Reg. No.															

B.Tech. DEGREE EXAMINATION, DECEMBER 2023
First Semester

18MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted from the academic year 2020-2021 to 2021-2022)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

		Marks	BL	CO	PO
1.	Find the eigen values of A^2 if $A=\begin{bmatrix}3&2&4\\0&2&6\\0&0&5\end{bmatrix}$ (A) 3, 2, 5 (B) 9, 4, 25 (C) 4, 1, 2 (D) 8, 2, 4	1	1	1	1
2.	The number of positive terms in the canonical form is called (A) Signature (B) Orthogonal (C) Index (D) Identity	1	1	1	1
3.	Write the quadratic form defined by the matrix $A=\begin{bmatrix}6&1&-7\\1&2&0\\-7&0&1\end{bmatrix}$ (A) $6x^2+2y^2+z^2+2xy-14xz$ (B) $6x^2+2y^2+z^2+2xy+14xz$ (C) $6x^2+y^2+z^2+xy+yz$ (D) $6x^2+6y^2+6z^2+xy+2xyz$	1	1	1	1
4.	Find the eigen values of the matrix $\begin{pmatrix}2&1\\1&2\end{pmatrix}$ (A) 2, 2 (B) 7, –3 (C) 4, 5 (D) 3, 1	1	1	1	1
5.	If $rt-s^2<0\ at\ (a,b)$, then the point is called (A) Maximum point (B) Minimum point (C) Saddle point (D) Bounded point	1	1	2	1
6.	If $z=x^3-3x^2y+3y^2$ then the value of $\frac{\partial z}{\partial y}$ is (A) $-3x^2-3y$ (B) $-3x^2+6y$ (C) $3x^2+4y+7x$ (D) $3x^2-4y-7x$	1	1	2	1

7. If $f(x,y)=e^{xy}$, then the value of $f_x(1,1)=$ 1 1 2 1

(A) $2e$ (B) $3e$
(C) e (D) $-e$

8. If $J_1=\frac{\partial(x,y)}{\partial(u,v)}$ *and* $J_2=\frac{\partial(u,v)}{\partial(x,y)}$ then $J_1J_2=$ 1 1 2 1

(A) -1 (B) 1
(C) -2 (D) 2

9. The solution of $\left(D^3+3D^2+3D+2\right)y=0$ is 1 1 3 1

(A) $y=C_1e^{-2x}+e^{-x/2}\left(C_2\cos\frac{\sqrt{3}}{2}x+C_3\sin\frac{\sqrt{3}}{2}x\right)$

(B) $y=C_1e^{-2x}+C_2e^{-3x}+C_3e^{-4x}$

(C) $y=\left(C_1+C_2x+C_3x^2\right)e^{-2x}$

(D) $y=C_1e^{-2x}+(C_2+C_3x)e^{-3x}$

10. The particular integral of $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=3e^{4x}$ is 1 1 3 1

(A) $\frac{2}{49}e^{4x}$ (B) $\frac{4}{49}e^{4x}$
(C) $\frac{3}{49}e^{4x}$ (D) $\frac{3}{49}e^{-4x}$

11. The complementary function of 1 1 3 1
$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\left[\log(1+x)\right]$ is

(A) $CF=C_1\cos z+C_2\sin z$ (B) $CF=C_1\cos 2z+C_2\sin 3z$
(C) $CF=C_1e^{2z}+C_2e^{3z}$ (D) $CF=C_1e^{z}+C_2e^{-z}$

12. The solution of $\left(D^2-2D+1\right)y=4$ is 1 1 3 1

(A) $y=(C_1+C_2x)e^{x}+4$ (B) $y=C_1e^{x}+C_2e^{-x}+4$
(C) $y=(C_1+C_2x)e^{-x}+4$ (D) $y=(C_1+C_2x)e^{-2x}+5$

13. The radius of curvature at any point on the curve $r=e^{\theta}$ is 1 1 4 1

(A) $\frac{\sqrt{2}}{r}$ (B) $\frac{r}{\sqrt{2}}$
(C) r (D) $\sqrt{2}\,r$

14. The equation of the circle of curvature at any point (x,y) with centre of curvature $\bar{x},\bar{y}$ and with radius of curvature ρ is 1 1 4 1

(A) $(x-\bar{x})^2+(y-\bar{y})^2=\rho^2$ (B) $(x-\bar{x})^2+(y-\bar{y})^2=\rho$
(C) $(x+\bar{x})^2+(y+\bar{y})^2=\rho^2$ (D) $(x+\bar{x})^2+(y+\bar{y})^2=\rho$

15. $\Gamma(n) =$ 1 1 4 1

(A) (n+1)!
(B) n!
(C) (n–1)!
(D) (n+2)!

