Ch-7

BINOMIAL THEOREM

* The expansion of a binomial for any positive integral n is given by Binomial Theorem, which is
$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} b + {}^nC_2\, a^{n-2} b^2 + \dots + {}^nC_{n-1}\, a \cdot b^{n-1} + {}^nC_n\, b^n$.

* The coefficients of the expansions are arranged in an array. This array is called Pascal's triangle.

Ex-7.1

Expand each of the expressions in Exercises 1 to 5.

1) $(1-2x)^5$

Ans-1) $(a-b)^n = {}^nC_0 a^{n-0} b^0 - {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 - {}^nC_3 a^{n-3} b^3 + {}^nC_4 a^{n-4} b^4 - {}^nC_5 a^{n-5} b^5$

$a = 1$, $b = 2x$, $n = 5$

$\rightarrow (1-2x)^5 = {}^5C_0 (1)^{5-0} (2x)^0 - {}^5C_1 (1)^{5-1} (2x)^1 + {}^5C_2 (1)^{5-2} (2x)^2 - {}^5C_3 (1)^{5-3} (2x)^3 + {}^5C_4 (1)^{5-4} (2x)^4 - {}^5C_5 (1)^{5-5} (2x)^5$

$\rightarrow (1-2x)^5 = 1 \times (1)^5 \times 1 - \frac{5}{1} \times (1)^4 \times 2x + \frac{5\times \cancel{4}^2}{\cancel{2}\times 1} \times (1)^3 \times (4x^2) - \frac{5\times \cancel{4}^2 \times \cancel{3}}{\cancel{3}\times\cancel{2}\times 1} \times (1)^2 \times (8x^3)$

$+ \frac{5\times\cancel{4}\times\cancel{3}\times\cancel{2}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1} \times 1 \times (16x^4) - 1 \times 1 \times 32x^5$

$\rightarrow (1-2x)^5 = 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$ Ans

2) $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

Ans-2) $(a-b)^n = {}^nC_0 a^{n-0} b^0 - {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 - {}^nC_3 a^{n-3} b^3 + {}^nC_4 a^{n-4} b^4 - {}^nC_5 a^{n-5} b^5$

$a = \frac{2}{x}$, $b = \frac{x}{2}$, $n = 5$

$\rightarrow \left(\frac{2}{x} - \frac{x}{2}\right)^5 = {}^5C_0 a^{5-0} b^0 - {}^5C_1 a^{5-1} b^1 + {}^5C_2 a^{5-2} b^2 - {}^5C_3 a^{5-3} b^3 + {}^5C_4 a^{5-4} b^4 - {}^5C_5 a^{5-5} b^5$

$\rightarrow \left(\frac{2}{x} - \frac{x}{2}\right)^5 = 1 \times \left(\frac{2}{x}\right)^5 \times 1 - \frac{5}{1} \times \left(\frac{2}{x}\right)^4 \times \frac{x}{2} + \frac{5\times 4}{2\times 1} \times \left(\frac{2}{x}\right)^3 \times \left(\frac{x}{2}\right)^2 - \frac{5\times 4\times 3}{3\times 2\times 1} \times \left(\frac{2}{x}\right)^2 \times \left(\frac{x}{2}\right)^3$

$+ \frac{5\times 4\times 3\times 2}{4\times 3\times 2\times 1} \times \frac{2}{x} \times \left(\frac{x}{2}\right)^4 - 1 \times \left(\frac{2}{x}\right)^0 \times \left(\frac{x}{2}\right)^5$

$\rightarrow \left(\frac{2}{x} - \frac{x}{2}\right)^5 = \frac{32}{x^5} - 5 \times \frac{\cancel{16}^8}{x^4} \times \frac{x}{\cancel{2}} + 10 \times \frac{\cancel{8}^2}{x^3} \times \frac{x^2}{\cancel{4}} - \cancel{10}^5 \times \frac{\cancel{4}}{x^2} \times \frac{x^3}{\cancel{8}_2} + \frac{5\times\cancel{2}}{x} \times \frac{x^4}{\cancel{16}_8}$

