30.	a. Find the Laplace transform of a periodic function $f(t)$, with period 2, given by $f(t) = \begin{cases} 1, & 0 < t < 1; \\ -1, & 1 < t < 2. \end{cases}$	12	3	3
	(OR)			
	b. Solve using Laplace transform method $y'' - 3y' + 2y = 2, y(0) = 0, y'(0) = 5$			
31.	a. If $u + v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$. Find $f(z)$ in terms of z.	12	4	4
	(OR)			
	b. (i) What is the region of the w-plane into which the rectangular region in the z-plane bounded by the lines $x = 0, y = 0, x = 1$ and $y = 2$ is mapped under the transformation $w = z + 2 - i$. [5 Marks] (ii) Find the bilinear transformation that maps the points $-1, 0, 1$ in the z-plane into the points $0, i, 3i$ in the w-plane. [7 Marks]			
32.	a. Find the Laurent's series expansion of $f(z) = \dfrac{1}{(z-1)(z-2)}$ valid in the region (i) $\lvert z\rvert < 1$ (ii) $1 < \lvert z\rvert < 2$ and (iii) $\lvert z\rvert > 2$	12	4	5
	(OR)			
	b. Evaluate using Contour integration $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 + 3\cos\theta}$			

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Reg. No

B.Tech. DEGREE EXAMINATION, JUNE 2023

Second Semester

18MAB102T - ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2018-2019 to 2021-2022)

Note:

i. **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40 minutes.

ii. **Part - B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours **Max. Marks: 100**

Part - A (20 × 1 Marks = 20 Marks)

Answer **All** Questions

		Marks	BL	CO
1.	The value of a double integral $\int_0^2 \int_0^1 4xy\,dx\,dy$ is (A) 3 (B) 2 (C) 4 (D) 6	1	2	1
2.	Area of the double integral in polar co-ordinates is equal to (A) $\iint_R dr\,d\theta$ (B) $\iint_R r^2 dr\,d\theta$ (C) $\iint_R (r+1) dr\,d\theta$ (D) $\iint_R r\,dr\,d\theta$	1	1	1
3.	The name of the curve $r = a(1 + \cos\theta)$ is (A) Leminiscate (B) Cardioid (C) Cycloid (D) Semicircle	1	1	1
4.	By changing the double integration $\int_0^a \int_y^a f(x,y)\,dx\,dy$ into polar coordinates is equal to (A) $\int_0^{\frac{\pi}{4}} \int_0^{\frac{a}{\cos\theta}} f(r,\theta) r\,dr\,d\theta$ (B) $\int_0^{\frac{\pi}{4}} \int_0^{\frac{a}{\cos\theta}} f(r,\theta) dr\,d\theta$ (C) $\int_0^{\frac{\pi}{2}} \int_0^{\frac{a}{\cos\theta}} f(r,\theta) dr\,d\theta$ (D) $\int_0^{\frac{\pi}{3}} \int_0^{\frac{a}{\cos\theta}} f(r,\theta) r\,dr\,d\theta$	1	2	1
5.	If $\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$ is solenoidal then the value of a is (A) 0 (B) 1 (C) 2 (D) -2	1	2	2
6.	The value of $curl grad\phi$ is (A) 0 (B) 1 (C) $\vec{a}$ (D) π	1	2	2
7.	If $\vec{F}$ is irrotational and C is a closed curve, then $\oint_C \vec{F}.d\vec{r} =$ (A) 1 (B) 2 (C) 0 (D) 3	1	1	2

8. The relation between a surface integral and a volume integral is known as — 1 1 2
(A) Green's theorem
(B) Stoke's theorem
(C) Gauss Divergence theorem
(D) workdone

9. If $L\{f(t)\} = F(s)$, then $L\{f(at)\} =$ — 1 1 3
(A) $sF(s)$
(B) $\frac{1}{s}F\left(\frac{s}{a}\right)$
(C) $\frac{1}{a}F\left(\frac{a}{s}\right)$
(D) $\frac{1}{a}F\left(\frac{s}{a}\right)$

10. If $L[f(t)] = F[s]$ then $L[tf(t)] =$ — 1 1 3
(A) $\frac{d}{ds}F(s)$
(B) $-\frac{d}{ds}F(s)$
(C) $(-1)^n \frac{d}{ds}F(s)$
(D) $-\frac{d^2}{ds^2}F(s)$

