Chapter-3

Trigonometric Functions

1° degree $= 60'$ minutes
$1'$ minutes $= 60''$ seconds

2π radian $= 360^\circ$

π radian $= \frac{360^\circ}{2}$

π radian $= 180^\circ$

Ex ⇒ (i) 20°

(Ans-i) $180^\circ = \pi$ radian

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$20^\circ = \frac{\pi}{\cancel{180}_9} \times \cancel{20}^1$

$20^\circ = \left(\frac{\pi}{9}\right)^c$

(ii) Radian to degree

$(\pi)^c = 180^\circ$

$1^c = \left(\frac{180}{\pi}\right)^\circ$

Ex - 3.1

(Que 1) Find the radian measures corresponding to the following degree measure :

i) 25°

Ans-i) $180^\circ = \pi^c$

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$25^\circ = \left(\frac{\cancel{25}^{5} \times \pi}{\cancel{180}_{36}}\right)^c$

$25^\circ = \left(\frac{5\pi}{36}\right)^c$

ii) $-47^\circ 30'$

Ans-ii) $-\left[47 + \frac{\cancel{30}^{1}}{\cancel{60}_{2}}\right]^\circ$

$-\left[\frac{94+1}{2}\right]$

$-\left[\frac{95}{2}\right]^\circ$

$180^\circ = \pi^c$

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$-\left[\frac{95}{2}\right]^\circ = -\left[\frac{\cancel{95}^{19}}{2} \times \frac{\pi}{\cancel{180}_{36}}\right]^c$

$-\left[\frac{95}{2}\right]^\circ = -\left[\frac{19\pi}{72}\right]^c$

iii) 240°

Ans-iii) $180^\circ = \pi^c$

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$240^\circ = \left(\cancel{240}^{4} \times \frac{\pi}{\cancel{180}_{3}}\right)^c$

$240^\circ = \left(\frac{4}{3}\pi\right)^c$

iv) 520°

Ans-iv) $180^\circ = \pi^c$

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$520^\circ = \left(\cancel{520}^{26} \times \frac{\pi}{\cancel{180}_{9}}\right)^c$

$520^\circ = \left(\frac{26\pi}{9}\right)^c$

(Que 2) Find the degree measures corresponding to the following radian measure (use $\pi = \frac{22}{7}$)

(i) $\left(\frac{11}{16}\right)^c$

Ans-i) $\pi^c = 180^\circ$

$(1)^c = \left(\frac{180}{\pi}\right)^\circ$

$\left(\frac{11}{16}\right)^c = \left(\frac{11}{16} \times \frac{180}{\pi}\right)^\circ$

$\left(\frac{11}{16}\right)^c = \left(\frac{\cancel{11}^{1}}{\cancel{16}_{4}} \times \frac{\cancel{180}^{45} \times 7}{\cancel{22}_{2}}\right)^\circ$

$\left(\frac{11}{16}\right)^c = \left(\frac{315}{8}\right)^\circ$

$= \left(39\frac{3}{8}\right)^\circ$

$= 39^\circ\left(\frac{3}{\cancel{8}_{2}} \times \cancel{60}^{15}\right)'$

$= 39^\circ\left(\frac{45}{2}\right)'$

$= 39^\circ\left(22\frac{1}{2}\right)'$

$= 39^\circ 22'\left(\frac{1}{\cancel{2}_{1}} \times \cancel{60}^{30}\right)''$

$= 39^\circ 22' 30''$ Ans

ii) $(-4)^c$

Ans-ii) $\pi^c = 180^\circ$

$1^c = \left(\frac{180}{\pi}\right)^\circ$

$(-4)^c = \left(-4^2 \times \frac{180 \times 7}{22_{11}}\right)^\circ$

$(-4)^c = \left(\frac{-2520}{11}\right)^\circ$

$= -\left(229\frac{1}{11}\right)^\circ$

$= -229^\circ\left(\frac{1}{11}\times 60\right)'$

$= -229^\circ\left(\frac{60}{11}\right)'$

$= -229^\circ\left(5\frac{5}{11}\right)'$

$= -229^\circ 5'\left[\frac{5}{11}\times 60\right]''$

$= -229^\circ 5'\left(\frac{300}{11}\right)''$

$= -229^\circ 5'\left(27\frac{3}{11}\right)''$

$= -229^\circ 5' 27''$ Approx.

(iii) $\left(\frac{5}{3}\pi\right)^c$

(Ans-iii) $\pi^c = 180^\circ$

$(1)^c = \left(\frac{180}{\pi}\right)^\circ$

$\left(\frac{5}{3}\pi\right)^c = \left(\frac{5}{3}\pi \times \frac{\cancel{180}^{60}}{\cancel{\pi}}\right)^\circ$

$\left(\frac{5}{3}\pi\right)^c = 300^\circ$

(iv) $\left(\frac{7}{6}\pi\right)^c$

(Ans-iv) $\pi^c = 180^\circ$

$1^c = \left(\frac{180}{\pi}\right)^\circ$

$\left(\frac{7\pi}{6}\right)^c = \left(\frac{7\cancel{\pi}}{\cancel{6}_1} \times \frac{\cancel{180}^{30^\circ}}{\cancel{6}\cancel{\pi}}\right)^\circ$

$\left(\frac{7\pi}{6}\right)^c = 210^\circ$

(Que 3) A wheel makes 360 revolution in one minute. Through how many radians does it turn in one second?

