

B.Tech. DEGREE EXAMINATION, DECEMBER 2023
Fourth Semester

18MAB206T – NUMERICAL METHODS AND ANALYSIS
(For the candidates admitted from the academic year 2020-2021 to 2021-2022)

Note:

- (i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
- (ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)Answer **ALL** Questions

- | | Marks | BL | CO | PO |
|--|-------|----|----|----|
| 1. Newton-Raphson method is also known as the method of _____
(A) Tangents (B) Secant
(C) Co-secant (D) Squares | 1 | 1 | 1 | 2 |
| 2. The order of convergence of Newton's method is
(A) 2 (B) 3
(C) 4 (D) 1 | 1 | 1 | 1 | 2 |
| 3. One of the direct method to solve system of simultaneous linear equation is
(A) Gauss-Jacobi (B) Gauss-Elimination
(C) Gauss-Seidel (D) Newton's method | 1 | 1 | 1 | 2 |
| 4. The convergence in the Gauss-Seidel method is roughly _____ as faster as in Jacobi's method.
(A) Three times (B) Five times
(C) Ten times (D) Two times | 1 | 1 | 1 | 2 |
| 5. The translation operator E is defined by
(A) $Ef(x) = f(x+h)$ (B) $Ef(x) = f(x)$
(C) $Ef(x+h) = f(x)$ (D) $Ef(x) = f(x-h)$ | 1 | 1 | 2 | 2 |
| 6. The relation between E and ∇ is
(A) $\nabla - E^{-1} = 1$ (B) $1 + \nabla = E^{-1}$
(C) $\nabla = 1 - E^{-1}$ (D) $\nabla = 1 + E^{-1}$ | 1 | 2 | 2 | 2 |
| 7. The $(n+1)^{th}$ and higher differences of a polynomial of degree 'n' are
(A) Quadratic (B) Parabolic
(C) Linear (D) Zeros | 1 | 1 | 2 | 2 |
| 8. $\delta E^{1/2} =$
(A) Δ (B) ∇
(C) δ (D) E | 1 | 2 | 2 | 2 |

9. The trapezoidal rule is 1 1 3 2
 (A) $h/2[\text{sum of the first and last ordinates} + 2(\text{sum of remaining ordinates})]$
 (B) $h/2[y_0 + y_1 + 2(y_2 + \dots y_n)]$
 (C) $h/2 [\text{sum of } y_0 \text{ and } y_{n-1} + 2 (\text{sum of remaining ordinates})]$
 (D) $h/2[y_0 + y_n + 2(y_2 + y_4 + \dots) + 4(y_1 + y_3 + \dots)]$
10. The error in Trapezoidal rule is 1 1 3 2
 (A) $|E| < \frac{(b-a)h^2}{12}$ (B) $|E| < \frac{(b-a)h^3}{12}$
 (C) $|E| < \frac{(b-a)h^3}{180}$ (D) $|E| < \frac{(b+a)h^3}{60}$
11. Simpson's Three-eight rule can be applied only when n is 1 1 3 2
 (A) Odd (B) Even
 (C) Prime (D) Multiple of 3
12. Simpson's one-third rule is also called 1 1 3 2
 (A) Parabolic rule (B) Hyperbolic rule
 (C) Elliptic rule (D) Trapezoidal rule
13. The improved Euler method is based on the averages of 1 1 4 2
 (A) Points (B) Slopes
 (C) Curves (D) Both points and slopes
14. Which of the following is a better method? 1 1 4 2
 (A) Taylor series method (B) R-K method
 (C) Euler's method (D) Modified Euler method
15. The error term in Milne's corrector formula is 1 1 4 2
 (A) $\frac{-h}{90}\Delta^4 y_0$ (B) $\frac{h}{90}\Delta^4 y_0$
 (C) $\frac{-h}{90}\Delta^3 y_0$ (D) $\frac{h}{90}\Delta^4 y_0$
16. Which of the following is Modified Euler's formula for $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ 1 1 4 1
 (A) $y_1 = y_0 + h^2 f(x_0, y_0)$
 (B) $y_1 = y_0 - h^2 f(x_0, y_0)$
 (C) $y_1 = y_0 + hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}hf(x_0, y_0)\right)$
 (D) $y_1 = y_0 + h$
17. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is called 1 1 5 2
 (A) Elliptic (B) Parabolic
 (C) Hyperbolic (D) Laplace equation

