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Fuzzy Control of a Pendulum-Driven Cart

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***Abstract*—This paper proposes a new fuzzy control algorithm for the pendulum-driven cart system. The pendulum-driven cart is a novel mechanism that gives a new driving concept of ground mobile robot which employs only internal thrust and static friction. The system has two degrees of freedom, i.e. the pendulum angle and the cart position, but one control actuator mounted on the pivot of the pendulum. It aims to drive the cart to track a desired trajectory by applying the control torque properly. The proposed fuzzy control algorithm simplifies the control approach from the other closed-loop control algorithms, and presents more effective simulation results. Realization of the algorithm is discussed. Simulation results are presented for comparative purposes in order to demonstrate the effectiveness of the proposed algorithm. Experimental results are also presented in the paper by using the LEGO Mindstorms kit and the ROBOLAB 2.5 Software. The experimental results show that the pendulum-driven cart system is realizable, and the proposed fuzzy control algorithm is effective.**

I. Introduction

The pendulum-driven cart (PDC) is a novel mechanism that proposes a new driving concept of ground mobile robot which employs only internal thrust and static friction [1][2]. The PDC system aims to drive the cart to track a desired trajectory by applying the control torque on the inverted pendulum properly. It is an underactuated mechanical system (UMS) with two degrees of freedom and one actuator. In contrast to the classical cart-pole system [3], the control input of the PDC system is on the pendulum pivot, instead of the force on the cart. The difference between the classical cart-pole system and the PDC system is that the former addresses a stabilization problem while the later addresses a tracking problem. Furthermore, an outstanding character that distinguishes the PDC system from the other UMSs is that friction plays a dominant role in the PDC system, while it can be simplified or omitted in the other UMSs. Based on the study in [6], the PDC system has conducted a new driving method of mobile robot.

A ground mobile robot is defined as a robotic device that has the ability to move on ground or rough terrain. Traditional mobile robots [7][8] employ wheel or leg as propulsive mechanism. However, the PDC system, considered as a particular mobile robot, utilizes internal force and static friction as means of propulsion. The advantage of this novel propulsive method is that the wheels of the PDC system are negligible and can be removed, which can increase its dexterity and flexibility.

As a consequence of the novel mechanism, a number of control challenge issues have been identified and studied in the literatures. The problem of the PDC system was first proposed by Li et al. [1]. A four-step motion and dynamic modelling of the PDC system are initially proposed in [1]. An open-loop control law with a six-step motion strategy is studied by Liu et al. [4]. In order to validate the system, Wane et al. conducted an initial experimental study using the open-loop control law [5]. In [6], a new concept of optimal control has been presented using the PDC system. Based on the dynamic model, Liu et al. proposed an open-loop control law, a closed-loop control (CLC) law, and a simple switch control law in [9], and further discussed the robust issue of the proposed control laws.

Recently, as an alternative approach to the classical nonlinear control approaches, the fuzzy control (FC) approach has been applied to the tracking control of UMSs [10][11][12]. As it is well-known, the FC approach can be heuristic-based or model-based. A heuristic-based FC algorithm [13][14] consists of a set of heuristic decision rules and is considered as a non-mathematical approach which has been proved to be efficient whenever the controlled systems cannot be well defined or modelled. Aside from the heuristics-based FC algorithm, a model-based FC algorithm, such as Takagi-Sugeno (T-S) model-based control algorithm [15][16], is another approach to improve the performance of a fuzzy system. In [15], a T-S fuzzy model was used for the task of the trajectory tracking of the Pendubot. Using this model, the linearized dynamics of the Pendubot around each operation point was represented by the T-S fuzzy local model, and the local fuzzy controller can ensure global stability and guarantee the optimal trajectory tracking of the system. A model-based FC algorithm was compared with a classical energy-based strategy on the real-time swing-up control of a cart-pole system in [3]. The control strategy based on the FC algorithm can swing the pendulum from any initial condition, and performs more effective than the energy-based strategy. In [16], the cart-pole system was approximated by a T-S fuzzy model in a small range of angle near its equilibrium state, and a fuzzy controller designed with the parallel distributed pole assignment scheme was adopted to position the pendulum and the cart at the desired states.

