Reg. No.															

B.Tech. DEGREE EXAMINATION, MAY 2022
Fourth Semester

18MAB204T – PROBABILITY AND QUEUEING THEORY
(For the candidates admitted from the academic year 2018-2019 to 2019-2020)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B** should be answered in answer booklet.

Time: 2½ Hours — Max. Marks: 75

PART – A (25 × 1 = 25 Marks)
Answer **ALL** Questions

Marks BL CO PO

1. If X is a discrete random variable such that $P(X=x_i)=p_i, i=1,2,3,......,$ then (x_i,p_i) is called — 1 1 1 1
   (A) Probability mass function (p.m.f) (B) Probability density function (p.d.f)
   (C) Probability distribution function (D) Moment generating function (M.G.F)

2. If X is a random variable discrete or continuous then $P(X \le x)$ is called — 1 1 1 1
   (A) Cumulative distribution function (B) Probability mass function (p.m.f)
   (C) Probability density function (p.d.f) (D) Moment generating function (M.G.F)

3. If X represents the outcome and M(t) is the M.G.F of X, then — 1 1 1 1
   (A) $E(X)=[M''(t)]_{t=0}$ (B) $E(X)=[M''(t)]$
   (C) $E(X)=[M'(t)]$ (D) $E(X)=[M'(t)]_{t=0}$

4. The probability distribution of a random variable is given by — 1 2 1 2

x:	0	1	2	3
p(x):	0.1	0.3	0.4	0.2

   Then E[X] is
   (A) 0 (B) 1
   (C) 1.7 (D) 2

5. The mean of the random variable X, if its probability density function is given by $f(x)=6x(1-x), 0 \le x \le 1$ — 1 2 1 2
   (A) 1/3 (B) 1/4
   (C) 1/5 (D) 1/2

6. Mean and variance of the Poisson distribution are — 1 1 2 1
   (A) λ and λ^2 (B) λ^2 and λ
   (C) λ and 1/λ (D) λ and λ

7. If X follows uniform distribution in (a,b) then its probability density function is given by 1 1 2 4

(A) $f(x)=\frac{1}{b-a}$ (B) $f(x)=\frac{1}{ab}$

(C) $f(x)=a-b$ (D) $f(x)=\frac{b}{a}$

8. The mean and variance of a binomial distribution are 4 and 4/3 respectively. Then the probability of success is 1 2 2 2

(A) 1/3 (B) 2/3

(C) 4/3 (D) 1/2

9. If the random variable X follows a Poisson distribution with mean 3, then P(X=1) is 1 2 2 2

(A) $3e^{-3}$ (B) $3e^{3}$

(C) e^{-3} (D) e^{-1}

10. If the probability of success in each trial is p then what is the probability that exactly 5 attempts are required to get 3 consecutive success. 1 2 2 2

(A) p^2q^3 (B) p^3q^2

(C) p^3q^3 (D) p^2q^2

11. The standard deviation of the sampling distribution is known as 1 1 3 1

(A) Sample median (B) Level of significance

(C) Standard error (D) Sample proportion

12. 95% confidence limits for the population proportion P is given by 1 1 3 1

(A) $\frac{|p-P|}{\sqrt{Pq/n}}\le 1.96$ (B) $\frac{|p-P|}{\sqrt{Pq/n}}\le 1.64$

(C) $\frac{|p-P|}{\sqrt{Pq/n}}\le 1.73$ (D) $\frac{|p-P|}{\sqrt{Pq/n}}\le 1.89$

13. The test statistic for the difference between sample mean and population mean is given by 1 1 3 1

(A) $Z=\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$ (B) $Z=\frac{\overline{X}-\mu}{\sigma\sqrt{n}}$

(C) $Z=\frac{\overline{X}-\mu}{\sigma}$ (D) $Z=\frac{\overline{X}-\mu}{\sigma n^2}$

14. If the null hypothesis is false, then which of the following is accepted? 1 2 3 2

(A) Null hypothesis (B) Positive hypothesis

(C) Negative hypothesis (D) Alterative hypothesis

15. The critical value of Z for a single tailed test (right or left) at LOS ‘α’ is the same as that for a two tailed test of LOS 1 2 3 2

(A) α/3 (B) α/2

(C) α (D) 2α

16. The distribution used to test the equality of the variance of the populations from which two samples (n<30) have been drawn 1 1 4 1

(A) Binomial distribution (B) F-distribution

(C) t-distribution (D) Chi-square distribution

17. Single server Poisson queue with finite capacity of Markovian model is 1 1 4 1

(A) (M/M/1):(∞/FIFO) (B) (M/M/S):(∞/FIFO)

(C) (M/M/1):(K/FIFO) (D) (M/M/S):(K/FIFO)

18. If the behaviour of the queuing system does not depend on time then the system is said to be in 1 2 4 2

(A) Transient state (B) Busy state

(C) Steady state (D) Idle state

19. In (M/M/1):(K/FIFO) if the arrival and service rates are respectively as 30 per hour and 20 per hour and the effective arrival rate of a customer is 20 per hour, then what is the probability of the idle? 1 2 4 2

(A) 0.5 (B) 0

(C) 1 (D) 0.7

20. A petrol pump with only one pump can accommodate 5 cars. The arrival of cars with a mean rate of 10 per hour. The service with a mean rate of 30 per hour. What is the effective arrival rate? 1 2 4 2

