b. Show that the vector field $\vec{F}=\left(x^2-y^2+x\right)\vec{i}-(2xy+y)\vec{j}$ is irrotational and hence find its scalar potential. — 8 4 2 2

23. a. Find the laplace transform of the function $f(t)=\begin{cases} t \text{ for } 0<t<a \\ 2a-t \text{ for } a<t<2a \end{cases}$ where $f(t+2a)=f(t)$. — 8 4 3 2

(OR)

b. Solve $\dfrac{d^2y}{dx^2}+6\dfrac{dy}{dx}+8y=2$ given $y(0)=1, y'(0)=1$. — 8 4 3 2

24. a. If $f(z)=u+iv$ is analytic find f(z) and v if $u=\dfrac{\sin 2x}{\cos 2x+\cosh 2y}$. — 8 3 4 1

(OR)

b. Find the bilinear transformation which maps the points z = –2, 0, 2 into the points *ω=0, i, –i* respectively. — 8 3 4 2

25. a.i. Using Cauchy integral formula evaluate $\int_c \dfrac{z}{z-2}dz$, where c is the circle $|z-2|=\frac{3}{2}$. — 4 3 5 1

ii. Evaluate $\int_c \dfrac{\sin \pi z^2+\cos \pi z^2}{(z-1)(z-2)}dz$ where C:|z|=3 by Cauchy's residue theorem. — 4 3 5 1

(OR)

b. Expand $f(z)=\dfrac{z^2-1}{(z+2)(z+3)}$ in a Laurent's series if — 8 4 5 2

(i) $|z|<2$
(ii) $|z|>3$
(iii) $2<|z|<3$

PART – C (1 × 15 = 15 Marks)

Answer **ANY ONE** Question

		Marks	BL	CO	PO
26.	Verify Gauss divergence theorem for $\vec{F}=\left(x^2-yz\right)\vec{i}+\left(y^2-zx\right)\vec{j}+\left(z^2-xy\right)\vec{k}$ taken over the rectangular parallelopiped enclosed by *x=0, x=a, y=0, y=b, z=0* and *z=c*.	15	4	2	2
27.	Evaluate $\int_0^{2\pi} \dfrac{d\theta}{13+5\sin\theta}$ using contour integration.	15	4	5	2

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Reg. No.															

B.Tech. / M.Tech (Integrated) DEGREE EXAMINATION, MAY 2023

Second Semester

21MAB102T – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted from the academic year 2021 - 2022 & 2022 - 2023)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours — Max. Marks: 75

PART – A (20 × 1 = 20Marks)

Answer **ALL** Questions

(Marks BL CO PO)

1. Evaluation of $\int_0^2\int_0^2 dxdy$ is — 1 2 1 1
   (A) 0 (B) 1
   (C) 4 (D) 2

2. Area of the double integral in Cartesian co-ordinate is — 1 1 1 2
   (A) $\iint dydx$ (B) $\iint xdxdy$
   (C) $\iint x^2dxdy$ (D) $\iint rdrd\theta$

3. The region of integration of the integral $\int_0^1\int_0^x f(x,y)dxdy$ is — 1 2 1 1
   (A) Square (B) Rectangle
   (C) Triangle (D) Circle

4. Evaluation of $\int_0^1\int_0^2\int_0^3 dxdydz$ is equal to — 1 2 1 1
   (A) 3 (B) 4
   (C) 2 (D) 6

5. If $\vec{r}$ is the position vector of the point *(x,y,z)* with respect to the origin then $\nabla.\vec{r}$ is — 1 1 2 1
   (A) 2 (B) 3
   (C) 0 (D) 1

6. If $\phi=xyz$ then $\nabla\phi$ is — 1 2 2 2
   (A) $yz\vec{i}+zx\vec{j}+xy\vec{k}$ (B) $xy\vec{i}+yz\vec{j}+zx\vec{k}$
   (C) $zx\vec{i}+xy\vec{j}+yz\vec{k}$ (D) 0

