Short-time quantum Fourier transform processing

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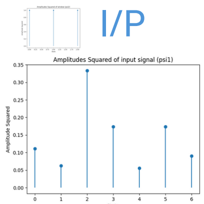
SCAN ME

Abstract - Short-time processing is crucial in both streamed and large-data contexts due to the quadratic cost of convolution-based approaches. We introduce the short-time quantum Fourier transform (STQFT) processing, bridging a key gap in quantum digital signal processing.

I/P

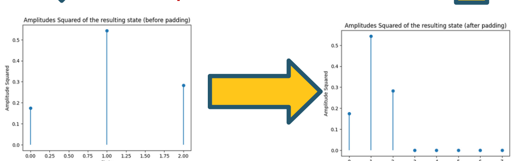


Quantum State

$$\nu = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}$$

01

Classical windowing and amplitude encoding to create the quantum state.

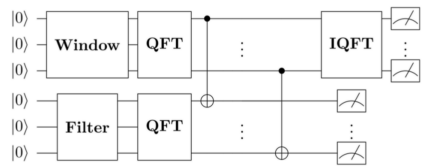


Amplitude Encoding:

Data is embedded into the coefficients of the quantum state vector. A quantum state of a system is represented by a column vector .The entries are complex numbers. The sum of the absolute values squared of the entries must equal 1.

Conclusion:

We presented a short-time quantum Fourier transform (STQFT) framework that combines two quantum filtering approaches (filter-as-register and filter-as-block-encoded-gate) with a novel quantum overlap-add technique. Despite current bottlenecks—including classical intermediates between subroutines—our method promises future applications in real-time signal processing (e.g., live streaming) and scalable handling of large datasets.

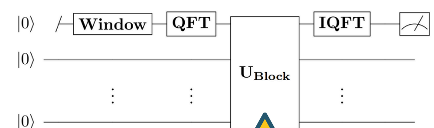


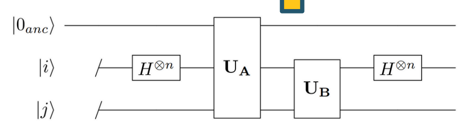
Filter as a Register

02

Quantum convolution can be performed via two filter encoding: as a register and as a gate

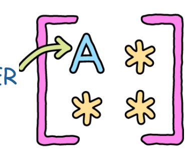
Filter as a unitary Gate





Block Encoding:

A technique for embedding a non-unitary matrix into a larger unitary matrix. We use matrix access oracle framework.



$$U_{Block} = \left[\begin{array}{cccccc|c} a_{1,1} & 0 & 0 & \cdots & 0 & 0 & \\ 0 & \ddots & 0 & \cdots & 0 & 0 & \\ 0 & 0 & a_{f_l,f_l} & \ddots & \vdots & \vdots & * \\ \vdots & \vdots & \ddots & 0 & 0 & 0 & \\ 0 & 0 & \cdots & 0 & \ddots & 0 & \\ 0 & 0 & \cdots & 0 & 0 & 0 & \\ \hline & & * & & & & * \end{array}\right]$$

The oracles' $\mathbf{U_A}$ and $\mathbf{U_B}$ operation on a quantum state is defined as follows:

$$U_A \left|0\right\rangle_{anc} \left|i\right\rangle \left|j\right\rangle = \left|A_{i,j}\right\rangle_{anc} \left|i\right\rangle \left|j\right\rangle ,$$
$$U_B \left|i\right\rangle \left|j\right\rangle = \left|j\right\rangle \left|i\right\rangle .$$

where $\left|A_{i,j}\right\rangle_{anc} \equiv A_{i,j} \left|0\right\rangle_{anc} + \sqrt{1 - \left|A_{i,j}\right|^2} \left|1\right\rangle_{anc}$

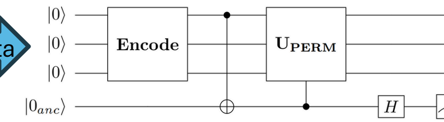
Convolution O/P

Classical Data

03

Quantum overlap-add is performed with a custom permutation gate.
Probabilistic algorithm

Quantum Overlap-Add (QOLA)



ENCODE
SPLIT
SLIDE
ADD
MEASURE

1. [a1, a2, a3, a4] [b1, b2, b3, b4]
2. [a1, a2, a3, a4, b1, b2, b3, b4]
3. [a1, a2, a3, a4, b1, b2, b3, b4 , 0, 0, 0, 0, 0, 0, 0 , 0]
4. [a1, a2, a3, a4, 0, 0, 0, 0, 0, 0, 0 , 0 b1, b2, b3, b4]
5. [a1, a2, a3, a4, 0, 0, 0, 0, 0, 0, b1, b2, b3, b4, 0 , 0]

Controlled Permutation matrix

6. [a1, a2, a3 + b1, a4 + b2, 0 + b3, 0 + b4, 0, 0, a1, a2, a3 - b1, a4 - b2, 0 - b3, 0 - b4, 0, 0]
7. [a1, a2, a3 + b1, a4 + b2, 0 + b3, 0 + b4, 0, 0]

Probability factor of 1/sqrt(2)
Post selection on 0

An arbitrary ancilla qubit controlled permutation matrix $(C - U_{PERM})$ reorganizes the coefficients in $\left|\phi_b\right\rangle$ to the desired overlap ratio configuration. We define a $2r \times 2r$ permuation matrix for l overlapping elements:

$$U_{PERM} = \begin{bmatrix} I_{r-l \times r-l} & \mathbf{0}_{r-l \times l} & \mathbf{0}_{r-l \times r} \\ \mathbf{0}_{r \times r-l} & \mathbf{0}_{r \times l} & I_{r \times r} \\ \mathbf{0}_{l \times r-l} & I_{l \times l} & \mathbf{0}_{l \times r} \end{bmatrix}_{2r \times 2r}$$