

Ch-2Relations And FunctionsCartesian Product of sets

Given two non-empty sets P and Q . The cartesian product $P \times Q$ is the set of all ordered pairs of elements from P and Q .

$$P \times Q = \{(p, q) : p \in P, q \in Q\}$$

Date: _____ Page No. _____

Ex-2.1

(Ques 1) If $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$, find the value of x and y .

(Ans 1) $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$

$$\frac{x}{3} + 1 = \frac{5}{3}, y - \frac{2}{3} = \frac{1}{3}$$

$$\frac{x+3}{3} = \frac{5}{3}, \frac{3y-2}{3} = \frac{1}{3}$$

$$x+3 = 5, 3y-2 = 1$$

$$\boxed{x=2}, 3y = 3$$

$$\boxed{y=1}$$

(Ques 2) If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of element in (A × B).

(Ans 2) ~~A = 3 elements~~

$$B = \{3, 4, 5\} = 3 \text{ elements}$$

$$A \times B = 9 \text{ elements}$$

(Ques 3) If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

(Ans 3) $G = \{7, 8\}$

$$H = \{5, 4, 2\}$$

$$G \times H$$

$$\{7, 8\} \times \{5, 4, 2\}$$

$$\Rightarrow \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$$

$$H \times G$$

$$\{5, 4, 2\} \times \{7, 8\}$$

$$\Rightarrow \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$$

(Ques 4) State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.

① If $P = \{m, n\}$ and $Q = \{n, m\}$, then $P \times Q = \{(m, n), (n, m)\}$.

(Ans-1) False , $P \times Q = \{(m, n), (m, m), (n, n), (n, m)\}$

② If A and B are non-empty sets, then $A \times B$ is a non-empty set of ordered pair (x, y) such that $x \in A$ and $y \in B$

(Ans-2) True

③ If $A = \{1, 2\}$, $B = \{3, 4\}$, then $A \times (B \cap \emptyset) = \emptyset$

(Ans-3) True

(Ques 4) If $A = \{-1, 1\}$, find $A \times A \times A$

(Ans-4) $A \times A \times A$

$$\{-1, 1\} \times \{-1, 1\} \times \{-1, 1\}$$

$$\{(-1, -1), (-1, 1), (1, -1), (1, 1)\} \times \{-1, 1\}$$

$$\{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$$

(Ques 5) If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B.

(Ans-5) $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

$$A = \{a, b\} \quad B = \{x, y\}$$

(Ques 6) Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that

① $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(Ans-6) ~~$A \times (B \cap C)$~~

$$A \times (\{1, 2, 3, 4\} \cap \{5, 6\}) = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\} \cap \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

$$A \times \emptyset = \emptyset$$

$$\emptyset = \emptyset$$

Hence verified

(ii) $A \times C$ is a subset of $B \times D$

(Ans-ii) $A \times C \subset B \times D$

$$\{1, 2\} \times \{5, 6\} \subset \{1, 2, 3, 4\} \times \{5, 6, 7, 8\}$$

$$\{(1, 5), (1, 6), (2, 5), (2, 6)\} \subset \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$$

Hence, verified

(Ques 8) Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have? List them.

(Ans-8) $A = \{1, 2\} \quad B = \{3, 4\}$

$$A \times B = \{1, 2\} \times \{3, 4\}$$

$$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

$$n(A \times B) = 2^{\underline{n}} 2^n$$

$$= 2^4$$

$$= 16$$

(Ques 9) Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B , where x, y and z are distinct elements.

(Ans-9) $n(A) = 3 \quad n(B) = 2$

$$A \times B = \{(x, 1), (y, 2), (z, 1)\}$$

$$A = \{x, y, z\} \quad B = \{1, 2\}$$

(Ques 10) The Cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.

$$(A \times A - 10) \quad A \times A = 9$$

$$A^2 = 9$$

A = 3 elements

$$A \times A = (-1, 0), (0, 1)$$

$$A = \{-1, 0, 1\}$$

$$A \times A$$

$$\{-1, 0, 1\} \times \{-1, 0, 1\}$$

$$\{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)\}$$

Remaining element

$$\Rightarrow (-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1) \cancel{\in}$$

Ex-2.2

(Que 1) Let $A = \{1, 2, 3, \dots, 14\}$. Define a relation R from A to A by $R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in A\}$. Write down its domain, codomain and range.

