Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2023

Third Semester

18MAB201T – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted from the academic year 2020-2021 to 2021-2022)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40^{th} minute.

(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours — Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

Q.	Question	Marks	BL	CO	PO
1.	The order of the PDE $(D+1)^2 y = x^3$ is (A) 1 (B) 2 (C) 3 (D) 4	1	1	1	1
2.	The notation of the partial derivative $\frac{\partial^2 z}{\partial x^2}$ is (A) p (B) q (C) r (D) t	1	1	1	2
3.	A solution got by giving particular values to the arbitrary constants in a complete integral is known as (A) Complete integral (B) Singular integral (C) General integral (D) Particular integral	1	1	1	2
4.	The PDE of the form $z = px + qy + f(p,q)$ is called (A) Lagrange's form PDE (B) Clairaut's form PDE (C) Poisson form PDE (D) Laplace equation	1	2	1	2
5.	cosx is a periodic function with period (A) $\pi/2$ (B) π (C) 2π (D) 3π	1	1	2	1
6.	Which of the following is an even function? (A) $x\sin x$ (B) $x\cos x$ (C) xe^x (D) $x\log x$	1	1	2	1
7.	If $x=\alpha$ is a point of continuity of $f(x)$, then the sum of the fourier series is (A) $f(0)$ (B) $f(\alpha)$ (C) $\frac{f(\alpha^-)+f(\alpha^+)}{2}$ (D) 0	1	2	2	2

8. The value of a_0 in the fourier series expansion of $f(x)=x$ in $(-\pi, \pi)$ is 1 2 2 2

(A) $\pi/2$ (B) π
(C) 2π (D) 0

9. The one dimensional wave equation is 1 1 3 1

(A) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ (B) $\alpha^2\frac{\partial^2 y}{\partial x^2}=\frac{\partial^2 y}{\partial t^2}$
(C) $\frac{\partial y}{\partial t}=\alpha\frac{\partial^2 y}{\partial x^2}$ (D) $\frac{\partial^2 y}{\partial x^2}=\alpha\frac{\partial^2 y}{\partial t^2}$

10. In one dimensional heat equation α^2 stands for 1 1 3 1

(A) K/ρ (B) T/m
(C) $K/\rho C$ (D) K/C

11. The classification of the PDE $2f_{xx}+f_{xy}-f_{yy}+u_x-u_y=0$ is 1 2 3 2

(A) Elliptic (B) Parabolic
(C) Hyperbolic (D) Both elliptic and parabolic

12. Heat flows from ____________ temperature. 1 1 3 1

(A) Lower to higher (B) Higher to lower
(C) Uniform (D) Constant

13. If $F[f(x)]=F(s)$, then $F\left[x^n f(x)\right]=$ __________ 1 2 4 2

(A) $(i)^n\frac{d^n}{ds^n}F(s)$ (B) $(-i)^n\frac{d^n}{ds^n}F(s)$
(C) $\frac{d^n}{dx^n}F(x)$ (D) $\frac{d^n}{ds^n}F(s)$

14. The Fourier transform of $f(x)=e^{-x^2/2}$ is 1 2 4 2

(A) $e^{x^2/2}$ (B) $e^{-x^2/2}$
(C) e^{-s^2} (D) $e^{-s^2/2}$

15. The value of $F_c[f(ax)]$ is 1 2 4 2

(A) $\frac{1}{a}F_s\left(\frac{s}{a}\right)$ (B) $\frac{1}{a}F_c\left(\frac{s}{a}\right)$
(C) $\frac{1}{s}F_s\left(\frac{a}{s}\right)$ (D) $\frac{1}{s}F_c\left(\frac{a}{s}\right)$

16. Fourier sine transform of $f(x)\sin ax$ is 1 1 4 2

(A) $\frac{1}{2}\left[F_c(s-a)+F_c(s+a)\right]$ (B) $\frac{1}{2}\left[F_c(s-a)-F_c(s+a)\right]$
(C) $\frac{1}{2}\left[F_s(s-a)+F_s(s+a)\right]$ (D) $\frac{1}{2}\left[F_s(s-a)-F_s(s+a)\right]$

