EECE 2560:
Fundamentals of Engineering Algorithms

Review (2)

Queues

- Implementations and applications of the Queue data structure

Trees

- Trees kinds, terminologies, and implementations.
- Trees traversals.
- Features and operations of the binary search trees.
- Tree sort

Heaps

- The array implementation of the heap.
- Heap operations: remove max value, add a new item, transform semiheap to maxheap, and transform an array to a heap.
- Heap sort.
- Priority queue with heap

Hash Tables

- Hash table and hash functions
- Hash collision resolution methods

Graphs

- Graphs types, terminologies, implementations, and operations.
- Graphs traversal methods and their implementation.
- Topological sorting.
- Spanning trees and their algorithms.
- Shortest paths algorithms.
- Euler and Hamilton circuits.
- The traveling-salesperson problem

Programming Problems

- Make sure to review all the programs given the slides covering the previous list of topics.

- Expect problems asking you to modify, extend, and/or explain those programs.

Binary Tree Total Nodes

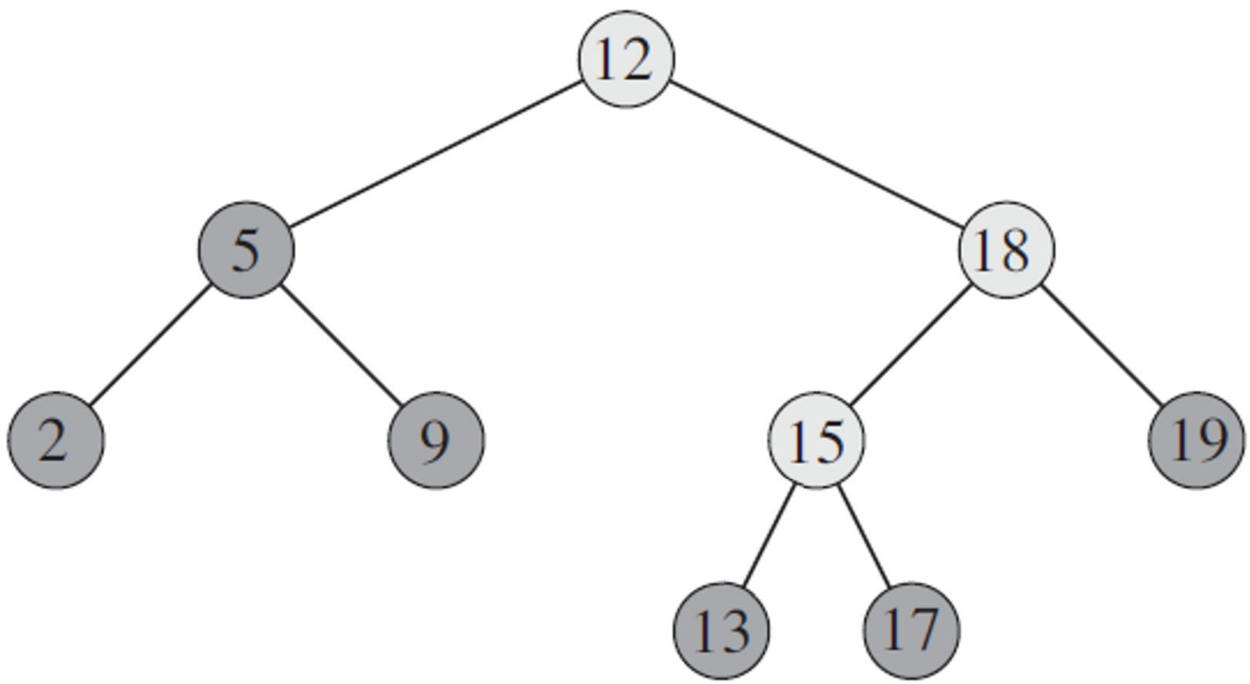
By using mathematical induction, prove that a full binary tree of height $h \geq 0$ has $2^h - 1$ nodes.

Binary Empty Subtrees

Prove by mathematical induction that a binary tree with n nodes has exactly $n + 1$ empty subtrees.