$- 1 \times \frac{x^5}{32}$

$\rightarrow \left(\frac{2}{x} - \frac{x}{2}\right)^5 = \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$ Ans

3) $(2x-3)^6$

Ans-3) $(a-b)^n = {}^nC_0\, a^{n-0} b^0 - {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 - {}^nC_3\, a^{n-3} b^3 + {}^nC_4\, a^{n-4} b^4 - {}^nC_5\, a^{n-5} b^5 + {}^nC_6\, a^{n-6} b^6$

$a = 2x \quad , b = 3 \quad , n = 6$

$\rightarrow (2x-3)^6 = {}^6C_0 (2x)^{6-0} (3)^0 - {}^6C_1 (2x)^{6-1} (3)^1 + {}^6C_2 (2x)^{6-2} (3)^2 - {}^6C_3 (2x)^{6-3} (3)^3 + {}^6C_4 (2x)^{6-4} (3)^4 - {}^6C_5 (2x)^{6-5} (3)^5 + {}^6C_6 (2x)^{6-6} (3)^6$

$\rightarrow (2x-3)^6 = 1 \times 64x^6 \times 1 - \frac{6}{1} \times 32x^5 \times 3 + \frac{\overset{3}{\cancel{6}} \times 5}{\cancel{2} \times 1} \times 16x^4 \times 9 - \frac{\cancel{6} \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1} \times 8x^3 \times 27 + \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4} \times \cancel{3}}{\cancel{4} \times \cancel{3} \times 2 \times 1} \times 4x^2 \times 81 - \frac{6 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times 2x \times 243 + 1 \times 1 \times 729$

$\rightarrow (2x-3)^6 = 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$ Ans

4) $\left(\frac{x}{3} + \frac{1}{x}\right)^5$

Ans-4) $(a+b)^n = {}^nC_0\, a^{n-0} b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + {}^nC_3\, a^{n-3} b^3 + {}^nC_4\, a^{n-4} b^4 + {}^nC_5\, a^{n-5} b^5$

$a = \frac{x}{3} \quad , b = \frac{1}{x} \quad , n = 5$

$\rightarrow \left(\frac{x}{3} + \frac{1}{x}\right)^5 = {}^5C_0 \times \left(\frac{x}{3}\right)^{5-0} \times \left(\frac{1}{x}\right)^0 + {}^5C_1 \times \left(\frac{x}{3}\right)^{5-1} \times \left(\frac{1}{x}\right)^1 + {}^5C_2 \times \left(\frac{x}{3}\right)^{5-2} \times \left(\frac{1}{x}\right)^2 + {}^5C_3 \times \left(\frac{x}{3}\right)^{5-3} \times \left(\frac{1}{x}\right)^3 + {}^5C_4 \times \left(\frac{x}{3}\right)^{5-4} \times \left(\frac{1}{x}\right)^4 + {}^5C_5 \times \left(\frac{x}{3}\right)^{5-5} \times \left(\frac{1}{x}\right)^5$

$\rightarrow \left(\frac{x}{3} + \frac{1}{x}\right)^5 = 1 \times \frac{x^5}{243} \times 1 + \frac{5}{1} \times \frac{x^4}{81} \times \frac{1}{x} + \frac{5 \times \overset{2}{\cancel{4}}}{\cancel{2} \times 1} \times \frac{x^3}{27} \times \frac{1}{x^2} + \frac{5 \times \cancel{4} \times \cancel{3}}{\cancel{3} \times 2 \times 1} \times \frac{x^2}{9} \times \frac{1}{x^3} + \frac{5 \times \cancel{4} \times \cancel{3} \times \cancel{2}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times \frac{x}{3} \times \frac{1}{x^4} + 1 \times 1 \times \frac{1}{x^5}$

