Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2023
Second Semester

18MAB102T – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted from the academic year 2018-2019 to 2021-2022)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

Marks BL CO

1. Evaluation of $\int_0^1\int_0^1 dxdy$ is — 1 1 1
   (A) 1 (B) 2
   (C) 0 (D) 4

2. The area of an ellipse is — 1 1 1
   (A) πa^2 (B) πab
   (C) $\pi a^2 b$ (D) $\pi a^2 b^2$

3. The region of integration of the integral $\int_{-b}^{b}\int_{-a}^{a} f(x,y)\,dxdy$ is — 1 1 1
   (A) Rectangle (B) Square
   (C) Triangle (D) Circle

4. $\int_0^1\int_0^2\int_0^3 dxdydz$ is equal to — 1 1 1
   (A) 2 (B) 3
   (C) 6 (D) 24

5. If $\phi = xyz$, then $\nabla\phi$ is — 1 1 2
   (A) $xy\vec{i} + yz\vec{j} + zx\vec{k}$ (B) $zx\vec{i} + xy\vec{j} + yz\vec{k}$
   (C) 0 (D) $yz\vec{i} + zx\vec{j} + xy\vec{k}$

6. If ϕ and ψ are scalar functions then $\nabla\phi\times\nabla\psi$ is — 1 1 2
   (A) Solenoidal (B) Irrotational
   (C) Constant vector (D) Both solenoidal and irrotational

7. The unit vector normal to the surface $x^2 + y^2 - z^2 = 1$ at $(1,1,1)$ is — 1 1 2
   (A) $\dfrac{\vec{i} + \vec{j} - \vec{k}}{\sqrt{3}}$ (B) $\dfrac{2\vec{i} + 2\vec{j} - 2\vec{k}}{\sqrt{2}}$
   (C) $\dfrac{3\vec{i} + 3\vec{j} - 3\vec{k}}{2\sqrt{3}}$ (D) $\dfrac{\vec{i} + \vec{j} - \vec{k}}{3\sqrt{2}}$

8. If $\vec{u}$ and $\vec{v}$ are irrotational, then $\vec{u} \times \vec{v}$ is 1 1 2

(A) Solenoidal (B) Irrotational
(C) Constant vector (D) Zero vector

9. The Laplace transform of a function f(t) exists if 1 1 3

(A) It is uniformly continuous of exponential order (B) It is continuous
(C) It is piecewise continuous of exponential order (D) It is not continuous

10. $L\left(te^{t}\right) =$ 1 1 3

(A) $\dfrac{1}{s^2-1}$ (B) $\dfrac{1}{s-1}$
(C) $\dfrac{1}{(s-1)^2}$ (D) $\dfrac{1}{(s+1)^2}$

11. Find $\lim\limits_{t \to \infty} f(t)$ where $f(t)=1+e^{-t}+t^2$ 1 1 3

(A) 1 (B) 2
(C) 0 (D) 3

12. $L^{-1}\left(\dfrac{1}{s+2}\right) =$ 1 1 3

(A) $-e^{-2t}$ (B) e^{2t}
(C) e^{-2t} (D) $-e^{2t}$

13. The function $f(z) = z^2 + z$ is 1 1 4

(A) Analytic (B) Not analytic
(C) Harmonic (D) Entire function

14. An analytic function f(z) with constant imaginary part is 1 1 4

(A) Constant (B) Dependent of $\overline{z}$
(C) Independent of $\overline{z}$ (D) Variable

15. Which one of the following are C-R equations in Cartesian coordinates? 1 1 4

(A) $u_x = u_y$ and $v_x = v_y$ (B) $u_y = v_x$ and $v_y = u_x$
(C) $u_x = v_y$ and $u_y = -v_x$ (D) $u_x = u_y$ and $v_y = -v_x$

16. For the mapping $\omega = \dot{z} + b$, if b=0 then the mapping is 1 1 4

(A) Rotation (B) Constant
(C) Identity (D) Contraction

17. The value of $\int_C \dfrac{dz}{z+2}, C: |z|=1$ is 1 1 5

(A) $2\pi i$ (B) $-2\pi i$
(C) $4\pi i$ (D) 0

18. The poles of csc z 1 1 5

(A) 0 (B) $n\pi$
(C) $n\pi/2$ (D) $1/n\pi$

19. The residue at z=0 of the functions $f(z)=e^{1/z}$ 1 1 5

(A) 1 (B) 2

(C) 0 (D) −1

20. What kind of singularity have the function $f(z)=\dfrac{1}{1-e^{z}}$? 1 1 5

(A) Essential singularity (B) Pole of order 2

(C) Simple pole (D) Removable singularity

PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions

		Marks	BL	CO	PO
21.	Evaluate $\int_0^1\int_1^2 x(x+y)\,dydx$.	4	1	1	
22.	Find the values of a, b, c if $\vec{F}=(x+2y+az)\vec{i}+(bx-3y-z)\vec{j}+(4x+cy+2z)\vec{k}$ is irrotational.	4	1	2	
23.	Find $L^{-1}\left[\dfrac{s}{(s-4)(s+5)}\right]$.	4	1	3	
24.	Find the constants, a, b, c if $f(z)=x+ay+i(bx+cy)$ is analytic.	4	1	4	
25.	Evaluate $\int_C \dfrac{\cos \pi z^2}{(z-1)(z-2)}dz$ where C is $\lvert z\rvert=3/2$.	4	1	5	
26.	Evaluate $\int_0^{\pi/2}\int_0^{\sin\theta} r\,dr\,d\theta$.	4	1	1	
27.	Find the image of the line x=k under the transformation w=1/z.	4	1	4	

PART – C (5 × 12 = 60 Marks)

Answer **ALL** Questions

		Marks	BL	CO	PO
28. a.	Evaluate by changing the order of integration $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$.	12	1	1	
	(OR)				
b.	Find the volume of sphere $x^2+y^2+z^2=a^2$ using triple integrals.	12	1	1	
29. a.	Verify Green's theorem for $\int_C (xy+y^2)dx+x^2dy$ where C is the boundary of the area between $y=x^2$ and $y=x$.	12	1	2	
	(OR)				

	b. Verify Stoke's theorem for $\vec{F}=(x^2+y^2)\vec{i}-2xy\vec{j}$ taken around the rectangle bounded by the lines $x=\pm a, y=0$ and $y=b$.	12	1	2
30.	a. Find $L^{-1}\left[\dfrac{2s^2-6s+5}{s^3-6s^2+11s-6}\right]$.	12	1	3
	(OR)			
	b. Solve $\dfrac{d^2y}{dt^2}-2\dfrac{dy}{dt}-8y=0,\ given\ y(0)=3, y'(0)=6,$ using Laplace transform.	12	1	3
31.	a. Prove that the function $v=e^{-x}(x\cos y+y\sin y)$ is harmonic and determine the corresponding analytic function $f(z)=u+iv$.	12	1	4
	(OR)			
	b. Find the bilinear transformation which maps $z=0,\ z=1,\ z=\infty$ into the points $w=i,\ w=1,\ w=-i$.	12	1	4
32.	a. Evaluate $\int_C \dfrac{dz}{(z^2+1)(z^2-4)}$ where C is $\lvert z\rvert=\dfrac{3}{2}$.	12	1	5
	(OR)			
	b. Evaluate $\int_0^{2x} \dfrac{d\theta}{2+\cos\theta}$ by contour integration.	12	1	5

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