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B.Tech DEGREE EXAMINATION, MAY 2024

Fifth Semester

18MAB304T - PROBABILITY AND APPLIED STATISTICS

(For the candidates admitted during the academic year 2018 - 2019 to 2021 - 2022)

Note:

i. **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

ii. **Part - B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours **Max. Marks: 100**

PART - A (20 × 1 = 20 Marks)

Answer **all** Questions

	Question	Marks	BL	CO
1.	In the simultaneous tossing of two perfect dice, the probability of obtaining 4 as the sum of the resultant faces is (A) $\frac{4}{12}$ (B) $\frac{3}{12}$ (C) $\frac{2}{12}$ (D) $\frac{1}{12}$	1	1	1
2.	If the occurrence of an event A is affected by the occurrence of another event B, then $P(A \cap B) =$ (A) $P(A)P(B)$ (B) $P(A) + P(B)$ (C) $P(A)P(B/A)$ (D) $P(A) + P(B) - P(A \cap B)$	1	2	1
3.	$Var\ (4x + 8)$ is (A) $12\ Var\ (x)$ (B) $4\ Var\ (x) + 8$ (C) $16\ Var\ (x) + 8$ (D) $16\ Var(x)$	1	3	1
4.	The Moment generating function of random variable X is given by $M_x(t) = \frac{e^t}{2-e^t}$. The mean of X is (A) 2 (B) 5 (C) 7 (D) 1	1	3	1
5.	If the probability that in a factory a worker skilled is 0.6 then the probability that out of 5 workers none will be skilled is (A) $5C_1 \left[\frac{6}{10}\right]^1 \left[\frac{4}{10}\right]^4$ (B) $5C_0 \left[\frac{6}{10}\right]^1 \left[\frac{4}{10}\right]^4$ (C) $5C_0 \left[\frac{6}{10}\right]^0 \left[\frac{4}{10}\right]^5$ (D) $5C_1 \left[\frac{6}{10}\right]^0 \left[\frac{4}{10}\right]^5$	1	3	2

6.	If the probability that for an army soldiers hit the target on any shot is $\frac{2}{10}$, then the probability mass function is (A) $\left(\frac{8}{10}\right)\left(\frac{2}{10}\right)^{x-1}$ (B) $\left(\frac{2}{10}\right)\left(\frac{8}{10}\right)^{x-1}$ (C) $\left(\frac{2}{10}\right)^{x-1}$ (D) $\left(\frac{8}{10}\right)^{x-1}$	1	2	2
7.	If the cumulative distribution function of the exponential distribution is $F(x) = \begin{cases} 1-e^{-\theta x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$, then the pdf is (A) $e^{-\theta x}$ (B) $e^{\theta x}$ (C) $\theta e^{-\theta x}$ (D) $\theta e^{\theta x}$	1	3	2
8.	The mean of the exponential distribution with pdf $\lambda e^{-\lambda x}, x > 0$ is (A) λ (B) $\frac{1}{\lambda}$ (C) $\frac{1}{\lambda^2}$ (D) 1	1	3	2
9.	An estimator t of θ is called asymptotically unbiased if (A) $\lim_{n\to\infty} E(\theta) = t$ (B) $\lim_{n\to\infty} E(t) = \theta$ (C) $\lim_{n\to\infty} E(t) = 1$ (D) $\lim_{n\to\infty} E(t) = 0$	1	2	3
10.	Maximum likelihood estimators are (A) Necessarily unbiased (B) Most inefficient (C) Most efficient (D) Most insufficient	1	1	3
11.	If consistent estimator converges to θ . if for each positive integer ε, the value of $\lim_{n\to\infty} P\{\|t_n - \theta\| < \varepsilon\}$ is (A) 0 (B) 1 (C) ∞ (D) -1	1	4	3
12.	Neymann's Factorization theorem used to check the existence of (A) Unbiasedness (B) Consistent estimator (C) Efficient estimator (D) Sufficient estimator	1	1	3

13.	Type I error in testing of hypothesis is, (A) Reject H_0 when it is false (B) Accept H_0 when it is false (C) Reject H_0 when it is true (D) Accept H_0 when it is true	1	1	4
14.	If the critical region is located equally in both sides of the normal curve of test statistic then the test is called (A) Level of significance (B) Level of confidence (C) One-tailed test (D) Two-tailed test	1	3	4
15.	The chi-square goodness of fit test can be used to test for (A) the significant difference between the theory and experiment. (B) Difference between population means (C) Normality (D) Probability	1	3	4
16.	If the null hypothesis is false then which of the following is accepted? (A) Null Hypothesis (B) Positive Hypothesis (C) Negative Hypothesis (D) Alternative Hypothesis.	1	1	4
17.	The two lines of regression are given $x + 2y - 5 = 0$, $2x + 3y - 8 = 0$, then the mean values of x and y are respectively given by: (A) $(3,1)$ (B) $(1,2)$ (C) $(1,-2)$ (D) $(-1,2)$	1	3	5
18.	The regression co-efficients are b_2 and b_1, then the correlation co-efficient r is (A) $\frac{b_1}{b_2}$ (B) $\frac{b_2}{b_1}$ (C) $b_1 b_2$ (D) $\pm\sqrt{b_1 b_2}$	1	5	5
19.	In two way classification the data are classified according to ________ different factors. (A) five (B) three (C) two (D) one	1	4	5
20.	The total sum of square is 58 and the sum of squares between samples is 7 then the sum of squares within samples is (A) 65 (B) 406 (C) 51 (D) 50	1	4	5