$\rightarrow \left(\frac{x}{3} + \frac{1}{x}\right)^5 = \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10}{27}x + \frac{10}{9}x + \frac{5}{3x^3} + \frac{1}{x^5}$

5) $\left(x+\frac{1}{x}\right)^6$

Ans-5) $(a-b)^n = {}^nC_0\, a^{n-0} b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + {}^nC_3\, a^{n-3} b^3 + {}^nC_4\, a^{n-4} b^4 + {}^nC_5\, a^{n-5} b^5 + {}^nC_6\, a^{n-6} b^6$

$a = x \;,\; b = \frac{1}{x} \;,\; n = 6$

$\rightarrow \left(x+\frac{1}{x}\right)^6 = {}^6C_0 \times (x)^{6-0} \times \left(\frac{1}{x}\right)^0 + {}^6C_1 \times (x)^{6-1} \times \left(\frac{1}{x}\right)^1 + {}^6C_2 \times (x)^{6-2} \times \left(\frac{1}{x}\right)^2 + {}^6C_3 \times (x)^{6-3} \times \left(\frac{1}{x}\right)^3 + {}^6C_4 \times (x)^{6-4} \times \left(\frac{1}{x}\right)^4 + {}^6C_5 \times (x)^{6-5} \times \left(\frac{1}{x}\right)^5 + {}^6C_5 \times (x)^{6-6} \times \left(\frac{1}{x}\right)^6$

$\rightarrow \left(x+\frac{1}{x}\right)^6 = 1 \times x^6 \times 1 + \frac{6}{1} \times x^5 \times \frac{1}{x} + \frac{\overset{3}{\cancel{6}} \times 5}{\cancel{2} \times 1} \times x^4 \times \frac{1}{x^2} + \frac{6 \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1} \times \cancel{x^3} \times \frac{1}{\cancel{x^3}} + \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4} \times \cancel{3}}{\cancel{4} \times \cancel{3} \times 2 \times 1} \times x^2 \times \frac{1}{x^4} + \frac{6 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times x \times \frac{1}{x^5} + 1$

$\rightarrow \left(x+\frac{1}{x}\right)^6 = x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$ Ans

Using binomial theorem, evaluate each of the following:

6) $(96)^3$

(Ans-6) $(96)^3 = (100-4)^3$

$(a-b)^n = {}^nC_0\, a^{n-0} b^0 - {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 - {}^nC_3\, a^{n-3} b^3$

$a = 100 \;,\; b = 4 \;,\; n = 3$

$\rightarrow (100-4)^3 = {}^3C_0 \times (100)^{3-0} \times (4)^0 - {}^3C_1 \times (100)^{3-1} \times (4)^1 + {}^3C_2 \times (100)^{3-2} \times (4)^2 - {}^3C_3 \times (100)^{3-3} \times (4)^3$

$\rightarrow (100-4)^3 = 1 \times (100)^3 \times 1 - \frac{3}{1} \times (100)^2 \times 4 + \frac{3 \times 2}{2 \times 1} \times 100 \times 16 - 1 \times (100)^0 \times 64$

$\rightarrow (100-4)^3 = 1000000 - 120000 + 4800 - 64$

$\rightarrow (100-4)^3 = 1004800 - 120064$

$\rightarrow (100-4)^3 = 884736$ Ans

7) $(102)^5$

Ans-7) $(102)^5 = (100+2)^5$

$$(a+b)^n = {}^nC_0\, a^{n-0} b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + {}^nC_3\, a^{n-3} b^3 + {}^nC_4\, a^{n-4} b^4 + {}^nC_5\, a^{n-5} b^5$$

