29. a. The joint probability mass function of two random variables X and Y is given by $P(x,y)=\frac{1}{27}(2x+y); x=0,1,2$ — 12 3 2 1

$y=0,1,2$

(i) Find marginal distribution of X
(ii) Find marginal distribution of Y
(iii) Find the conditional distribution of X given Y
(iv) Find the conditional distribution of Y given X

(OR)

b. If the joint pdf of (X,Y) is given by $f_{XY}(x,y)=x+y; 0\le x,y\le 1$. Find the pdf of U=XY. — 12 3 2 2

30. a. If X denotes the sum of the numbers obtained when 2 dices are thrown. Obtain an upper bound for $P\{|X-7|\geq 4\}$. Compare with the exact probability. — 12 4 3 2

(OR)

b. A random sample of size 100 is taken from a population whose mean is 60 and variance 400 using central limit theorem, what probability can be assert that the mean of the sample will not differ from $\mu=60$ by more than 4? — 12 4 3 2

31. a. Given a random variable Y with characteristic function $\phi(\omega)=E\left(e^{i\omega Y}\right)$ and a random process $X(t)=\cos(\lambda t+Y)$. Show that X(t) is stationary in the WSS if $\phi(1)=\phi(2)=0$. — 12 3 4 1

(OR)

b. Consider two random variables $X(t)=3\cos(\omega t+\theta)$ *and* $X(t)=2\cos\left(\omega t+\theta-\frac{\pi}{2}\right)$, where θ is RV uniformly distributed in (0, 2π). Prove that $\sqrt{R_{XX}(0).R_{YY}(0)}\geq |R_{XY}(\tau)|$. — 12 3 4 1

32. a. Given the PSD of a continuous process as $S_{XX}(\omega)=\frac{\omega^2+9}{\omega^4+5\omega^2+4}$, find the mean square value of the process. — 12 4 5 2

(OR)

b. A random process X(t) with auto correlation function $R_{XX}(\tau)=e^{-2|\tau|}$ is the input to a linear system whose impulse response is $h(t)=2e^{-t}, t\geq 0$. Find the power spectral density $S_{YY}(\omega)$. — 12 4 5 2

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Reg. No.															

B.Tech. DEGREE EXAMINATION, MAY 2023
Fourth Semester

18MAB203T – PROBABILITY AND STOCHASTIC PROCESSES
(For the candidates admitted from the academic year 2018-2019 to 2021-2022)
(Statistical tables to be provided)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40^{th} minute.
(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours — Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

Marks BL CO PO

1. If $M_x(t)=\frac{3}{3-t}$, then the MGF (Moment Generating Function) of $M_{2x+5}(t)$ is — 1 2 1 1
(A) $e^{5t}\frac{3}{3-2t}$ (B) $\frac{3}{3-2t}$
(C) $e^{5t}\frac{3}{3-t}$ (D) $e^{t}\frac{3}{3-2t}$

2. If k is constant, then — 1 1 1 1
(A) $E(k)=1, \text{var}(k)=1$ (B) $E(k)=1, \text{var}(k)=0$
(C) $E(k)=k, \text{var}(k)=0$ (D) $E(k)=k, \text{var}(k)=k^2$

3. If X is a random variable with pdf $f(x)=\begin{cases} c(1-x^2); -1<x<1 \\ 0 ; \quad otherwise \end{cases}$ then the value of c is — 1 2 1 2
(A) 1/4 (B) 3/4
(C) 1/2 (D) 3/2

4. Which one of the following distribution satisfies memoryless property? — 1 1 1 1
(A) Uniform distribution (B) Binomial distribution
(C) Exponential distribution (D) Poison distribution

5. The marginal probability function of Y from $f_{XY}(x,y)$ is — 1 1 2 1
(A) $\int f(x,y)dx$ (B) $\int f(x,y)dy$
(C) $\iint f(x,y)dxdy$ (D) $\iint yf(x,y)dxdy$

6. If X and Y are independent random variables with density functions $f_X(x)=e^{-x}, x\geq 0$ *and* $f_Y(y)=e^{-y}, y\geq 0$ then the joint pdf of (X,Y) is — 1 1 2 1
(A) $f_{XY}(x,y)=e^{-x}+e^{-y}$ (B) $f_{XY}(x,y)=e^{-x}-e^{-y}$
(C) $f_{XY}(x,y)=e^{-x}e^{-y}$ (D) $f_{XY}(x,y)=e^{-x}/e^{-y}$

7. Two discrete jointly distributed random variables x and y are independent, if — 1 1 2 1
(A) $p_{ij}=p_{.j}/p_{i.}$ (B) $p_{ij}=p_{i.}\times p_{.j}$
(C) $p_{ij}=p_{i.}-p_{.j}$ (D) $p_{ij}=p_{i.}+p_{.j}$

