Reg. No															

B.Tech/M.Tech(Integrated) DEGREE EXAMINATION, NOVEMBER 2023

Second Semester

21MAB102T - ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2022-2023 onwards)

Note:

i. **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40^{th} minute.

ii. **Part - B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours **Max. Marks: 75**

PART - A (20 × 1 = 20 Marks)

Answer **all** Questions

	Question	Marks	BL	CO
1.	Evaluation of $\int_0^1 \int_0^1 dxdy$ is (A) 1 (B) 0 (C) 2 (D) 4	1	1	1
2.	Area of the double integral in Cartesian co-ordinate is equal to (A) $\iint_R dydx$ (B) $\iint_R rdrd\theta$ (C) $\iint_R xdxdy$ (D) $\iint_R x^2dxdy$	1	1	1
3.	Change the order of integration in $\int_0^a \int_0^x dxdy$ is (A) $\int_0^a \int_0^x dxdy$ (B) $\int_0^a \int_0^x xdydx$ (C) $\int_0^a \int_y^a dxdy$ (D) $\int_0^a \int_0^y dxdy$	1	2	1
4.	$\int_0^1 \int_0^2 \int_0^3 dxdydz$ is equal to (A) 3 (B) 4 (C) 2 (D) 6	1	2	1

5. The unit vector normal to the surface $x^2+y^2-z^2=1$ at (1,1,1,) is 1 2 2

(A) $\dfrac{\vec{i}+\vec{j}-\vec{k}}{\sqrt{3}}$

(B) $\dfrac{2\vec{i}+2\vec{j}-2\vec{k}}{\sqrt{2}}$

(C) $\dfrac{3\vec{i}+3\vec{j}-3\vec{k}}{2\sqrt{3}}$

(D) $\dfrac{\vec{i}+\vec{j}-\vec{k}}{3\sqrt{2}}$

6. If $\vec{r}$ is the position vector of the point (x, y, z) w.r.to the origin, then $\nabla\cdot\vec{r}$ is 1 2 2

(A) 1
(B) 2
(C) 3
(D) 4

7. The connection between a surface integral and a volume integral is known as 1 2 2

(A) Green's theorem
(B) Gauss Divergence theorem
(C) Cauchy's theorem
(D) Stoke's theorem

8. If $\phi = xyz$,then $\nabla\phi$ is 1 1 2

(A) $yz\,\vec{i}+zx\,\vec{j}+xy\,\vec{k}$

(B) $xy\,\vec{i}+yz\,\vec{j}$

(C) $xy\,\vec{i}+yz\,\vec{j}+zx\,\vec{k}$

(D) $zx\,\vec{i}+xy\,\vec{j}+yz\,\vec{k}$

9. $L(t^4)=$ 1 2 3

(A) $\dfrac{4!}{s^5}$

(B) $\dfrac{3!}{s^4}$

(C) $\dfrac{4!}{s^4}$

(D) $\dfrac{5!}{s^4}$

10. $L(\cosh t)=$ 1 1 3

(A) $\dfrac{s}{s^2+1}$

(B) $\dfrac{s}{s^2-1}$

(C) $\dfrac{1}{s^2+1}$

(D) $\dfrac{1}{s^2-1}$

11. Using the initial value theorem, find the value of the function $f(t)=1+e^{-t}+t^2$ 1 1 3

(A) 0
(B) 1
(C) 2
(D) 3

12. The period of $\tan t$ is 1 1 3

(A) 0
(B) π
(C) $\frac{\pi}{2}$
(D) $\frac{\pi}{4}$

13. The function $f(z) = u + iv$ is analytic if 1 2 4

(A) $u_x = v_y, u_y = -v_x$
(B) $u_x = -v_y, u_y = v_x$
(C) $u_x + v_y = 0, u_y - v_x = 0$
(D) $u_y = v_y, u_x = v_x$

14. the transformation $w = cz$ where c is real constant known as 1 1 4

(A) rotation
(B) reflection
(C) magnification
(D) translation

15. The real part of $f(z) = e^{2z}$ is 1 1 4

(A) $e^x \cos y$
(B) $e^x \sin y$
(C) $e^{2x} \cos 2y$
(D) $e^{2x} \sin 2y$

16. The invariant points of the transformation $w = -\left(\frac{2z+4i}{iz+1}\right)$ are 1 1 4

(A) $z = 4i, -i$
(B) $z = 2i, i$
(C) $z = -4i, i$
(D) $z = -2i, i$

17. If f(z) is analytic inside and on c , the value of $\int_c f(z)\,dz$,where c is the simple closed curve, *is* 1 1 5

(A) 0
(B) f(a)
(C) $2\pi i$f(a)
(D) πif(a)

18. If f(z) is analytic inside and on c , the value of $\int_c \frac{f(z)}{(z-a)^2}\,dz$,where c is the simple closed curve *and a is any point within c , is* 1 1 5

(A) f(a)
(B) $2\pi i$f(a)
(C) πif(a)
(D) $2\pi i f'(a)$

19. Let $C_1: |z-a| = R_1$ and $C_2: |z-a| = R_2$ be two concentric circles ($R_2 < R_1$), the f(z) can be expanded as a Laurent's series if 1 1 5

(A) f(z) is analytic within C_2
(B) f(z) is not analytic within C_2
(C) f(z) is analytic in the annular region
(D) f(z) is not analytic in the annular region

20. The residue of f(z) $= \frac{z}{(z-2)}$ is 1 1 5

(A) 1
(B) 2
(C) 3
(D) 4

PART - B (4 × 10 = 40 Marks)

Answer **any 4** Questions

		Marks	BL	CO
21.	Change to polar coordinates and hence evaluate $\int_0^a \int_y^a \frac{x^2\,dx\,dy}{\sqrt{x^2+y^2}}$	10	3	1
22.	Using Green's theorem, evaluate $\int_C (x^2-y^2)dx+2xy\,dy$ Where C is the closed curve of the region bounded by $y=x^2$ and $y^2=x$.	10	3	2
23.	Solve the equation by Laplace transform $y''+9y=6\cos 3t,\ y(0)=2,\ y'(0)=0.$	10	3	3
24.	Find the analytic function $f(z)=u+iv$ if $u-v=\frac{\sin 2x}{\cosh 2y-\cos 2x}$	10	4	4
25.	Find the Laurent's series of $f(z)=\frac{z}{(z^2+1)(z^2+4)}$ in the region $1<\lvert z\rvert<2.$	10	3	5
26.	Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$	10	3	5

PART - C (1 × 15 = 15 Marks)

Answer **any 1** Questions

		Marks	BL	CO
27.	Find the area lying inside the circle $r = a\sin\theta$ and outside the cardioid $r=a(1-\cos\theta)$.	15	4	1
28.	Prove that $\operatorname{div}(r^n\bar{r})=(n+3)r^n$. Deduce that $r^n\bar{r}$ is solenoidal if and only if n= -3	15	4	2

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