(Ans-3) Wheel revolution in one minutes = 360

Wheel revolution in one second = ~~360~~ $\frac{360}{60}$ = ~~60~~ 6

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Angle of 1 revolution of wheel $= 2\pi$
Angle of 6 revolutions of wheel $= 2\pi \times 6$
$= 12\pi$ Ans

(Que 4) Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm. (use $\pi = \frac{22}{7}$)

(Ans-4) Given:

Find: $\theta = ?$

legth of arc $= 22$ cm
radius $= 100$ cm

Solution:

$$\theta = \frac{l}{r} \Rightarrow \frac{22}{100} \Rightarrow \frac{11}{50} \text{ radian}$$

$$\pi^c = 180^\circ$$
$$(1)^c = \left(\frac{180}{\pi}\right)^\circ$$

$$\left(\frac{11}{50}\right)^c = \left(\frac{11}{50} \times \frac{180}{\pi}\right)^\circ$$

$$\left(\frac{11}{50}\right)^c = \left(\frac{\cancel{11}^{1}}{\cancel{50}_{5}} \times \frac{\cancel{180}^{9} \times 7}{\cancel{22}_{2}}\right)^\circ$$

$$\left(\frac{11}{50}\right)^c = \left(\frac{63}{5}\right)^\circ$$

$$= \left(12\frac{3}{5}\right)^\circ$$

$$= 12^\circ \left(\frac{3 \times \cancel{60}^{12}}{\cancel{5}}\right)'$$

$$= 12^\circ\, 36' \text{ Ans}$$

(Que 5) In a circle of diameter 40cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Given;

(Ans-5) Daimeter $= 40cm$

$2r = 40cm$

$r = 20cm$

Chord $= 20cm$

ΔOAB

$OA = OB = AB = 20cm$

$\theta = \angle AOB = 60^\circ$

Solution:

$180^\circ = \pi^c$

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$60^\circ = \left(\frac{60 \times \pi}{180}\right)^c$

$60^\circ = \left(\frac{\pi}{3}\right)^c$

$\theta = \frac{l}{r}$

$\left(\frac{\pi}{3}\right)^c = \frac{l}{20}$

$\frac{20\pi}{3} = l$

$l = \frac{20\pi}{3}$ cm

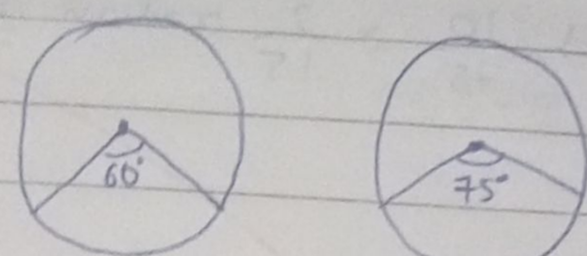
length of minor arc $= \frac{20\pi}{3}$ cm.

Que6) If in two circles, arcs of the same length subtend angles 60° and 75° at the centre. Find the ratio of their radii.

Ans-6) Given:

$\theta_1 = 60^\circ$, $\theta_2 = 75^\circ$

$l_1 = l_2$



Find:

$r_1 : r_2$

Solution:

$$180^\circ = (\pi)^c$$
$$1^\circ = \left(\frac{\pi}{180}\right)^c$$
$$60^\circ = \left(60 \times \frac{\pi}{180_3}\right)^c$$
$$60^\circ = \left(\frac{\pi}{3}\right)^c$$

$$180^\circ = (\pi)^c$$
$$1^\circ = \left(\frac{\pi}{180}\right)^c$$
$$75^\circ = \left(\cancel{75}^{5} \times \frac{\pi}{\cancel{180}_{12}}\right)^c$$
$$75^\circ = \left(\frac{5\pi}{12}\right)^c$$

$l_1 = l_2$ [Given]

$\theta_1 \times r_1 = \theta_2 \times r_2$

$\frac{\pi}{3} \times r_1 = \frac{5\pi}{12} \times r_2$

$\frac{r_1}{r_2} = \frac{5\cancel{\pi}}{\cancel{12}_4} \times \frac{\cancel{3}^1}{\cancel{\pi}}$

$\frac{r_1}{r_2} = \frac{5}{4}$

$r_1 : r_2 = 5 : 4$ Ans

(Que 7) Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describe an arc of length,

(i) 10 cm

(Ans-i) $\theta = \frac{l}{r} \Rightarrow \frac{10}{75} \Rightarrow \frac{2}{15}$ radian Ans

$\pi^c = 180°$

$1^c = \frac{180°}{\pi}$

$\left(\frac{2}{15}\right)^c = \frac{180}{\pi} \times \frac{2}{15} \Rightarrow \frac{7 \times \cancel{180}^{12}}{\cancel{22}_{11}} \times \frac{\cancel{2}^{1}}{\cancel{15}_{1}} \Rightarrow \frac{84}{11}$

$180° = \pi$

(ii) 15 cm

(Ans-ii) $\theta = \frac{l}{r} \Rightarrow \frac{15}{75} = \frac{1}{5}$ radian Ans