(Ans-1) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

Here in relation $\Rightarrow 3x - y = 0$
 $3x = y$

So, $R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

Domain of $R = \{1, 2, 3, 4\}$

Range of $R = \{3, 6, 9, 12\}$

Codomain of $R = \{1, 2, 3, \dots, 14\}$

(Que 2) Define a relation R on the set N of natural numbers by $R = \{(x, y) : y = x+5, x \text{ is a natural number less than } 4; x, y \in N\}$. Depict this relationship using roster form. Write down the domain and the range.

(Ans-2) $R = \{(x, y) : y = x+5, x > 1, x, y \in N\}$

$R = \{(1, 6), (2, 7), (3, 8)\}$

Domain = $\{1, 2, 3\}$

Range = $\{6, 7, 8\}$

(Que 3) $A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by $R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$. Write R in roster form.

(Ans-3) $A = \{1, 2, 3, 5\} \quad B = \{4, 6, 9\}$

$R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$

$R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

(Que 4) The Fig. 2.7 shows a relationship between the sets P and Q. Write this relation

(i) in set-builder form

(Ans-i) $R = \{(x, y) : x = y + 2, 5 \leq x \leq 7, x \in \mathbb{N}\}$

(ii) roster form

(Ans-ii) $R = \{(5, 3), (6, 4), (7, 5)\}$

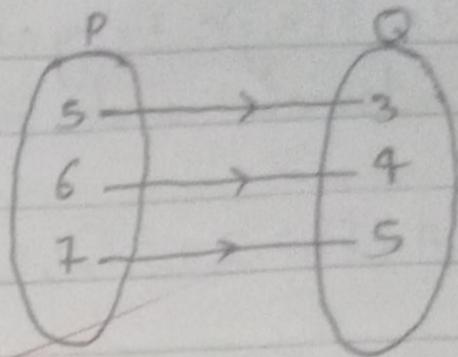


Fig 2.7

What is its domain and range?

Domain = {5, 6, 7}

Range = {3, 4, 5}

(Que 5) Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$.

i) Write R in roster form

(Ans-i) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$

ii) Find the domain of R

(Ans-ii) Domain = {1, 2, 3, 4, 6}

iii) Find the range of R

(Ans-iii) Range = {1, 2, 3, 4, 6}

(Que 6) Determine the domain and range of the relation R defined by $R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$.

Date _____
Page no. _____

d Q. (Ans-6) $R = \{(n, n+5) : n \in \{0, 1, 2, 3, 4, 5\}\}$
 $R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$
Domain = $\{0, 1, 2, 3, 4, 5\}$
Range = $\{5, 6, 7, 8, 9, 10\}$

(Ans-7) Write the relation $R = \{(n, n^3) : n \text{ is a prime number less than } 10\}$ in roster form.

(Ans-7) ~~Given~~ $R = \{(n, n^3) : n \text{ is a prime number less than } 10\}$
Prime number less than 10 $\Rightarrow 2, 3, 5, 7$
 $R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

(Ans-8) Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A to B.

(Ans-8) $A = \{x, y, z\}$ $B = \{1, 2\}$
 ~~$A \times B = \{(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)\}$~~

Since $n(A \times B) = 6$, the number of subsets of $A \times B$ is 2^6 . Therefore, the number of relations from A into B will be 2^6 .

(Ans-9) Let R be the relation on Z defined by $R = \{(a, b) : a, b \in Z, a-b \text{ is an integer}\}$. Find the domain and range of R.

(Ans-9) ~~$R = \{(a, b) : a, b \in Z, a-b \text{ is an integer}\}$~~

Here, $a-b$ is an integer. Therefore

Domain = Z

Range = Z

Ex-2.3

(Que 1) Which of the following relations are functions? Give reasons.
If it is a function, determine its domain and range.

i) $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

(Ans-i) Yes, because every element of set A has one and only one image in set B. Domain = $\{2, 5, 8, 11, 14, 17\}$ Range = $\{1\}$

ii) $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$

(Ans-ii) Yes, because every element of set A has one and only one image in set B. Domain = $\{2, 4, 6, 8, 10, 12, 14\}$ Range = $\{1, 2, 3, 4, 5, 6, 7\}$

iii) $\{(1, 3), (1, 5), (2, 5)\}$.

