Reg. No.															

B.Tech. DEGREE EXAMINATION, MAY 2024

Third Semester

18MAB201T – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted from the academic year 2018-2019 to 2021-2022)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

		Marks	BL	CO	PO
1.	The degree of the PDE $\frac{d^2y}{dx^2}+2y=\left(\frac{dy}{dx}\right)^3$ (A) 1 (B) 2 (C) 3 (D) 4	1	1	1	1
2.	The partial derivative $\frac{\partial^2 z}{\partial x \partial y}$ can be denoted as (A) q (B) r (C) s (D) t	1	1	1	2
3.	In a solution, the number of arbitrary constants is equal to the number of independent variables is known as (A) Particular integral (B) Singular integral (C) General integral (D) Complete integral	1	1	1	2
4.	The PDE $f(x,p)=g(y,q)$ is called as (A) Lagrange's equation (B) Clairaut's equation (C) Separable equation (D) Laplace equation	1	2	1	2
5.	Sinx is a periodic function with period (A) $\pi/2$ (B) π (C) 2π (D) 3π	1	1	2	1
6.	Which of the following function is an odd function? (A) $x\ sinx$ (B) $x\ cosx$ (C) $x\ e^x$ (D) $x\ logx$	1	1	2	1
7.	If $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$, then the function f(x) is (A) Odd (B) Even (C) Neither odd nor even (D) Periodic	1	1	2	2

8. The value of b_n in the Fourier series expansion of $f(x)=x^2$ in $(-1,1)$ is 1 1 2 2

(A) 1/3 (B) $\pi/3$

(C) π (D) 0

9. How many initial and boundary conditions are required to solve $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ 1 1 3 1

(A) Two (B) Three

(C) Four (D) Five

10. One dimensional wave equation is used to find 1 1 3 1

(A) Temperature (B) Time

(C) Displacement (D) Mass

11. The classification of the PDE $u_{xx}+3u_{xy}+4u_{yy}=\sin x$ is 1 2 3 1

(A) Elliptic (B) Parabolic

(C) Hyperbolic (D) Deterministic

12. The tension T caused by stretching the string before fixing it at the end points is 1 1 3 1

(A) Large (B) Decreasing

(C) Constant (D) Zero

13. If $F\left[f(x)\right]=F(s)$ and a>0 then F[f(ax)]=_____ 1 2 4 2

(A) $\frac{1}{a}F\left(\frac{a}{s}\right)$ (B) $\frac{1}{a}F\left(\frac{s}{a}\right)$

(C) $\frac{1}{s}F\left(\frac{s}{a}\right)$ (D) $\frac{1}{s}F\left(\frac{a}{s}\right)$

14. If $f(x)=e^{-ax}, a>0$, *then* $F_s\left[e^{-ax}\right]$ is 1 2 4 2

(A) $\sqrt{\frac{\pi}{2}}\left(\frac{a}{s^2+a^2}\right)$ (B) $\sqrt{\frac{2}{\pi}}\left(\frac{a}{s^2+a^2}\right)$

(C) $\sqrt{\frac{\pi}{2}}\left(\frac{s}{s^2+a^2}\right)$ (D) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2+a^2}\right)$

15. The Fourier sine transform of *1/x* is 1 2 4 2

(A) $\sqrt{\frac{2}{\pi}}$ (B) $\sqrt{\frac{4}{\pi}}$

(C) $\sqrt{\frac{\pi}{2}}$ (D) $\sqrt{\frac{\pi}{4}}$

16. Fourier cosine transform of f(x)sinax is 1 2 4 2

(A) $\frac{1}{2}\left[F_c(a-s)+F_c(a+s)\right]$ (B) $\frac{1}{2}\left[F_c(a-s)-F_c(a+s)\right]$

(C) $\frac{1}{2}\left[F_c(a-s)+F_s(a+s)\right]$ (D) $\frac{1}{2}\left[F_s(a-s)-F_s(a+s)\right]$