(iii) 21 cm

(Ans-iii) $\theta = \frac{l}{r} \Rightarrow \frac{21}{75} \Rightarrow \frac{7}{25}$ radian Ans

$90^\circ = \frac{\pi}{2}$

II - Quadrant	I - Quadrant
$90^\circ + \theta$	$90^\circ - \theta$
$180^\circ - \theta$	$360^\circ + \theta$
[$\sin\theta$, $\csc\theta$] +	[All] +

$180^\circ = \pi$ — 0°, $360^\circ = 2\pi$

III - Quadrant	IV - Quadrant
$180^\circ + \theta$	$270^\circ + \theta$
$270^\circ + \theta$	$360^\circ - \theta$
	$(-\theta)$
[$\tan\theta$, $\cot\theta$] +	[$\cos\theta$, $\sec\theta$] +

$270^\circ = \frac{3\pi}{2}$

[90°/270°] change	[180°/360°] no change
$\sin\theta \longleftrightarrow \cos\theta$	$\sin\theta \longleftrightarrow \sin\theta$
$\tan\theta \longleftrightarrow \cot\theta$	$\cos\theta \longleftrightarrow \cos\theta$
$\csc\theta \longleftrightarrow \sec\theta$	$\tan\theta \longleftrightarrow \tan\theta$

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

$\sin^2\theta = 1 - \cos^2\theta$
$\cos^2\theta = 1 - \sin^2\theta$
$\tan^2\theta = \sec^2\theta - 1$
$\sec^2\theta = 1 + \tan^2\theta$
$\cot^2\theta = 1 - \operatorname{cosec}^2\theta$
$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D.
$\operatorname{cosec}\theta$	N.D.	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D.
$\cot\theta$	N.D.	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

☆ Formula :-

i) $\sin(A+B) = \sin A \cdot \cos B + \sin A \cdot \cos A$

ii) $\sin(A-B) = \sin A \cdot \cos B + \sin B \cdot \cos A$

iii) $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

iv) $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

v) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

vi) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

vii) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

viii) $\sin C \mp \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

ix) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

x) $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan\theta}$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

Ex-3.2

Find the value of other five trigonometric functions in Exercises 1 to 5.

1) $\cos x = -\frac{1}{2}$, x lies in third quadrant.

Ans-1) $\cos x = -\frac{1}{2}$

$\sec x = \frac{1}{\cos x} = \frac{1}{-\frac{1}{2}} = \boxed{-2}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$

$\sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$

$\sin x = \pm \frac{\sqrt{3}}{2}$

Since x lies in the 3^{rd} quadrant

So, $\boxed{\sin x = -\frac{\sqrt{3}}{2}}$

$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{-\frac{\sqrt{3}}{2}} = \boxed{-\frac{2}{\sqrt{3}}}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \boxed{\sqrt{3}}$

$\cot x = \frac{1}{\tan x} = \boxed{\frac{1}{\sqrt{3}}}$

So, the other values are

$\sec x = -2$, $\sin x = -\frac{\sqrt{3}}{2}$, $\operatorname{cosec} x = \frac{-2}{\sqrt{3}}$

$\tan x = \sqrt{3}$, $\cot x = \frac{1}{\sqrt{3}}$

2) $\sin x = \frac{3}{5}$, x lies in second quadrant.

Ans-2) $\sin x = \frac{3}{5}$

$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\frac{3}{5}} = \boxed{\frac{5}{3}}$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos^2 x = 1 - \left(\frac{3}{5}\right)^2$

$\cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$

$\cos x = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$

Since x lies in the 2^{nd} quadrant

So, $\boxed{\cos x = -\frac{4}{5}}$

$\sec x = \frac{1}{\cos x} = \frac{1}{-\frac{4}{5}} = \boxed{-\frac{5}{4}}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{3}{5}}{-\frac{4}{5}} = \boxed{-\frac{3}{4}}$

$\cot x = \frac{1}{\tan x} = \boxed{-\frac{4}{3}}$

So, the other value are →

$\operatorname{cosec} x = \frac{5}{3}$, $\cos x = -\frac{4}{5}$, $\sec x = -\frac{5}{4}$, $\tan x = \frac{-3}{4}$

$\cot x = \frac{-4}{3}$

3) $\cot x = \frac{3}{4}$, x lies in third quadrant

Ans 3) $\cot x = \frac{3}{4}$

$\tan x = \frac{1}{\cot x} = \frac{1}{\frac{3}{4}} = \boxed{\frac{4}{3}}$

$1 + \tan^2 x = \sec^2 x$

$1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$

$1 + \frac{16}{9} = \sec^2 x$

$\frac{25}{9} = \sec^2 x$

$\sec x = \pm \frac{5}{3}$

Since, x lies in 3^{rd} quadrant,

So, $\boxed{\sec x = -\frac{5}{3}}$

$\cos x = \frac{1}{\sec x} = \frac{1}{-\frac{5}{3}} = \boxed{-\frac{3}{5}}$

$\tan x = \frac{\sin x}{\cos x}$

$\frac{4}{3} = \frac{\sin x}{-\frac{3}{5}} \Rightarrow \sin x = \frac{4}{3} \times -\frac{3}{5} \Rightarrow \boxed{-\frac{4}{5}}$