PART - B (5 × 4 = 20 Marks)

Answer any 5 Questions

		Marks	BL	CO
21.	The distribution function of a random variable X is given by $F(x) = 1 - (1 + x)e^{-x}$, $x \geq 0$. Find the density function, mean and variance.	4	3	1
22.	Let X be a random variable with probability density function $f(x) = \frac{1}{3}e^{-x/3}$, $x > 0$. Find the moment generating function of x and the mean.	4	3	1

23.	A travel company has two cars for hiring. The demand for a car on each day is distributed as Poisson variate, with mean 1.5. Calculate the proportion of days on which (i) neither cars were used (ii) some demand is refused.	4	4	2
24.	If X is uniformly distributed over $(0,10)$. Find (i) $P(x < 4)$ (ii) $P(2 < x < 5)$	4	3	2
25.	Prove that $t = \frac{\Sigma x}{n+1}$ is a biased estimator of the population mean μ.	4	4	3
26.	The number of air-craft accidents that occurred during the various days of a week are given below. Test whether the accidents are uniformly distributed over the week.	4	4	4

Day	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	15	19	13	12	16	15

27.	The following data were available $\bar{x} = 970, \bar{y} = 18, \sigma_X = 38, \sigma_Y = 2$. Correlation coefficient $r = 0.6$. Find the line of regression and obtain the values of X and $Y = 20$.	4	5	5

PART - C (5 × 12 = 60 Marks)
Answer **all** Questions

Marks BL CO

28. (a) A random variable X has the following probability distribution. 12 3 1

X	0	1	2	3	4	5	6	7	9
$P(x)$	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) Find the value of a .
(ii) Find $P(-2 < x < 2)$ and $P(x < 2)$.
(iii) Evaluate the mean and variance of X.
(iv) Find the cumulative distribution function of X.

(OR)

(b) (i) A box contains 2000 components of which 15% are defective. A second box contains 5000 components of which 25% are defective. Two other boxes contain 1000 components each with 10% defective components. A box is chosen at random and an item selected was found to the defective. Find the probability that this has come from the first box and second box [6 Marks].

(ii) The density function of a random variable X is given by $f(x) = kx(2-x), 0 < x < 2$. Find k , mean, variance and r[th] moment. [6 Marks].

29. (a) Fitting a binomial distribution for the following values 12 4 2

X	0	1	2	3	4	5	6
Frequency	5	18	28	12	7	6	4

Find the theoretical frequencies.

(OR)

(b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and S.D.

30. (a) (i) Show that sample mean $\bar{x}$ is a consistent estimator of the population μ [6 Marks]. 12 4 3

(ii) Find out a sufficient estimator for σ^2 in $N(x : \sigma^2)$ [6 Marks].

(OR)

(b) If $x_1, x_2, \dots x_n$ are random observations from a population with mean θ and variance σ^2 . Verify whether the following estimator are unbiased and consistent of θ :

(i) $t_1 = \frac{3x_1+x_2}{4}$ (ii) $t_2 = \frac{2x_1+3x_2}{10}$

31. (a) (i) In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? 12 4 4

[6 Marks]

(ii) A sample of 900 members is found to have a mean 3.5 cm. Can it be reasonable regarded as a simple sample from a large population whose mean is 3.38 cm and a standard deviation 2.4 cm? [6 Marks]

(OR)

(b) The nicotine contents in two independent samples of tobacco are given below.

Sample I	21	24	25	26	27	-
Sample II	22	27	28	30	31	36

Can you say that the two samples come from same normal population?

32. (a) Marks obtained by 10 students in Mathematics (x) and Statistics (y) are given below. Find (i) The Regression equations (ii) Also find y when $x = 55$ 12 5 5

Marks in Maths (x)	60	34	40	50	45	40	22	43	42	64
Marks in Statistics (y)	75	32	33	40	45	33	12	30	34	51

(OR)

(b) An Experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following cleanness readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different model of engines.

	Engine 1	**Engine 2**	**Engine 3**
Detergent A	45	43	51
Detergent B	47	46	52
Detergent C	48	50	55
Detergent D	42	37	49

Looking on the detergents of treatments and the Engines at blocks, obtain the appropriate anova table and test at 1% level of significance. Perform Two way classification.

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