b. Find the Fourier series upto second harmonics from the following data. 12 3 2 2

x:	0	1	2	3	4	5
$f(x)$:	9	18	24	28	26	20

30. a. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $30\sin\dfrac{\pi x}{l}$, find the displacement function $y(x,t)$. 12 4 3 1

(OR)

b. A rod, 30 cm. long, has its ends A and B kept at 20°C and 80°C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function $u(x,t)$. 12 4 3 1

31. a. Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2, & |x|\le a \\ 0 & ,|x|>a \end{cases}$ and hence deduce that 12 3 4 2

$$\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)dt=\frac{\pi}{4}.$$

(OR)

b. Evaluate the following by using transforms methods. 12 3 4 2

(i) $\displaystyle\int_0^\infty \frac{x^2}{\left(x^2+a^2\right)\left(x^2+b^2\right)}dx$

(ii) $\displaystyle\int_0^\infty \frac{dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$

32. a. Find inverse Z-transform of 12 3 5 2

$F(z)=\dfrac{3z^2-18z+26}{(z-2)(z-3)(z-4)}$ by the method of partial fraction.

(OR)

b. Solve using Z-transform $y_{n+2}-3y_{n+1}+2y_n=2^n$, *given* $y_0=y_1=0$. 12 3 5 2

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Reg. No.														

B.Tech. DEGREE EXAMINATION, JUNE 2023

Third Semester

18MAB201T – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted from the academic year 2018-2019 to 2021-2022)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

Marks BL CO PO

1. The order of the partial differential equation $\dfrac{d^2y}{dx^2}+2y=\left(\dfrac{dy}{dx}\right)^3$ is 1 1 1 1
(A) 1 (B) 2
(C) 3 (D) 4

2. The notation of the partial derivative $\dfrac{\partial^2 z}{\partial y^2}$ is 1 1 1 2
(A) p (B) q
(C) r (D) t

3. A solution which is obtained from complete integral by eliminating arbitrary constants is known as 1 2 1 2
(A) Complete integral (B) Singular integral
(C) General integral (D) Particular integral

4. The PDE of the form Pp+Qq=R is known as 1 1 1 2
(A) Laplace equation (B) Lagrange's equation
(C) Clairaut's equation (D) Higher order equation

5. tanx is a periodic function with period 1 1 2 1
(A) $\pi/2$ (B) π
(C) 2π (D) 3π

6. If $\int_{-a}^{a} f(x)dx=0$, then the function $f(x)$ is 1 2 2 2
(A) Odd (B) Even
(C) Neither odd nor even (D) Periodic

7. If $f(x+T)=f(x)$, for all x then the function $f(x)$ is called 1 1 2 2
(A) Odd (B) Even
(C) Bounded (D) Periodic

8. The value of a_n in the Fourier series expansion of $f(x)=x^3$ in $(-l, l)$ is 1 2 2 2
(A) $\pi/2$ (B) $l/2$
(C) π (D) 0

9. The one dimensional heat equation is 1 1 3 2

(A) $\frac{\partial^2 y}{\partial t^2}=\alpha^2\frac{\partial^2 y}{\partial x^2}$ (B) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

(C) $\frac{\partial^2 u}{\partial t^2}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ (D) $\frac{\partial^2 y}{\partial x^2}=\alpha^2\frac{\partial^2 y}{\partial t^2}$

