Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2024

Fourth Semester

18MAB206T – NUMERICAL METHODS AND ANALYSIS

(For the candidates admitted from the academic year 2018-2019 to 2021-2022)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

		Marks	BL	CO	PO
1.	Newton's method is not applicable for finding a root of $f(x)=0$ with the initial approximation $x=x_0$, if (A) $f(x_0)=0$ (B) $f'(x_0)=0$ (C) $f(x_0)\neq 0$ (D) $f'(x_0)\neq 0$	1	1	1	2
2.	A root of the equation $x^4-x^3-x^2-1=0$ lies between (A) 0 and 1 (B) 1 and 2 (C) 2 and 3 (D) 3 and 4	1	2	1	2
3.	Consider the equation f(x)=0 with f(a)<0 and f(b)>0, then by Regula Falsi method (A) $x_1=\frac{af(b)-bf(a)}{b-a}$ (B) $x_1=\frac{bf(b)-af(a)}{b-a}$ (C) $x_1=\frac{bf(b)-af(a)}{f(b)-f(a)}$ (D) $x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}$	1	1	1	2
4.	The equation $x=\cos x$ is (A) A linear equation (B) A quadratic equation (C) A transcendental equation (D) An algebraic equation	1	1	1	2
5.	If $f(x)=x^2$ *and* $h=2$, then the value of Ef(x) is (A) x^2+2x+1 (B) x^2+2x+2 (C) x^2+2x+4 (D) x^2+4x+4	1	2	2	1
6.	If f(x) is a polynomial of degree n, then Δf(x) is a polynomial of degree (A) n–2 (B) n–1 (C) n (D) n+1	1	1	2	2
7.	If $y_0=0, y_1=3$ *and* $y_2=5$, then the value of ∇y_1 is (A) 3 (B) 2 (C) 1 (D) 0	1	2	2	2
8.	The averaging operator μ is defined as (A) $\mu f(x)=f\left(x+\frac{h}{2}\right)-f\left(x-\frac{h}{2}\right)$ (B) $\mu f(x)=f\left(x+\frac{h}{2}\right)+f\left(x-\frac{h}{2}\right)$ (C) $\mu f(x)=\frac{1}{2}\left[f\left(x+\frac{h}{2}\right)-f\left(x-\frac{h}{2}\right)\right]$ (D) $\mu f(x)=\frac{1}{2}\left[f\left(x+\frac{h}{2}\right)+f\left(x-\frac{h}{2}\right)\right]$	1	1	2	1

9. If $y_0 = 2$ *and* $y_1 = 4$, then by Newton's forward formula, the value of $\left(\dfrac{dy}{dx}\right)_{x=x_0}$ is | 1 | 2 | 3 | 1

(A) 4 (B) 3
(C) 2 (D) 0

10. If $y_0 = a$ *and* $y_1 = b$ then by Newton's backward formula, the value of $\left(\dfrac{d^2y}{dx^2}\right)_{x=x_1}$ is | 1 | 2 | 3 | 1

(A) 0 (B) 1
(C) b–a (D) b+a

11. Let y=f(x) be a given function such that f(0)=3 and f(a)=7, then by Trapezoidal rule with h=a, the value of $\int_0^a f(x)dx$ is | 1 | 2 | 3 | 1

(A) 10/a (B) 5/a
(C) 10a (D) 5a

12. Which of the following method is useful for finding a numerical integration? | 1 | 1 | 3 | 1

(A) Newton's method (B) Runge-Kutta method
(C) Simpson's (3/8) rule (D) Euler's method

13. Given that $\dfrac{dy}{dx} = f(x,y), y(1) = 1$. If h=1, then by Euler's method | 1 | 2 | 4 | 1

(A) $y(2) = 1 + f(0,1)$ (B) $y(2) = 1 + f(1,1)$
(C) $y(2) = 1 + \dfrac{1}{2} f(1,1)$ (D) $y(2) = 1 + f(2,2)$

14. Given that $\dfrac{dy}{dx} = x + y, y(0) = 1$, then by Euler's method, with h=1. | 1 | 2 | 4 | 1

(A) y(1)=1 (B) y(1)=1.5
(C) y(1)=2 (D) y(1)=2.5

15. If k_1 and k_2 are usual constants for finding a numerical solution of the initial value problem $\dfrac{dy}{dx} = f(x,y), y(x_0) = y_0$ by Runge-Kutta fourth order method. Then | 1 | 1 | 4 | 2

(A) $k_2 = hf(x_0, y_0)$ (B) $k_2 = hk_1 f(x_0, y_0)$
(C) $k_2 = hf(x_0 + h, y_0 + k_1)$ (D) $k_2 = hf\left(x_0 + \dfrac{h}{2}, y_0 + \dfrac{k_1}{2}\right)$

16. Consider the initial value problem $\dfrac{dy}{dx} = f(x,y), y(x_0) = y_0$. If $h = x_1 - x_0$, then by Taylor series method, the value of $y(x_1)$ is | 1 | 1 | 4 | 2