This paper studies a new FC algorithm for the PDC system, and compares with the CLC law in [2]. The proposed FC algorithm aims to drive the cart forward as further as possible within one stroke. For this purpose, it simply uses the

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pendulum angle and the angular velocity as feedback signals, and deduces 6 control rules according to the T-S inference model. In particular, an adaptive control law is used in the inference model in order to swing the pendulum smoothly. The proposed algorithm presents a better robust performance than the CLC law under parameter uncertainties. To verify the proposed algorithm, experimental work is implemented using the LEGO Mindstorms kit [18]. The experimental results demonstrate the effectiveness of the proposed algorithm.

This paper is organized as follows. In section II, the system model and the CLC law are introduced. In section III, the proposed FC algorithm is studied. Simulation results and comparison with the CLC law are presented in section IV. Experimental implementation is presented in section V. Finally, concluding remarks are given in section VI.

II. The System Model and Motion Control

A. Dynamic Modelling

The PDC system is shown in Figure 1. The inverted pendulum mounted on the pivot is attached with a DC motor. The motor can generate torque in order to rotate the pendulum. The cart has four passive wheels which make it move horizontally on the ground. The parameters of the system are defined as follows. M is the cart mass, m is the ball mass, l is the length from the pivot to the mass center of the ball, μ is the friction coefficient between the cart and the ground, θ is the pendulum angle from vertical, x is the cart position, and τ is the torque applied to the pivot by the DC motor.

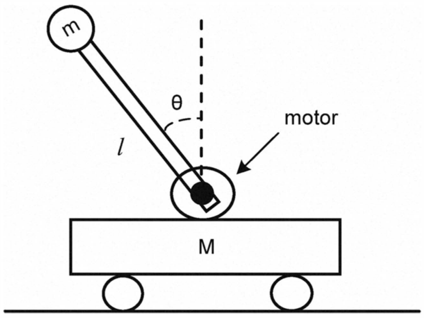

Figure 1 The PDC system

The dynamical model of the PDC system [2] is given as below

$$(M+m)\ddot{x} - ml(\cos\theta + \mu\sin\theta\,\mathrm{sgn}(\dot{x}))\ddot{\theta} + ml(\sin\theta - \mu\cos\theta\,\mathrm{sgn}(\dot{x}))\dot{\theta}^2 + \mu(M+m)g\,\mathrm{sgn}(\dot{x}) = 0 \quad (1)$$

$$(-ml\cos\theta)\ddot{x} + (ml^2)\ddot{\theta} - mgl\sin\theta = \tau \quad (2)$$

Let q_1=x and q_2=θ. Equations (1) and (2) can be written into a general compact form:

$$\begin{cases} D_{11}\ddot{q}_1 + D_{12}\ddot{q}_2 + h_1(q_1,\dot{q}_1,q_2,\dot{q}_2) = 0 \\ D_{21}\ddot{q}_1 + D_{22}\ddot{q}_2 + h_2(q_1,\dot{q}_1,q_2,\dot{q}_2) = u \end{cases}$$

where $D_{11} = M+m$, $D_{12} = -ml(\cos q_2 + \mu\sin q_2\,\mathrm{sgn}(\dot{q}_1))$, $D_{21} = -ml\cos q_2$, $D_{22} = ml^2$, $h_1 = ml(\sin q_2 - \mu\cos q_2\,\mathrm{sgn}(\dot{q}_1))\dot{q}_2^2 + \mu(M+m)g\,\mathrm{sgn}(\dot{q}_1)$, $h_2 = -mgl\sin q_2$

The control objective of the PDC system is to drive the cart towards one direction only by applying the control torque τ. Note that the control objective of the classical cart-pole system is to make the origin globally asymptotically stable [3].

B. Motion Generation

From the system dynamics, it is known that the system is an UMS which the pendulum is directly controlled by the DC motor while the cart is driven by internal force and static friction. As a consequence, in order to drive the cart towards one direction by applying the control torque only, two phases proposed in [2] are given as below.

Fast motion phase: moving the pendulum anti-clockwise fast leads to $F_x >> f_{max}$, where F_x is the internal force on horizontal, f_{max} is the maximal static friction on the cart wheels, thus this will lead to the cart moving forward.

Slow motion phase: moving the pendulum clockwise slowly which leads to $|F_x|<|f_{max}|$, this will keep the cart in still.

