ii. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda=1/2$. 6 3 2 1,2
(1) What is the probability that the repair time exceeds 2 hrs?
(2) What is the conditional probability that a repair takes atleast 10 hrs given that its duration exceeds 9 hrs?

30. a.i. A salesman in a departmental store claims that atmost 60 percent of the shoppers entering the store leave without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Are these sample results consistent with the claim of the salesman? Use an LOS of 0.05. 6 3 3 1,2

ii. A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm and the SD is 10 cm? 6 3 3 1,2

(OR)

b. The following data represent the biological values of protein from cow's milk and buffalo's milk at a certain level. 12 4 3 1,2
Cow's milk : 1.82 2.02 1.88 1.61 1.81 1.54
Buffalo's milk : 2.00 1.83 1.86 2.03 2.19 1.88
Examine if the average values of protein in the two samples significantly differ.

31. a. The following data show defective articles produced by 4 machines: 12 4 4 1,2

Machine :	A	B	C	D
Production time :	1	1	2	3
No. of defectives :	12	30	63	98

Do the figures indicate a significant difference in the performance of the machines?

(OR)

b. Arrivals at a telephone booth are considered to be Poisson with an average time of 10 min between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 min. 12 4 4 1,2
(a) Find the average number of persons waiting in the system
(b) What is the probability that a person arriving at the booth will have to wait in the queue?
(c) What is the probability that it will take him more than 10 min altogether to wait for phone and complete his call?
(d) Estimate the fraction of the day when the phone will be in use

32. a. There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process a marble is selected from each urn and the 2 marbles selected are interchanged. Let the state a_i of the system be the number of red marbles in A after i changes. What is the probability that there are 2 red marbles in A after 3 steps? In the long run, what is the probability that there are 2 red marbles in urn A? 12 4 5 1,2

(OR)

b. Find the nature of the states of the Markov chain with the tpm 12 4 5 1,2

$$P=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

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Reg. No. | | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, MAY 2023

Fourth & Fifth Semester

18MAB204T – PROBABILITY AND QUEUEING THEORY

(For the candidates admitted from the academic year 2018-2019 to 2021-2022)

(Statistical table to be provided (normal table, t-table, F-table, Chi-square table))

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

Marks BL CO PO

1. If $p(X=x_i)=p_i$ is called the probability mass function of the random variable X then 1 1 1 1
(A) $p_i \neq 0$ (B) $p_i < 0$
(C) $p_i \geq 0$ (D) $p_i \leq 1$

2. Variance of a discrete or continuous random variable X, is defined by 1 1 1 1
(A) $Var(X)=E(X^2)-[E(X)]^2$ (B) $Var(X)=E(X^2)-E(X)$
(C) $Var(X)=[E(X)]^2-E(X^2)$ (D) $Var(X)=E(X)-E(X^2)$

3. Moment generating function of a random variable X is 1 1 1 1
(A) $M_X(t)=E(e^t)$ (B) $M_X(t)=E(e^{tX})$
(C) $M_X(t)=E(e^{tY})$ (D) $M_X(t)=E(e^{at})$

4. If $f_X(x)$ is a pdf of X and Y=T(X) then the pdf of Y is 1 1 1 1
(A) $f_Y(y)=f_X(x).\left|\frac{dy}{dx}\right|$ (B) $f_Y(y)=f_X(x).\left|\frac{dx}{dy}\right|$
(C) $f_Y(y)=f_X(x).2x$ (D) $f_Y(y)=f_X(x)$

5. Moment generating function of binomial distribution is 1 1 2 1
(A) $(q+pe^t)^n$ (B) $(p+qe^t)^n$
(C) $(qe^t+p)^n$ (D) $(q+p)^n$

6. Mean and variance of Poisson distribution are 1 1 2 1
(A) λ and $1/\lambda$ (B) λ and λ
(C) λ and λ^2 (D) λ and 0

7. The probability density function of an uniform random variable in (–3, 3) is 1 2 2 1
(A) 1/2 (B) 1/3
(C) 1/6 (D) 1/8

8. The standard normal variate Z is given by 1 1 2 1
(A) $Z=\frac{X-\mu}{\sigma}$ (B) $Z=X-\mu$
(C) $Z=\frac{X+\mu}{\sigma}$ (D) $Z=\frac{X-\mu}{\sigma}$

9. The standard deviation of the sampling distribution of a statistic is called 1 1 3 1
(A) Standard error (B) Sampling error
(C) Standard deviation (D) Correlation coefficient

