PART – B (5 × 10 = 50 Marks)
Answer **ALL** Questions

Q. No.	Question	Marks	BL	CO	PO
26. a.	Evaluate by changing the order of integration in $\int_0^4\int_{x^2/4}^{2\sqrt{x}} dydx$.	10	3	1	2
	(OR)				
b.	Find the volume of the sphere $x^2+y^2+z^2=a^2$ by using triple integration.	10	4	1	1
27. a.	Verify Gauss divergence theorem for $\vec{F}=4xz\vec{i}-y^2\vec{j}+yz\vec{k}$ taken over the cube bounded by the plane $x=y=z=0,\ z=2,\ y=2,\ x=2$.	10	3	2	1
	(OR)				
b.	Show that the vector $\vec{F}=\left(6xy+z^3\right)\vec{i}+\left(3x^2-z\right)\vec{j}+\left(3xz^2-y\right)\vec{k}$ is irrotational and hence find the scalar potential.	10	4	2	2
28. a.	Solve $\left(D^2-4D+8\right)y=e^{2t},\ given\ y(0)=2, y'(0)=-2$ using Laplace transform.	10	3	3	1
	(OR)				
b.	Find the Laplace transform of $f(t)=\begin{cases} t, & 0\le t\le 2 \\ 4-t, & 2\le t\le 4 \end{cases}$ where $f(t+4)=f(t)$.	10	4	3	2
29. a.	If $u-v=e^x(\cos y-\sin y)$, find the analytic function interms of z.	10	3	4	1
	(OR)				
b.	Find the bilinear transformation which maps the points $z_1=0, z_2=1, z_3=\infty$ into $\omega_1=i, \omega_2=-1, \omega_3=-i$ respectively.	10	3	4	2
30. a.	Expand $f(z)=\dfrac{1}{(z-1)(z-2)}$ as a Laurent's series valid in the region (i) $\lvert z\rvert<1$ (ii) $1<\lvert z\rvert<2$ (iii) $\lvert z\rvert>2$	10	3	5	1
	(OR)				
b.	Evaluate, using contour integration $\int_0^{2\pi}\dfrac{d\theta}{13+5\sin\theta}$.	10	3	5	1

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Reg. No.															

B.Tech. DEGREE EXAMINATION, MAY 2022
Second Semester

18MAB102T – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted from the academic year 2018-2019 to 2019-2020)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B** should be answered in answer booklet.

Time: 2½ Hours Max. Marks: 75

PART – A (25 × 1 = 25 Marks)
Answer **ALL** Questions

Q. No.	Question	Marks	BL	CO	PO
1.	Find the value of $\int_0^1\int_0^1 dxdy$ is (A) 1 (B) 2 (C) 0 (D) 4	1	1	1	1
2.	Change the order of integration in $\int_0^a\int_0^x dydx$ is (A) $\int_0^a\int_0^x dxdy$ (B) $\int_0^a\int_0^x xdydx$ (C) $\int_0^a\int_y^a dxdy$ (D) $\int_0^a\int_0^y dxdy$	1	2	1	1
3.	What is the value of $\int_0^1\int_0^x dxdy$ (A) ½ (B) −1 (C) 1 (D) 1/3	1	1	1	2
4.	The name of the curve $r=a(1+\cos\theta)$ is (A) Cycloid (B) Hypocycloid (C) Cardioid (D) Hemicircle	1	1	1	1
5.	Find the value of $\int_0^{\pi/2}\int_0^{\sin\theta} r^4 dr\, d\theta$ (A) 8/75 (B) 6/75 (C) 8/65 (D) 4/75	1	1	1	2
6.	If $\phi(x,y,z)=xyz\ then\ \nabla\phi$ is (A) $yz\vec{i}+zx\vec{j}+xy\vec{k}$ (B) $xy\vec{i}+yz\vec{j}+zx\vec{k}$ (C) $zx\vec{i}+xy\vec{j}+y\vec{k}$ (D) 0	1	2	2	1
7.	Find curl (grad ϕ) is (A) −1 (B) 1 (C) 0 (D) ϕ	1	1	2	2