$\operatorname{cosec} x = \frac{1}{\sin x} = \boxed{-\frac{5}{4}}$

So, the other value are → $\tan x = \frac{4}{3}$, $\sec x = -\frac{5}{3}$, $\cos x = -\frac{3}{5}$, $\sin x = -\frac{4}{5}$, $\operatorname{cosec} x = -\frac{5}{4}$

4) $\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Ans 4) $\sec x = \frac{13}{5}$

$\cos x = \frac{1}{\sec x} = \frac{1}{\frac{13}{5}} = \boxed{\frac{5}{13}}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - \left(\frac{5}{13}\right)^2$

$\sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$

$\sin x = \pm \frac{12}{13}$

Since x lies in 4^{th} quadrant,

So, $\boxed{\sin x = -\frac{12}{13}}$

$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{-\frac{12}{13}} = \boxed{-\frac{13}{12}}$

$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{12}{13}}{\frac{5}{13}} = \boxed{-\frac{12}{5}}$

$\cot x = \frac{1}{\tan x} = \frac{1}{-\frac{12}{5}} = \boxed{-\frac{5}{12}}$

So, the other values are →

$\cos x = \frac{5}{13}$, $\sin x = \frac{-12}{13}$, $\operatorname{cosec} x = \frac{-13}{12}$,

$\tan x = \frac{-12}{13}$, $\cot = \frac{-5}{12}$

5) $\tan x = -\frac{5}{12}$, x lies in second quadrant

(Ans-5) $\tan x = \frac{-5}{12}$

$\cot x = \frac{1}{\tan x} = \frac{1}{-\frac{5}{12}} = \boxed{-\frac{12}{5}}$

$1 + \tan^2 x = \sec^2 x$

$1 + \left(\frac{-5}{12}\right)^2 = \sec^2 x$

$1 + \frac{25}{144} = \sec^2 x$

$\frac{169}{144} = \sec^2 x$

$\sec^2 x = \pm \frac{13}{12}$

Since x lies in the 2^{nd} quadrant

$\sec x = \boxed{-\frac{13}{12}}$

$\cos x = \frac{1}{\sec x} = \frac{1}{-\frac{13}{12}} = \boxed{-\frac{12}{13}}$

$\tan x = \frac{\sin x}{\cos x}$

$-\frac{5}{12} = \frac{\sin x}{-\frac{12}{13}}$

$\sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \boxed{\frac{5}{13}}$

$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\frac{5}{13}} = \boxed{\frac{13}{5}}$

Since, the other values are → $\cot x = -\frac{12}{5}$, $\sec x = \frac{-13}{12}$,

$\cos x = \frac{-12}{13}$, $\sin x = \frac{5}{13}$ and $\operatorname{cosec} x = \frac{13}{5}$.

Find the values of the trigonometric functions in Exercises 6 to 10.

6) $\sin 765^\circ$

Ans-6) $\sin 765^\circ$

$\sin(2\times360^\circ+45^\circ)$

$\sin 45^\circ$

$\frac{1}{\sqrt{2}}$ Ans

7) $\operatorname{cosec}(-1410^\circ)$

Ans-7) $\operatorname{cosec}(-1440^\circ+30^\circ)$

$\operatorname{cosec}(-4\times360^\circ+30^\circ)$

$\operatorname{cosec} 30^\circ$

2 Ans

8) $\tan \frac{19\pi}{3}$

Ans-8) $\tan \frac{19\pi}{3}$

$\tan 6\frac{1}{3}\pi$

$\tan\left(6\pi+\frac{\pi}{3}\right)$

$\tan \frac{\pi}{3}$

$\tan 60^\circ = \sqrt{3}$ Ans

9) $\sin\left(-\frac{11\pi}{3}\right)$

Ans-9) $\sin\left(-\frac{11}{3}\pi\right)$

$\sin\left(\frac{-11\times \cancel{180}^{60}}{\cancel{3}}\right)$

$\sin(-660) = -\sin(660)$

$= -\sin(2\times 360 - 60)$

$= -[-\sin 60]$

$= \frac{\sqrt{3}}{2}$

10) $\cot\left(-\frac{15\pi}{4}\right)$

Ans-10) $\cot\left(\frac{-15\times \cancel{180}^{45}}{\cancel{4}}\right)$

$\cot(-675) = -\cot(675^\circ)$

$= -\cot(2\times 360 - 45)$

$= -[\cot 45^\circ]$

$= 1$

Ex-3.3

Prove that:

1) $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$

(Ans-1) $~~\sin^2\left(\frac{180}{6}\right) + \cos^2\left(\frac{180}{3}\right)~~$

$\sin^2\left(\frac{\pi}{6} \times \frac{180}{\pi}\right) + \cos^2\left(\frac{\pi}{3} \times \frac{180}{\pi}\right) - \tan^2\left(\frac{\pi}{4} \times \frac{180}{\pi}\right) = -\frac{1}{2}$

LHS $\Rightarrow$

$= \sin^2 30^\circ + \cos^2 60^\circ - \tan^2 45$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$