10. In large sample test, the test statistic for the difference between sample mean and population mean is 1 1 3 1
(A) $Z=\dfrac{\overline{X}-\mu}{s/\sqrt{n}}$ (B) $Z=\dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}}$
(C) $Z=\dfrac{s-\sigma}{\sigma/\sqrt{2n}}$ (D) $Z=\dfrac{s-\mu}{\sigma/\sqrt{2n}}$

11. The mean of t-distribution is 1 1 3 1
(A) 0 (B) 1
(C) n (D) ∞

12. In general, the number of degrees of freedom is given by 1 1 3 1
(A) $\gamma=n-k$ (B) $\gamma=n-2$
(C) $\gamma=n-1$ (D) $\gamma=n-3$

13. χ^2-test is valid if each individual frequency O_i is 1 1 4 1
(A) ≥ 10 (B) ≥ 20
(C) ≥ 30 (D) ≥ 40

14. In (M/M/1:K/FIFO) model, P_n is equal to ________ when $0\leq n\leq k-1$. 1 2 4 1
(A) $\left(\dfrac{\lambda}{\mu}\right)^{n-1}P_0$ (B) $\left(\dfrac{\lambda}{\mu}\right)^{n}P_0$
(C) $\left(\dfrac{\mu}{\lambda}\right)^{n}P_0$ (D) $\left(\dfrac{\mu}{\lambda}\right)^{n-1}P_0$

15. In (a/b/c):(d/e), the symbolic representation of queueing model, 'd' stands for 1 1 4 1
(A) Number of customers (B) Number of servers
(C) Capacity of the system (D) Queue discipline

16. The most common queue discipline is 1 1 4 1
(A) FIFO (B) SIRO
(C) MIFO (D) LIFO

17. If the period of the state 'i' is greater than 1 then the state 'i' is called 1 1 5 1
(A) Periodic (B) Aperiodic
(C) Ergodic (D) Recurrent

18. If a Markov chain is finite irreducible then all its states are 1 1 5 1
(A) Null persistent (B) Non-null persistent
(C) Recurrent (D) Persistent

19. A non-null persistent and aperiodic state is called 1 1 5 1
(A) Recurrent (B) Persistent
(C) Periodic (D) Ergodic

20. The tpm of a Markov chain is a ________ since the sum of all the elements of any row is 1. 1 1 5 1
(A) Square matrix (B) Stochastic matrix
(C) Zero matrix (D) Row matrix

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

Marks BL CO PO

21. A continuous RV has a pdf $f(x)=3x^2, 0\leq x\leq 1$. Find a and b such that 2 1 1,2
(i) $P(X\leq a)=P(X>a)$ and 2
(ii) $P(X>b)=0.05$ 2

22. Find the moment generating function of a random variable which is uniformly distributed over (–1, 2) and hence find its mean and variance. 4 2 1 1,2

23. The number of monthly breakdowns of a computer is a RV having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month. 4 2 2 1,2
(i) without a breakdown
(ii) with only 1 breakdown and
(iii) with atleast one breakdown

24. Find the MGF of geometric distribution and hence find its mean. 4 3 2 1,2

25. The mean value of a random sample of 60 items was found to be 145, with an SD of 40. Find the 95% confidence limits for the population mean. 4 3 3 1,2

26. In the usual notation of a (M/M/1:∞/FIFO) queue system if $\lambda=12$ per hour and $\mu=24$ per hour, find the average number of customers in the system and in the queue. 4 3 4 1,2

27. If the initial state probability distribution of a Markov chain is $p^{(0)}=\left(\dfrac{5}{6},\dfrac{1}{6}\right)$ and the tpm of the chain is $\begin{pmatrix}0 & 1\\ 1/2 & 1/2\end{pmatrix}$. Find the probability distribution of the chain after 2 steps. 4 3 5 1,2

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

Marks BL CO PO

28. a. A discrete RV X has the following probability distribution. 12 4 1 1,2

x:	0	1	2	3	4	5	6	7	8
P(x):	a	3a	5a	7a	9a	11a	13a	15a	17a

Find the value of 'a', P(X<3), variance and distribution function of X.

(OR)

b. A random variable X has pdf $f(x)=e^{-x}, x\geq 0$. Use Tchebycheff's inequality to show that $P\{|X-1|>1\}<1/4$ and show also that the actual probability is e^{-3}. 12 4 1 1,2

29. a. Fit a Poisson distribution for the following distribution: 12 4 2 1,2

x:	0	1	2	3	4	5	Total
f:	142	156	69	27	5	1	400

(OR)

b.i. State and prove the memoryless property of exponential distribution. 6 3 2 1,2