(A) $y_0 + hy_0' + \dfrac{h^2}{2} y_0'' + \dfrac{h^3}{3} y_0''' + \ldots$ (B) $y_0 + hy_0' + \dfrac{h^2}{2!} y_0'' + \dfrac{h^3}{3!} y_0''' + \ldots$
(C) $y_0 - hy_0' + \dfrac{h^2}{2} y_0'' - \dfrac{h^3}{3} y_0''' + \ldots$ (D) $y_0 - hy_0' + \dfrac{h^2}{2!} y_0'' - \dfrac{h^3}{3!} y_0''' + \ldots$

17. The equation $u_{xx} + u_{yy} = f(x, y)$ is known as 1 1 5 2

(A) Poisson equation
(B) Laplace equation
(C) Heat equation
(D) Wave equation

18. The equation $a\dfrac{\partial^2 u}{\partial x^2} + b\dfrac{\partial^2 u}{\partial y^2} + c\dfrac{\partial^2 u}{\partial x \partial y} = 0$ is hyperbolic, if 1 1 5 2

(A) $b^2 - 4ac < 0$
(B) $b^2 - 4ac > 0$
(C) $c^2 - 4ab > 0$
(D) $c^2 - 4ab < 0$

19. By Standard five point formula, the value of $u_{1,1}$ is 1 2 5 1

(A) $\dfrac{1}{4}\left[u_{0,1} + u_{2,1} + u_{1,0} + u_{1,2}\right]$
(B) $\dfrac{1}{2}\left[u_{0,1} + u_{2,1} + u_{1,0} + u_{1,2}\right]$
(C) $\dfrac{1}{4}\left[u_{0,1} + u_{1,0} + u_{2,1} + u_{2,2}\right]$
(D) $\dfrac{1}{2}\left[u_{0,0} + u_{1,0} + u_{0,1} + u_{2,2}\right]$

20. If $u_{0,0} = u_{0,2} = u_{2,0} = u_{2,2} = 5$, then by Diagonal five point formula, the value of $u_{1,1}$ is 1 2 5 1

(A) 5/4
(B) 5
(C) 10
(D) 20

PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions

Marks BL CO PO

21. Solve by Jacobi method up to three iterations. 4 3 1 1.2

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 35$$

22. Find a root of the following equation lying between 0 and 4 by Bisection method, $x^3 - x^2 + x - 1 = 0$. 4 3 1 1.2

23. Express $f(x) = x^3 + x^2 + x$ in terms of factorial polynomial. 4 4 2 1.2

24. Find a polynomial satisfying the points (0, 1), (1,2) and (2,5) using Newton's or Largange's formula for interpolation. 4 4 2 1,2

25. Find the value of $\left(\dfrac{dy}{dx}\right)_{x=1}$ by Newton's forward formula using the table below: 4 4 3 1.2

x	1	2	3
y	1	4	9

26. Compute y at x=0.25 by Modified Euler's method given $\dfrac{dy}{dx} = 2xy, y(0) = 1$ *and* $h = 0.25$. 4 4 4 1.2

27. Classify the partial differential equation 4 4 5 1.2

$$u_{xx} + 4u_{xy} + (x^2 + 4y^2)u_{yy} = \sin(x + y)$$

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

	Marks	BL	CO	PO
28. a. Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method.	12	3	1	1.2
(OR)				
b. Solve the system by Gauss Jordan method. $2x+3y-z=5$, $4x+4y-3z=3$, $2x-3y+2z=2$	12	3	1	1.2
29. a. Find the missing terms of the sequence 1, 8, ?, 64, ?, 216.	12	4	2	1.2
(OR)				
b. Prove that (i) $\nabla\Delta = \Delta - \nabla = \delta^2$ (ii) $\left(\frac{\Delta^2}{E}\right)e^x.\left(\frac{Ee^x}{\Delta^2 e^x}\right) = e^x$	12	4	2	1.2
30. a. Find an approximate value of $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ from the integral $\int_0^2 \frac{dx}{1+x+x^2}$ taking n=6, using (i) Trapezoidal rule (ii) Simpson's $\left(\frac{1}{3}\right)$ rule	12	4	3	1.2
(OR)				
b. Find the first two derivatives of $x^{1/3}$ at x=56 given the table below.	12	4	3	1.2

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

	Marks	BL	CO	PO
31. a. Solve $\frac{dy}{dx} = x + y, y(1) = 0$ *and get* $y(1.1), y(1.2)$ by Taylor series method.	12	4	4	1.2
(OR)				
b. Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1$ and get y(0.2) by Runge-Kutta fourth order method.	12	4	4	1.2
32. a. Solve $u_t = u_{xx}$ subject to $u(0,t) = (1,t) = 0$ *and* $u(x,0) = \sin \pi x, 0 < x < 1$.	12	4	5	1.2
(OR)				
b. Solve $\nabla^2 u = 8x^2y^2$ for square Mesh given u=0 on the four boundaries dividing the square into 16 sub-squares of length 1 unit.	12	4	5	1.2

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