11. Find the inverse Laplace transform of $\frac{2}{s^2}$ — 1 2 3
(A) t^2
(B) $3t$
(C) 2
(D) $2t$

12. Find the inverse Laplace transform of $\frac{1}{(s-1)^2+1} =$ — 1 2 3
(A) $e^{-t}\cos t$
(B) $e^{t}\cos t$
(C) $e^{t}\sin t$
(D) $e^{-t}\sin t$

13. The Cauchy-Riemaan equation in polar co-ordinates is — 1 1 4
(A) $ru_r = v_\theta, rv_r = -u_\theta$
(B) $ru_r = -v_\theta, rv_r = u_\theta$
(C) $ru_r = -v_\theta, rv_r = -u_\theta$
(D) $ru_r = v_\theta, rv_r = u_\theta$

14. An analytic function with constant modulus is — 1 1 4
(A) zero
(B) analytic
(C) harmonic
(D) constant

15. The transformation $w = cz$ where c is real constant represents — 1 1 4
(A) rotation
(B) reflection
(C) magnification
(D) magnification and rotation

16. The fixed points of the transformation $w = \frac{2z+6}{z+7}$ — 1 2 4
(A) $-6, 1$
(B) $6, -1$
(C) $-6, -1$
(D) $6, 1$

17. The value of $\oint_c \frac{z^2}{z-2}dz$ where c is a circle $|z| = 1$ is — 1 2 5
(A) 0
(B) 2
(C) $\frac{\pi}{2}$
(D) π

18. If $f(z) = \frac{z}{(z-1)^2(z-2)}$ then — 1 1 5
(A) $z = 1$ is a pole of order 2 and $z = 2$ is a pole of order 1
(B) $z = 1$ is a pole of order 1 and $z = 2$ is a pole of order 2
(C) $z = 1$ is a pole of order 2 and $z = 2$ is a pole of order 2
(D) $z = 1$ is a pole of order 1 and $z = 2$ is a pole of order 1

19. If $f(z) = \frac{\sin z}{z}$ then — 1 2 5
(A) $z = 0$ is a pole of order 3
(B) $z = 0$ is a pole of order 2
(C) $z = 0$ is a removable singularity
(D) $z = 0$ is a essential singularity

20. The residue of $f(z) = \frac{e^{2z}}{z+1}$ at $z = -1$ is — 1 1 5
(A) e^{-2}
(B) $-2e^{-2}$
(C) -1
(D) $2e^{-2}$

Part - B (5 × 4 Marks = 20 Marks)

Answer **any 5** Questions

Q. No.	Question	Marks	BL	CO
21.	Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\,d\theta\,dr$.	4	3	1
22.	Find a unit normal vector to the level surface $x^2 + 2y^2 + z^2 = 7$ at the point $(1, -1, 2)$.	4	3	2
23.	Find Laplace Transform of $e^{2t}\cos 2t$	4	3	3
24.	Analyze whether the function $u = e^{-2x}\sin 2y$ is harmonic.	4	4	4
25.	Using Cauchy's integral formula, evaluate $\int_C \frac{e^{-z}}{z+1}dz$ where C is a circle $\lvert z\rvert = 2$	4	4	5
26.	Using partial fraction method, find the inverse Laplace transform of $\frac{1}{s(s+3)}$	4	3	3
27.	Find the area of the region bounded by the parabolas $y = x^2$ and $x = y^2$	4	4	1

Part - C (5 × 12 Marks = 60 Marks)

Answer **All** Questions

Q. No.	Question	Marks	BL	CO
28.	a. Change the order of integration and evaluate $\int_0^{a}\int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ **(OR)** b. Find the volume of a sphere $x^2 + y^2 + z^2 = a^2$ by using triple integrals.	12	4	1
29.	a. i. Find the angle between the surfaces $x^2 + yz = 2$ and $x + 2y - z = 2$ at $(1, 1, 1)$ [6 Marks] ii. Show that the vector field $\vec{F}$ given by $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational and hence find its scalar potential. [6 Marks] **(OR)** b. Verify Stoke's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ over the surface of a cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the XOY plane.	12	4	2