Pendulum on a Cart using LEGO NXT Kits

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October 17, 2014

LEGO Pendulum on a Cart

Using

1. LEGO Mindstorms NXT Kit
2. HiTechnic Angle Sensor
3. Simulink (with Hardware Support Package for LEGO NXT)

Figure : LEGO Pendulum on a Cart

Objective: To rapidly damp the oscillation of the pendulum on a moving cart caused by external disturbances and/or jerk in the cart movement.

Building the Pendulum and the Cart

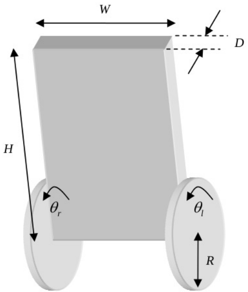

Figure : Cart powered by a single motor

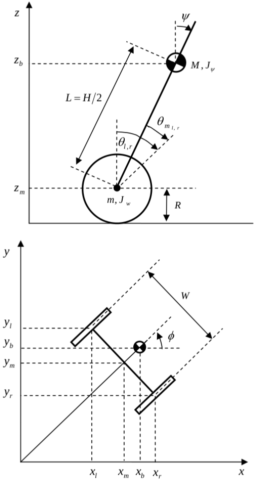

Figure : Pendulum and Angle sensor Mount

Mathematical Modelling - Equations of Motion

The equations of motion are derived using Lagrangian dynamics.
The Lagrangian takes the form

$$L = T - U$$

The Euler-Lagrange equations of motions for the robot are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \qquad i = 1, 2, \ldots, n$$

where $q = (\theta, \phi, \psi)$ are the generalized coordinates.

Mathematical Modelling - State Equations of the System

On solving the Euler-Lagrange equations, a non-linear model of the system is obtained. On linearizing the model about the equilibrium point($\psi = 0$) and neglecting second order terms like $\dot{\psi}^2$ and $\dot{\phi}^2$, we obtain:

$$\left[(2m + M)R^2 + 2J_w + 2n^2 J_m\right]\ddot{\theta} + \left(MLR - 2n^2 J_m\right)\ddot{\psi} = F_\theta$$

$$\left(MLR - 2n^2 J_m\right)\ddot{\theta} + \left(ML^2 + J_\psi + 2n^2 J_m\right)\ddot{\psi} - MgL\psi = F_\psi$$

$$\left[\frac{1}{2}mW^2 + J_\phi + \frac{W^2}{2R^2}\left(J_w + n^2 J_m\right)\right]\ddot{\phi} = F_\phi$$

Mathematical Modelling - State Space Representation

Here, we consider x_1, x_2 as states and u as input and thus the state space representation of the linearized model is of the form:

$$\dot{\mathbf{x_1}} = A_1\mathbf{x_1} + B_1\mathbf{u}$$
$$\dot{\mathbf{x_2}} = A_2\mathbf{x_2} + B_2\mathbf{u}$$

where,

$$\mathbf{x_1} = [\theta, \psi, \dot{\theta}, \dot{\psi}]^T, \mathbf{x_2} = [\phi, \dot{\phi}]^T, \mathbf{u} = [v_l, v_r]^T$$

A Servo (PID type) controller is employed to track the reference signal. An integral gain is inserted into the feedback loop in order to regulate out steady state errors.

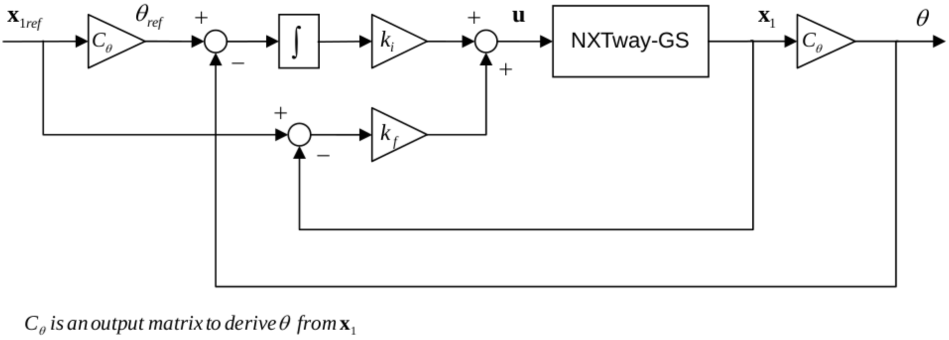

Figure : Servo Controller Block diagram

Controller Design

Additionally,

- P control is used for wheel synchronization when moving straight. This is required since two DC motors don't move at the same speed for a given voltage.
- Rotation is achieved by giving higher power to the left motor
- Remote controlled via Bluetooth by altering the $\dot{\theta}$ reference for speed and rotation is achieved as explained above.

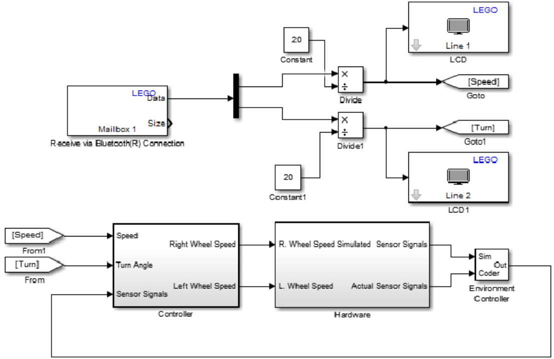

Figure : Simplified Simulink Model of NXTway-GS

Simulink Model - Remote

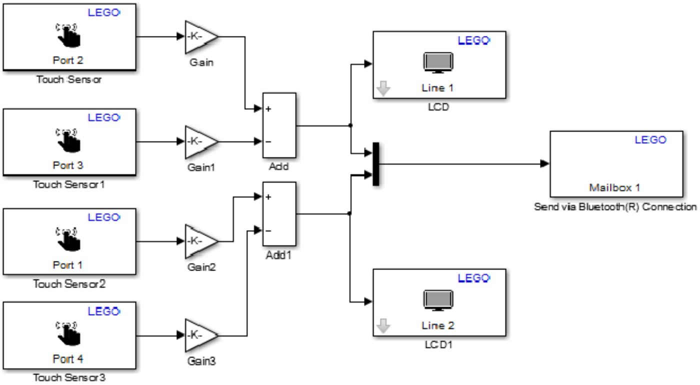

Figure : Simplified Simulink Model of Remote

Demonstration

References

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Control of self-balancing two-wheeled robot built with LEGO Mindstorms NXT.

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Gyro Sensor (NGY1044)
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Two wheeled balancing LEGO robot

Thank You!