Reg. No.														

B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, MAY 2024

Fourth Semester

21MAB204T – PROBABILITY AND QUEUEING THEORY

(For the candidates admitted during the academic year 2021-2022 , 2022-2023 & 2023-2024)

(Statistical tables to be provided)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours | Max. Marks: 75

PART – A (20 × 1 = 20Marks)

Answer **ALL** Questions

Q.	Question	Marks	BL	CO	PO
1.	The first moment of a random variable X about its mean is always (A) 1 (B) 0 (C) −1 (D) ±1	1	1	1	1
2.	If a random variable X has the pdf $f(x)=2x;0<x<1$, then the CDF F(x) is (A) $x^2;0<x<1$ (B) $x^3;0<x<1$ (C) $2x^2;0<x<1$ (D) $\frac{x^2}{2};0<x<1$	1	1	1	1
3.	If X and Y are two random variables then Var(aX)=_________ (A) aVar(X) (B) a^2Var(X) (C) Var(X) (D) Var(a^2X^2)	1	1	1	1
4.	If a random variable X has the MGF $M(t)=\frac{3}{3-t}$, then the mean of X is (A) 3 (B) 1/3 (C) 1/2 (D) 1	1	1	1	1
5.	The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean = 1.8, then P(X=1)=_____ (A) 0.5 (B) 0.1653 (C) 0.2975 (D) 0.6065	1	1	2	1
6.	The MGF of the binomial distribution is (A) $(qe^t+p)^n$ (B) $(pe^t+q)^n$ (C) $(pe^t-q)^n$ (D) $(qe^t-p)^n$	1	1	2	1
7.	The mean of the exponential distribution is (A) $1/\lambda$ (B) $2/\lambda^2$ (C) λ (D) $\lambda^2/2$	1	1	2	1

8. The mean and variance of the standard normal distribution are 1 1 2 1

(A) 1 and 0 (B) 0 and 1
(C) μ and σ^2 (D) μ and σ

9. $F(\infty, \infty)$ is equal to 1 1 3 1

(A) 0 (B) 1
(C) 1/2 (D) ∞

10. The marginal probability function of X from $f_{XY}(x,y)$ is 1 1 3 1

(A) $\int f(x,y)dx$ (B) $\int f(x,y)dy$
(C) $\iint_R f(x,y)dxdy$ (D) $\frac{d}{dx}f(x,y)$

11. If X and Y are independent random variables with probability density function $f_X(x)=e^{-x};x\geq 0$ *and* $f_Y(y)=e^{-y};y\geq 0$, then joint pdf of (X,Y) is 1 2 3 1

(A) $e^{-x}+e^{-y}$ (B) e^{-x}/e^{-y}
(C) e^{-y}/e^{-x} (D) $e^{-(x+y)}$

12. If mean=1000 and SD=100 for each of n independent random variables with n=60, then by central limit theorem, we have 1 2 3 1

(A) $N\left(1000,\frac{100}{\sqrt{60}}\right)$ (B) $N(1000,100)$
(C) $N\left(1000,100\sqrt{60}\right)$ (D) $N\left(1000,\sqrt{\frac{100}{60}}\right)$

13. The symbolic notation of queueing model is represented by 1 1 4 1

(A) Euler (B) Fisher
(C) Neumann (D) Kendall

14. In queueing model (M/M/I):(∞/FIFO), probability of no customer in the system is 1 1 4 1

(A) $P_0=1-\frac{\lambda}{\mu}$ (B) $P_0=1+\frac{\lambda}{\mu}$
(C) $P_0=\frac{\lambda}{\mu}$ (D) $P_0=\frac{\mu}{\lambda}$

15. In queueing model (M/M/I):(K/FIFO), the overall effective arrival rate is 1 2 4 1

(A) $\mu(1+P_0)$ (B) μP_0
(C) $\mu(1-P_0)$ (D) $\mu+P_0$

16. The cumulative distribution function is F(x,y), then the probability density function f(x,y)= 1 2 4 1

(A) $\frac{\partial}{\partial x}F(x,y)$ (B) $\frac{\partial}{\partial y}F(x,y)$
(C) $\iint_R F(x,y)dxdy$ (D) $\frac{\partial^2}{\partial x\partial y}F(x,y)$

