

24. a. Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes. 8 3 4 2

- Find the average number of customers in the shop
- Find the average number of customers in the queue
- What is the probability that the server is idle?
- Find the average time a customer spends in the shop

(OR)

- b. A petrol pump has 4 pumps. The service time follows an exponential distribution at the rate of 10 cars per hour and cars arrive for service in a Poisson process at the rate of 30 cars per hour. 8 4 4 3

- Find the average waiting time in the queue
- Find the average time spent in the system
- Find the average number of cars in the system

25. a. The transition probability matrix of a Markov chain $\{X_n\}$ $n=1,2,\dots$ having 3 states 1,2,3 is 8 3 5 3

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \text{ and the initial distribution}$$

$$P^{(0)} = (0.7, 0.2, 0.1).$$

$$\text{Find, (i) } P(X_2 = 3, X_1 = 3, X_0 = 2) \text{ (ii) } P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$$

(OR)

- b. Suppose that the probability of a dry day following a rainy day is $1/3$ and that the probability of a rainy day following a dry day is $1/2$. Given that May 1 is a dry day, find the probability that 8 4 5 2

- May 3 is also a dry day
- May 5 is also a dry day

PART - C (1 × 15 = 15 Marks)

Answer ANY ONE Question

26. Obtain the lines of regression from the data given below. 15 3 3 3

X	62	64	65	69	70	71	72	74
Y	126	125	139	145	165	152	180	208

27. Find the nature of the states of the Markov chain with the transition probability matrix. 15 3 5 2

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

Reg. No.

B.Tech. / M.Tech (Integrated) DEGREE EXAMINATION, MAY 2023
Fourth Semester

21MAB204T – PROBABILITY AND QUEUEING THEORY
(For the candidates admitted from the academic year 2021 - 2022 & 2022 - 2023)
(Data books, tables, graph sheet to be provided)

Note:

- Part - A should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
- Part - B and Part - C should be answered in answer booklet.

Time: 3 Hours

Max. Marks: 75

PART - A (20 × 1 = 20 Marks)

Answer ALL Questions

1. A random variable X has the following probability function. Find 'a' 1 1 1 1

X	0	1	2	3	4
P(X)	a	2a	5a	7a	9a

- 1/24
- 21/24
- 7/12
- 2/24

2. The probability of the impossible event is 1 1 1 2

- 2
- 1
- 3
- 0

3. If C is a constant (non random variable) then Var(C) is 1 2 1 1

- 0
- C
- Var(C)
- 1

4. The distribution function of a random variable X is given by 1 2 1 1

$$F(x) = 1 - e^{-x}, x \geq 0 \text{ then the density function is}$$

- $f(x) = e^x$
- $f(x) = xe^x$
- $f(x) = xe^{-x}$
- $f(x) = e^{-x}$

5. The variance of the Binomial distribution is 1 1 2 2

- np
- npq
- $1/\lambda$
- λ

6. The variance of the Poisson distribution is 1 1 2 2

- λ
- λ^2
- $1/\lambda$
- $1/\lambda^2$

7. If X is exponentially distributed with parameter λ then for any two positive integers s and t $P(X > s + t / X > s) =$ 1 1 2 1

- $P(X > t)$
- $P(X < t)$
- $P(X > s)$
- $P(X < s)$

8. The standard normal distribution is represented by
 (A) $N(0,1)$ (B) $N(1,1)$
 (C) $N(1,0)$ (D) $N(0,0)$
9. The conditional probability density function of Y given X is
 (A) $f(x,y)f(x)$ (B) $f(x,y)f(y)$
 (C) $\frac{f(x,y)}{f(x)}$ (D) $\frac{f(x,y)}{f(y)}$
10. If X and Y are independent random variables then $\text{Cov}(x,y)$ is equal to
 (A) 0 (B) 1
 (C) 2 (D) 3
11. If X and Y have joint probability density function

$$f(x,y) = \begin{cases} kxy, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

 Find the value of k.
 (A) 2 (B) 1
 (C) 4 (D) 3
12. The coefficient of correlation lies between
 (A) -1 and 1 (B) 0 and 1
 (C) 0 (D) -1
13. What stands for 'e' in the queue model (a/b/c:d/e)
 (A) Queue discipline (B) System capacity
 (C) Maximum queue size (D) Service time
14. The symbolic notation of queueing model is represented by
 (A) Kendall (B) Euler
 (C) Fisher (D) Neumann
15. The average number of customers in the system (MM/1: ∞ /FIFO) model is
 (A) $\frac{\lambda}{\mu - \lambda}$ (B) $\frac{\mu}{\lambda - \mu}$
 (C) $\frac{\lambda}{\mu + \lambda}$ (D) $\frac{\mu}{\lambda + \mu}$
16. Which term refers to "A customer who leaves the queue because the queue is too long"?
 (A) Balking (B) Reneging
 (C) Jockeying (D) Leaving
17. If P is the transition probability matrix of a homogenous Markov chain, then n step tpm is
 (A) $P^{(n^2)}$ (B) P^n
 (C) P (D) N

18. A state i is said to be periodic with period d_i if
 (A) $d_i < 1$ (B) $d_i = 1$
 (C) $d_i > 1$ (D) $d_i = 0$
19. A non-null persistent and aperiodic state is called
 (A) Empty (B) Finite
 (C) Ergodic (D) 1
20. If the one step transition probability does not depend on the step, then the Markov chain is called a
 (A) n step (B) Regular
 (C) Homogeneous (D) Non homogeneous

PART - B (5 × 8 = 40 Marks)

Answer ALL Questions

21. a. A discrete random variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7	8
P(X)	k	3k	5k	7k	9k	11k	13k	15k	17k

- (i) Find the value of k
 (ii) Find $P(X < 3)$
 (iii) Find $P(0 < X < 3)$
 (iv) Find the distribution function of X

(OR)

- b. A fair die is tossed 720 times. Use Tchebycheff inequality to find a lower bound for the probability of getting 100 to 140 sixes.

22. a. Fit a Poisson distribution for the following data:

X	0	1	2	3	4	5	Total
f	142	156	69	27	5	1	400

Finding the probability mass function and then finding the theoretical frequencies.

(OR)

- b. If X is normally distributed with mean 12 and SD 4. Find out the following

- (i) $P(X \geq 20)$
 (ii) $P(X \leq 20)$
 (iii) $P(0 \leq X \leq 12)$

23. a. Find the coefficient of correlation between industrial production and export using the following data.

Production	55	56	58	59	60	60	62
Export	35	38	37	39	44	43	44

(OR)

- b. Given the following probability distribution of (X,Y).

Find (i) $P(X \leq 1)$ (ii) $P(Y \leq 3)$ (iii) $P(X \leq 1, Y \leq 3)$ (iv) $P(X \leq 1 | Y \leq 3)$

Y \ X	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64