8. If $x = uv, y = u(1-v)$ then $J\left(\frac{x,y}{u,v}\right)$ is — 1 2 2 2

(A) 1–u (B) –u

(C) uv (D) v

9. If $s_n \sim N(200,5)$ then $p[200 < s_n < 210]$ is equal to — 1 1 3 1

(A) $p(0<z<2)$ (B) $p(-2<z<2)$

(C) $p(0<z<5)$ (D) $p(-2<z<1.6)$

10. ________provides a simple method for computing approximate probabilities for sum of independent random variables. — 1 1 3 1

(A) Central limit theorem (B) Tchebycheff's inequality

(C) Jensen's inequality (D) Markov inequality

11. If X is a random variable $\mu_x = 3, \sigma_x^2 = 16$ then the upper bound for $P[|X-3| \geq 3]$ is — 1 2 3 1

(A) 16/9 (B) 11/3

(C) 7/3 (D) 19/25

12. If $P[X \geq a] \leq \frac{\sigma^2}{\sigma^2 + a^2}, a > 0$ then the given inequality is — 1 1 3 1

(A) Tchebycheff's inequality (B) Jenson's inequality

(C) Markov's inequality (D) One-sided Tchebycheff's inequality

13. The first order densities of an SSS process are — 1 1 4 1

(A) Dependent on time (B) Independent of time

(C) Continuous (D) Discrete

14. R(τ) is maximum at — 1 1 4 1

(A) τ=0 (B) τ=1

(C) τ=–1 (D) τ=2

15. $R_{XX}(0)$ is equal to — 1 1 4 1

(A) $E[X(t)]$ (B) $Var[X(t)]$

(C) $E[X^2(t)]$ (D) $[E(X(t))]^2$

16. A stationary process has ACF given $R(\tau) = 2 + 4e^{-2|\tau|}$, then the mean square value is — 1 2 4 2

(A) 6 (B) 8

(C) 4 (D) 2

17. If the unit impulse response function h(t) is absolutely integrable from $\int_{-\infty}^{+\infty} |h(t)|\, dt < \infty$, then the system is said to be — 1 1 5 1

(A) Stable (B) Linear

(C) Causal (D) Memoryless

18. The average of power of random process {X(t)} is defined as — 1 1 5 1

(A) $R_{XX}(0)$ (B) $R_{XX}(\tau)$

(C) $R_{XX}(-\tau)$ (D) $S_{XX}(0)$

19. Unit impulse response for a causal system h(t) is zero when — 1 1 5 1

(A) t>0 (B) t<0

(C) t=0 (D) t≥0

20. The average power of waveform $X(t) = A\cos(\omega_0 t + \theta)$ is — 1 2 5 2

(A) $\frac{A^2}{2}$ (B) $\frac{A^2}{4}$

(C) A^2 (D) $2A^2$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

Marks BL CO PO

21. A continuous random variable has the pdf of $f(x) = 5x^4, -1 < x < 0$. Find $P(x > -1/2 / x < -1/4)$. — 4 2 1 1

22. Show that the function $f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$ is a joint p.d.f of X and Y. — 4 1 2 1

23. Derive the Chernoff bounds for the Poisson variate. — 4 2 3 2

24. A random process X(t) is defined as $X(t) = \begin{cases} A & for\ 0 \leq t \leq 1 \\ 0 & otherwise \end{cases}$ where A is a random variable that is uniformly distributed from θ to –θ. Prove that auto correlation function of X(t) is $\frac{\theta^2}{3}.[R_{XX}(t_1,t_2)]$. — 4 2 4 1

25. The power spectral density function of a WSS is given by — 4 2 5 2

$$S_{XX}(\omega) = \begin{cases} 1 : |\omega| < \omega_0 \\ 0 : \text{otherwise} \end{cases}$$

Find the auto-correlation function of the process.

26. A random variable 'X' has the following probability function — 4 1 1 1

$X=x_i$	0	1	2	3	4
$P(X=x_i)$	k	3k	5k	7k	9k

(i) Find k (ii) P[X≥3]

27. Show that $R_{XX}(-\tau) = R_{XX}(\tau)$. — 4 1 4 1

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

Marks BL CO PO

28. a.i. A continuous random variable X has a pdf $f(x) = kx^2 e^{-x}; x > 0$. Find k, mean and variance. — 6 3 1 1

ii. Find the probability that at most 4 defective fuses will be found in a box of 200 fuses if experience shows that 2% of such fuses are defective. — 6 3 1 2

(OR)

b. In a distribution exactly normal 7% of the items are under 35 and 89% are under 63. What are the mean and S.D of the distribution. — 12 3 1 2