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B.Tech DEGREE EXAMINATION, NOVEMBER 2023

Fifth Semester

18AIE339T - MATRIX THEORY FOR ARTIFICIAL INTELLIGENCE

(For the candidates admitted during the academic year 2020 - 2021 & 2021 - 2022)

Note:

i. **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40^{th} minute.
ii. **Part - B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours **Max. Marks: 100**

PART - A (20 × 1 = 20 Marks)

Answer **all** Questions

	Question	Marks	BL	CO
1.	If the determinant of the coefficient matrix in a system of linear equations is zero, which of the following statements about the system is accurate? (A) It has no solution (B) It has a unique solution (C) It has infinitely many solutions (D) It has exactly two solutions	1	1	1
2.	Which term is used to describe a collection of vectors that satisfy certain properties in linear algebra? (A) Linear Space (B) Vector Pool (C) Matrix Set (D) Scalar Field	1	2	1
3.	In linear algebra, what is the rank of a matrix? (A) It refers to the number of columns in the matrix. (B) It denotes the number of linearly independent columns (or rows) in the matrix. (C) It signifies the total number of elements in the matrix. (D) It represents the determinant of the matrix.	1	2	1
4.	In a vector space, what is the maximum number of vectors in a basis? (A) It is always zero (B) It is equal to the number of rows in the matrix (C) It is equal to the number of linearly independent columns in the matrix (D) It is equal to the dimension of the vector space	1	1	1
5.	In matrix calculus, what is the derivative of a vector with respect to a matrix? (A) A scalar (B) A vector (C) A matrix (D) An array	1	2	2
6.	What is the main goal of LU decomposition? (A) Factorizing a matrix into lower and upper triangular matrices (B) Finding the eigenvalues of a matrix (C) Computing the determinant of a matrix (D) Solving systems of linear equations	1	1	2
7.	What is the range of a linear transformation represented by a matrix? (A) The span of the column vectors of the matrix (B) The determinant of the matrix (C) The set of all real numbers (D) The set of all complex numbers	1	1	2
8.	In Eigen decomposition, what does a matrix A decompose into? (A) A matrix of eigenvalues and a matrix of eigenvectors (B) A diagonal matrix and its inverse (C) A lower triangular matrix and an upper triangular matrix (D) A permutation matrix	1	2	2

9. In what context, the Jacobian matrix is commonly used? 1 2 3

(A) Linear algebra, for matrix operations
(B) Calculus, especially for multivariable functions
(C) Number theory, for prime factorization
(D) Probability theory, for conditional probabilities

10. What is the convergence rate of the standard gradient descent method for convex functions? 1 1 3

(A) Quadratic
(B) Exponential
(C) Linear
(D) It depends on the specific function

11. What is the purpose of employing the graphical method in linear programming? 1 2 3

(A) Finding the optimal solution to linear programming problems
(B) Visualizing the feasible region and identifying the optimal solution
(C) Solving non-linear programming problems
(D) Analyzing the dual problem of linear programming

12. In quadratic programming, what is a positive semidefinite matrix? 1 1 3

(A) A matrix that has at least one positive eigenvalue
(B) A matrix that has only positive entries
(C) A matrix that has no negative eigenvalues
(D) A matrix that has no zero entries

13. In Gauss elimination, what is the role of pivot elements? 1 1 4

(A) They determine the number of variables in the system
(B) They are the coefficients of the linear equations
(C) They are used to perform row operations for elimination
(D) They represent the constants in the equations

14. When is the Least Square Method particularly useful in solving linear systems? 1 2 4

(A) When the system of equations is overdetermined
(B) When the system of equations is underdetermined
(C) When the system of equations has no solutions
(D) When the system of equations is inconsistent

15. In numerical optimization, why is the computation of gradients important? 1 1 4

(A) Gradients determine the number of variables in the system
(B) Gradients provide information about the local rate of change of the objective function
(C) Gradients directly give the optimal solution to an optimization problem
(D) Gradients are used to perform row operations in matrix factorization