$a=100$, $b=2$, $n=5$

$$\rightarrow (100+2)^5 = {}^5C_0 \times (100)^{5-0} \times (2)^0 + {}^5C_1 \times (100)^{5-1} \times (2)^1 + {}^5C_2 \times (100)^{5-2} \times (2)^2 + {}^5C_3 \times (100)^{5-3} \times (2)^3 + {}^5C_4 \times (100)^{5-4} \times (2)^4 + {}^5C_5 \times (100)^{5-5} \times (2)^5$$

$$\rightarrow (100+2)^5 = 1 \times (100)^5 \times 1 + \frac{5}{1} \times (100)^4 \times 2 + \frac{5\times\cancel{4}^{2}}{\cancel{2}\times 1} \times (100)^3 \times 4 + \frac{5\times\cancel{4}^{2}\times\cancel{3}}{\cancel{3}\times\cancel{2}\times 1} \times (100)^2 \times 8 + \frac{5\times\cancel{4}\times\cancel{3}\times\cancel{2}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1} \times 100 \times 16 + 1 \times 1 \times 32$$

$$\rightarrow (100+2)^5 = 1000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$$

$$\rightarrow (100+2)^5 = 11040508032$$ Ans

8) $(101)^4$

Ans-8) $(100+1)^4 = (101)^4$

$$(a+b)^n = {}^nC_0\, a^{n-0} b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + {}^nC_3\, a^{n-3} b^3 + {}^nC_4\, a^{n-4} b^4$$

$a=100$, $b=1$, $n=4$

$$\rightarrow (100+1)^4 = {}^4C_0 \times (100)^{4-0} \times (1)^0 + {}^4C_1 \times (100)^{4-1} \times (1)^1 + {}^4C_2 \times (100)^{4-2} \times (1)^2 + {}^4C_3 \times (100)^{4-3} \times (1)^3 + {}^4C_4 \times (100)^{4-4} \times (1)^4$$

$$\rightarrow (100+1)^4 = 1 \times (100)^4 \times 1 + \frac{4}{1} \times (100)^3 \times 1 + \frac{\cancel{4}^{2}\times 3}{\cancel{2}\times 1} \times (100)^2 \times 1 + \frac{4\times\cancel{3}\times\cancel{2}}{\cancel{3}\times\cancel{2}\times 1} \times 100 \times 1 + 1 \times 1 \times 1$$

$$\rightarrow (100+1)^4 = 100000000 + 4000000 + 60000 + 400 + 1$$

$$\rightarrow (100+1)^4 = 104060401$$ Ans

9) $(99)^5$

Ans-9) $(99)^5 = (100-1)^5$

$(a+b)^n = {}^nC_0\, a^{n-0} b^0 - {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 - {}^nC_3\, a^{n-3} b^3 + {}^nC_4\, a^{n-4} b^4 - {}^nC_5\, a^{n-5} b^5$

$a = 100, \quad b = 1, \quad n = 4$

$\rightarrow (100-1)^5 = {}^5C_0 \times (100)^{5-0} \times (1)^0 - {}^5C_1 \times (100)^{5-1} \times (1)^1 + {}^5C_2 \times (100)^{5-2} \times (1)^2 - {}^5C_3 \times (100)^{5-3} \times (1)^3 +$
${}^5C_4 \times (100)^{5-4} \times (1)^4 - {}^5C_5 \times (100)^{5-5} \times (1)^5$

$\rightarrow (100-1)^5 = 1 \times (100)^5 \times 1 - \frac{5}{1} \times (100)^4 \times 1 + \frac{5 \times \cancel{4}^2}{\cancel{2} \times 1} \times (100)^3 \times 1 - \frac{5 \times \cancel{4}^2 \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 1} \times (100)^2 \times 1 +$

$\frac{5 \times \cancel{4 \times 3 \times 2}}{\cancel{4 \times 3 \times 2} \times 1} \times 100 \times 1 - 1 \times 1 \times 1$

$\rightarrow (100-1)^5 = 10000000000 - 500000000 + 10000000 - 100000 + 500 - 1$

$\rightarrow (100-1)^5 = 10010000500 - 500100001$

$\rightarrow (100-1)^5 = 9509900499$ Ans

Que 10) Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Ans-10) $(1.1)^{10000} = (1+0.1)^{10000}$