BST Insertion

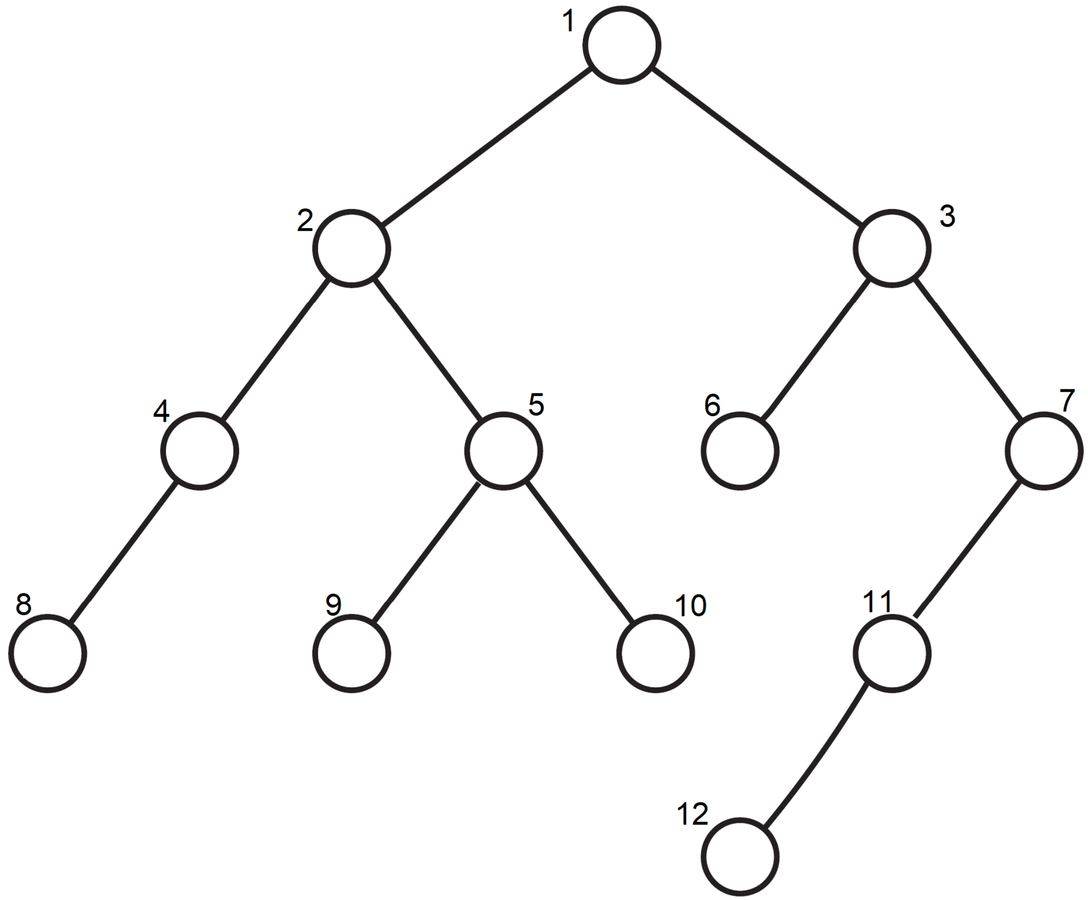
Where will node 16 be inserted in the following BST?



BST Node Removal

The numbers on the shown BST simply label the nodes so that you can reference them; they do not indicate the contents of the nodes.

Explain how can we remove node #1 from the tree.



Valid BST Sequence

- Suppose that we have numbers between 1 and 1000 in a binary search tree, and we want to search for the number 363. Which of the following sequences could not be the sequence of nodes examined?

```
a.   2, 252, 401, 398, 330, 344, 397, 363.
b. 924, 220, 911, 244, 898, 258, 362, 363.
c. 925, 202, 911, 240, 912, 245, 363.
d.   2, 399, 387, 219, 266, 382, 381, 278, 363.
e. 935, 278, 347, 621, 299, 392, 358, 363.
```

BST Recursive Min and Max

- Write recursive versions of TREE-MINIMUM and TREE-MAXIMUM

```
TREE-MINIMUM(x)
while x.left ≠ NIL
    x = x.left
return x
```

```
TREE-MAXIMUM(x)
while x.right ≠ NIL
    x = x.right
return x
```

BST with Minimum Height

Arrange nodes that contain the letters

A, C, E, F, L, V, Z

into a BST with a minimum height.

Heaps

1) What are the leaves of the heap represented by an array with values:
   `[23, 17, 14, 6, 13, 10, 1, 5, 7, 12]`?

2) Is the heap represented by the following array a max-heap?
   `[23, 17, 14, 6, 13, 10, 1, 5, 7, 12]` ?

