Stats 295, Homework 3

Due date: **November 15**

Programming exercises can be completed in any language of your choice.

1. Consider a discrete-time single-type branching process X_n with a Poisson progeny distribution and the corresponding generating function $Q(s) = \mathrm{E}\left(s^{X_1}\right) = e^{\lambda(s-1)}$.

   (a) Write a function to simulate X_n starting from $X_0 = 1$.

   (b) Estimate $q_n = \Pr(X_n = 0)$ for $n = 5, 15, 30$ and $\lambda = 1.4$ by simulating the branching process $N = 1000$ times.

   (c) Compare your simulation-based estimate with a numerical solution that uses the Newton's method to solve equation $Q(s) = s$.
   (Recall that Newton's method for solving an algebraic equation $f(x) = 0$ prescribes a recursion $x_{n+1} = x_n - f(x_n)/f'(x_n)$.)

2. Suppose we are interested in estimating the mean of the progeny distribution μ of a discrete-time single-type branching process.

   (a) One way of going about this task is to equate the number of particles present at time n, X_n, with the corresponding expectation $\mathrm{E}(X_n \mid X_0 = 1)$. Show that the resulting estimate $\tilde{\mu}$ underestimates μ on average. In other words, prove that $\mathrm{E}(\tilde{\mu}) \leq \mu$.

   (b) Another method of moments uses equation $\mathrm{E}(X_n) = \mathrm{E}(X_{n-1})\mu$ to arrive at an estimate

   $$\hat{\mu} = \begin{cases} \frac{X_n}{X_{n-1}} & \text{if } X_{n-1} > 0 \\ 1 & \text{if } X_{n-1} = 0. \end{cases} \tag{1}$$

   Show that $\mathrm{E}(\hat{\mu}) = \mu \Pr(X_{n-1} > 0) + \Pr(X_{n-1} = 0)$.

   (c) Calculate $\mathrm{E}(\hat{\mu})$ for the Poisson progeny distribution in the problem 1(a) using $\lambda = 1.1$ and $n = 20$.

3. Download and install an `EpiEstim` R package. Use the provided `get_oc_data.R` script to download SARS-CoV-2 cases in Orange County, CA. Estimate changes in the effective reproductive R_t in Orange County. You may find this demo helpful to get you started: `https://cran.r-project.org/web/packages/EpiEstim/vignettes/demo.html`.