$= \frac{1}{4} + \frac{1}{4} - \frac{1}{1}$

$= \frac{1+1-4}{4}$

$= \frac{-2}{4} \Rightarrow \frac{-1}{2}$ Hence, LHS = RHS

2) $2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

(Ans-2) LHS $\Rightarrow 2\sin^2\left(\frac{180}{6}\right) + \operatorname{cosec}^2\left(\frac{7 \times 180}{6}\right) \cdot \cos^2\left(\frac{180}{3}\right)$

$\Rightarrow 2\sin^2 30^\circ + \operatorname{cosec}^2 210^\circ \cdot \cos^2 60^\circ$

$\Rightarrow 2 \times \left(\frac{1}{2}\right)^2 + \frac{1}{\sin^2 210^\circ} \cdot \left(\frac{1}{2}\right)^2$

$\Rightarrow 2 \times \frac{1}{4} + \frac{1}{\sin^2(180+30)} \cdot \left(\frac{1}{4}\right)$

$\Rightarrow \frac{1}{2} + \frac{1}{-\sin^2 30} \cdot \frac{1}{4}$

$\Rightarrow \frac{1}{2} + (-2)^2 \times \frac{1}{4} \Rightarrow \frac{1}{2} + 4 \times \frac{1}{4} \Rightarrow \frac{1}{2} + 1 \Rightarrow \frac{3}{2}$ Hence LHS = RHS

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Q. 3) $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

LHS $\Rightarrow$

Ans 3) $\cot^2\left(\frac{180}{6}\right) + \operatorname{cosec}\left(\frac{5\times \cancel{180}^{30}}{\cancel{6}}\right) + 3\tan^2\left(\frac{180}{6}\right)$

$\Rightarrow \cot^2 30^\circ + \operatorname{cosec} 150^\circ + 3\tan^2 30^\circ$

$= (\sqrt{3})^2 + \frac{1}{\sin 150^\circ} + 3\times\left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + \frac{1}{\sin(90+60)} + \cancel{3}^{1}\times\frac{1}{\cancel{3}_{1}}$

$\Rightarrow 3 + \frac{1}{\cos 60^\circ} + 1$

$\Rightarrow 3 + 2 + 1 \Rightarrow 6$ Hence, LHS = RHS

4) $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

(Ans-4) LHS $\Rightarrow 2\sin^2\left(\frac{3\times \cancel{180}^{45}}{\cancel{4}}\right) + 2\cos^2\left(\frac{\cancel{180}^{45}}{\cancel{4}}\right) + 2\sec^2\left(\frac{\cancel{180}^{60}}{\cancel{3}}\right)$

$= 2\sin^2 135^\circ + 2\cos^2 45^\circ + 2\sec^2 60^\circ$

$= 2\sin^2(90+45)^\circ + 2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 2\times\frac{1}{\cos^2 60^\circ}$

$= 2\cos^2 45^\circ + \cancel{2}^{1}\times\frac{1}{\cancel{2}_{1}} + 2\times(2)^2$

$= 2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 2\times 4$

$= \cancel{2}^{1}\times\frac{1}{\cancel{2}_{1}} + 1 + 8$

$= 1 + 1 + 8 = 10$

Hence, LHS = RHS

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5) Find the value of:

i) $\sin 75^\circ$

Ans-i) $\sin \cancel{(60-15)}(45+30)^\circ$

$=$ Let $45 = A$ and $30 = B$

$= \sin(A+B)$

$= \sin A . \cos B + \cos A . \sin B$ $\left\{ \begin{array}{l} \sin(A+B) \\ = \sin A . \cos B + \cos A . \sin B \end{array} \right\}$

$= \sin 45^\circ . \cos 30^\circ + \cos 45^\circ . \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} . \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \Rightarrow \frac{\sqrt{3}+1}{2\sqrt{2}}$ Ans

ii) $\tan 15^\circ$

Ans-ii) $\tan(45-30)^\circ$

$=$ Let $45 = A$ and $30 = B$

$= \tan(A-B)$

$= \frac{\tan A - \tan B}{1 + \tan A . \tan B}$ $\left\{ \begin{array}{l} \tan(A-B) \\ = \frac{\tan A - \tan B}{1 + \tan A . \tan B} \end{array} \right\}$

$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ . \tan 30^\circ}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$

$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$\Rightarrow \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2} \Rightarrow \frac{3+1-2\sqrt{3}}{3-1} \Rightarrow \frac{4-2\sqrt{3}}{2} \Rightarrow \frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}} \Rightarrow 2-\sqrt{3}$ Ans

Prove the following:

6) $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

LHS $\Rightarrow$

Ans-6) Let, $\frac{\pi}{4}-x=A$ and $\frac{\pi}{4}-y=B$

$\Rightarrow \cos A\cdot\cos B-\sin A\cdot\sin B$ $\quad\{\cos A\cdot\cos B-\sin A\cdot\sin B=\cos(A-B)\}$