According to the two phases, a detailed pendulum velocity profile is presented as below. The control objective of the CLC law [2] is to track the desired velocity profile by using the partial feedback linearization technique. The optimization of the profile parameters are given in [9].

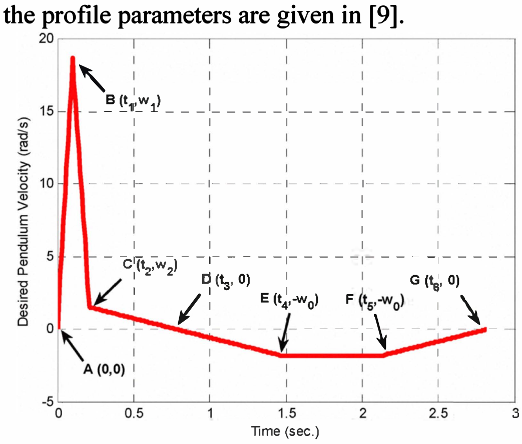

Figure 2 Desired pendulum velocity profile

C. Closed-loop Control using Partial Feedback Linearization

According to the dynamic model in equation (1), the cart acceleration can be obtained as below

$$\ddot{x} = \frac{1}{M+m}[ml(\cos\theta + \mu\sin\theta)\ddot{\theta} - ml(\sin\theta - \mu\cos\theta)\dot{\theta}^2 - \mu(M+m)g] \quad (3)$$

Putting equations (3) in (2), and then due to Spong [17], there exists an invertible change of control input

$$\tau = \alpha(\theta)u + \beta(\theta,\dot{\theta}) \quad (4)$$

where

$$\alpha = ml^2 - \frac{m^2l^2}{M+m}\cos\theta(\cos\theta + \mu\sin\theta),$$

$$\beta = \frac{ml\cos\theta}{M+m}[ml(\sin\theta - \mu\cos\theta)\dot{\theta}^2 + \mu(M+m)g] - mgl\sin\theta$$

Define

$$\tilde{\theta} = \theta - \theta_d$$

$$\dot{\tilde{\theta}} = \dot{\theta} - \dot{\theta}_d$$

and let

$$u = \ddot{\theta}_d - K_v\dot{\tilde{\theta}} - K_p\tilde{\theta}$$

where θ_d is the desired pendulum angle and $\dot{\theta}_d$ is the desired pendulum velocity. Applying the control law (4) to (2) gives the linear subsystem

$$\ddot{\tilde{\theta}} + K_v\dot{\tilde{\theta}} + K_p\tilde{\theta} = 0 \quad (5)$$

which is asymptotically stable by properly selection of the linear gains K_v and K_p.

III. Fuzzy Control

In order to simplify the control approach (4), this section will propose a T-S model-based control algorithm. The proposed FC rules are developed based on the desired profiles shown in Figure 3. According to the figures, three control regions (A, B, and C) can be identified in one cycle. The region A is an accelerating process which aims to rotate the pendulum fast. As shown from the figure of the desired pendulum velocity, the pendulum will be accelerated to the maximal speed, and the cart will be fully driven in the region A. The region B is a decelerating process which aims to stop the pendulum immediately. In this process, it is seen from the figures of the desired pendulum velocity and the desired cart velocity that the pendulum will be decelerated rapidly, and the cart will be stopped. The region C is a smooth process which aims to rotate the pendulum back to the original position slowly. The figure of the desired cart velocity shows that the cart will be kept still in this process. According to the description above, each control region should be given a consequent control law. In other words, when the continuous state hits the switching boundary, a consequent control law will be applied to the pendulum.