7. The unit normal vector to the surface $x^2+y^2-z^2=1$ at $(1,1,1)$ is 1 2 2 1

(A) $(\vec{i}+\vec{j}-\vec{k})/\sqrt{3}$ (B) $(2\vec{i}+2\vec{j}-2\vec{k})/\sqrt{2}$
(C) $(3\vec{i}+3\vec{j}-3\vec{k})/2\sqrt{3}$ (D) $(\vec{i}+\vec{j}-\vec{k})/3\sqrt{2}$

8. If $\vec{u}$ and $\vec{v}$ are irrotational then $\vec{u}\times\vec{v}$ is 1 1 2 1

(A) Solenoidal (B) Irrotational
(C) Constant vector (D) Zero vector

9. Find $L\left[e^{-at}\right]$ 1 1 3 1

(A) $\frac{1}{s+1}$ (B) $\frac{1}{s-1}$
(C) $\frac{1}{s+a}$ (D) $\frac{1}{s-a}$

10. If $L[f(t)]=F(s)$ then $L[t\,f(t)]=$ 1 1 3 2

(A) $\frac{d}{ds}F(s)$ (B) $-\frac{d}{ds}F(s)$
(C) $(-1)^n\frac{d}{ds}F(s)$ (D) $-\frac{d^2}{ds^2}F(s)$

11. Find $L^{-1}\left[\frac{1}{(s-1)^2}\right]$ 1 1 3 2

(A) te^{-t} (B) $t\,e^t$
(C) t^2e^t (D) t

12. Using the final value theorem, find the value of the function $f(t)=1+e^{-t}(\sin t+\cos t)$ 1 2 3 2

(A) 1 (B) 0
(C) ∞ (D) -2

13. If a function $u(x,y)$ satisfies $u_{xx}+u_{yy}=0$ then u is 1 1 4 1

(A) Analytic (B) Harmonic
(C) Differentiable (D) Continuous

14. The points at which the function $f(z)=\frac{1}{z^2+1}$ fails to be analytic is 1 1 4 1

(A) $z=\pm 1$ (B) $z=0$
(C) $z=\pm i$ (D) $z=\pm 2$

15. An analytic function with constant modulus is 1 1 4 1

(A) Zero (B) Analytic
(C) Constant (D) Harmonic

16. The transformation ω=z+c where c is a complex constant represents 1 1 4 1

(A) Rotation (B) Magnification
(C) Translation (D) Magnification and rotation

17. The value of $\int_c \frac{zdz}{z-2}$, where c is the circle |z|=1 is 1 2 5 2

(A) 0 (B) $\frac{\pi}{2}i$
(C) π/2 (D) 2

18. Let $C_1:|z-a|=R_1$ and $C_2:|z-a|=R_2$ be two concentric circles ($R_2<R_1$), the annular region is defined as 1 1 5 1

(A) Within C_1 (B) Within C_2
(C) Within C_2 and outside C_1 (D) Within C_1 and outside C_2

19. If $f(z)=\frac{\sin z}{z}$, then 1 1 5 1

(A) z=0 is a simple pole (B) z=0 is a pole of order 2
(C) z=0 is a removable singularity (D) z=0 is a zero of f(z)

20. If $f(z)$ is analytic inside and on C, the value of $\int_c \frac{f(z)}{z-a}dz$ where C is the simple closed curve and 'a' is any point within C is 1 1 5 2

(A) $f(a)$ (B) $2\pi i\,f(a)$
(C) $2\pi i\,f'(a)$ (D) 0

PART – B (5 × 8 = 40 Marks)
Answer **ALL** Questions

		Marks	BL	CO	PO
21. a.	By changing the order of integration, evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$.	8	4	1	2
	(OR)				
b.	Evaluate $\int_0^a\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2-y^2}} \frac{dzdydx}{\sqrt{a^2-x^2-y^2-z^2}}$.	8	4	1	2
22. a.i.	Find the directional derivative of $\phi=x^2yz+4xz^2+xyz$ at $(1,2,3)$ in the direction of $2\vec{i}+\vec{j}-\vec{k}$.	4	3	2	1
ii.	Find the angle between the surfaces $z=x^2+y^2-3$ and $x^2+y^2+z^2=9$ at $(2,-1,2)$.	4	3	2	1
	(OR)				