$\Rightarrow \cos(A-B)$

$\Rightarrow \cos\left(\frac{\pi}{4}-x+\frac{\pi}{4}-y\right)$

$\Rightarrow \cos\left\{\frac{2\pi}{4}-(x+y)\right\}$

$\Rightarrow \cos\left\{\frac{\pi}{2}-(x+y)\right\}$

$\Rightarrow \cos\left\{\frac{180}{2}-(x+y)\right\}$

$\Rightarrow \cos\{90-(x+y)\}$

$\Rightarrow \sin(x+y)$ $\quad\{\cos(90^\circ-\theta)=\sin\theta\}$

Hence, LHS = RHS

7) $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

(Ans-7) LHS $\Rightarrow \dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$

$$=\dfrac{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\times\tan x}}{\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}$$

$$= \frac{\frac{1+\tan x}{1-\tan x}}{\frac{1-\tan x}{1+\tan x}}$$

$$= \frac{1+\tan x}{1-\tan x} \times \frac{1+\tan x}{1-\tan x}$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)^2$$ Hence, LHS = RHS

8) $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos(\frac{\pi}{2}+x)} = \cot^2 x$

Ans LHS $\Rightarrow \frac{\cos(180+x) \cdot \cos(-x)}{\sin(180-x) \cdot \cos(90+x)}$

$$= \frac{-\cos x \cdot \cos(-x)}{\sin x \cdot (-\sin x)}$$

$$= \frac{\cancel{-}\cos x \cdot \cos x}{\cancel{-}\sin x \cdot \sin x}$$

$$= \frac{\cos^2 x}{\sin^2 x}$$

$$= \cot^2 x$$

Hence LHS = RHS

9) $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right] = 1$

LHS $\Rightarrow$

Ans 9) $\cos\left(\frac{3\times180}{2}+x\right)\cos(360+x)\left[\cot\left(\frac{3\times180}{2}-x\right)+\cot(360^\circ+x)\right]$

$\Rightarrow \cos(270^\circ+x)\cos(360+x)\left[\cot(270-x)+\cot(360+x)\right]$

$\Rightarrow \sin x \cdot \cos x\ (\tan x + \cot x)$

$\Rightarrow \sin x \cdot \cos x \times \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$

$\Rightarrow \cancel{\sin x \cdot \cos x} \times \frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}$

$\Rightarrow \sin^2 x + \cos^2 x$ $\{\sin^2\theta + \cos^2\theta = 1\}$

$\Rightarrow 1$ Hence, LHS = RHS

(Que 10) $\sin(n+1)x \ \sin(n+2)x + \cos(n+1)x \ \cos(n+2)x = \cos x$

(Ans-10) LHS ⇒ Let, $(n+1)x = A$ and $(n+2)x = B$

$= \sin A . \sin B + \cos A . \cos B$

$= \cos A . \cos B + \sin A . \sin B$

$= \cos(A - B)$ $\quad\{\cos A . \cos B + \sin A . \sin B = \cos(A+B)\}$

$= \cos\{(n+1)x - (n+2)x\}$

$= \cos\{x(\cancel{n}+1-\cancel{n}-2)\}$

$= \cos(x \times -1)$

$= \cos(-x)$

$= \cos x$

Hence, LHS = RHS

(Que 11) $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2}\sin x$

(Ans-11) LHS⇒ $\cos\left(\frac{3 \times 180}{4} + x\right) - \cos\left(\frac{3 \times 180}{4} - x\right)$

$= \cos(135 + x) - \cos(135 - x)$

Let $135 + x = C$ $\quad 135 - x = D$

$\Rightarrow \cos C - \cos D$

$\Rightarrow -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$ $\quad\left\{\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right) . \sin\left(\frac{C-D}{2}\right)\right\}$

$= -2\sin\left(\frac{135 + \cancel{x} + 135 - \cancel{x}}{2}\right)\sin\left(\frac{135 + x - 135 + x}{2}\right)$

$= -2\sin\left(\frac{270}{2}\right)\sin\left(\frac{2x}{2}\right)$

$= -2\sin 135 . \sin x$

$\Rightarrow -2\sin(180 - 45)^\circ . \sin x$

$\Rightarrow -2\sin 45^\circ . \sin x$

$= -2 \times \frac{1}{\sqrt{2}} . \sin x \Rightarrow -\sqrt{2}\sin x$

Hence, proved LHS = RHS

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(Que 12) $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

(Ans-12) LHS ⇒ $\sin^2 6x - \sin^2 4x$

$= (\sin 6x - \sin 4x)(\sin 6x + \sin 4x)$ $\{a^2 - b^2 = (a+b)(a-b)\}$

$=$ let $6x = C$ and $4x = D$

$\Rightarrow (\sin C - \sin D)(\sin C + \sin D)$

$\Rightarrow 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \cdot 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$ $\left\{\begin{array}{l} \sin C - \sin D \equiv 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C+D}{2}\right) \\ \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \end{array}\right.$

$\Rightarrow 2\cos\left(\frac{6x+4x}{2}\right) \sin\left(\frac{6x-4x}{2}\right) \cdot 2\sin\left(\frac{6x+4x}{2}\right) \cos\left(\frac{6x-4x}{2}\right)$

$\Rightarrow 2\cos\left(\frac{10x}{2}\right) \sin\left(\frac{2x}{2}\right) \cdot 2\sin\left(\frac{10x}{2}\right) \cos\left(\frac{2x}{2}\right)$

$\Rightarrow 2\cos 5x \cdot \sin x \cdot 2\sin 5x \cdot \cos x$

$\Rightarrow 2\sin 5x \cdot \cos 5x \cdot 2\sin x \cdot \cos x$ $\{2\sin x \cdot \cos x = \sin 2x\}$

