**Reg. No.**

## B.Tech. DEGREE EXAMINATION, DECEMBER 2023
Fourth Semester

## 18MAB203T – PROBABILITY AND STOCHASTIC PROCESSES
*(For the candidates admitted from the academic year 2020-2021 to 2021-2022)*
(Statistical tables to be provided)

**Note:**
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours — Max. Marks: 100

### PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

Marks BL CO PO

1. A discrete random variables takes ________ values. — 1 1 1 1
   (A) Positive (B) Finite
   (C) Countably infinite (D) Finite or countably infinite

2. If the probability density function of a random variable X is given by $f(x) = kx, 0 < x < 1$, find k. — 1 2 1 2
   (A) 0 (B) 1
   (C) 2 (D) 1/2

3. If the pdf of a RV X is $f(x) = 2x, 0 < x < 1, \textit{then } P(X > 0.5)$ equal to — 1 2 1 2
   (A) 1/2 (B) 2/3
   (C) 3/4 (D) 4/5

4. The first moment of X about its mean is always — 1 1 1 1
   (A) 1 (B) −1
   (C) 0 (D) ±1

5. The joint pdf of (X, Y) is $f(x,y) = \begin{cases} 2x & 0 < x, y < 2 \\ 0 & otherwise \end{cases}$ then the marginal pdf of X is — 1 2 2 1
   (A) *2(1−x)* (B) *2(1−y)*
   (C) *4x* (D) *2xy*

6. Correlation coefficient *r* lies in the interval — 1 1 2 1
   (A) $-1 \le r \le 1$ (B) $0 \le r < 1$
   (C) $r < -1$ (D) $r > 1$

7. The joint pdf of (X,Y) is given by $f(x,y) = \begin{cases} xy & 0 < x < 1, 0 < y < 2 \\ 0 & otherwise \end{cases}$ then the value of E(XY) is — 1 2 2 2
   (A) 8/3 (B) 8/9
   (C) 1/3 (D) 1/9

8. If $f_{XY}(x,y)$ is the joint pdf of (X,Y) then *f(x/y)* — 1 1 2 1
   (A) $f(x,y)/f_X(x)$ (B) $f(x,y)/f_Y(y)$
   (C) $f_X(x)/f_Y(y)$ (D) $f_Y(y)/f_X(x)$

9. If X is a random variable with mean 0 and finite variance $\sigma^2$ then for any a>0, $P(X \geq a) \leq$________ 1 1 3 1

(A) $\dfrac{\sigma}{\sigma^2+a^2}$  (B) $\dfrac{\sigma^2}{\sigma^2-a^2}$

(C) $\dfrac{\sigma^2}{\sigma^2+a^2}$  (D) $\dfrac{\sigma}{\sigma^2-a^2}$

10. A random variable X has a mean of 9 and a variance of 3. Then an upper bound for $P\{|X-9| \geq 3\}$ is 1 2 3 2

(A) 3  (B) 9

(C) 1/9  (D) 1/3

11. $P\{|X-\mu| \leq C\} \geq$________where C>0 1 1 3 1

(A) $1-\dfrac{\sigma^2}{C^2}$  (B) $1-\dfrac{\sigma}{C^2}$

(C) $\dfrac{\sigma^2}{C^2}$  (D) $1+\dfrac{\sigma^2}{C^2}$

12. If mean=1000 and S.D=100 for each of n independent random variables with n=60 then by central limit theorems, we have 1 1 3 1

(A) $N\left(1000, \dfrac{100}{\sqrt{60}}\right)$  (B) $N(1000,100)$

(C) $N\left(1000, 100\sqrt{60}\right)$  (D) $N\left(1000, \sqrt{\dfrac{100}{60}}\right)$

13. The first and second order densities of an SSS process are 1 1 4 1

(A) Dependent on time  (B) Independent of time

(C) Continuous  (D) Discrete

14. $R_{XX}(-\tau)$ is equal to 1 1 4 1

(A) $-R_{XX}(\tau)$  (B) $R_{XX}(\tau)$

(C) $\tau R_{XX}(\tau)$  (D) $R_{XY}(\tau)$

15. A stationary process has auto correlation function given by $R(\tau)=\dfrac{25\tau^2+36}{6.25\tau^2+4}$, then the mean value of the process is 1 2 4 2

(A) 9  (B) 3

(C) 4  (D) 2

16. $|R_{XY}(\tau)| \leq$ 1 1 4 1

(A) $R_{XX}(0)R_{YY}(0)$  (B) $\sqrt{R_{XX}(0)R_{YY}(0)}$

(C) $\sqrt{R_{XX}(0)+R_{YY}(0)}$  (D) $\sqrt{R_{XX}(0)-R_{YY}(0)}$

17. The convolution form of the output of linear time invariant system is 1 1 5 1

(A) $Y(t)=\int_{-\infty}^{+\infty} h(u)X(t-u)\,du$  (B) $Y(t)=\int_{0}^{\infty} h(u)X(t-u)\,du$

(C) $Y(t)=\int_{-\infty}^{+\infty} h(t)X(t-u)\,du$  (D) $Y(t)=\int_{0}^{\infty} h(t)X(t-u)\,du$