17. The value of $z\left[\sqrt{2}\right]$ is

1 2 5 2

(A) $\frac{z}{z-1}$ (B) $\frac{z}{z-\sqrt{2}}$

(C) $\frac{\sqrt{2}z}{z-1}$ (D) $\frac{\sqrt{2}z}{z-\sqrt{2}}$

18. z-transform of $f(n)=\frac{1}{9^n}$ is

1 2 5 2

(A) $\frac{9z}{z-1}$ (B) $\frac{z}{z-9}$

(C) $\frac{9z}{9z-1}$ (D) $\frac{9z}{9z+1}$

19. The value of $z^{-1}[e^{1/z}]$ is

1 2 5 2

(A) $\frac{1}{n+1}$ (B) $\frac{1}{(n+1)!}$

(C) $\frac{1}{n}$ (D) $\frac{1}{n!}$

20. Poles of $F(z)=\frac{z}{(z+1)(z-2)}$ are

1 2 5 2

(A) z = 1, –2 (B) z = –1, 2

(C) z = –1, –2 (D) z = 1,2

PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions

	Marks	BL	CO	PO
21. Find the complete integral of $z = px+qy+\sqrt{pq}$.	4	3	1	2
22. Find the root mean square value of $f(x)=x^2$ *in* $(-\pi,\pi)$.	4	3	2	2
23. What are the assumptions to be made to derive the one dimensional wave equation?	4	3	3	1
24. Find the Fourier sine transform of $f(x)=xe^{-ax}$.	4	3	4	2
25. If $F(z)=\frac{10z}{(z-1)(z-2)}$, then find f(0).	4	3	5	2
26. Find the complete integral of $p^2+q^2=x+y$.	4	3	1	2
27. A bar, 10 cm long with insulated sides, has its ends A and B kept at 20°C and 90°C until steady state conditional prevail, find u(x).	4	3	3	2

PART – C (5 × 12 = 60 Marks)

Answer **ALL** Questions

		Marks	BL	CO	PO
28. a.	Solve $\left(D^2-2DD'\right)z=x^3y+e^{2x}$.	12	3	1	2
	(OR)				
b.	Solve $x\left(z^2-y^2\right)p+y\left(x^2-z^2\right)q=z\left(y^2-x^2\right)$.	12	3	1	2
29. a.	Find the Fourier series of $f(x)=x^2$ in $(-\pi, \pi)$ and hence find the sum of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}....$	12	3	2	2
	(OR)				
b.	Prove that $f(\theta)=0.75+0.37\cos\theta-1.004\sin\theta$ from the following data, where $\theta=\frac{2\pi x}{T}$.	12	3	2	

x	0	T/6	T/3	T/2	2T/3	5T/6	T
$f(x)$:	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

30. a.	A string is stretched and fixed to two points $x=0$ and $x=l$ apart. Motion is started by displacing the string into the form $y=k\left(lx-x^2\right)$ from which it is released at time t=0. Find the displacement $y(x,t)$.	12	4	3	1
	(OR)				
b.	Solve $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ subject to (i) $u(0,t)=0$, *for* $t\geq 0$ (ii) $u(l,t)=0$, *for* $t\geq 0$ (iii) $u(x,0)=\pi x$, $0<x<l$	12	4	3	1
31. a.	Prove that $e^{-x^2/2}$ is a self-reciprocal function under Fourier transform.	12	3	4	2
	(OR)				
b.	Find the Fourier sine transform of $f(x)=e^{-ax}$ and hence evaluate (i) $\int_0^\infty \frac{x\sin mx}{x^2+a^2}dx$ (ii) $\int_0^\infty \frac{x^2}{\left(x^2+a^2\right)^2}dx$	12	3	4	2
32. a.	Find $Z^{-1}\left(\frac{z^2-3z}{(z-5)(z+2)}\right)$ by using residue theorem.	12	3	5	2
	(OR)				
b.	Solve $y(n+2)-3y(n+1)-10y(n)=0$, given $y(0)=1, y(1)=0$ using z-transform.	12	3	5	2

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