$\Rightarrow \sin 2(5x) \cdot \sin 2(x)$

$\Rightarrow \sin 10x \cdot \sin 2x$ Ans

(Que 13) $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

(Ans-13) LHS ⇒ $\cos^2 2x - \cos^2 6x$

Let, ~~cos~~ $2x = A$ and $6x = B$

$\cos^2 A - \cos^2 B$

$\sin(B+A) \cdot \sin(B-A)$ $\{\cos^2 A - \cos^2 B = \sin(B+A) \cdot \sin(B-A)\}$

$\sin(6x+2x) \cdot \sin(6x-2x)$

$\sin 8x \cdot \sin 4x$

$\sin 4x \cdot \sin 8x$

Hence, LHS = RHS

(Que 14) $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

(Ans-14) LHS $\Rightarrow 2\sin 4x + \sin 2x + \sin 6x$

let, $2x = C$ and $6x = D$

$= 2\sin 4x + \sin C + \sin D$

$= 2\sin 4x + 2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)$ $\quad \left\{\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\right\}$

$= 2\sin 4x + 2\sin\left(\frac{2x+6x}{2}\right)\cdot\cos\left(\frac{2x-6x}{2}\right)$

$= 2\sin 4x + 2\sin 4x \cdot \cos(-2x)$

$= 2\sin 4x + 2\sin 4x \cdot \cos 2x$

$= 2\sin 4x\,(1 + \cos 2x)$

$= 2\sin 4x\,(\cancel{1} + 2\cos^2 x \cancel{-1})$ $\quad \{\cos(2x) = 2\cos^2 A - 1\}$

$= 2\sin 4x \cdot 2\cos^2 x$

$= 4\cos^2 x \cdot \sin 4x$ Hence, LHS = RHS

(Que 15) $\cot 4x\,(\sin 5x + \sin 3x) = \cot x\,(\sin 5x - \sin 3x)$

(Ans-15) LHS $\Rightarrow$ Let $5x = C$ and $3x = D$

$\Rightarrow \cot 4x\left\{2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\right\} = \cot x\left\{2\cos\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right)\right\}$

$\Rightarrow \cot 4x\left\{2\sin\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)\right\} = \cot x\left\{2\cos\left(\frac{5x+3x}{2}\right)\cdot\sin\left(\frac{5x-3x}{2}\right)\right\}$

$\Rightarrow \cot 4x\,(2\sin 4x \cdot \cos x) = \cot x\,(2\cos 4x \cdot \sin x)$

$\Rightarrow \frac{\cos 4x}{\cancel{\sin 4x}} \cdot 2\cancel{\sin 4x} \cdot \cos x = \frac{\cos x}{\cancel{\sin x}} \times \cancel{\sin x} \cdot 2\cos 4x$

$\Rightarrow 2\cos 4x \cdot \cos x = \cos x \times 2\cos 4x$

$\Rightarrow 2\cos x \cdot \cos 4x = 2\cos x \cdot \cos 4x$

Hence, LHS = RHS

(Que 16)

(Ans-16) LH

(Que 17) sin

(Ans-17) LHS le

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(Que 16) $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

(Ans-16) LHS $\Rightarrow \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$\Rightarrow \frac{-2\sin\left(\frac{9x+5x}{2}\right)\cdot\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\cdot\sin\left(\frac{17x-3x}{2}\right)}$ $\left\{\begin{array}{l}\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\\ \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\end{array}\right.$

$\Rightarrow \frac{-\cancel{2}\,\cancel{\sin 7x}\cdot\sin 2x}{\cancel{2}\cos 10x\cdot\cancel{\sin 7x}}$

$\Rightarrow -\frac{\sin 2x}{\cos 10x}$

Hence, LHS = RHS

Que 17) $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

(Ans-17) ~~LHS~~ let, $5x = C$ and $3x = D$

LHS $\Rightarrow \frac{\sin C + \sin D}{\cos C + \cos D}$

$\Rightarrow \frac{2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)}{2\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)}$ $\left\{\begin{array}{l}\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\\ \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\end{array}\right\}$

$\Rightarrow \frac{2\sin\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)}$

$\Rightarrow \frac{\cancel{2}\sin 4x\cdot\cancel{\cos x}}{\cancel{2}\cos 4x\cdot\cancel{\cos x}}$

$\Rightarrow \frac{\sin 4x}{\cos 4x}$

$\Rightarrow \tan 4x$

Hence, LHS = RHS

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(Que 18) $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$

(Ans-18) LHS ⇒ $\frac{\sin x - \sin y}{\cos x + \cos y}$

$\Rightarrow \frac{2\cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)}$

$\left\{ \begin{array}{l} \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \\ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \end{array} \right.$

$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$

$= \tan\left(\frac{x-y}{2}\right)$

Hence, LHS = RHS

(Que 19) $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

LHS

(Ans-19) let $3x = C$ and $x = D$

$\Rightarrow \frac{\sin 3x + \sin x}{\cos 3x + \cos x}$

$\Rightarrow \frac{\sin C + \sin D}{\cos C + \cos D}$

$\Rightarrow \frac{2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)}{2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)}$

$\left\{ \begin{array}{l} \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \\ \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \end{array} \right\}$

