Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2023

Fourth, Fifth and Sixth Semester

39

18MAB302T – DISCRETE MATHEMATICS FOR ENGINEERS

(For the candidates admitted from the academic year 2020-2021 & 2021-2022)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

Question	Marks	BL	CO
1. The cardinality of the power set of the set {0, 1, 2} is (A) 8 (B) 6 (C) 7 (D) 9	1	1	1
2. If A is a non-empty set with n elements, then the number of possible relations on the set A is (A) 2^n (B) 2^{n-1} (C) 2^{n^2} (D) 2^{n+1}	1	1	1
3. A relation R on set A is defined as if $x\mathbb{R}y, y\mathbb{R}x$, then $x = y \quad \forall x, y \in A$, then $\mathbb{R}$ is _____ (A) Symmetric (B) Asymmetric (C) Antisymmetric (D) Transitive	1	2	1
4. A function f: A→B is said to be __________ if for every y∈B there exist at least one element x∈A such that f(x)=y. (A) Surjective (B) Bijective (C) Injective (D) Automorphism	1	2	1
5. If the object A is chosen in M ways and B in N ways then either A or B is chosen in _________ ways. (A) M/N (B) MN (C) M+N (D) M–N	1	1	2
6. Assuming that repetitions are not permitted, how many four-digit numbers are less than 4000, can be formed from the six digits 1, 2, 3, 5, 7, 8? (A) 125 (B) 124 (C) 63 (D) 180	1	2	2
7. Every integer n>1 can be represented uniquely as a product of (A) Prime members (B) Composite numbers (C) Even numbers (D) Odd numbers	1	1	2

8. For any positive integers a and 3 there exists unique integers q and r such that a=3q+r where r must satisfy 1 2 2

(A) $1 < r < 3$ (B) $0 < r < 3$
(C) $0 \le r < 3$ (D) $0 < r \le 3$

9. The biconditional is conjunction of two __________ statement. 1 1 3

(A) Negation (B) Compound
(C) Connective (D) Conditional

10. Negation of $P \to (P \vee \neg Q)$ is

(A) $\neg P \to (\neg P \vee Q)$ (B) $P \wedge (\neg P \wedge Q)$
(C) $\neg P \vee (\neg P \vee \neg Q)$ (D) $\neg P \to (\neg P \to Q)$

11. $P \to Q$ is logically equivalent to _________. 1 2 3

(A) $\neg P \vee \neg Q$ (B) $P \vee \neg Q$
(C) $\neg P \vee Q$ (D) $\neg P \wedge Q$

12. A premises may be introduced at any point in the derivation is called 1 1 3

(A) Rule T (B) Rule US
(C) CP rule (D) Rule P

13. Fourth root of unity namely 1, −1, i,−i form a group with respect to 1 2 4

(A) Addition (B) Subtraction
(C) Multiplication (D) Division

14. If G is a finite group and order of group is m, then for all $a \in G$ 1 1 4

(A) $a^m \neq e$, an identity (B) $a^m = e$, an identity
(C) $a^m = a$ (D) $a^m = a^{-1}$

15. A finite integral domain is a ___________. 1 1 4

(A) Subfield (B) Vector
(C) Field (D) Ring

16. If in a ring R, the exist an elements a, b such that a*b =0 implies either a = 0 or b = 0 or both a = 0 and b = 0 then R is 1 2 4

(A) Ring with unit element (B) Ring with zero divisor
(C) Ring without zero divisor (D) Boolean ring

17. A graph G is said to be a simple graph 1 1 5

(A) G has no loops (B) There is exactly one edge between any given pair of vertices
(C) Both (A) and (B) (D) It contains only parallel edges

18. If G is an undirected graph with 12 edges. Also, it is given that two vertices are of degree 2, two are of degree 3, and one of degree 4 and remaining are of degree 5. How many total vertices are there in G? 1 2 5

(A) 8 (B) 7
(C) 9 (D) 10

19. A path in a connected graph G=V, E) is called Hamilton path if — 1 1 5

(A) It includes every edge exactly once (B) It includes every vertex exactly once

(C) It includes every edge exactly twice (D) It includes every vertex exactly twice

20. A degree of pendent vertex is — 1 2 5

(A) 0 (B) 1

(C) 2 (D) 3

PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions — Marks BL CO

21. Prove that $(A-C)\cap(C-B)=\varnothing$ analytically where A, B and C are sets. — 4 3 1

22. Draw the Hasse diagram for (D12, |) where D_{12} is the set of positive integers divisor of 12. — 4 4 1

23. Find the number of ways of preparing a garland with 3 yellow, 4 pink and 2 red roses of different sizes such that the two red roses come together. — 4 3 2

24. Construct the truth table for the following $(P\to Q)\leftrightarrow(\neg Q\to\neg P)$ — 4 4 3

25. Prove the following implication without using truth table $P\Rightarrow(Q\to P)$. — 4 3 3

26. If α, β are elements of the symmetric group S_4 given by — 4 3 4

$$\alpha=\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 2 & 1\end{pmatrix} \text{ and } \beta=\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 3. & 1\end{pmatrix}$$

Find αβ, βα, α^2 and α^{-1}.