Division Hash Function

For hashing by division method, among the following numbers, which number is the best choice for modulus *m*?

- A 31
- B 32
- C 40
- D 43

Hash with Linear Probing

Given a hash table with size m = 13, and a hash function $h(k) = k \bmod m$.

For $k = 24$, state the hash position (home slot) and the following **three** positions if linear probing is used for collision resolution.

Connected Undirected Graph

A connected undirected graph that has n vertices must have at least _______ edges

A. n

B. $n-1$

C. $n/2$

D. $2n$

Graph Cycles

A connected undirected graph that has n vertices and more than $n - 1$ edges ...

- **A** cannot contain a cycle
- **B** must contain at least one cycle
- **C** can contain at most two cycles
- **D** must contain at least two cycles

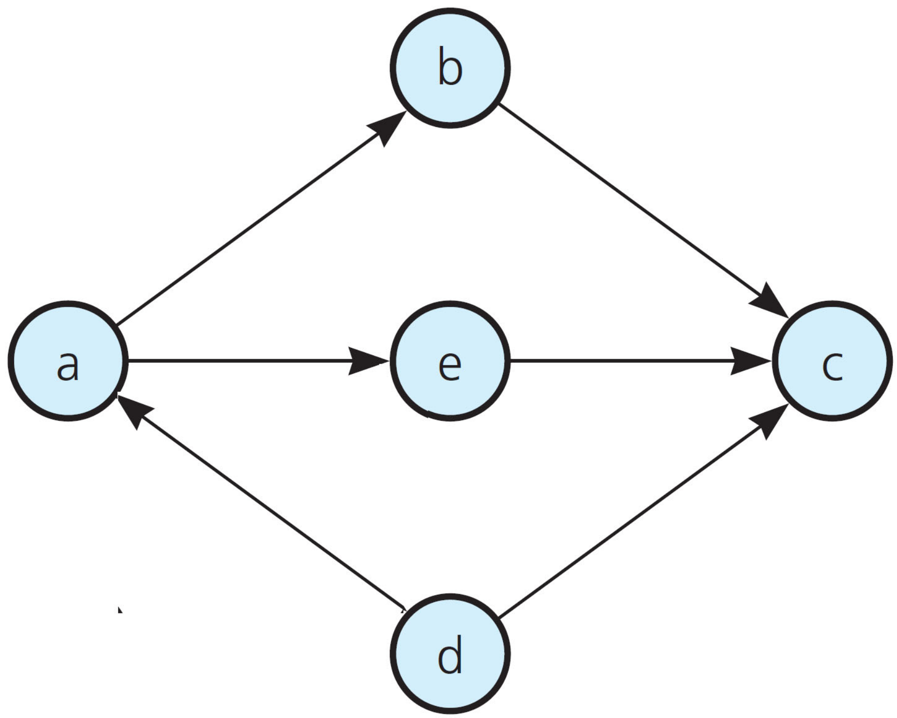
Trees Edges

A tree with n nodes must contain ______ edges

- A n
- B $n-1$
- C $n/2$
- D $2n$

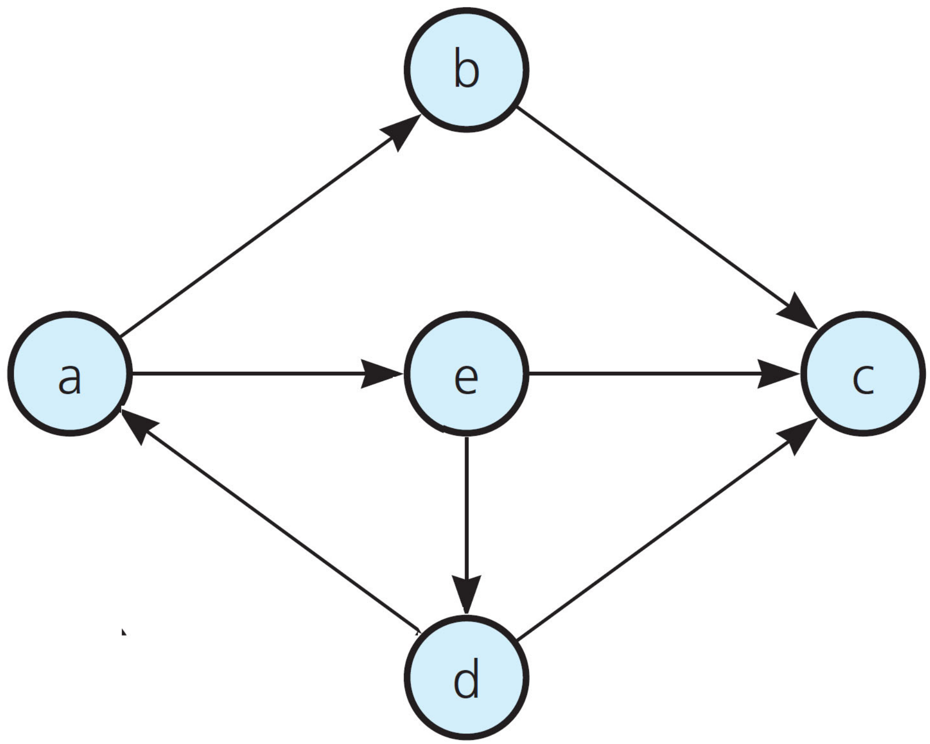
Topological Sorting (1)

