Reg. No.															

M.Sc. DEGREE EXAMINATION, MAY 2022

Fourth Semester

18PMA402 – INTEGRAL EQUATIONS AND TRANSFORMATION TECHNIQUES

(For the candidates admitted during the academic year 2018-2019 onwards)

Time: Three hours | Max. Marks: 100

PART – A (5 × 5 = 25 Marks)

Answer **ANY FIVE** Questions

1. Define Fredholm integral equations of first and second kind.

2. Convert the differential equation
   $y''(x)-3y'(x)+2y(x)=5\sin x,$
   $y(0)=1, y'(0)=-2$
   into an integral equation.

3. Using Fredholm determinants, find B_1 and B_2 for $K(x,ty)=xe^{t}$.

4. If F(s) is the complex Fourier transform of f(x), then show that $F\{f(x-a)\}=e^{isa}F(s)$.

5. Find
   $Fs\left(xe^{-ax}\right)$ and $Fc\left(xe^{-ax}\right)$

6. Evaluate
   $$L\left\{\int_0^t \frac{e^{-t}\sin t}{t}dt\right\}$$

7. Verify the initial value theorem for the function 3–2 cost.

8. Find Z-transform of $a^n \cos n\theta$ and $a^n \sin n\theta$.

PART – B (5 × 15 = 75 Marks)

9. a. Solve Fredholm integral equation

$$y(x) = 1 + \lambda \int_0^1 xt\, y(t)dt$$ by successive approximation method.

(OR)

b. Solve $y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\, y(t)\, dt$ by Kernel separable method.

10. a. Find the resolvent Kernel of Kernel $K(x,t) = 2x - t$, $0 \le x \le 1$, $0 \le t \le 1$, using Fredholm determinants.

(OR)

b. Find the Volterra solution of Fredholm Integral equation

$$y(x) = f(x) + \int_0^1 e^{x-t} f(t)\, dt.$$

11. a. Find the Fourier transform of

$$f(x) = \begin{cases} (1-x^2), & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$

Hence evaluate

$$\int_0^{\infty} \left(\frac{x\cos x - \sin x}{x^3} \right) \cos\frac{x}{2}\, dx$$

(OR)

b. Find the Fourier sin and cosine transform of $e^{-ax}, a > 0$. And also using Parseval's identities, prove that

i. $$\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$$

ii. $$\int_0^\infty \frac{t^2}{(t^2+1)^2} = \frac{\pi}{4}$$

12. a. Find the inverse transform of

$$\frac{5s+3}{(s-1)(s^2+2s+5)}$$

(OR)

b. Use Laplace Transform method to solve

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t \text{ with } x = 2, \frac{dx}{dt} = -1 \, at \, t = 0.$$

13. a. Using the inversion integral method, find the inverse Z transform of

$$\frac{2z}{(z-1)(z^2+1)}$$

(OR)

b. Using the Z transform, solve

$u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0, u_1 = 1$

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