17. In a transition probability matrix, the sum of all elements in any row is 1 1 5 1

(A) 0 (B) 1
(C) 2 (D) 3

18. In a Markov chain, if every state can be reached from every other state, then the Markov chain is called 1 1 5 1

(A) Reducible (B) Irreducible
(C) Periodic (D) Non-periodic

19. If P is the tpm of the regular chain, then 1 1 5 1

(A) $\pi P=\pi$ (B) $\pi=P$
(C) $\pi+P=\pi$ (D) $\pi-P=\pi$

20. A state i is said to be periodic with period d_i of 1 1 5 1

(A) $d_i<1$ (B) $d_i>1$
(C) $d_i=1$ (D) $d_i=0$

PART – B (5 × 8 = 40 Marks)

Answer **ALL** Questions

	Marks	BL	CO	PO
21. a.	8	3	1	2
21. b.	8	3	1	2
22. a.	8	4	2	2
22. b.	8	4	2	2
23. a.	8	4	3	2
23. b.	8	4	3	2

21. a. If the CDF of a random variable is given by

$$F(x)=0 \quad for \quad x<0$$
$$=\frac{x^2}{16} \quad for \quad 0<x<4$$
$$=1 \quad for \quad 4\le x$$

Find pdf and P(X>1/X<3). Also find mean and variance.

(OR)

b. If X denotes the sum of the numbers obtained when two dice are thrown, obtain an upper bound for P(|X–7|≥3).

22. a. An irregular 6-faced dice is such that the probability of giving 3 odd numbers in 7 throws is twice the probability of giving 4 odd numbers in 7 throws. How many sets of exactly 7 trials can be expected to give no odd number out of 5000 sets?

(OR)

b. In a normal distribution, 70% of the items are under 35, and 89% are under 63. What are the mean and standard deviation of the distribution.

23. a. The joint PDF of a two dimensional random variable (X,Y) is given by

$$f(x,y)=\begin{cases} \dfrac{6}{5}\begin{pmatrix} x+y^2; 0\le x\le 1 \\ 0\le y\le 1 \end{pmatrix} \\ 0 \quad ;otherwise \end{cases}$$

Find (i) the marginal PDF of X and marginal pdf of Y (ii) $P\left(\frac{1}{2}\le X\le\frac{3}{4}/Y>\frac{1}{2}\right)$.

(OR)

b. The life time of a certain brand of an electric bulb may be considered as a random variable with mean 1200 hours and standard deviation 250 hrs. Find the probability using central limit theorem, that the average life time of 60 bulbs exceeds 1250 hours.

Question	Marks	BL	CO	PO
24. a. Customers arrive at a one man barbershop according to a Poisson process with a mean inter arrival time of 12 minutes customer spend an average of 10 minutes in the barber's chair. (i) What is the expected number of customers in the barbershop and in the queue? (ii) How much time can a customer expected to spend in the barber's shop? (iii) Calculate the percentage of time an arrival can walk straight into barber's chair without having to wait (iv) What is the average time customer spends in the queue?	8	3	4	2
(OR)				
b. Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour. (i) Find the effective arrival rate at the clinic (ii) What is the probability that an arriving patient will not wait? (iii) What is the expected waiting time until a patient is discharged from the clinic?	8	3	4	1
25. a. The t.p.m of a Markov chain with three states 0,1,2 is $P=\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ and the initial state distribution of the chain is $P[X_0=i]=\frac{1}{3}; i=0,1,2$ find (*i*) $P[X_2=2]$ (*ii*) $P[X_3=1, X_2=2, X_1=1, X_0=2]$	8	3	5	1
(OR)				
b. Find the nature of the states of the Markov chain with the tpm. $P=\begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix}$	8	3	5	2

PART – C (1 × 15 = 15 Marks)

Answer **ANY ONE** Questions

Question	Marks	BL	CO	PO
26. There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour. (i) What fraction of the time all the typists will be busy? (ii) What is the average number of letters waiting to be typed? (iii) What is the average time a letter has to spend for waiting and for being typed?	15	3	4	2
27. A gambler has ₹ 2. He bets ₹ 1 at a time and wins ₹ 1 with probability ½. He stops playing if he losses ₹ 2 or wins ₹ 4. (i) What is the tpm of the Markov chain? (ii) What is the probability that he has lost his money at the end of 5 days? (iii) What is the probability that the game lasts more than 7 plays?	15	3	5	2

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