8. What is the condition for $\vec{F}$ to be conservative 1 1 2 1
(A) Solenoidal vector (B) Irrotational vector
(C) Rotational vector (D) Neither solenoidal nor irrotational

9. Find the value of $\nabla\left(r^n\right)$ is 1 2 2 2
(A) $n\vec{r}$ (B) $n\vec{r}\ r^n$
(C) $nr^{n-2}\ \vec{r}$ (D) nr^n

10. If $\vec{r}$ is the position vector of the point *(x,y,z)* with respect to the origin, then div $\vec{r}$ is 1 2 2 1
(A) 2 (B) 3
(C) 0 (D) 1

11. Find L (cos2t) 1 1 3 2
(A) $\frac{s}{s^2+4}$ (B) $\frac{s}{s+2}$
(C) $\frac{2}{s^2+2}$ (D) $\frac{4}{s^2+4}$

12. If $L(f(t))=F(s)$ then $L\left(e^{-at}f(t)\right)=$ 1 1 3 1
(A) $F(s+a)$ (B) $F(s-a)$
(C) $F(s)$ (D) $\frac{1}{a}F(s/a)$

13. Find inverse Laplace transform of $\frac{1}{s^2-a^2}$ is 1 1 3 2
(A) $\frac{\sin at}{a}$ (B) $\frac{\sinh at}{a}$
(C) $\sin at$ (D) $\sinh at$

14. $L\left(t^4\right)$ is equal to 1 1 3 1
(A) $\frac{3!}{s^4}$ (B) $\frac{4!}{s^4}$
(C) $\frac{4!}{s^5}$ (D) $\frac{5!}{s^4}$

15. Find the value of $L\left[e^{-3t}\right]=$ 1 2 3 2
(A) $\frac{1}{s+3}$ (B) $\frac{1}{s-3}$
(C) $\frac{2}{s+3}$ (D) $\frac{3}{s-3}$

16. Find the critical point of transformation $\omega=z^2$ is 1 1 4 1
(A) $z=2$ (B) $z=0$
(C) $z=1$ (D) $z=-2$

17. The function *f(z)=logz* is 1 1 4 1
(A) Differentiable (B) Analytic everywhere
(C) Analytic (D) Analytic everywhere except at the origin

18. An analytic function with constant modulus is 1 1 4 1
(A) Function of x (B) Function of y
(C) Function of z (D) Constant

19. Relate a function u(x,y) satisfies $u_{xx}+u_{yy}=0$ then u is 1 1 4 2
(A) Analytic (B) Harmonic
(C) Differential (D) Continuous

20. If $u+iv$ is analytic, then the curve $u=c_1$ and $v=c_2$. 1 1 4 1
(A) Intersect each other (B) Cut orthogonally
(C) Parallel (D) Coincides

21. Find the value of $\int_c \frac{z}{(z-1)^2}dz$, where c is the circle |z|=2 is 1 1 5 2
(A) πi (B) $2\pi i$
(C) $4\pi i$ (D) 0

22. If $f(z)=\frac{\sin z}{z}$, *then* 1 2 5 1
(A) z=0 is a simple pole (B) z=0 is a pole of order 2
(C) z=0 is a removable singularity (D) z=0 is a zero of f(z)

23. If f(z) is analytic inside and on c, the value of $\int_c f(z)dz$, where c is the simple closed curve and 'a' is any point within c, is 1 1 5 1
(A) *f(a)* (B) $2\pi i\, f(a)$
(C) $\pi i\, f(a)$ (D) 0

24. The annular region for the function $f(z)=\frac{1}{z(z-1)}$ is 1 1 5 2
(A) $0<|z|<1$ (B) $1<|z|<2$
(C) $1<|z|<0$ (D) $|z|<1$

25. Find the residue of *f(z)=cot z* is 1 1 5 1
(A) π (B) 1
(C) −1 (D) 0