Reg. No.															

B.Tech. DEGREE EXAMINATION, JUNE 2024
First Semester

18MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted from the academic year 2018-2019 to 2021-2022)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

Q.	Question	Marks	BL	CO	PO
1.	The matrix of the quadratic form x^2+xy is (A) $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ (B) $\begin{pmatrix}0&4\\5&1\end{pmatrix}$ (C) $\begin{pmatrix}1&1/2\\1/2&0\end{pmatrix}$ (D) $\begin{pmatrix}1&3/2\\5/2&1\end{pmatrix}$	1	1	1	1
2.	Two Eigen values of the matrix $A=\begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix}$ are 1 and 2. Find the third eigen value. (A) 3 (B) 2 (C) 1 (D) 0	1	1	1	1
3.	Find the eigen values of A^3 if $A=\begin{bmatrix}3&1&2\\0&1&4\\0&0&2\end{bmatrix}$ (A) 3, 1, 2 (B) 9, 1, 4 (C) 27, 1, 8 (D) 81, 1, 16	1	1	1	1
4.	Find the nature of the quadratic form $2x^2+3y^2+2z^2+2xy$ (A) Positive definite (B) Semi positive definite (C) Negative definite (D) Indefinite	1	1	1	1
5.	If $z=x^2+y^2+3xy$, then $\frac{\partial z}{\partial x}$ is (A) $x+y$ (B) x^2+y^2 (C) $2x+3y$ (D) $2x+4xy$	1	1	2	1
6.	If $f(x,y)=e^x\cos y$, then the value of $f_{xxx}(0,0)$ is (A) 1 (B) 2 (C) 3 (D) 0	1	1	2	1

7. If f(x,y) is an implicit function, then $\frac{dy}{dx} =$ 1 1 2 1

(A) $-\frac{\partial f}{\partial x}\Big/\frac{\partial f}{\partial y}$ (B) $\frac{\partial f}{\partial x}\Big/\frac{\partial f}{\partial y}$

(C) $\frac{\partial x}{\partial f}\Big/\frac{\partial y}{\partial x}$ (D) $-\frac{\partial f}{\partial y}\Big/\frac{\partial f}{\partial x}$

8. If u and v are functionally dependent, then their Jacobian value is 1 1 2 1

(A) 0 (B) 1

(C) 2 (D) 3

9. The general solution of $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=0$ is 1 1 3 1

(A) $y=c_1e^{x}+c_2e^{-4x}$ (B) $y=c_1e^{-2x}+c_2e^{-3x}$

(C) $y=(c_1+c_2x)e^{-x}$ (D) $y=c_1e^{-x}+c_2e^{-2x}$

10. The particular integral of $(D^2+6D+8)y=e^{-2x}$ is 1 1 3 1

(A) $\frac{x}{2}e^{-2x}$ (B) $\frac{x}{2}e^{-x}$

(C) $\frac{x}{3}e^{x}$ (D) $\frac{x}{3}e^{2x}$

11. The complementary function of $(x^2D^2+4xD+2)y=x+\frac{1}{x}$ is 1 1 3 1

(A) $CF=(C_1+C_2z)e^{z}$ (B) $CF=C_1e^{-z}+C_2e^{-2z}$

(C) $CF=C_1e^{-2z}+C_2e^{4z}$ (D) $CF=C_1e^{-3z}+C_2e^{5z}$

12. The general solution of $(D^2+1)y=5$ is 1 1 3 1

(A) $y=C_1\cos x+C_2\sin x$ (B) $y=(C_1e^{-x}+C_2e^{x})+5$

(C) $y=(C_1\cos x+C_2\sin x)+5$ (D) $y=(C_1+C_2x)e^{x}+5$

13. The curvature of the straight line is 1 1 4 1

(A) 0 (B) 1

(C) 2 (D) 3

14. The envelope of the family of curves $A\alpha^2+B\alpha+C=0$ (α is a parameter) is 1 1 4 1

(A) $B^2+4AC=0$ (B) $B^2-AC=0$

(C) $B^2+AC=0$ (D) $B^2-4AC=0$

15. The radius of curvature of the curve $y=4\sin x\ at\ x=\frac{\pi}{2}$ is 1 1 4 1

(A) 1/2 (B) –1/2

(C) 1/4 (D) –1/4

16. The evolute of the cycloid $x=a(\theta-\sin\theta), y=a(1-\cos\theta)$ is 1 1 4 1

(A) Astroid (B) Parabola

(C) Cycloid (D) Circle

17. The sequence $S_n = \left(\frac{n}{n-1}\right)^2$, converges to — 1 1 5 1

(A) 1 (B) 2
(C) 3 (D) 4

18. Let $\sum_{n=1}^{\infty} u_n$ be a series of positive terms such that $\lim_{n\to\infty} n \log \frac{u_n}{u_{n+1}} = l$ then the series $\sum u_n$ is convergent if — 1 1 5 1

(A) $l<1$ (B) $l>1$
(C) $l=1$ (D) $l=0$

19. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is — 1 1 5 1

(A) Convergent (B) Divergent
(C) Oscillatory (D) Monotonically increasing

20. Let $\sum u_n$ be a series of positive terms such that $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) = l$. Then the series $\sum u_n$ is divergent if — 1 1 5 1

(A) $l=1$ (B) $l>1$
(C) $l<1$ (D) $l=2$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

	Question	Marks	BL	CO	PO
21.	Find the constants a and b such that $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ matrix has 3 and –2 as its eigen values.	4	1	1	1
22.	If $x = r\cos\theta, y = r\sin\theta$, then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.	4	1	2	1
23.	Solve $\left(D^2 + 6D + 9\right)y = 3e^{4x}$.	4	2	3	1
24.	Find the envelope of the straight line $x\cos\alpha + y\sin\alpha = a\sin\alpha\cos\alpha, \alpha$ being the parameter.	4	2	4	1
25.	Examine the convergence of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$	4	1	5	1
26.	Determine the nature of the quadratic form without reducing to canonical form $2x_1x_2 + 2x_2x_3 - 2x_3x_1$.	4	2	1	1
27.	If $u = \sin\left(\frac{x}{y}\right), x = e^t, y = t^2$, then find $\frac{du}{dt}$.	4	1	2	1

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

Q. No.		Question	Marks	BL	CO	PO
28.	a.	Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.	12	2	1	2
		(OR)				
	b.	Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and hence find A^4.	12	2	1	2
29.	a.	Expand $x^2y+3y-2$ in powers of (x–1) and (y+2) using Taylor's series upto terms of third degree.	12	2	2	2
		(OR)				
	b.	A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions in order that the total surface area is minimum.	12	2	2	2
30.	a.	Solve $\left(D^2-5D+6\right)y=x^2+3x-1$.	12	2	3	2
		(OR)				
	b.	Solve $\frac{d^2y}{dx^2}+y=\sec x$ by the method of variation of parameters.	12	2	3	2
31.	a.	Find the equation of the circle of curvature of the parabola $y^2=12x$ at the point (3,6).	12	2	4	2
		(OR)				
	b.	Show that the evolute of the cycloid $x=a(\theta-\sin\theta), y=a(1-\cos\theta)$ is another cycloid.	12	2	4	2
32.	a.	Test the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.6}+....$	12	2	5	2
		(OR)				
	b.	Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{n^3+1}{2^n+1}$.	12	2	5	1

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