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B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, MAY 2024

Third Semester

21MAB206T - NUMERICAL METHODS AND ANALYSIS

(For the candidates admitted during the academic year 2022-2023 onwards)

Note:

i. **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

ii. **Part - B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours **Max. Marks: 75**

PART - A (20 × 1 = 20 Marks)

Answer **all** Questions

		Marks	BL	CO
1.	What is the range in which the negative root of the equation $e^x = 2 + x$ lies between (A) -1.6 and -1.7 (B) -1.7 and -1.8 (C) -1.8 and -1.9 (D) -1.9 and -2.0	1	2	1
2.	What is the condition for convergence of a function $x = \phi(x)$ in the iterative method ? (A) $\lvert\phi(x)\rvert < 1$ (B) $\lvert\phi(x)\rvert > 1$ (C) $\lvert\phi'(x)\rvert > 1$ (D) $\lvert\phi'(x)\rvert < 1$	1	1	1
3.	What is the order of convergence of Newton's method? (A) 1 (B) 2 (C) 3 (D) 4	1	1	1
4.	What is the condition of convergence for iterative methods of solving a system of simultaneous linear equations? (A) The coefficient matrix should be upper triangular (B) The coefficient matrix should be diagonally dominant (C) The coefficient matrix should be singular (D) The coefficient matrix should be rectangular matrix	1	1	1
5.	What is the definition of the central difference operator δ ? (A) $\delta f(x) = f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)$ (B) $\delta f(x) = f(x) - f\left(x+\frac{h}{2}\right)$ (C) $\delta f(x) = f(x) + f\left(x+\frac{h}{2}\right)$ (D) $\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$	1	1	2
6.	What is the relation between E and ∇? (A) $\nabla - E^{-1} = 2$ (B) $1 + \nabla = E^{-1}$ (C) $\nabla = 1 - E^{-1}$ (D) $\nabla = 1 + E^{-1}$	1	1	2

7. $\Delta \tan^{-1} x = ?$ 1 2 2

(A) $\tan^{-1}(x+h) - \tan^{-1} x$ (B) $\tan^{-1}(x+h) + \tan^{-1} x$

(C) $\tan^{-1} x + \tan^{-1}(x+h)$ (D) $\tan^{-1} x - \tan^{-1}(x+h)$

8. What is the name of the interpolation when the first three terms in the Newton-Gregory forward interpolation formula are used? 1 1 2

(A) Linear interpolation (B) Parabolic interpolation
(C) Elliptic interpolation (D) Hyperbolic interpolation

9. Simpson's three-eight rule can be applied only when n is 1 1 3

(A) odd (B) even
(C) prime (D) multiple of 3

10. Simpson's one-third rule is also called 1 1 3

(A) Parabolic rule (B) Hyperbolic rule
(C) Elliptic rule (D) Trapezoidal rule

11. The error in Trapezoidal rule is 1 1 3

(A) $|E| < \frac{(b-a)h^2}{12}$ (B) $|E| < \frac{(b-a)h^3}{12}$

(C) $|E| < \frac{(b-a)h^3}{180}$ (D) $|E| < \frac{(b-a)h^4}{12}$

12. Newton's backward difference formula to get the first derivative of y(x) at any x is 1 1 3

(A) $\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \left(\frac{2v+1}{2}\right)\nabla^2 y_n + \ldots\right]$ (B) $\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \left(\frac{2u-1}{2}\right)\nabla^2 y_0 + \ldots\right]$

(C) $\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_n + \left(\frac{2v+1}{2}\right)\Delta^2 y_n + \ldots\right]$ (D) $\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \left(\frac{v+1}{2}\right)\nabla^2 y_n + \ldots\right]$

13. Taylor's series method is 1 1 4

(A) Single step method (B) Multi step method
(C) Indirect method (D) Trial and error method

14. The improved Euler method is based on the averages of 1 1 4

(A) Points (B) Slopes
(C) Curves (D) Points and Curves

15. Which of the following formula is a particular case of Runge-Kutta formula of second order? 1 1 4

(A) Taylor's formula (B) Euler's modified formula
(C) Euler's improved formula (D) Euler's simplified formula