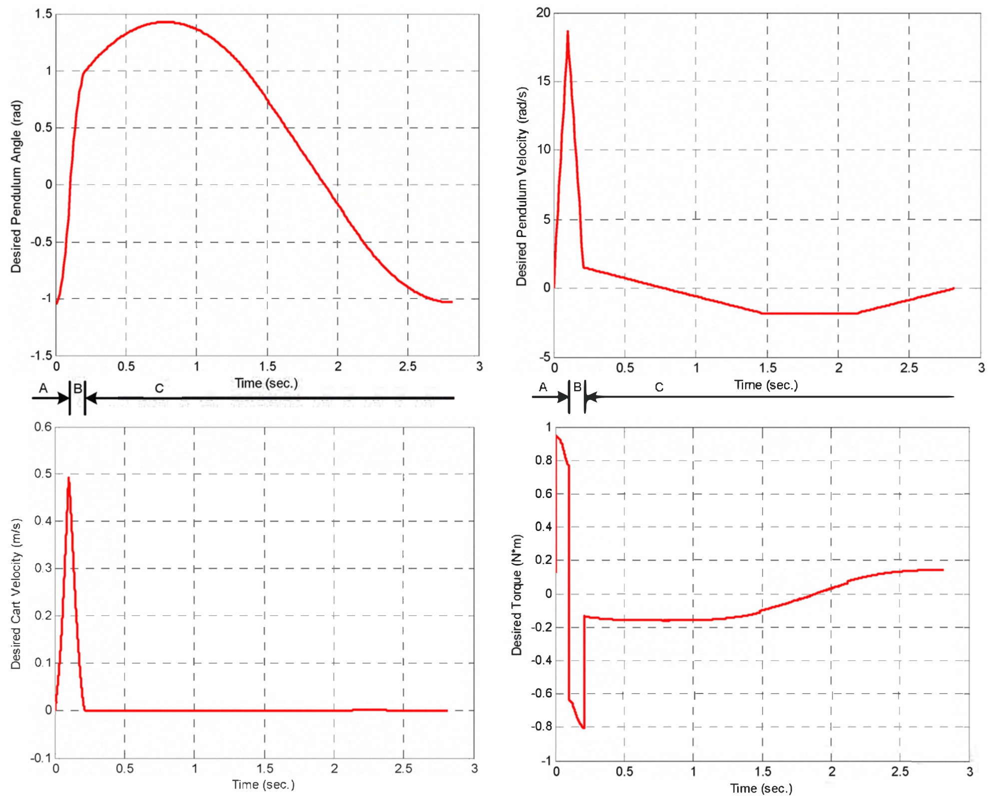

Figure 3 Desired profiles using the open-loop control law in [4] (A: fast acceleration; B: fast deceleration; C: smooth rotation)

Based on this philosophy described above, the switching boundaries shown in Figure 4 can be defined as follows. First, let consider anti-clockwise rotation of the pendulum ($\dot{\theta} > 0$). Due to inertia, to actuate the still pendulum, a pre-accelerating control region Ω_1: $[-\pi/2, -\theta_s)$ is defined, where $\theta_s > 0$. The area Ω_2: $[-\theta_s, \theta_s]$ is a general accelerating control region. When the pendulum is in Ω_2, it will be fully actuated. A decelerating control region Ω_3: $(\theta_s, \pi/2]$ is defined in order to stop the swinging pendulum rapidly. Compared with Figure 3, the control regions Ω_1 and Ω_2 correspond to the region A, and the control region Ω_3 corresponds to the region B. When the pendulum rotates in clockwise ($\dot{\theta} < 0$), the control region $[\pi/2, -\pi/2]$ corresponds to the region C. To rotate the pendulum smoothly, the following gravitational control law is adopted

$$\tau_g = -mgl\sin\theta + \tau_s \quad (6)$$

where τ_s is a small constant torque selected for smooth rotation.