27. Prove that the number of edges in a bipartite graph with n vertices is atmost $n^2/4$. — 4 4 5

PART – C (5 × 12 = 60 Marks)

Answer **ALL** Questions — Marks BL CO

28. a.i. R is the relation on the set of integers such that (a, b) ∈R if 3a+4b=7n for some integer n, prove R is equivalence relation? — ~~12~~ 6 3 1

ii. If A={1, 2, 3, 4, 5}, B={1, 2, 3, 8, 9} and f:A→B and g:A→A are defined by f={(1, 8), (3,9), (4,3), (2, 1), (5,2)} and g={(1, 2), (3, 1), (2, 2), (4, 3), (5, 2)}. Find $f\circ g, g\circ f, f\circ f, g\circ g$ if they exist. — ~~12~~ 6 3 1

(OR)

b. Let $A=\{1,2,3,4\}$ and $R=\{(1,2),(2,3),(2,1),(3,4)\}$ using Warshall's algorithm, find the transitive closure of R. — 12 3 1

29. a.i. If there are 5 points inside a square of side length 2. Prove that two of the points are within a distance of $\sqrt{2}$ of each other — ~~12~~ 6 4 2

ii. In a group of 72 students, 47 have background in electronics, 59 have background in mathematics and 42 have background in both the subject, how many students do not have background in any of the subjects? — ~~12~~ 6 4 2

(OR)

b. Use the Euclidean algorithm to find gcd(1819, 3587) and also express linear combination of the given number. 12 4 2

30. a.i. Prove the following by using direct method $P\vee Q, Q\rightarrow R, P\rightarrow S, \neg S \Rightarrow R\wedge(P\vee Q)$ 6 ~~12~~ 4 3

ii. Show that $P\rightarrow Q, P\rightarrow R, Q\rightarrow \neg R$ and P are inconsistent. 6 ~~12~~ 4 3

(OR)

b.i. Use indirect method of proof to show that $R\rightarrow \neg Q, R\vee S, S\rightarrow \neg Q, P\rightarrow Q \Rightarrow \neg P$ 6 ~~12~~ 4 3

ii. Use mathematical induction to show that $n! \geq 2^{n-1} \quad \forall n\geq 1$. 6 ~~12~~ 4 3

31. a.i Let $Q^+ = \{\text{set of all postive rational number}\}$, let $*$ be defined on Q^+ by $a*b = \dfrac{ab}{3} a,b\in Q^+$, prove that $(Q^+,*)$ is an abelian group. ~~12~~ 6 4 4

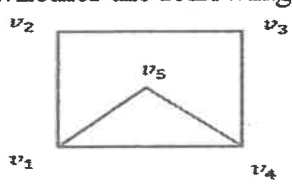
ii. Prove that the intersection of two subgroups of a group is also a subgroup of a group. ~~12~~ 6 4 4

(OR)

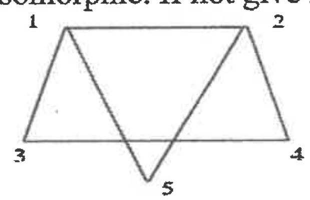
b. Find the code words generated by the parity check matrix 12 4 4

$$H=\begin{bmatrix}1&1&1\\1&0&1\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$ when the encoding function is $e:B^3\rightarrow B^6$.

32. a.i. Check whether the following graphs are isomorphic. If not give reason. 6 ~~12~~ 3 3



$G_1(V_1, E_1)$

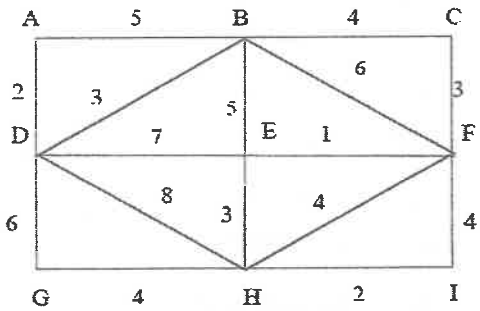


$G_2(V_2, E_2)$

ii. Prove that a tree with n vertices has n-1 edges. 6 ~~12~~ 3 3

(OR)

b. Use Kuskal's algorithm to find a minimum spanning tree for the weighted graph. 12 4 4



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