Write the topological order of the vertices for the following graph.



Topological Sorting (2)

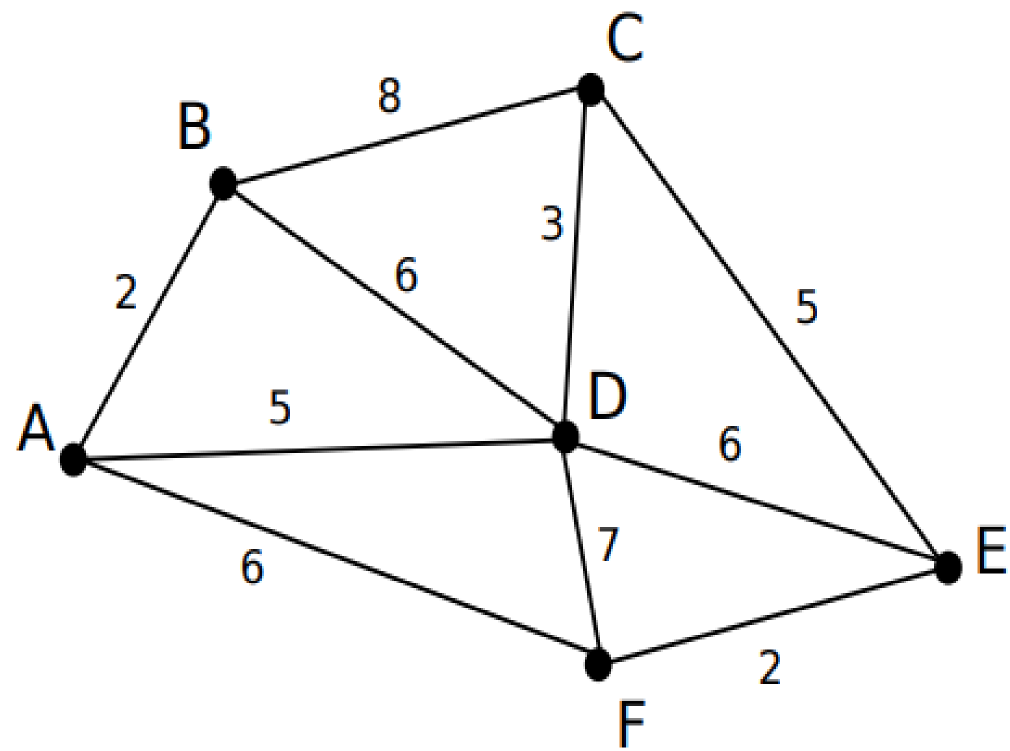
Write the topological order of the vertices for the following graph.



Minimum Spanning Trees (1)