- $(a+b)^n = {}^nC_0\, a^{n-0} b^0 + {}^nC_1\, a^{n-1} b^1 + \dots$
- $a = 1, \quad b = 0.1, \quad n = 10000$

$\Rightarrow 1 \times (1)^{10000-0} \times (0.1)^0 + {}^{10000}C_1 \times (1)^{10000-1} \times (0.1)^1 + \dots$

$\Rightarrow 1 + 10000 \times 1 \times 0.1 + \dots$

$\Rightarrow 1 + 1000 + \dots$

$\Rightarrow 1001 + \dots$

Therefore $1000 < (1.1)^{10000}$

Que11) Find $(a+b)^4-(a-b)^4$. Hence, evaluate $(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4$.

(Ans-11) $(a+b)^4 = {}^4C_0\, a^{4-0}b^0 + {}^4C_1\, a^{4-1}b^1 + {}^4C_2\, a^{4-2}b^2 + {}^4C_3\, a^{4-3}b^3 + {}^4C_4\, a^{4-4}b^4$

$(a+b)^4 = 1\times a^4\times 1 + \frac{4}{1}\times a^3b^1 + \frac{4\times 3}{2\times 1}\times a^2b^2 + \frac{4\times 3\times 2}{3\times 2\times 1}\times ab^3$

$+ 1\times a^0\times b^4$

$(a+b)^4 = \cancel{a^4} + 4a^3b + \cancel{6a^2b^2} + 4ab^3 + \cancel{b^4}$

$-(a-b)^4 = \cancel{a^4} \mp 4a^3b \pm \cancel{6a^2b^2} \mp 4ab^3 \pm \cancel{b^4}$

$(a+b)^4-(a-b)^4 = 8a^3b + 8ab^3$

$\Rightarrow (a+b)^4-(a-b)^4 = 8ab(a^2+b^2)$

$\Rightarrow (\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4 = 8\times\sqrt{3}\times\sqrt{2}\,[(\sqrt{3})^2+(\sqrt{2})^2]$

$\Rightarrow (\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4 = 8\sqrt{6}\,(3+2)$

$\Rightarrow (\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4 = 8\sqrt{6}\times 5$

$\Rightarrow (\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4 = 40\sqrt{6}$ Ans

Que12) Find $(x+1)^6+(x-1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$.

(Ans-12) $(x+1)^6 = {}^6C_0\, a^{6-0}b^0 + {}^6C_1\, a^{6-1}b^1 + {}^6C_2\, a^{6-2}b^2 + {}^6C_3\, a^{6-3}b^3 + {}^6C_4\, a^{6-4}b^4$

$+ {}^6C_5\, a^{6-5}b^5 + {}^6C_6\, a^{6-6}b^6$

$(x+1)^6 = 1\times a^6\times 1 + 6\times a^5b + \frac{6\times 5}{2\times 1}a^4b^2 + \frac{6\times 5\times 4}{3\times 2\times 1}\times a^3b^3 +$

$\frac{6\times 5\times 4\times 3}{4\times 3\times 2\times 1}\times a^2b^4 + \frac{6\times 5\times 4\times 3\times 2}{5\times 4\times 3\times 2\times 1}\times ab^5 + 1\times a^0\times b^6$

$(x+1)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

$+\ (x-1)^6 = a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6a^5 + b^6$

$(x+1)^6+(x-1)^6 = 2a^6 + 30a^4b^2 + 30a^2b^4 + 2b^6$

$= 2a^4(a^2+15b^2) + 2b^4(15a^2+b^2)$

Que 13) Show that $9^{n+1}-8n-9$ is divisible by 64, whenever n is a positive integer.