Q	Question	Marks	BL	CO	PO
17.	z-transform of 10 is (A) $\frac{z}{z-1}$ (B) $\frac{z}{z-10}$ (C) $\frac{10z}{z-1}$ (D) $\frac{10z}{z-10}$	1	1	5	2
18.	Z-transform of $f(n)=(-2)^n$ is (A) $\frac{z}{z-1}$ (B) $\frac{z}{z+1}$ (C) $\frac{z}{z-2}$ (D) $\frac{z}{z+2}$	1	2	5	2
19.	The value of $z^{-1}\left(\frac{z}{(z-1)^2}\right)$ is (A) n (B) (n+1) (C) n^2 (D) $(n+1)^2$	1	2	5	2
20.	If $F(z)=\frac{z^2(z+1)^3}{(z-2)^7}$, then the order of the pole at z=2 is (A) 1 (B) 2 (C) 3 (D) 7	1	2	5	2

PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions

Q	Question	Marks	BL	CO	PO
21.	Form the PDE by eliminating the arbitrary constants from $z=\left(x^2+a^2\right)\left(y^2+b^2\right)$.	4	3	1	2
22.	State Dirchilet's conditions.	4	3	2	2
23.	Write down the possible solutions of one dimensional wave equation.	4	3	3	1
24.	If $F[f(x)]=F(s)$, then prove that $F[f(x-a)]=e^{ias}F(s)$.	4	3	4	2
25.	Find Z-transform of $f(n)=n^2$.	4	3	5	2
26.	Solve pq=4.	4	3	1	2
27.	A rod is 20 cm long has its ends A and B at temperature 10°C and 50°C until steady state condition prevails. Find the steady state temperature distribution function in the rod.	4	3	3	1

PART – C (5 × 12 = 60 Marks)

Answer **ALL** Questions

	Question	Marks	BL	CO	PO
28. a.	Solve $\left(D^2+5DD'+6D'^2\right)z=\cos(x+2y)+x^2y.$	12	3	1	2
	(OR)				
b.	Solve $(3z-4y)p+(4x-2z)q=2y-3x.$	12	3	1	2
29. a.	Find the half range fourier cosine series of $f(x)=x(\pi-x)\ in\ (0,\pi)$ and hence find the sum of the series $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+....$	12	4	2	2
	(OR)				
b.	Find Fourier series upto first two harmonics from the following data.	12	3	2	2

x:	0	π/3	2π/3	π	4π/3	5π/3	2π
f(x):	1	1.4	1.9	1.7	1.5	1.2	1

	Question	Marks	BL	CO	PO
30. a.	A string is stretched and fastened between two points *x=0* and *x=l* apart. The motion is started by displacing the string into the form $y=100\sin\frac{\pi x}{l}$ from which it is released at the time t=0. Find *y(x,t)*.	12	4	3	1
	(OR)				
b.	A rod of length *l* has its ends A and B kept at 0°C and 100°C respectively until steady-state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so, while that of A is maintained, find *u(x,t)*.	12	4	3	1
31. a.	Find the Fourier transform of $f(x)=\begin{cases}1 & ,if\,\lvert x\rvert\le a\\ 0 & ,\ otherwise\end{cases}$ and hence evaluate $\int_0^{\infty}\frac{\sin^2 t}{t^2}dt$.	12	3	4	2
	(OR)				
b.	Find the fourier cosine transform of $f(x)=e^{-ax}$ and hence evaluate (i) $\int_0^{\infty}\frac{\cos mx}{x^2+a^2}dx$ (ii) $\int_0^{\infty}\frac{dx}{\left(x^2+a^2\right)^2}$	12	3	4	2
32. a.	Find $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$ using convolution theorem.	12	3	5	2
	(OR)				
b.	Solve $y(n+2)+6y(n+1)+9y(n)=2^n$, given $y(0)=y(1)=0$ using Z-transform.	12	4	5	2

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