16. Using Euler's method, the value of y (0.01) given $y' = -y$, y (0) = 1 by taking h = 0.01 is 1 2 4

(A) 0.06 (B) 2.06
(C) -0.99 (D) 0.99

17. If $B^2 - 4AC = 0$, then the second order PDE is known as — 1 1 5

(A) Parabolic (B) Elliptic
(C) Hyperbolic (D) Simple

18. The nature of the PDE $-2f_{xy} - f_x = 0$ is — 1 1 5

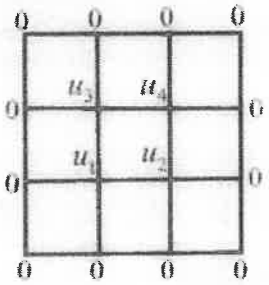
(A) Hyperbolic (B) Elliptic
(C) Parabolic (D) Poisson

19. The solution of hyperbolic equation $u_{tt} = a^2 u_{xx}$ is stable only if — 1 1 5

(A) $\lambda = 1$ (B) $\lambda < \frac{1}{a}$

(C) $\lambda = \frac{1}{a}$ (D) $\lambda > \frac{1}{a}$

20. Using Poisson difference equation $u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = -10(i^2 + j^2 + 10)$ at $u_1(i = 1, j = 1)$ in the diagram given below, we get the equation — 1 1 5



(A) $u_1 + u_4 - 4u_3 = -120$ (B) $u_2 + u_3 - 4u_1 = -120$

(C) $u_1 + u_4 - 4u_2 = -120$ (D) $u_1 + u_3 - 4u_2 = -120$

PART - B (5 × 8 = 40 Marks) — Marks BL CO

Answer **all** Questions

21. (a) Find a positive root of $xe^x = 2$ by the method of False position. — 8 1 1

(OR)

(b) Solve the following system of equations by Gauss Elimination method.
$2x + 3y - z = 5$; $4x + 4y - 3z = 3$; $2x - 3y + 2z = 2$.

22. (a) Estimate the production for 1964 and 1966 from the following data : 8 4 2

Year	:	1961	1962	1963	1964	1965	1966	1967
Production	:	200	220	260	–	350	–	430

(OR)

(b) From the following data, find θ at $x = 43$

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

23. (a) Find the minimum value of y from the following table using numerical differentiation. 8 4 3

x	0.60	0.65	0.70	0.75
y	0.6221	0.6155	0.6138	0.6170

(OR)

(b) Evaluate $\int_{-3}^{3} x^4 dx$ by using (1) Trapezoidal rule (2) Simpson's rule. Verify your results by actual integration.

24. (a) Apply the fourth order Runge-Kutta method to find $y(0 \cdot 2)$ given that $y' = x + y, y(0) = 1$ by taking $h = 0.1$. 8 3 4

(OR)

(b) Solve numerically $y' = y + e^x, y(0) = 0$ for $x = 0.2, 0.4$ by Improved Euler method.

25. (a) Solve the Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, y = 0, x = 3, y = 3$ with $u = 0$ on the boundary and mesh length 1 unit. 8 3 5

(OR)

(b) Given that $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$ with the boundaries $u(0, t) = 0, u(4, t) = 0$, $u(x, 0) = x(4 - x)$. Assume $h = 1$. Find the values of u upto $t = 5$.

PART - C (1 × 15 = 15 Marks) Marks BL CO

Answer **any 1** Questions

26. The population of a certain town is given below. Find the rate of growth of the population in 1931, 1941, 1961 and 1971. 15 3 3

Year	x	1931	1941	1951	1961	1971
Population n thousands	y	40 · 62	60 · 80	79.95	103.56	132.65

27. Solve $u_{xx} + u_{yy} = 0$ correct upto one decimal place over the square mesh of side 4 units with the following boundary conditions: 15 4 5

(i) $u(0, y) = 0$ for $0 \le y \le 4$

(ii) $u(4, y) = 12 + y$, for $0 \le y \le 4$

(iii) $u(x, 0) = 3x$ for $0 \le x \le 4$

(iv) $u(x, 4) = x^2$ for $0 \le x \le 4$

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