Reg. No.															

M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

Fourth Semester

18PMA402 – INTEGRAL EQUATIONS AND TRANSFORMATION TECHNIQUES

(For the candidates admitted during the academic year 2018-2019 onwards)

Time: Three hours Max. Marks: 100

PART – A (5 × 5 = 25 Marks)

Answer **ANY FIVE** Questions

1. Define Volterra integral equations of first and second kind.

2. Solve the integral using Laplace Transform

$$y(x) = 3x^2 + \int_0^x y(t)\sin(x-t)\,dt .$$

3. State Fredholm's First fundamental theorem.

4. If $F\{f(x)\} = F(s)$, then show that

$$F\{x^n f(x)\} = (-i)^n \frac{d^x}{ds^x} F(s) .$$

5. If $F\{f(x)\} = F(s)$, then show that

$$F\{f(x)\cos ax\} = \frac{1}{2}\left[F(s+a) + F(s-a)\right].$$

6. Evaluate

$$L\left\{e^{-t}\int_0^t \frac{\sin t}{t}\,dt\right\}$$

7. Verify the initial value theorem for the function $1+e^{-t}\left(\sin t+\cos t\right)$.

8. Find Z-transform ne^{an}.

PART – B (5 × 15 = 75 Marks)

9. a. Solve the Volterra integral equation $y\left(x\right)=1+x+\int_{0}^{x}\left(x-t\right)y(t)dt$ by successive approximation method.

(OR)

b. Solve $y\left(x\right)=e^{x}+\int_{0}^{\pi}e^{x-t}y\left(t\right)dt$ by resolvent Kernel method.

10. a. Determine $D\left(\lambda\right)$ and $D\left(x,t:\lambda\right)$ and hence solve the integral equation

$$y\left(x\right)=e^{x}+\lambda\int_{0}^{1}2e^{x}e^{t}y\left(t\right)dt$$

(OR)

b. Show that the integral equation

$y\left(x\right)=f\left(x\right)+\frac{1}{\pi}\int_{0}^{2\pi}\sin\left(x+t\right)y\left(t\right)dt$ Possesses no solution for $f\left(x\right)=x$ but that it possesses infinitely many solutions when $f\left(x\right)=1$.

11. a. Find the Fourier transform of

$$f(x)=\begin{cases}1-|x|, & |x|<1\\ 0, & |x|>1\end{cases}$$

Hence show that

i. $\displaystyle\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$

ii. $\displaystyle\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$

(OR)

b. Find the Fourier transform of $e^{-a^2x^2}$, $a>0$. Hence deduce that $e^{-x^2/2}$ is self reciprocal with respect to Fourier transform.

12. a. Find the Laplace transform of

i. $t\,e^{-3t}\sin t$

ii. $\dfrac{\cos at-\cos bt}{t}$

iii. $\dfrac{l-e^{t}}{t}$

(OR)

b. State and prove convolution theorem and using this theorem find

$$L-1\left\{\frac{S^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right\}$$

13.a.i. Find the inverse Z transform of

$$\frac{2z^2+3z}{(z+2)(z-4)}$$ by partial fractions method. (8 Marks)

ii. Using Convolution theorem to find

$$Z^{-1}\left\{\frac{Z^2}{(z-a)(z-b)}\right\}$$ (7 Marks)

(OR)

b. Solve $y_{n+2}+6y_{n+1}+9y_n=2^n$ with $y_0=y_1=0$, using Z-transforms.

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