \mid x(\omega) = x\}}$$

$$= \frac{f_{(x,y)}(x, y_2)}{f_x(x)}$$

Definition: x and y are related variables over Ω if $f_{y|x}(\cdot|x)$ changes as x changes.

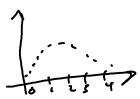
- The "form" of the relationship between x and y is given by how $f_{y|x}(\cdot|z)$ changes as x_2 changes.

eg. Ω = 1^{st} year students at U of T

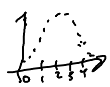
y = GPA as of Dec 31, 2015.

x = gender

$f_{y|x}(\cdot|m)$



$f_{y|x}(\cdot|F)$



- often simplifying assumptions are introduced.
- regression assumption: $f_{y|x}(\cdot|x)$ changes at most though its mean as x changes, $E(y|x)(x)$

$$= \frac{1}{\# \{\omega : x(\omega) = x\}} \sum_{\{\omega : x(\omega) = x\}} y(\omega)$$

$$\stackrel{Ex}{=} \sum_y y\, f_{y|x}(y|x)$$