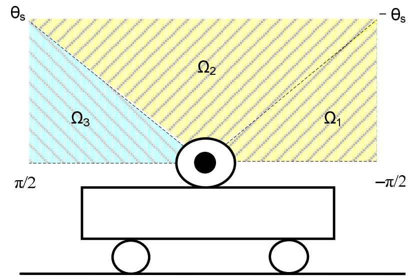

Figure 4 The switching boundaries

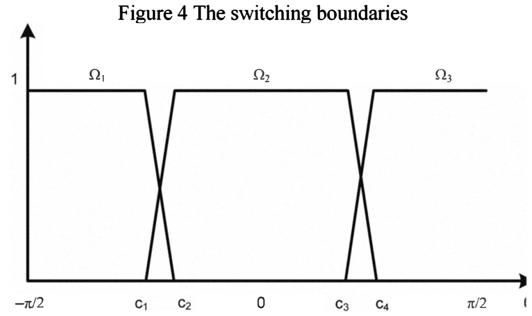

Figure 5 Membership functions of the input θ

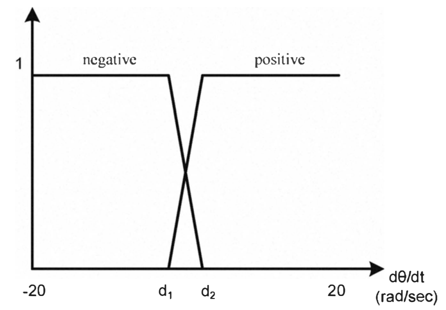

Figure 6 Membership functions of the input dθ/dt

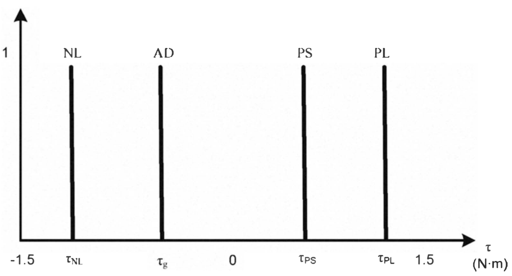

Figure 7 Membership functions of the output τ

Summarizing above, the FC algorithm which is based on the T-S inference model has two inputs which are the pendulum angle (θ) and the angular velocity (dθ/dt) of the pendulum, and one output which is the control torque (τ). Let the input fuzzy variable θ be partitioned into three fuzzy sets: Ω_1, Ω_2 and Ω_3, and let the input fuzzy variable dθ/dt be partitioned into two fuzzy sets: Positive and Negative. The membership functions for the input fuzzy sets of the T-S inference model are shown in Figure 5 and Figure 6, where c_i (i=1, … 4) and d_j (j=1,2) are parameters of the input fuzzy sets. For the output fuzzy sets, the output variable τ is partitioned into four sets: Negative Large, Gravitational Control, Positive Small, and Positive Large. The output membership functions are defined as shown in Figure 7, where τ_{NL}, τ_g, τ_{PS}, and τ_{PL} are parameters of the output fuzzy sets.

The idea of driving the cart to one desired direction is to rapidly rotate the pendulum in one anti-clockwise stroke, and to slowly rotate the pendulum to its initial position in one clockwise stroke. For example, if the state of the pendulum angle is in Ω_1, the pendulum should be fully accelerated using Positive Large. If the state of the pendulum angle is in Ω_3 and the state of the pendulum velocity is Positive, a Negative Large control action will be provided to decelerate the pendulum rapidly. Also, if the state of the pendulum angle is in Ω_2 or Ω_3 and the state of the pendulum velocity is Negative, the Gravitational Control action will be utilized to smoothly rotate the pendulum to its initial position.

Summarizing above, the FC algorithm is presented as below. Alternatively, the complete fuzzy rules of the algorithm are indicated in Table 1.

The FC algorithm:

If *θ is in Ω_1* then *τ is Positive Large (PL)*
If *θ is in Ω_2* and *dθ/dt is Positive* then *τ is Positive Small (PS)*
If *θ is in Ω_3* and *dθ/dt is Positive* then *τ is Negative Large (NL)*
If *θ is in Ω_2* or *Ω_3* and *dθ/dt is Negative* then *τ is Gravitational Control (GC)*

θ / dθ/dt	Ω_1	Ω_2	Ω_3
Positive	PL	PS	NL
Negative	PL	GC	GC

Table 1 The fuzzy rule base for the FC algorithm

IV. SIMULATION STUDY

This section presents the simulation studies of the proposed FC algorithm. The system parameters are specified as follows: *M=0.5kg, m=0.05kg, L=0.3m, μ=0.01, g=9.81m/s^2, T_s=1ms.* The initial position of the pendulum is set to $\theta_0=\pi/3$ in radian, and the switching boundary is chosen at $\theta_s=\pi/4$ in radian. The parameters of the input and the output fuzzy sets in the FC algorithm are given in Table 2, where c_i are given in radian, d_j are given in radian per second, and $\tau_{(*)}$ are given in N·m. The control surface of the FC algorithm is given in Figure 8. Because the GC action in Algorithm 4.3 is not constant, it changes along with the pendulum angle. The relationship between the pendulum angle and the GC action is presented in Figure 9.

c_1	–0.79
c_2	–0.76
c_3	0.76
c_4	0.79
d_1	–1
d_2	1
τ_{PL}	0.95
τ_{PS}	0.19
τ_{NL}	–0.95
τ_s	–0.005

