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B.Tech DEGREE EXAMINATION, NOVEMBER 2023

Fourth and Fifth Semester

18MAB204T - PROBABILITY AND QUEUEING THEORY

(For the candidates admitted during the academic year 2020 - 2021 & 2021 - 2022)

(Statistical tables to be provided)

Note:

i. **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

ii. **Part - B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours **Max. Marks: 100**

PART - A (20 × 1 = 20 Marks)

Answer **all** Questions

		Marks	BL	CO
1.	If $f(x) = \frac{1}{10}, x = 10$, then $E(x)$ is (A) 0 (B) 2 (C) 1 (D) -1	1	2	1
2.	$Var(4x + 8)$ is (A) $16Var(x)$ (B) $4Var(x)$ (C) $8Var(x)$ (D) $12Var(x)$	1	2	1
3.	A random variable X has the following probability function: The value of k is (A) 5/24 (B) 3/24 (C) 1/24 (D) 7/24	1	2	1
4.	$F(\infty) =$ (A) 0 (B) 1/2 (C) ∞ (D) 1	1	1	1
5.	Which of the following distribution must satisfy the fact that the mean and variance always equal? (A) Binomial (B) Poisson (C) Geometric (D) Exponential	1	1	2
6.	The mean of the uniform distribution defined in (a,b) is (A) a+b (B) a-b (C) 1/(a+b) (D) (a+b)/2	1	1	2
7.	Normal distribution is the limiting form of _____ distribution under suitable statistical conditions (A) Binomial (B) Poisson (C) Geometric (D) Uniform	1	1	2
8.	If X is exponentially distributed with mean 10 then the pdf is (A) $(1/10)e^{-(1/10)x}$ (B) $10e^{-10x}$ (C) $(1/10)e^{(1/10)x}$ (D) $10e^{-(1/10)x}$	1	2	2
9.	The null and alternative hypotheses divide all possibilities into: (A) two sets that overlap (B) two non-overlapping sets (C) two sets that may or may not overlap (D) as many sets as necessary to cover all possibilities	1	1	3

x	0	1	2	3	4
P(x)	k	2k	5k	7k	9k

10.	Which of the following values is not typically used for level of significance? (A) 0.01 (B) 0.05 (C) 0.10 (D) 0.25	1	1	3
11.	What is the standard deviation of a sampling distribution called? (A) Sampling error (B) Sample error (C) Standard error (D) Simple error	1	1	3
12.	The number of independent values in a set of values is called (A) Test-statistic (B) Degrees of freedom (C) Level of significance (D) Confidence level	1	1	3
13.	The chi-square test is not very effective if the sample is: (A) large (B) heterogenous (C) irregular (D) small	1	1	4
14.	The abbreviation of SIRO is (A) Service In Retail Order (B) Service In Regular Order (C) Service In Rationale Order (D) Service In Random Order	1	1	4
15.	Notation for queuing system is followed from (A) Gantrowich's Notation (B) Kendall's Notation (C) Fermat's Notation (D) Taylor's Notation	1	1	4
16.	F statistic should be taken like (A) denominator is greater than the numerator (B) numerator is lesser than the denominator (C) numerator is greater than the denominator (D) numerator equal to denominator	1	1	4
17.	Sum of the row entries in a TPM (Transition Probability Matrix) equals (A) 1 (B) 0.5 (C) 2 (D) 1/5	1	1	5
18.	Periodicity with period 1 is called (A) non-periodic (B) aperiodic (C) normal periodic (D) quasi-periodic	1	1	5
19.	Steady State Probability of Markov chain is (A) $\pi P = \pi$ (B) $\pi^2 P = \pi$ (C) $\pi P = \pi^2$ (D) $\pi P = 2\pi$	1	1	5
20.	In Markov Chain (A) X_n depends on $X_0, X_1, \ldots, X_{n-1}$ (B) X_n depends only on X_{n-1} (C) X_{n-1} depends on X_n (D) X_{n+1} depends on $X_0, X_1, \ldots, X_{n-1}$	1	1	5

PART - B (5 × 4 = 20 Marks)

