Reg. No															

B.Tech DEGREE EXAMINATION, JUNE 2024

Second Semester

18MAB102T - ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2018 - 2019 to 2021 - 2022)

Note:

i. **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
ii. **Part - B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours **Max. Marks: 100**

PART - A (20 × 1 = 20 Marks)

Answer **all** Questions

	Question	Marks	BL	CO
1.	Evaluation of $\int_0^1\int_0^1 dxdy$ is (A) 1 (B) 2 (C) 0 (D) 4	1	1	1
2.	The area of an ellipse is (A) $\pi a^2 b$ (B) πab^2 (C) πr^2 (D) πab	1	1	1
3.	The curve $y^2 = 4x$ is a (A) parabola (B) hyperbola (C) straight line (D) ellipse	1	1	1
4.	The name of the curve $r = a(1+\cos\theta)$ is (A) cycloid (B) lemniscate (C) cardioid (D) parabola	1	1	1
5.	The condition for $\vec{F}$ to be Conservative is, $\vec{F}$ should be (A) solenoidal vector (B) irrotational vector (C) rotational (D) neither solenoidal nor irrotational	1	1	2
6.	If $\vec{r}$ is the position vector of the point (x, y, z) w. r. to the origin, then $\nabla \cdot \vec{r}$ is (A) 3 (B) 0 (C) 2 (D) 1	1	2	2

7.	If $\vec{a}$ is a constant vector and $\vec{r}$ is the position vector of the point (x, y, z) w. r. to the origin then $curl(\vec{a} \times \vec{r})$ is (A) 1 (B) 0 (C) $2\vec{a}$ (D) $2\vec{r}$	1	3	2
8.	If $\phi = xyz$, then $\nabla\phi$ is (A) $zx\,\vec{i} + xy\,\vec{j} + yz\,\vec{k}$ (B) $xy\,\vec{i} + yz\,\vec{j} + zx\,\vec{k}$ (C) 0 (D) $yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$	1	2	2
9.	$L(1) =$ (A) s (B) $\frac{1}{s^2}$ (C) $\frac{1}{s}$ (D) 1	1	1	3
10.	$L(\cos 2t) =$ (A) $\frac{s}{s^2+2}$ (B) $\frac{s}{s^2+4}$ (C) $\frac{2}{s^2+2}$ (D) $\frac{4}{s^2+4}$	1	1	3
11.	$L(t^4) =$ (A) $\frac{5!}{s^4}$ (B) $\frac{4!}{s^5}$ (C) $\frac{3!}{s^4}$ (D) $\frac{4!}{s^4}$	1	3	3
12.	Inverse Laplace transform of $\frac{1}{s^2-a^2}$ is (A) $\frac{\sinh at}{a}$ (B) $\frac{\sin at}{a}$ (C) $\sinh at$ (D) $\sin at$	1	1	3

13. The function $f(z)=u+iv$ is analytic if 1 1 4

(A) $u_x=-v_y, u_y=v_x$ (B) $u_y=v_y, u_x=v_x$

(C) $u_x=v_y, u_y=-v_x$ (D) $u_x+v_y=0, u_y-v_x=0$

14. The transformation $w=cz$ where c is real constant represents 1 1 4

(A) rotation (B) magnification

(C) reflection (D) magnification and rotation

15. If a function $u(x,y)$ satisfies $u_{xx}+u_{yy}=0$, then u is 1 1 4

(A) differentiable (B) harmonic

(C) continuous (D) analytic

16. The critical point of transformation $w=z^2$ is 1 1 4

(A) $z=-2$ (B) $z=2$

(C) $z=1$ (D) $z=0$

17. The value of $\int_c \frac{z\,dz}{z-2}$ where c is the circle $|z|=1$ is 1 1 5

(A) 0 (B) $\frac{\pi}{2}i$

(C) $\frac{\pi}{2}$ (D) 2

18. If $f(z)$ is analytic inside and on c, the value of $\int_c \frac{f(z)}{z-a}dz$, where c is the simple closed curve and a is any point within c, is 1 1 5

(A) $f(a)$ (B) $2\pi i f(a)$

(C) $\pi i f(a)$ (D) 0

19. A curve which does not cross itself is called a 1 1 5

(A) simple closed curve (B) curve

(C) multiple curve (D) closed curve

20. The part $\sum_{n=1}^{\infty} b_n(z-a)^{-n}$ consisting of negative integral powers of $(z-a)$ is called as 1 1 5

(A) The analytic part of the Laurent's series (B) The principal part of the Laurent's series

(C) The real part of the Laurent's series (D) The imaginary part of the Laurent's series

PART - B (5 × 4 = 20 Marks)

Answer **any 5** Questions

Marks BL CO

21. Evaluate $\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^4 \sin\emptyset\, dr\, d\emptyset\, d\theta$ 4 2 1

22.	Find grad ϕ for the following functions. (i) $\phi = 3x^2y - y^3z^2$ at the point $(1, -2, 1)$. (ii) $\phi = \log(x^2 + y^2 + z^2)$ at the point $(1, 2, 1)$.	4	2	2
23.	Verify final value theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$	4	2	3
24.	Show that $u = 3x^2y - y^3$ is harmonic function.	4	3	4
25.	Evaluate $\oint_c \frac{e^{-z}}{z+1} dz$, where c is the circle $\lvert z \rvert = 2$.	4	2	5
26.	Test whether $f(z) = \bar{z}$ is analytic.	4	2	4
27.	In what direction from $(3, 1, -2)$ is the directional derivative of $\phi = x^2y^2z^4$ maximum? Find also the magnitude of this maximum.	4	2	2

PART - C (5 × 12 = 60 Marks)
Answer **all** Questions

		Marks	BL	CO
28.	(a) Change the order of integration and hence evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\,dx\,dy$. (OR) (b) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$, using triple integral.	12	3	1
29.	(a) Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ (OR) (b) Verify Gauss divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	12	3	2

30. (a) 12 1 3

Find the Laplace transform of a periodic function $f(t)$, with period 2, given by

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ -1, & 1 < t < 2 \end{cases}$$

(OR)

(b)

Solve: $x'' - 2x' + x = e^{-t}$, given that $x(0) = 2, x'(0) = 1$.

31. (a) 12 1 4

Find the bilinear mapping which maps $-1, 0, 1$ of the z-plane onto $i, 1, -i$ of the w-plane.

(OR)

(b)

Show that the function $u = e^{x} \cos y$ is harmonic and find the harmonic conjugate of u.

32. (a) 12 1 5

Find the Laurent's series expansion to represent the function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$, where $(i)\ |z| < 2,\ (ii)\ 2 < |z| < 3, (iii)\ |z| > 3$.

(OR)

(b)

Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$ by using contour integration.

* * * * *