Table 2 The parameters of the FC algorithm

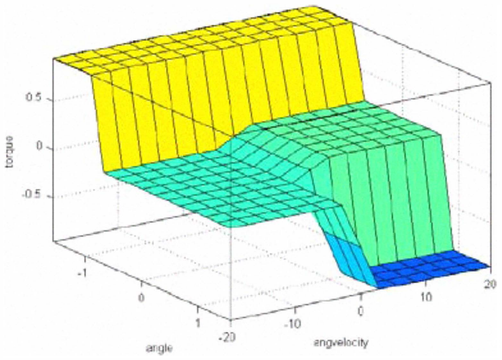

Figure 8 The control surface of the FC algorithm

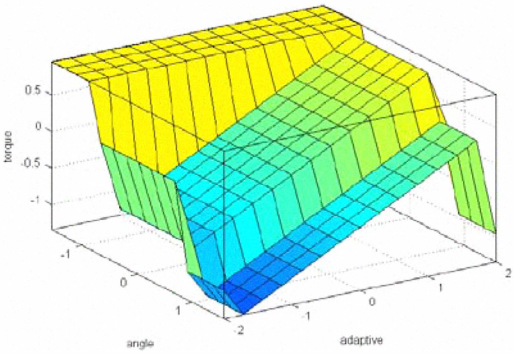

Figure 9 The control surface of the pendulum angle vs. the GC action

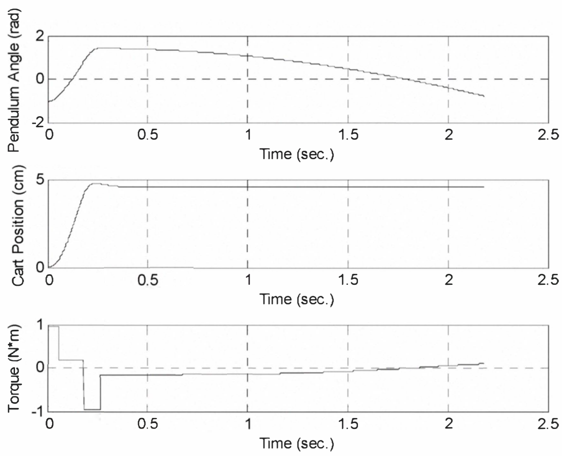

Figure 10 The FC algorithm in one cycle

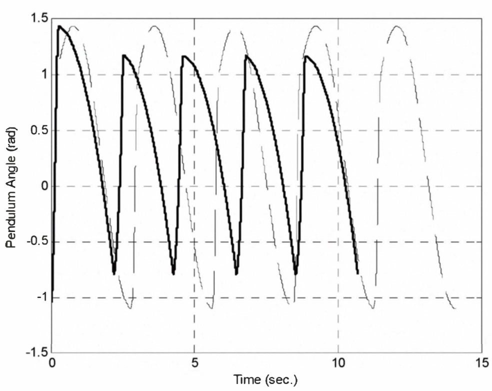

Figure 11 The pendulum angle in five cycles (solid line: FC; dash line: CLC)

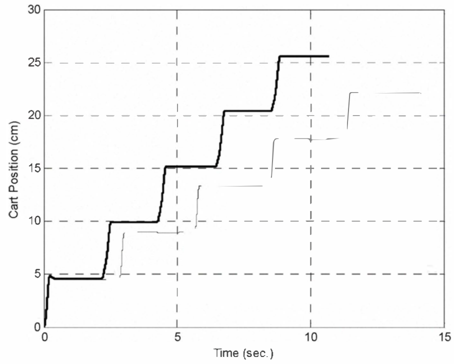

Figure 12 The cart position in five cycles (solid line: FC; dash line: CLC)

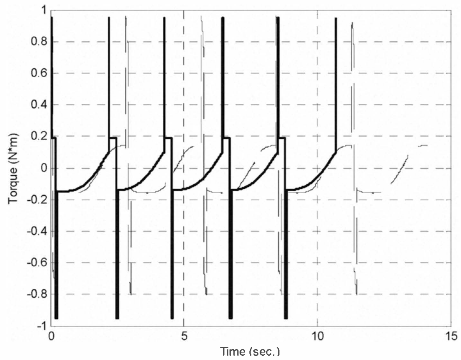

Figure 13 The control torque in five cycles (solid line: FC; dash line: CLC)

The simulation results in one cycle are shown in Figure 10. From the results, it can be found that it costs 2.18 seconds for one cycle. The cart has a backward movement in the fast motion, and stays at 4.54 cm in the end. The simulation results using the FC algorithm and the CLC algorithm in five cycles are presented in Figure 11, Figure 12, and Figure 13. In Figure 12, it is seen that, using the FC algorithm, the cart is driven forward 25.6 cm in 10.7 seconds, which gives the average speed at 2.39 cm/second. By using the CLC algorithm, the cart is driven forward 22.17 cm in 14.1 seconds, which gives the average speed at 1.57 cm/second.