Ans-13) 9^{n+1}

$(1+8)^{n+1} = {}^{n+1}C_0 \times (1)^{n+1-0} \times 8^0 + {}^{n+1}C_1 \times (1)^{n+1-1} \times 8^1 + {}^{n+1}C_2 \times (1)^{n+1-2} \times 8^2 + {}^{n+1}C_3 \times (1)^{n+1-3} \times 8^3 + {}^{n+1}C_4 \times (1)^{n+1-4} \times 8^4 + \dots$

$(1+8)^{n+1} = 1 \times 1 \times 1 + (n+1) \times 1 \times 8 + 8^2 \left[{}^{n+1}C_2 \times 1 + {}^{n+1}C_3 \times 1 \times 8 + {}^{n+1}C_4 \times 1 \times 8^2 + \dots\right]$

$(1+8)^{n+1} = 1 + 8n + 8 + 64\left[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + {}^{n+1}C_4 \times 8^2 + \dots\right]$

$9^{n+1} = 8n + 9 + 64\left[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + {}^{n+1}C_4 \times 8^2 + \dots\right]$

$9^{n+1} - 8n - 9 = 64\left[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + {}^{n+1}C_4 \times 8^2 + \dots\right]$

Que 12) Find $(x+1)^6 + (x-1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$.

Ans-12) $(x+1)^6 = {}^6C_0 \times x^{6-0} \times 1^0 + {}^6C_1 \times x^{6-1} \times (1)^1 + {}^6C_2 \times x^{6-2} \times (1)^2 + {}^6C_3 \times x^{6-3} \times (1)^3 + {}^6C_4 \times x^{6-4} \times (1)^4 + {}^6C_5 \times x^{6-5} \times (1)^5 + {}^6C_6 \times x^{6-6} \times (1)^6$

$(x+1)^6 = 1 \times x^6 \times 1 + 6 \times x^5 \times 1 + \frac{\overset{3}{\cancel{6}} \times 5}{\cancel{2} \times 1} \times x^4 \times 1 + \frac{\cancel{6} \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1} \times x^3 \times 1 +$

$\frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4} \times \cancel{3}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times x^2 \times 1 + \frac{6 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times x \times 1 + 1 \times 1 \times 1$

$(x+1)^6 = x^6 + \cancel{6x^5} + 15x^4 + \cancel{20x^3} + 15x^2 + \cancel{6x} + 1$

$+ (x-1)^6 = x^6 - \cancel{6x^5} + 15x^4 - \cancel{20x^3} + 15x^2 - \cancel{6x} + 1$

$(x+1)^6 + (x-1)^6 = 2x^6 + 30x^4 + 30x^2 + 2$

$(x+1)^6 + (x-1)^6 = 2(x^6 + 15x^4 + 15x^2 + 1)$ Ans

$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = 2\left((\sqrt{2})^6 + 15 \times (\sqrt{2})^4 + 15 \times (\sqrt{2})^2 + 1\right)$

$= 2(8 + 60 + 30 + 1)$

$= 2 \times 99$

$= 198$ Ans

(Que 14) Prove that $\sum_{r=0}^{n} 3^r \; {}^nC_r = 4^n$.

(Ans-14) $\sum_{r=0}^{n} 3^r \; {}^nC_r = 4^n$

$= {}^nC_0 3^0 + {}^nC_1 3^1 + {}^nC_2 3^2 + {}^nC_3 3^3 + \dots\dots + {}^nC_n 3^n$

$= {}^nC_0 (1)^{n-0} 3^0 + {}^nC_1 (1)^{n-1} 3^1 + {}^nC_2 (1)^{n-2} 3^2 + {}^nC_3 (1)^{n-3} 3^3 + \dots\dots + {}^nC_n (1)^{n-n} 3^n$

$= (1+3)^n \qquad \{ (1+b)^n = {}^nC_0 b^0 + {}^nC_1 b^1 + {}^nC_2 b^2 + \dots + {}^nC_n b^n \}$

$= 4^n$

Hence proved