(Ans-iii) No, because every element of set A has ^{not} one and only one image in set B.

(Que 2) Find the domain and range of the following real functions.

i) $f(x) = -|x|$

(Ans-i) Here, if we put any real number on the place of x then its equation going to be correct.

And after solving this equation our answer is only going to be negative real number.

Therefore,

$$\text{Domain} = \mathbb{R}$$

$$\text{Range} = \mathbb{R}^-$$

ii)
Ans-ii)

(Que 3)

i)

(Ans-i)

(Que 4)

i)

(Ans-i)

(ii) $f(x) = \sqrt{9-x^2}$

Answ)

$$0 \leq 9-x^2$$

$$0 \leq 3^2-x^2$$

$$0 \leq (3+x)(3-x)$$

$$0 \leq 3+x \quad 0 \leq 3-x$$

~~$x \geq -3 \quad 3 \geq x$~~

$$\text{Domain} \Rightarrow [-3, 3]$$

$$y = \sqrt{9-x^2}$$

$$y = \sqrt{9-(-3)^2}$$

$$y = \sqrt{9-9}$$

$$y = \sqrt{0}$$

$$y = 0$$

$$y = \sqrt{9-x^2}$$

$$y = \sqrt{9-0}$$

$$y = \sqrt{9}$$

$$y = 3$$

$$\text{Range} \Rightarrow 0 \leq y \leq 3 \Rightarrow$$

(Ques 3) A function f is defined by $f(x) = 2x-5$. Write down the value of

i) $f(0)$

(Ans-i) $f(0) = 2 \times 0 - 5$

$$= 0 - 5$$

$$= -5$$

ii) $f(7)$

(Ans-ii) $f(7) = 2 \times 7 - 5$

$$= 14 - 5$$

$$= 9$$

iii) $f(-3)$

(Ans-iii) $f(-3) = 2 \times -3 - 5$

$$= -6 - 5$$

$$= -11$$

(Ques 4) The function t which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$

Find -

i) $t(0)$

(Ans-i) $t(0) = \frac{9 \times 0}{5} + 32$

$$= \frac{0}{5} + 32$$

$$= 32$$

ii) $t(28)$

(Ans-ii) $t(28) = \frac{9 \times 28}{5} + 32$

$$= \frac{252}{5} + 32$$

$$= \frac{252+160}{5} \Rightarrow \frac{412}{5}$$

(iii) $f(-10)$

$$(\text{Ans-iii}) f(-10) = \frac{9x-10}{5} + 32$$

$$= -\frac{90}{5} + 32$$

$$= -18 + 32$$

$$= 14 \quad (\text{Ans})$$

(iv) The value of c , when $f(c) = 212$

$$(\text{Ans-iv}) f(c) = \frac{9c}{5} + 32$$

$$212 = \frac{9c}{5} + 32$$

~~$$212 - 32 = \frac{9c}{5}$$~~

$$180 = \frac{9c}{5}$$

$$\frac{180 \times 5}{9} = c$$

$$100 = c$$

(Ques 5) Find

 $f(x) =$ (ii) $f(x)$ [Ans-ii] $x^2 \geq$ $x^2 + 2$ $f(x)$

Range

(iii) $f(x)$

(Ques 5) Find the range of each of the following functions:

(Ans-iii) Here

$$\textcircled{i} \quad f(x) = 2 - 3x, x \in \mathbb{R}, x > 0.$$

$$x > 0$$

$$3x > 0 \times 3$$

$$3x > 0$$

$$-3x < 0$$

$$-3x < 0$$

$$(2 - 3x) < 0 + 2$$

$$f(x) < 2$$

Range $(-\infty, 2)$ f

Then

(Ans) Find the range of each of the following functions:

i) $f(x) = 2 - 3x$, $x \in \mathbb{R}$, $x > 0$

ii) $f(x) = x^2 + 2$, x is a real number

Ans-ii) $x^2 \geq 0$

$x^2 + 2 \geq 0 + 2$

$f(x) \geq 2$

Range $[2, \infty)$

iii) $f(x) = x$, x is a real number

Ans-iii) Here, x is a real number. Therefore domain is
real number.

$f(x) = x$

Therefore, Range = real number