V. EXPERIMENT

In this section, experimental set-up and results are given. The prototype of the PDC system is implemented by using LEGO Mindstorms kit [18]. The ROBOLAB 2.5 [19] is used to connect the LEGO RCX controller and the computer.

A. Experimental set-up

The objective of this experiment is to demonstrate the proposed system and the FC algorithm. The prototype of the PDC system is shown in Figure 14. Two DC motors are fixed at the center of the cart in order to rotate the pendulum simultaneously. The DC motor shown has a maximal torque at 0.89 N·m. Two rotation sensors are used, which one is directly attached to the wheel, and another is connected to one motor by using gears. The rotation sensor has a maximal rotation velocity at 5000 rpm and 16 positions per revolution. The RCX controller is a simple controller that has three sensor inputs and three motor ports. It can download the control program and transmit data to the computer via infrared communication.

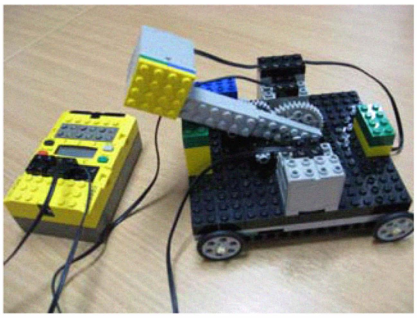

Figure 14 Experimental rig of the PDC system

B. *Experimental results*

For simplicity of the control process in real experiment, let the input fuzzy variable θ be partitioned into two fuzzy sets: Positive and Negative, the input fuzzy variable dθ/dt be partitioned into two fuzzy sets: Positive and Negative, and the output variable τ be partitioned into three fuzzy sets: Positive Large, Negative Large, and Negative Small. Therefore, the FC algorithm is implemented as follows:

If *θ is Negative* and *dθ/dt is Positive* then *τ is Positive Large*;
If *θ is Positive* and *dθ/dt is Positive* then *τ is Negative Large*;
If *dθ/dt is Negative* then *τ is Negative Small*.

The experimental results in eight cycles are presented in Figure 15 using a sequence of images from the video clips. In order to observe the progress of the cart clearly, a black line is marked on the table. From the figure, it is seen that the cart made great effort moving from the starting line to the end line.

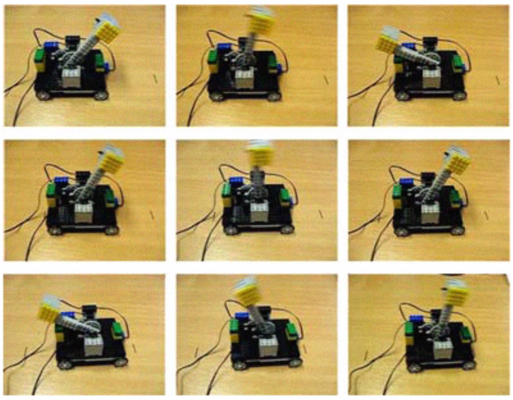

Figure 15 Experimental results in eight cycles

VI. CONCLUSION

In this paper a new FC algorithm is proposed for the PDC system. The proposed fuzzy system is based on the T-S inference model. Based on the model, 6 control rules have been defined. The control parameters used in the fuzzy rules are selected based on the control torque used in the CLC algorithm. The selecting procedure is studied. Compared with the CLC algorithm, the FC algorithm presents a more simplified control approach. The simulation results show that the proposed FC algorithm is more effective than the CLC algorithm. Finally, the experimental demonstration is presented. The experiment is developed by using LEGO Mindstorms kit and the ROBOLAB 2.5. A simple FC algorithm is implemented on the real system. From the experimental results, the proposed FC algorithm is demonstrated, and the locomotion of the PDC system is validated.