18. The power spectral density of a random signal with autocorrelation function $R_{XX}(\tau)=e^{-2|\tau|}$ is 1 2 5 1

(A) $\frac{4}{4+\omega^2}$ (B) $\frac{2}{2+\omega^2}$

(C) $\frac{4}{4-\omega^2}$ (D) $\frac{2}{2-\omega^2}$

19. The power spectral density of a WSS process is always 1 1 5 1

(A) Finite (B) Zero

(C) Non-negative (D) Negative

20. If $X(t)$ *and* $Y(t)$ are orthogonal then 1 1 5 1

(A) $S_{XY}(\omega)=1$ (B) $S_{XY}(\omega)=0$

(C) $S_{XY}(\omega)\le 1$ (D) $S_{XY}(\omega)\ne 0$

## PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions

**Marks BL CO PO**

21. State and prove memory less property of exponential distribution. 4 3 1 1

22. If X and Y are have joint pdf of $f_{XY}(x,y)=\begin{cases} x+y & 0<x<1,0<y<1 \\ 0 & \text{otherwise}\end{cases}$ check whether X and Y are independent. 4 3 2 2

23. State and prove Cauchy Schwartz inequality. 4 4 3 2

24. Consider the random process $X(t)=\cos(t+\varnothing)$, where ∅ is a random variable with density function f(∅)=1/π, –π/2<∅<π/2, check whether the process is stationary or not. 4 3 4 1

25. If $R(\tau)=e^{-2\lambda|\tau|}$ is the auto correlation function of a random process X(t), obtain the spectral density of X(t). 4 3 5 1

26. A random variable X has the following probability function. 4 3 1 1

| *x* | –2 | –1 | 0 | 1 |
|---|---|---|---|---|
| *P(x)* | 0.4 | k | 0.2 | 0.3 |

Find the value of k and the mean value of X.

27. If the joint pdf of (X,Y) is given by $f(x,y)=\begin{cases} 2-x-y, 0\le x\le 1, 0\le y\le 1 \\ 0 \quad \textit{otherwise}\end{cases}$ 4 3 2 1

Find E(x).

## PART – C (5 × 12 = 60 Marks)

Answer **ALL** Questions

**Marks BL CO PO**

28. a. A discrete random variable X has the following probability distribution. 16 3 1 1

| *x* | 0 | 1 | 2 | 3 | *4* | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| *P(X=x)* | a | 3a | 5a | 7a | *9a* | 11a | 13a | 15a | 17a |

(i) Find a (ii) mean (iii) variance, (iv) find F(x).

**(OR)**

b.i. Out of 2000 families with 4 children each, how many would you expect to have (1) atleast 1 boy (2) 2 boys (3) 1 or 2 girls (4) no girls. 8 4 1 2

ii. State and prove memoryless property of geometric distribution. 8 3 1 1

29. a. If the joint pdf of (X,Y) is given by $f(x,y)=k\left(x^3y+xy^3\right)0\le x\le 2, 0\le y\le 2$. 12 3 2 1

Find
(i) the value of k
(ii) P(1/2≤Y≤3/4)
(iii) P(1/4≤X≤1/2)
(iv) f(X/Y)

**(OR)**

b. The joint probability mass function of X and Y is given below. 12 3 2 1

| X<br>Y | −1 | 1 |
|---|---|---|
| 0 | 1/8 | 3/8 |
| 1 | 2/8 | 2/8 |

Find the correlation coefficient of X and Y.

30. a. If $X_1, X_2, \ldots X_n$ are independent Poison variables with parameter λ=2, use central limit theorem to estimate P(120<$s_n$<160), where $S_n=X_1+X_2\ldots+X_n$ and n=75. 12 4 3 2

**(OR)**

b. A random variable X is exponentially distributed with parameter 1. Use Tchebycheff's inequality to show that $P(-1\le X\le 3)\ge 3/4$. Find the actual probability also. 12 4 3 2

31. a. Show that the process $X(t)=A\cos\lambda t+B\sin\lambda t$ (where A and B are random variables) is WSS, if A and B are un-correlated independent uncorrelated random variables with zero mean and having same variances. 12 3 4 1

**(OR)**

b.i. If $Y(t)=X(t+a)-X(t-a)$, where X(t) is a WSS process then show that $R_{YY}(\tau)=2R_{XX}(\tau)-R_{XX}(\tau-2a)-R_{XX}(\tau+2a)$. 6 3 4 1

ii. Prove that $|R_{XY}(\tau)|\le\sqrt{R_{XX}(0)-R_{YY}(0)}$. 6 3 4 1

32. a. If the power spectral density of a WSS process is given by 12 3 5 1

$$S_{XX}(\omega)=\begin{cases}\frac{b}{a}(a-|\omega|) & if\ |\omega|\le a\\ 0 & if\ |\omega|>a\end{cases}$$

Find the auto-correlation function of the process.

**(OR)**

b. A random process X(t) is the input to a linear system whose impulse response is $n(t)=2e^{-t}, t>0$, if the auto correlation function of the process is $R_{XX}(\tau)=e^{-2|\tau|}$. Find the power spectral density of the output process Y(t). 12 3 5 2

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