Reg. No.															

M.Sc. DEGREE EXAMINATION, MAY 2023

Fourth Semester

18PMA402 – INTEGRAL EQUATIONS AND TRANSFORMATION TECHNIQUES

(For the candidates admitted during the academic year 2018-2019)

Time: Three hours Max. Marks: 100

PART – A (5 × 5 = 25 Marks)

Answer **ANY FIVE** Questions

1.i. Define Fredholm first and second kind integral equations with examples.

ii. Define Voltenna first and second kind integral equations with examples.

2. Convert the following boundary value problem (BVP) into Fredholm integral equation

$\frac{d^2u}{dx^2}+u=x,\quad 0<x<\pi/2$ with the boundary conditions $u(0)=0$ and $u\left(\pi/2\right)=\pi$

3. Find the resolvent Kernel $R(x,t;\lambda)$ of $K(x,t)=xe^t$, $0\le x\le 1,\quad 0\le t\le 1$ by the method of Fredholm's determinant.

4. Find the Fourier transform of

$$f(x)=\begin{cases} x & for \quad |x|\le a \\ 0 & for \quad |x|>a \end{cases}$$

5. State and prove convolution theorem for Fourier transform.

6. Verify the final value theorem for the function $1-e^{-t}$.

7. If $L\{f(t)=F(s)$, then show that

$L\{\sinh(at)f(t)\}=\frac{1}{2}\left[F(s-a)-F(s+a)\right]$. Hence evaluate $\sinh(2t).\sin(3t)$.

8. Find the inverse Z transform of

$$F(z)=\frac{z}{z^2-6z+8}$$

PART – B (5 × 15 = 75 Marks)

9. a. If $\frac{d^2u}{dx^2}+\lambda u=0,\ 0<x<\ell\ and\ u(x)$ satisfies the boundary conditions $u(0)=0$ and $u(\ell)=0$, then, show that

$u(x)=\lambda\int_0^1 K(x,t)u(t)dt$ where

$$K(x,t)=\begin{cases} t/\ell\,(\ell-x); & 0<t<x \\ x/\ell\,(\ell-t); & x<t<\ell \end{cases}$$

(OR)

b. Find the resolvent Kernel of the Fredholm integral equation $u(x)=e^x+\frac{1}{2}\int_0^1 e^{x-t}\ u(t)dt$ using this resolvent Kernel, find the solution $u(x)$ of the given integral equation.

10. a. Find the eigenvalues and eigenfunctions of the homogeneous Fredholm integral equation of second kind

$$u(x)=\lambda\int_0^{2\pi}\sin(x+t)u(t)dt.$$

(OR)

b. Determine $D(\lambda)$ and $D(x,t;\lambda)$ and hence solve the Fredholm integral equation

$$u(x)=e^x+\lambda\int_0^1 e^x e^t\,u(t)\,dt$$

11. a.i. Show that $F_s\left[f''(x)\right]=-s^2F_s(s)+sf(0)$ if $f(x)$ and $f'(x)\to 0$ as $x\to\infty$.

ii. Show that $F_c\left[f''(x)\right]=-S^2F_c(s)-f'(0)$ if $f(x)$ and $f'(x)\to 0$ as $x\to\infty$.

(OR)

b. Find the Fourier transform of

$$f(x)=\begin{cases} a-|x| & if \quad |x|<a \\ 0 & if \quad |x|>a \end{cases}$$

Hence show that

i. $\displaystyle\int_0^\infty\left(\frac{\sin x}{x}\right)^2 dx=\frac{\pi}{2}$

ii. $\displaystyle\int_0^\infty\left(\frac{\sin x}{x}\right)^4 dx=\frac{\pi}{3}$

12. a.i. Show that $L^{-1}\left\{\frac{1}{s(s-a)}\right\}=\frac{1}{a}\left(e^{at}-1\right)$.

ii. Show that

$$L^{-1}\left\{\frac{1}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right\}=\frac{1}{b^2-a^2}\left(\frac{\sin at}{a}-\frac{\sin bt}{b}\right)$$

(OR)

b.i. Use initial value theorem to find $f(0)$ and $f'(0)$ from the following function

$$F(s)=\frac{s}{s^2-5s+12}$$

ii. Use final value theorem to find $f(\infty)$, if it exists, from the following function

$$F(s)=\frac{1}{s\left(s^2+as+b\right)}$$

13. a. Using convolution theorem, find the inverse of $\frac{z^2}{(z-a)(z-b)}$. Hence evaluate the inverse of $\frac{8z^2}{(2z-1)(4z-1)}$.

(OR)

b. Using the Z transform, solve the following difference equations

i. $f_{n+1}+3f_n=n,\ f_0=1$

ii. $f_{n+2}+3f_{n+1}+2f_n=0,\ f_0=1, f_1=2$

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