REFERENCES

[1] H. Li, K. Furuta, and F. L. Chernousko, "A pendulum-driven cart via internal force and static friction," Proceedings of Conference on Physics and Control, St. Petersburg, Russia, 2005.

[2] H. Yu, Y. Liu, and T. C. Yang, "Closed-loop Tracking Control of a Pendulum-driven Cart-pole Underactuated System", *Proc. IMechE, Part I: J. Systems and Control Engineering*, 2008, 222(I2), 109-125.

[3] N. Muskinja, B. Tovornik, ''Swinging Up and Stabilization of a Real Inverted Pendulum,'' IEEE Transactions on Industrial Electronics, Vol.53, No.2, April 2006.

[4] Y. Liu, H. Yu, and B. Burrows, "Optimization and Control of a Pendulum-driven Cart-pole System," IEEE Int. Conf. on Networking, Sensing and Control, London, UK, April 2007.

[5] S. O. Wane, H. Yu, and T. C. Yang, "Development of a Reaction Drive for a Propulsion Mechanism," IEEE Int. Conf on Networking, Sensing and Control, London, UK, 15-17 April 2007.

[6] H. Yu, T. C. Yang, Y. Liu, and S. O. Wane, "A Further Study of Control for a Pendulum Driven Cart", *Int. J. Advanced Mechatronic Systems*, 1(1), 2008, pp. 44-52.

[7] M. C. Deng, A. Inoue, Y. Shibata, K. Sekiguchi and N. Ueki, ''An obstacle avoidance method for two wheeled mobile robot'', Proc. of the IEEE Int. Conf. on Networking, Sensing and Control, London, UK, 2007.

[8] J. Velagic, B. Lacevic, and N. Osmic, ''Nonlinear motion control of mobile robot dynamic model'' in Motion Planning, X. J. Jing, InTech Education and Publishing, June 2008.

[9] Y. Liu, H. Yu, and T. C. Yang, "On Tracking Control of a Pendulum-driven Cart-pole Underactuated System", *International Journal of Modeling, Identification and Control*, Vol. 4, No. 4, pp. 357-372, 2008.

[10] M. I. EI-Hawwary, A. L. Elshafei, H. M. Emara, and H. A. Abdel Fattah, "Adaptive Fuzzy Control of the Inverted Pendulum Problem", IEEE Trans. on Control System Technology, vol.14, no.6, pp.1135-1144, 2006.

[11] C. L. Hwang, H. M. Wu, and C. L. Shih, "Fuzzy Sliding-mode Underactuated Control for Autonomous Dynamic Balance of an Electrical Bicycle", IEEE Trans. on Control System Technology, vol.17, no.3, pp.658-670, 2009.

[12] C. C. Kung, T. H. Chen, and L. C. Huang, "Adaptive Fuzzy Sliding Mode Control for a Class of Underactuated Systems", Proc. of IEEE Int. Conf. on Fuzzy Systems, Jeju Island, Korea, 2009.

[13] A. Sala, T. M. Guerra, R. Babuska, "Perspectives of Fuzzy System and Control", J. Fuzzy Sets Systems, vol.156. issue 3, pp.432-444, 2009.

[14] K. Michels, F. Klawonn, R. Kruse, and A. Numberger, *Fuzzy Control: Fundamentals, Stability and Design of Fuzzy Controller*, Spinger-Verlag, New York, 2006.

[15] Z. Cai and C. Y. Su, "Real-time Tracking Control of Underactuated Pendubot using Takagi-Sugeno Fuzzy Systems", IEEE Int. Symposium on Computational Intelligence in Robotics and Automation, Kobe, Japan, 2003.

[16] C. W. Tao, J. S. Taur, T. W. Hsieh, and C. L. Tsai, "Design of a Fuzzy Controller with Fuzzy Swing-up and Parallel Distributed Pole Assignment Schemes for an Inverted Pendulum and Cart System", IEEE Trans. Control System Technology, vol.16, no.6, pp.1277-1288, 2008.

[17] M. W. Spong, ''Partial Feedback Linearization of Underactuated Mechanical Systems'', Proc. of IEEE/RSJ/GI Int. Conf. on Intelligent Robots and Systems, Munich, Germany, 1994.

[18] The LEGO Group, available: http://www.lego.com

[19] LEGO Education, avialable: http://www.lego.com/eng/education/mindstorms/