(A) 9.99 (B) 8.88

(C) 7.77 (D) 6.66

21. A Markov chain is said to be 'aperiodic' if 1 1 5 1

(A) $d_i=0$ (B) $d_i=1$

(C) $d_i<1$ (D) $d_i>1$

22. If P is the TPM of the Markov chain, then 1 1 5 1

(A) $\pi P=0$ (B) $\pi P=\pi$

(C) $\pi(P+1)=0$ (D) $\pi P=1$

23. If the one-step transition probability does not depend on the step then the Markov chain is 1 1 5 1

(A) Reducible (B) Regular

(C) Homogeneous (D) Non-homogeneous

24. If he studies (S) one night, he is 70% sure not to study (N) the next night. On the other hand, if he does not study (N) one night, he is 60% sure not to study (N) the next as well. The TPM is 1 2 5 2

(A)
$$P=\begin{array}{cc} & \begin{array}{cc} S & N \end{array} \\ \begin{array}{c} S \\ N \end{array} & \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$

(B)
$$P=\begin{array}{cc} & \begin{array}{cc} S & N \end{array} \\ \begin{array}{c} S \\ N \end{array} & \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$

(C)
$$P=\begin{array}{cc} & \begin{array}{cc} S & N \end{array} \\ \begin{array}{c} S \\ N \end{array} & \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix} \end{array}$$

(D)
$$P=\begin{array}{cc} & \begin{array}{cc} S & N \end{array} \\ \begin{array}{c} S \\ N \end{array} & \begin{bmatrix} 0 & 1 \\ 0.6 & 0.4 \end{bmatrix} \end{array}$$

25. The state 'I' is said to be non-null persistent if its mean recurrence time is (A) Empty (B) Infinite (C) Finite (D) 1	1	2	5	2

PART – B (5 × 10 = 50 Marks)

Answer **ALL** Questions

	Marks	BL	CO	PO
26. a. The amount of bread (in hundreds of pounds) X that a certain bakery is able to sell in a day is found to be a numerical valued phenomenon, with a probability function specified by the probability density function. $f(x) \textit{given by}$ $f(x)=\begin{cases} Kx & , \quad 0\le x\le 5 \\ K(10-x), & \quad 5\le x\le 10 \\ 0 & , \quad \textit{otherwise} \end{cases}$				
(i) Find the value of 'K' such that *f(x)* is a probability density function	4	3	1	1,2
(ii) What is the probability that the number of pounds of bread that will be sold tomorrow is (1) More than 500 pounds, (2) Less than 500 pounds, (3) Between 250 and 750 pounds	6	3	1	1,2
(OR)				
b. A discrete RV X can take the values −1, 0, 1 with probabilities 1/8, 3/4, 1/8 respectively. Apply Tchebycheff's inequality to compute $P\{\mid X \mid\ge 2\sigma\}$ and compare it with the exact probability.	10	4	1	1,2
27. a.i. A room has three camp sockets. From a collection of 10 light bulbs, only 6 are good. A person selects 3 at random and puts them in the sockets. What is the probability that room will have light?	4	4	2	1,2
ii. Buses arrive at a specified stop at 15 minutes interval starting at 7a.m that is, they arrive at 7, 7.15, 7.30, 7.45 and so on. If a passenger arrives at a stop at a random time that is uniformly distributed between 7 and 7.30, estimate the probability that he waits for (1) Less than 5 minutes for a bus (2) More than 10 minutes for a bus	6	4	2	1,2
(OR)				
b.i. The amount of time that a watch can run without having to be reset is a random variable having exponential distribution, with mean 120 days. Estimate the probability that such a watch will (1) have to be reset in less than 24 days (2) not have to be reset for atleast 180 days	4	4	2	1,2
ii. The marks obtained by a number of students in a certain subject are assumed to be approximately normally distributed with mean value 65 are with standard deviation 5. If 3 students are taken at random from this set, what is the probability that exactly 2 of them will have marks over 70?	6	3	2	1,2

28. a.i. A manufacturer of light bulbs claims that on the average 2% of the bulbs manufactured by his firm are defective. A random sample of 400 bulbs contained 13 defective bulbs. On the basis of this sample, can you support the manufacture's claim at 5% LOS? 5 4 3 1,2

ii. A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm and the SD is 10 cm? 5 4 3 1,2

(OR)

b. The following data relates to the marks obtained by 11 students in 2 tests, one held at the beginning of a year and the other at the end of the year after intensive coaching. 10 3 3 1,2

Test 1:	19	23	16	24	17	18	20	18	21	19	20
Test 2:	17	24	20	24	20	22	20	20	18	22	19

Do the data indicate that the students have benefitted by the coaching?

29. a. A survey of 320 families with 5 children revealed the following distribution. 10 4 4 1,2

No. of boys:	0	1	2	3	4	5
No. of girls:	5	4	3	2	1	0
No. of families:	12	40	88	110	56	14

Is this result consistent with the hypothesis that male and female births are equally probable?

(OR)

b. If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 minutes before the picture starts and if it takes exactly 1.5 minutes to reach the correct seat after purchasing the ticket, 10 3 4 1,2

(i) Can he expect to be seated for the start of the picture?

(ii) What is the probability that he will be seated for the start of the picture?

(iii) How early must be arrive in order to be 99% sure of being seated for the start of the picture?

30. a. A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However, if he sells either in B or C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady state? 10 4 5 1,2

(OR)

b. The three-state Markov chain is given by the tpm. 10 3 5 1,2

$$p=\begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

Prove that the chain is irreducible and all the states are aperiodic and non-null persistent. Find also the steady state distribution of the chain.

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