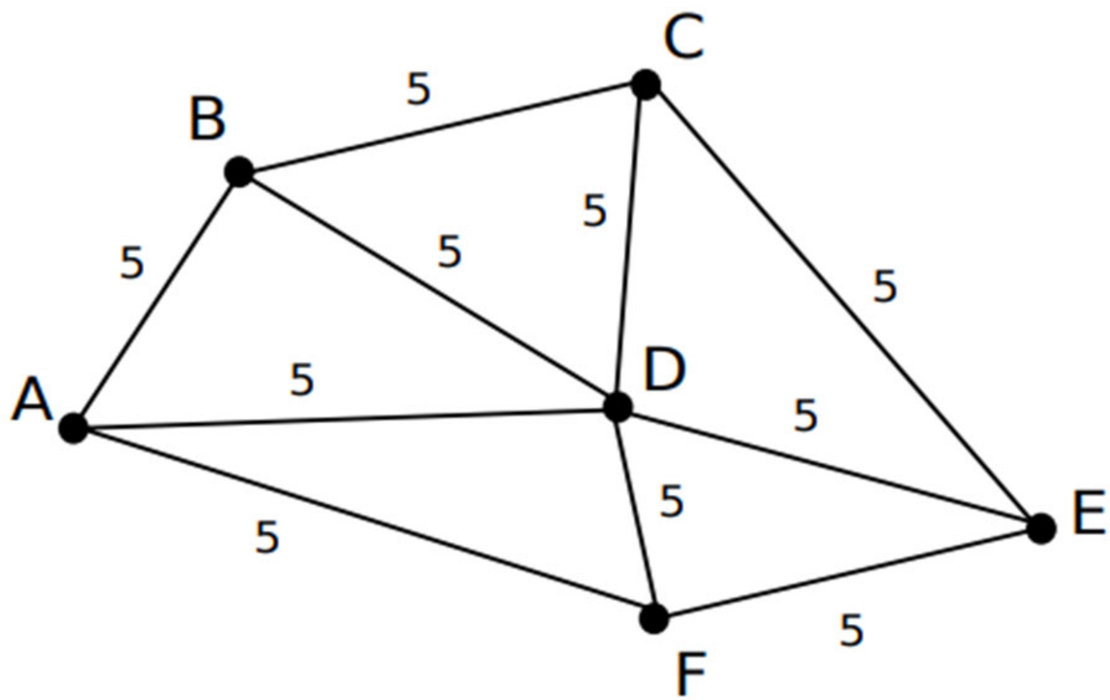
Consider the shown undirected graph. To find the MST, we started by adding edge (A, B) to our initially empty MST. Which edge will be added to the tree next?

using Kruskal's Algorithm and using Prim's Algorithm.



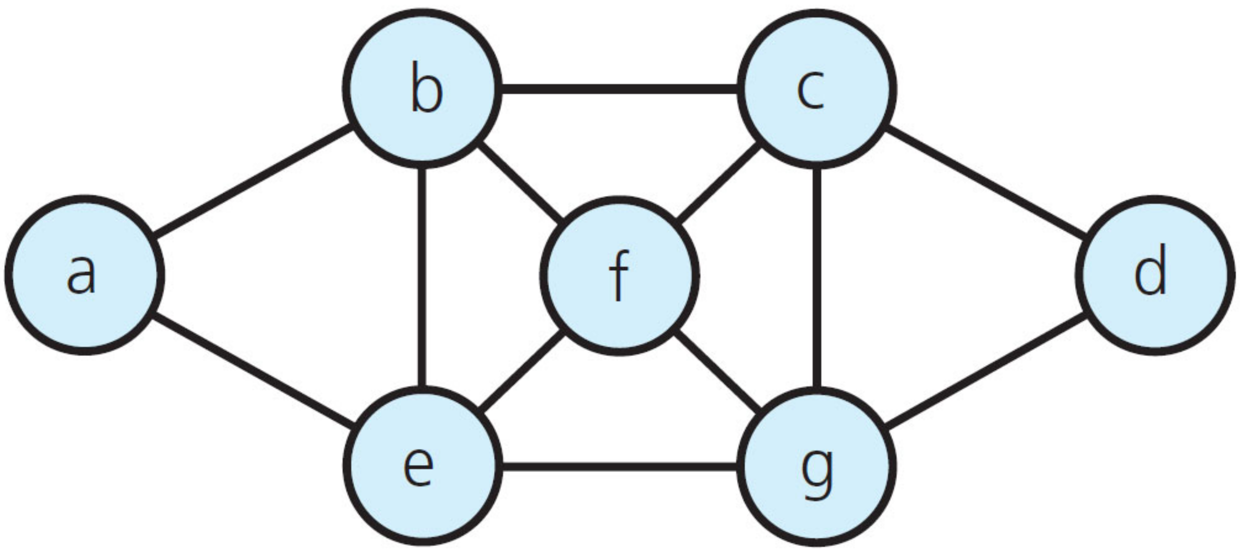
Minimum Spanning Trees (2)

Consider the shown undirected-weighted graph. To find its MST, which of the following algorithms is(are) the most efficient to use? BFS – DFS – Prim - Kruskal