10. In one dimensional wave equation, α^2 stands for 1 1 3 1

(A) $K/\rho C$ (B) T/m

(C) $\rho C/K$ (D) K/ρ

11. The classification of the PDE $u_{xx}-2u_{xy}+u_{yy}=x+y$ is 1 2 3 1

(A) Elliptic (B) Parabolic

(C) Hyperbolic (D) Both parabolic and hyperbolic

12. How many initial and boundary conditions are required to solve one dimensional wave equation 1 2 3 1

(A) Two (B) Three

(C) Four (D) Five

13. If $F[f(x)]=F(s)$, then $F[f(x+a)]=$__________ 1 2 4 2

(A) $e^{-iax}F(x)$ (B) $e^{+iax}F(x)$

(C) $e^{+ias}F(s)$ (D) $e^{-ias}F(s)$

14. The Fourier cosine transform of $e^{-ax}, a>0$ is 1 2 4 2

(A) $\sqrt{\frac{\pi}{2}}\left(\frac{a}{s^2+a^2}\right)$ (B) $\sqrt{\frac{2}{\pi}}\left(\frac{a}{s^2+a^2}\right)$

(C) $\sqrt{\frac{\pi}{2}}\left(\frac{s}{s^2+a^2}\right)$ (D) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2+a^2}\right)$

15. The value of $F_s[f(ax)]$ is 1 1 4 2

(A) $\frac{1}{a}F_s\left(\frac{s}{a}\right)$ (B) $\frac{1}{a}F_c\left(\frac{s}{a}\right)$

(C) $\frac{1}{a}F_s\left(\frac{a}{s}\right)$ (D) $\frac{1}{s}F_c\left(\frac{a}{s}\right)$

16. The value of $F_c[f(x)\cos ax]$ 1 2 4 2

(A) $\frac{1}{2}[F_s(s+a)+F_s(s-a)]$ (B) $\frac{1}{2}[F_s(s+a)-F_s(s-a)]$

(C) $\frac{1}{2}[F_c(s+a)+F_c(s-a)]$ (D) $\frac{1}{2}[F_c(s+a)-F_c(s-a)]$

17. Z-transform of 5 is 1 2 5 2

(A) $\frac{z}{z-1}$ (B) $\frac{z}{z-5}$

(C) $\frac{5z}{z-1}$ (D) $\frac{5z}{z-5}$

18. Z-transform of $f(n)=a^n$ is 1 2 5 2

(A) $\frac{z}{z-a}$ (B) $\frac{z}{z+a}$

(C) $\frac{az}{z-1}$ (D) $\frac{az}{z-a}$

19. Z-transform of $f(t)=t$ is 1 2 5 2

(A) $\frac{z}{(z-1)^2}$ (B) $\frac{Tz}{(z-1)^2}$

(C) $\frac{z}{z-T}$ (D) $\frac{Tz}{z-T}$

20. The inverse Z-transform $F(z)=\frac{z^2+z}{(z-1)^3}$ 1 2 5 2

(A) n (B) n^2

(C) n(n–1) (D) n(n+1)

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

		Marks	BL	CO	PO
21.	Form the PDE by eliminating the arbitrary function from $z=f\left(x^2+y^2\right)$.	4	3	1	2
22.	Find the Root Mean Square value of $f(x)=x$ in $(-l,l)$.	4	3	2	2
23.	What are the possible solutions of one dimensional heat equation?	4	3	3	1
24.	State and prove change of scale property for Fourier transform.	4	3	4	2
25.	If $F(z)=\frac{5z^2}{(z+1)(5z+1)}$, then find f(0).	4	3	5	2
26.	Solve $p^2+q^2=npq$.	4	3	1	2
27.	A rod, 30 cm long has its ends A and B kept at 40°C and 70°C respectively, until steady state conditions prevail. Find u(x) in the rod.	4	3	3	1

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

		Marks	BL	CO	PO
28. a.	Solve $\left(D^2+5DD'+6D'^2\right)z=\cos(x+2y)+x^2y$.	12	3	1	2
	(OR)				
b.	Solve $(mz-ny)p+(nx-lz)q=ly-mx$.	12	3	1	2
29. a.	Find the Fourier series of $f(x)=\begin{cases}1+\frac{2x}{l}, & -l\le x\le 0\\ 1-\frac{2x}{l}, & 0\le x\le l\end{cases}$ in (–l, l) and deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$.	12	3	2	2

(OR)