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Chapter-4

Complex Numbers And Quadratic Equations

Complex number → A number of the form $a+ib$, where a and b are real numbers, is defined to be a complex number.

Ex → $2+i3$, $(-1)+i\sqrt{3}$, $4+i\left(\frac{-18}{11}\right)$, etc.

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$z^{-1} = \frac{\bar{z}}{|z|^2}$

$|z| = \sqrt{(a)^2+(b)^2}$

$\bar{z} = \overline{a+ib}$

Ex-4.1

Express each of the complex number given in the Exercises 1 to 10 in the form $a+ib$.

1) $(5i)\left(-\frac{3}{5}i\right)$

(Ans-1) $(5i)\left(-\frac{3}{5}i\right)$

$5 \times -\frac{3}{5} i^2$

-3×-1

3 Ans $= 3+0i$

2) $i^9 + i^{19}$

Ans-2) $i^8 \times i + i^{16} \times i^3$

$(i^4)^2 \times i + (i^4)^4 \times (-i)$

$(1)^2 \times i + (1)^4 \times (-i)$

$1i - 1i$

$0 + 0i$ Ans

3) i^{-39}

(Ans-3) $\frac{1}{i^{39}}$

$\Rightarrow \frac{1}{i^{36} \times i^3}$

$\Rightarrow \frac{1}{(i^4)^9 \times (-i)}$

$\Rightarrow \frac{1}{(1)^9 \times (-i)}$

$\Rightarrow \frac{1}{-1i}$

$\Rightarrow \frac{1}{-1i} \times \frac{1i}{1i}$

$\Rightarrow \frac{1i}{-1i^2}$

$\Rightarrow \frac{1i}{1}$

$\Rightarrow 0+1i$ Ans

4) $3(7+i7)+i(7+i7)$

Ans-4) $3(7+i7)+i(7+i7)$

$21+21i+7i+7i^2$

$21+28i-7$

$14+28i$ Ans

5) $(1-i)-(-1+i6)$

(Ans-5) $1-i+1-i6$

$1+1-i6-i$

$2-7i$ Ans

6) $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Ans-6) $\frac{1}{5}+i\frac{2}{5}-4-i\frac{5}{2}$

$\left(\frac{1}{5}-\frac{4}{1}\right)+\left(\frac{2i}{5}-\frac{5i}{2}\right)$

$\left(\frac{1-20}{5}\right)+i\left(\frac{4-25}{10}\right)$

$\frac{-19}{5}+i\left(\frac{-21}{10}\right)$

$-\frac{19}{5}-\frac{21}{10}i$ Ans

7) $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Ans-7) $\left[\frac{1}{3}+\frac{7i}{3}+\frac{4}{1}+\frac{1i}{3}\right]+\frac{4}{3}-\frac{i}{1}$

$\left[\frac{1+12}{3}+\frac{7i+1i}{3}\right]+\frac{4}{3}-\frac{i}{1}$

$\frac{13}{3}+\frac{4}{3}+\frac{8i}{3}-\frac{i}{1}$

$\frac{17}{3}+\frac{8i-3i}{3}$

$\frac{17}{3}+\frac{5i}{3}$ Ans

8) $(1-i)^4$

(Ans 8) $\{(1-i)^2\}^2$

$(1+i^2-2i)^2$

$(1-1-2i)^2$

$(-2i)^2$

$4i^2$

4×-1

-4 Ans

$-4+0i$

9) $\left(\frac{1}{3}+3i\right)^3$

using identity $\{(a+b)^3 = a^3+b^3+3a^2b+3ab^2\}$

(Ans-9) $\frac{1}{27}+27i^3+\left(3\times\frac{1}{3}\times3i\right)+\left(3\times\frac{1}{3}\times9i^2\right)$

$\frac{1}{27}+27i^3+i+9i^2$

$\frac{1}{27}+(-27i)+i-9$

$\frac{1}{27}-\frac{9}{1}+i-27i$

$\frac{1-243}{27}-26i$

$-\frac{242}{27}-26i$ Ans

10) $\left(-2-\frac{1}{3}i\right)^3$

Ans-10) $\left\{(-1)\left(2+\frac{1}{3}i\right)\right\}^3$

$(-1)^3 \times \left(2+\frac{1}{3}i\right)^3$ $\quad \{(a\times b)^3 = a^3 \times b^3\}$

$-1 \times \left(8+\frac{1}{27}i^3+8\times4\times\frac{1}{3}i+3\times2\times\frac{1}{3}i^2\right)$ $\quad \{(a+b)^3 = a^3+b^3+3a^2b+3ab^2\}$

$-\left(8+\frac{1}{27}i^3+4i+\frac{2}{3}i^2\right)$

$-\left(8-\frac{1}{27}i+4i-\frac{2}{3}\right)$

$-\left(\frac{24-2}{3}+\frac{108i-1i}{27}\right)$

$-\left(\frac{22}{3}+\frac{107}{27}i\right)$

$-\frac{22}{3}-\frac{107}{27}i$

Find the multiplicative inverse of each of the complex numbers given in the Exercises 11 to 13.

11) $4-3i$

Ans-11) $|z| = \sqrt{(4)^2+(-3)^2}$
$= \sqrt{16+9}$
$= \sqrt{25}$
$= 5$

$\bar{z} = \overline{4-3i}$
$= 4+3i$

$z^{-1} = \frac{\bar{z}}{|z|^2}$
$= \frac{4+3i}{25}$
$= \frac{4}{25}+\frac{3i}{25}$

12) $\sqrt{5} + 3i$

Ans-12) $|z| = \sqrt{(\sqrt{5})^2 + (3)^2}$

$= \sqrt{5+9}$

$= \sqrt{14}$

$\bar{z} = \overline{\sqrt{5} + 3i}$

$= \sqrt{5} - 3i$

$z^{-1} = \frac{\bar{z}}{|z|^2}$

$= \frac{\sqrt{5} - 3i}{14}$

$= \frac{\sqrt{5}}{14} - \frac{3i}{14}$ Ans

13) $-i$

Ans-13) $|z| = \sqrt{(0)^2 + (-1)^2}$

$= \sqrt{0+1}$

$= \sqrt{1}$

$= 1$

$\bar{z} = \overline{0 + -1i}$

$= 0 + 1i$

$z^{-1} = \frac{\bar{z}}{|z|^2}$

$= \frac{0+1i}{1}$

$= 0 + 1i$ Ans

Que 14) Express the following expression in the form of $a+ib$:

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

(Ans-14) $\Rightarrow \frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+i\sqrt{2}}$ $\left\{(a+b)(a-b)=a^2-b^2\right\}$

$\Rightarrow \frac{9-5i^2}{\sqrt{2}i+\sqrt{2}i}$

$\Rightarrow \frac{9+5}{2\sqrt{2}i}$

$\Rightarrow \frac{14}{2\sqrt{2}i}$

$\Rightarrow \frac{7}{\sqrt{2}i}$

$\Rightarrow \frac{7}{\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i}$

$\Rightarrow \frac{7\sqrt{2}i}{2i^2}$

$\Rightarrow \frac{7\sqrt{2}i}{-2}$

$\Rightarrow 0-\frac{7\sqrt{2}i}{2}$ Ans

Akshay
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