Euler Circuit

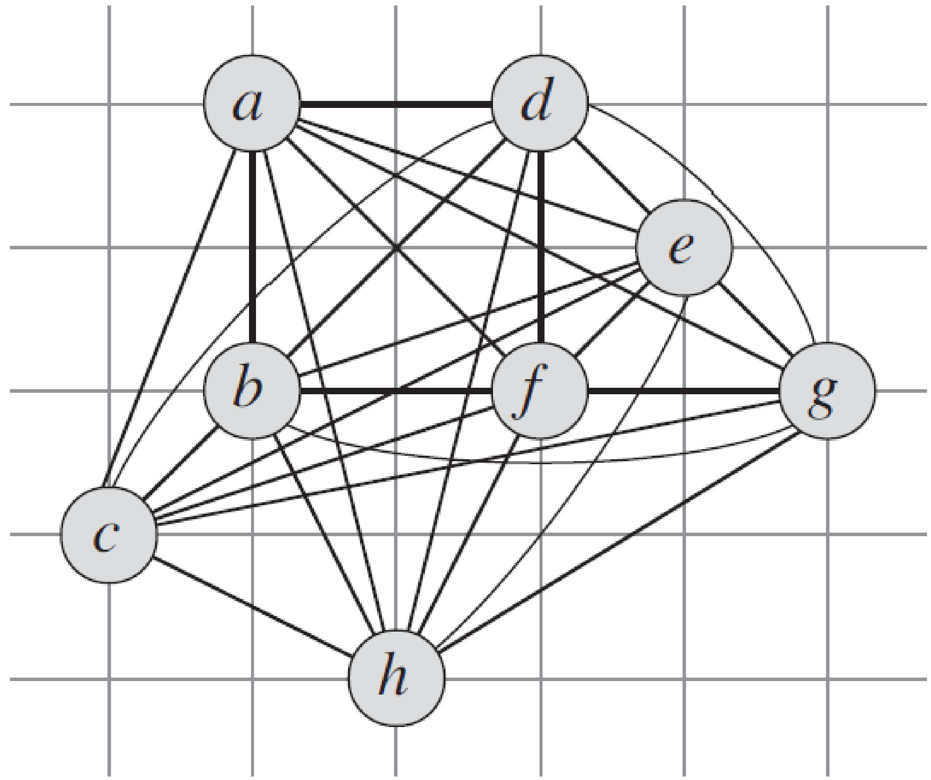
Determine an Euler circuit for the following graph.



TSP Naïve Algorithm

Describe a naïve algorithm that can solve the Traveling-Salesperson Problem starting from *a*.

What is the efficiency of your algorithm?



Graph Transpose

- The *transpose* of an unweighted directed graph $G = (V, E)$, is the graph $G^{\mathsf{T}} = (V, E^{\mathsf{T}})$, where

$$E^{\mathsf{T}} = \{(v, u) \in V \times V, (u, v) \in E\}.$$

Thus, G^{T} is G with all its edges reversed. Describe efficient in-place algorithm for computing G^{T} from G for adjacency-matrix representation of G. Analyze the running time of your algorithm.

Homework 5 – Problem 1

Assume we have a hash table consisting of *m* = 11 slots, and suppose a nonnegative integer key *k* is hashed into the table using the hash function *h1*(*k*), which is defined as follows:

```
int h1(int key) {
  int x = (key + 5) * (key + 5);
  x = x / 16;
  x = x + key;
  x = x % 11;
  return x;
}
```

The sequence of 12 integer key values listed below are to be inserted in the table, in the order given. Suppose that collisions are resolved by using linear probing. Show the probe sequence (the sequence of slots to be probed until an empty one, if any, is available) for each key. In addition, show the final contents of the hash table after all keys are inserted.

Homework 5 – Problem 2

Assume that a heap is implemented using an array called **`items`** where the root of the heap is stored in **`items[0]`**. Provide the mathematical proof of the following formulas that calculate the indices of the children and parent of any node **`items[`i`]`**.

- Left child of **`items[`i`]`** is **`items[`$2\times i + 1$`]`**
- Right child of **`items[`i`]`** is **`items[`$2\times i + 2$`]`**
- Parent of **`items[`i`]`** is **`items[`$\lfloor (i-1)/2 \rfloor$`]`**

Final Notes

This review lecture is not comprehensive as you still need to review and practice all the examples/problems provided in the lectures and the homework assignments.