$\Rightarrow \frac{2\sin\left(\frac{3x+x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right)}{2\cos\left(\frac{3x+x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right)}$

$\Rightarrow \frac{\cancel{2}\sin 2x \cdot \cancel{\cos x}}{\cancel{2}\cos 2x \cdot \cancel{\cos x}}$

$\Rightarrow \frac{\sin 2x}{\cos 2x} \Rightarrow \tan 2x$

Hence, LHS = RHS

(Que 20) $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

(Ans-20) LHS ⇒ ~~Sin sin~~ $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$\Rightarrow \frac{-1 \times (\sin x - \sin 3x)}{-1 \times (\sin^2 x - \cos^2 x}$

$\Rightarrow \frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x}$

Let, $3x = C$ and $x = D$

$\Rightarrow \frac{\sin C - \sin D}{\cos 2x}$ $\{\cos^2\theta - \sin^2\theta = \cos 2\theta\}$

$\Rightarrow \frac{2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)}{\cos 2x}$ $\left\{\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)\right\}$

$\Rightarrow \frac{2\cos\left(\frac{3x+x}{2}\right) \cdot \sin\left(\frac{3x-x}{2}\right)}{\cos 2x}$

$\Rightarrow \frac{2\cancel{\cos 2x} \cdot \sin x}{\cancel{\cos 2x}}$

$\Rightarrow 2\sin x$

Hence, LHS = RHS

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(Que 21) $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

(Ans-21) LHS ⇒ let $4x = C$ and $2x = D$

$\Rightarrow \dfrac{\cos 3x + \cos C + \cos D}{\sin C + \sin 3x + \sin D}$

$\Rightarrow \dfrac{\cos 3x + 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)}{\sin 3x + 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)}$ $\left\{ \begin{array}{l} \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \\ \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \end{array} \right\}$

$\Rightarrow \dfrac{\cos 3x + 2\cos\left(\frac{4x+2x}{2}\right) \cdot \cos\left(\frac{4x-2x}{2}\right)}{\sin 3x + 2\sin\left(\frac{4x+2x}{2}\right) \cdot \cos\left(\frac{4x-2x}{2}\right)}$

$\Rightarrow \dfrac{\cos 3x + 2\cos 3x \cdot \cos x}{\sin 3x + 2\sin 3x \cdot \cos x}$

$\Rightarrow \dfrac{\cos 3x(1 + 2\cos x)}{\sin 3x(1 + 2\cos x)}$

$\Rightarrow \dfrac{\cos 3x}{\sin 3x} \Rightarrow \cot 3x$ Hence, LHS = RHS

(Que 22) $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

(Ans-22) LHS ⇒ $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$

$\Rightarrow \cot x \cot 2x - \cot 3x(\cot 2x + \cot x)$

$\Rightarrow \cot x \cdot \cot 2x - \cot(2x + x)(\cot 2x + \cot x)$

$\Rightarrow \cot x \cot 2x - \left[\dfrac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$ $\left\{ \cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B} \right\}$

$\Rightarrow \cot x \cot 2x - (\cot 2x \cot - 1)$

$\Rightarrow \cot x \cot 2x - \cot 2x \cot + 1$

$\Rightarrow 1$

Hence, LHS = RHS

(Que 23) $\tan 4x = \dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

(Ans-23) LHS ⇒ $\tan 2\cdot(2x)$

$\Rightarrow \dfrac{2\tan 2x}{1-\tan^2 2x}$ $\left\{\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}\right\}$

$\Rightarrow \dfrac{2\times\left(\dfrac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$ $\left\{\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}\right\}$

$\Rightarrow \dfrac{\dfrac{4\tan x}{1-\tan^2 x}}{1-\left(\dfrac{2\tan}{1-\tan^2 x}\right)^2}$

$\Rightarrow \dfrac{\dfrac{4\tan x}{1-\tan^2 x}}{\dfrac{(1-\tan^2 x)^2-(2\tan x)^2}{(1-\tan^2 x)^2}}$

$\Rightarrow \dfrac{4\tan x(1-\tan^2 x)}{(1)^2+(\tan^2 x)^2-2\tan^2 x-4\tan^2 x}$

$\Rightarrow \dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$ Hence, LHS = RHS

(Que 24) $\cos 4x = 1-8\sin^2 x\cos^2 x$

(Ans-24) LHS ⇒ $\cos 2\times 2x$

$1-2\sin^2 2x$ $\{\cos 2x = 1-2\sin^2 x\}$

$1-2(2\sin x\cos x)^2$ $\{\sin 2A = 2\sin A\cdot\cos A\}$

$1-2\times 4\sin^2 x\cdot\cos^2 x$

$1-8\sin^2 x\cdot\cos^2 x$

Hence, LHS = RHS

(Que 25) $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

(Ans-25) LHS $\Rightarrow \cos 6x$

$\Rightarrow \cos 3(2x)$

$= 4\cos^3 2x - 3\cos 2x$ $\quad \{\cos 3A = 4\cos^3 A - 3\cos A\}$

$= 4\{(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)\}$ $\quad \{\cos 2x = 2\cos^2 x - 1\}$

$= 4\{(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x)\} - 6\cos^2 x + 3$

$= 4(8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x) - 6\cos^2 x + 3$

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Hence, LHS = RHS

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