16. $\beta(m,n) =$ 1 1 4 1

(A) $\dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$
(B) $\Gamma(m) / \Gamma(n)$
(C) $\Gamma(n) / \Gamma(m)$
(D) $\Gamma(m+n) / \Gamma(m)\Gamma(n)$

17. The series $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ is convergent if 1 1 5 1

(A) p=1
(B) p=0
(C) p<1
(D) p>1

18. Let $\sum_{n=1}^{\infty} u_n$ be a series of positive terms such that $\lim_{n\to\infty} \dfrac{u_n}{u_{n+1}} = l$. Then the series is convergent if 1 1 5 1

(A) $l>1$
(B) $l<1$
(C) $l=1$
(D) $l=0$

19. The sequence $S_n = \dfrac{2n^3+7n}{5n^3+3n^2}$ converges to 1 1 5 1

(A) 1
(B) 0
(C) 1/5
(D) 2/5

20. If $\sum_{n=1}^{\infty} u_n$ is a series of positive terms and $\lim_{n\to\infty} u_n^{1/n} = l$, then $\sum u_n$ is divergent if 1 1 5 1

(A) $l<1$
(B) $l>1$
(C) $l=1$
(D) $l=0$

PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions

		Marks	BL	CO	PO
21.	Two eigen values of the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ are 3 and 6. Find the eigen values of A^{-1}.	4	1	1	1
22.	Find the nature of the quadratic form $x_1^2+3x_2^2+6x_3^2+2x_1x_2+2x_2x_3+4x_3x_1$ without reducing to canonical form.	4	2	1	1
23.	Find $\dfrac{dy}{dx}$, if $xe^{-y}-2ye^{x}=1$.	4	1	2	1

		Marks	BL	CO	PO
24.	Examine for extreme values of $f(x,y)=x^2+y^2+6x+12$.	4	1	2	1
25.	Solve $\left(D^2+3D+2\right)y=e^{-2x}$.	4	2	3	1
26.	Find its envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$, where a and b are connected by the relation $a+b=c$, where c being a constant.	4	2	4	1
27.	Test the convergence of $\left\{\frac{n-2}{3n}\right\}$.	4	1	5	2

PART – C (5 × 12 = 60 Marks)

Answer **ALL** Questions

		Marks	BL	CO	PO
28. a.	Reduce the quadratic form $x_1^2+5x_2^2+x_3^2+2x_1x_2+2x_2x_3+6x_3x_1$ to a canonical form using orthogonal transformation.	12	2	1	1
	(OR)				
b.	Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{bmatrix}$ and hence find A^{-1}.	12	2	1	1
29. a.	Find the Taylor series expansion of e^{xy} at (1,1) up to the third degree terms.	12	2	2	2
	(OR)				
b.	Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.	12	2	2	1
30. a.	Solve $\left(D^2+5D+6\right)y=x^2+4e^{3x}$.	12	2	3	2
	(OR)				
b.	Solve $\frac{d^2y}{dx^2}+y=\operatorname{cosec} x$ by the method of variation of parameters.	12	2	3	2
31. a.	Find the evolute of the parabola $y^2=4ax$.	12	2	4	2
	(OR)				
b.	Find the radius of curvature at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ of the curve $x^3+y^3=3axy$.	12	2	4	2
32. a.	Test the convergence or divergence of the series $\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots\infty, x>0$.	12	2	5	1
	(OR)				
b.	Test the convergence of the series $x+\frac{1}{2}\frac{x^3}{3}+\frac{1.3}{2.4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+\ldots\infty, x>0$.	12	2	5	1

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