Answer **any 5** Questions

		Marks	BL	CO
21.	Let X be a random variable with pdf $f(x) = \frac{x}{12}$, $1 < x < 5$. Find the pdf of the random variable $Y = 2X - 3$.	4	2	1
22.	State and prove memoryless property of exponential distribution.	4	3	2
23.	Define the following: (i) Type I and Type II error (ii) Null and Alternative hypothesis	4	1	3
24.	A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variance?	4	3	4
25.	A student's study habits are as follows: If he studies one night, 70% sure that he will not study next night. On the other hand, if he does not study one night 60% sure that he will not study the next night also. Find the steady state probability.	4	4	5

26. A random variable X has the pmf $p(x) = \frac{1}{2^x}$, $x = 1, 2, 3, \ldots$. Find the MGF(Moment Generating Function). 4 3 1

27. It is known that probability that an item produced by a certain machine will be defective is 0.01. By applying Poisson approximation, show that the probability that a random sample of 100 items selected at random from the total output will contain not more than one defective item is $\frac{2}{e}$ 4 3 2

PART - C (5 × 12 = 60 Marks) Marks BL CO

Answer **all** Questions

28. (a) A random variable X has the following distribution 12 3 1

x	0	1	2	3	4	5	6	7	8
P[X=x]	k	3k	5k	7k	9k	11k	13k	15k	17k

Find (i) the value of k,(ii) the Distribution Function (CDF) (iii) $P(0 < X < 3/X > 2)$ and (iv) the smallest value of α for which $P(X \leq \alpha) > \frac{1}{2}$.

(OR)

(b)
$$f(x) = \begin{cases} kx, & \text{when } 0 \leq x \leq 1 \\ k, & \text{when } 1 \leq x \leq 2 \\ 3k - kx, & \text{when } 2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

Find (a) the value of k (b) the CDF of X (c) $P(1.5 < X < 3.2/0.5 < X < 1.8)$

29. (a) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. 12 4 2

(OR)

(b) Fit a binomial distribution for the following data and hence find the theoretical frequencies:

x	0	1	2	3	4
f	5	29	36	25	5

30. (a) (i) In a large city A, 20 percent of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5 percent of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? 12 3 3
(ii) A random sample of 200 tins of coconut oil gave an average weight of 4.95 Kgs with a standard deviation 0.21 Kg. Do we accept the hypothesis of net weight 5 Kgs per tin at 1% level. Table value of z at 1% level is 2.58.

(OR)

(b) Two independent samples of size 8 and 7 contained the following values:

Sample 1	19	17	15	21	16	18	16	14
Sample 2	15	14	15	19	15	18	16	

Is the difference between the sample means significant?

31.	(a) In a group of people, 1872 were men and 1887 were women. 2257 individuals were in favor of the proposal and 917 opposed to it. 243 men were undecided and 442 women were opposed to the proposal. Do you justify or contradict the hypothesis there is no association between gender and attitude?	12	4	4
	(OR)			
	(b) Customers arrive at a one-man barbershop according to a Poisson process with a mean interarrival time of 12 minutes. Customers spend an average of 10 minutes in the barber's chair. (i) What is the expected number of customers in the barbershop and in the queue? (ii) How much time can a customer expect to spend in the barber's shop? (iii) What is the average time customers spend in the queue? (iv) What is the percentage of customers who have to wait before getting into the barber's chair? (v) What is the probability that the waiting time in the system is greater than 30 min? (vi) What is the probability that more than 3 customers are in the system.			
32.	(a) Three boys A, B, C are throwing a ball to each other. A always throw the ball to B and B always throws to C but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the tpm and classify the states.	12	4	5
	(OR)			
	(b) A person went to a gambling casino with Rs.3. He wins Rs.1 at each round with a probability 1/2. Being a very cautious player, the person has decided to stop playing when he doubles his initial amount (i.e., when he has a total of Rs.6) or when he loses all his money. i. Give the state-transition diagram of the process. ii. What is the probability that he stops after being ruined (i.e., he lost all his money)? iii. What is the probability that he stops after he has doubled his original amount?			

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