16. In what scenarios would you prefer using the Gauss-Seidel method over Gauss elimination? 1 2 4

(A) When dealing with non-linear systems of equations
(B) When exact solutions are not required, and quick approximations are sufficient
(C) When the system of equations is underdetermined
(D) When solving systems with large sparse matrices

17. How are matrices commonly used in image processing applications of AI? 1 2 5

(A) To store only grayscale images
(B) To represent convolutional filters
(C) To apply random noise to images
(D) To perform text recognition

18. In natural language processing (NLP), how are matrices utilized for word embedding. 1 1 5

(A) To represent the semantic similarity between words
(B) To visualize the frequency of words in a document
(C) To encode words as vectors in a continuous space
(D) To determine the syntactic structure of sentences

19.	In an AI-driven recommendation system, which type of optimization problem is typically encountered? (A) Minimizing user engagement (B) Minimizing recommendation diversity (C) Maximizing recommendation relevance (D) Minimizing computational resources	1	1	5
20.	What are the difficulties associated with attaining a global minimum in a non-convex optimization problem? (A) Non-convex problems always have multiple global minima (B) Non-convex problems may have multiple local minima, making it hard to distinguish them from the global minimum (C) Non-convex problems can never have a global minimum (D) Non-convex problems are always easier to solve than convex problems	1	2	5

PART - B (5 × 4 = 20 Marks)
Answer **any 5** Questions

		Marks	BL	CO
21.	How do you determine if a given set of vectors forms a linearly independent system? Provide an example.	4	2	1
22.	Define a vector space and discuss the properties that must be satisfied for a set to be considered a vector space.	4	1	1
23.	Explain Cramer's Rule for solving a system of linear equations. Provide an example.	4	2	2
24.	Describe the Cholesky decomposition method and explain when it is used to solve systems of linear equations.	4	2	2
25.	Discuss the process of computing differentials of real matrices and their applications.	4	2	3
26.	Discuss the advantages of using Gauss elimination with partial pivoting compared to the basic Gauss elimination method. Provide an example illustrating the need for partial pivoting.	4	2	4
27.	Define univariate, bivariate, and multivariate optimization. Provide examples of each and discuss how the number of variables affects the complexity of the optimization problem.	4	1	5

PART - C (5 × 12 = 60 Marks)
Answer **all** Questions

		Marks	BL	CO
28.	(a) Explain the concept of linear mapping in the context of neural networks. How this concept is utilized in feedforward and backpropagation processes? **(OR)** (b) Provide an example of a real-world problem that can be modeled as a system of linear equations. Illustrate how matrices are used to represent and solve this problem.	12	2	1
29.	(a) Describe the process of Singular Value Decomposition (SVD) and its significance in data compression and dimensionality reduction. Provide an example. **(OR)** (b) Compare and contrast LU decomposition and QR decomposition methods. When would you choose one over the other for solving linear systems?	12	3	2
30.	(a) Explain how optimization techniques are applied in robotics and autonomous systems. Provide examples of how Linear Algebra is used to model robot motion and control. **(OR)** (b) Explain the concept of Complex Gradient Matrices and their applications in non-convex optimization problems. Provide a real-world example.	12	2	3

31.	(a) Explain how the Least Square Method is applied in regression analysis. Provide a real-world example and discuss its advantages.	12	2	4
	(OR)			
	(b) Describe the Conjugate Gradient method for solving linear systems. How does it differ from direct methods like Gauss elimination?			
32.	(a) Describe how matrices are used in natural language processing (NLP) applications. Provide examples of how matrices are used for text representation and analysis.	12	2	5
	(OR)			
	(b) Explain how techniques like momentum and adaptive learning rates are used to enhance gradient descent algorithms. Provide an example where these techniques are beneficial.			

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