18. Bender-Schmidt recurrence equation is valid only if
- (A) $k = \frac{h^2}{2}$ (B) $k = \frac{ah^2}{2}$
- (C) $k = \frac{2}{ah^2}$ (D) $k = \frac{2}{h^2}$

19. The equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$ is also called
- (A) Laplace equation (B) Poisson equation
- (C) One dimensional heat equation (D) Two dimensional heat equation

20. Bender-Schmidt scheme converges for
- (A) $\lambda=1$ (B) $\lambda=1/2$
- (C) $\lambda=3$ (D) $\lambda=3/2$

PART – B (5 × 4 = 20 Marks)

Answer ANY FIVE Questions

- | | Marks | BL | CO | PO |
|---|-------|----|----|----|
| 21. Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 4 decimal places. | 4 | 3 | 1 | 2 |
| 22. Find the 7 th term of the sequence 1, 4, 10, 20, 35, 56... | 4 | 3 | 2 | 2 |
| 23. Evaluate $\int_1^2 \frac{dx}{1+x^2}$ taking $h=0.2$ using trapezoidal rule. | 4 | 3 | 3 | 2 |
| 24. Solve $\frac{dy}{dx} = x + y, y(1) = 0$ to obtain $y(1.1)$ by Taylor's method taking $h=0.1$. | 4 | 3 | 4 | 2 |
| 25. Using Euler's method, solve numerically the equation $y' = x + y, y(0) = 1$, for $x = 0.2$ and 0.4 . | 4 | 3 | 4 | 2 |
| 26. Classify the partial differential equation $(x+1)u_{xx} - 2(x+2)u_{xy} + (x+3)u_{yy} = 0$. | 4 | 3 | 5 | 2 |
| 27. State modified Euler formula and improved Euler formula. | 4 | 3 | 4 | 2 |

PART – C (5 × 12 = 60 Marks)

Answer ALL Questions

- | | Marks | BL | CO | PO |
|--|-------|----|----|----|
| 28. a. Solve the following system of equation of using Gauss-Seidel method correct to three decimal places
$8x - 3y + 2z = 20$
$4x + 11y - z = 33$
$6x + 3y + 12z = 35$ | 12 | 3 | 1 | 2 |
| (OR) | | | | |
| b. Solve the system of equations by Gauss-Jordan method.
$x + 2y + z = 3$
$2x + 3y + 3z = 10$
$3x - y + 2z = 13$ | 12 | 3 | 1 | 2 |

29. a. The population of a town is as follows.

12 3 2 2

x	1941	1951	1961	1971	1981	1991
y	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976.

(OR)

- b. Use Lagrange's interpolation formula to fit a polynomial to the data.

12 3 2 2

x	0	1	3	4
y	-12	0	6	12

And hence find the value of y when x=2.

30. a. Find the first two derivative of $(x)^{1/3}$ at $x=50$ and $x=56$ given the table below.

12 3 5 2

x	50	51	52	53	54	55
$y=x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030

(OR)

- b. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using trapezoidal and Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rules, taking $h=1$.

12 3 5 2

31. a. Using R-K method of fourth order compute $y(0.1)$, $y(0.2)$ given $\frac{dy}{dx} + y + xy^2 = 0$, $y(0) = 1$, correct to 4 decimals.

12 3 4 2

(OR)

- b. Using modified Euler's method, find $y(0.1)$ and $y(0.2)$ for the differential equation, $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$.

12 4 4 2

32. a. Find the Liebmann's method the values at the interior lattice points of the square region of the harmonic function u whose boundary values are as shows below.

12 3 5 1

		11.1	17.0	19.7	
0					18.6
0					21.9
0					21.0
0					17.0
0					9.0
	8.7	12.1	12.8		

(OR)

- b. Solve $u_t = 4u_{xx}$, $u(0,t) = 0$, $u(8,t) = 0$ and $u(x,0) = 4x - \frac{1}{2}x^2$, taking $h=1$, $k=1/8$ using Bender-Schmidt formula upto $t=5$.

12 3 5 2

* * * * *