

Statistical Inference
(EECE5612)
Spring 2022

Homework 7
(Assigned March 28, due April 4, 2022.)

Objective: The objective of this exercise is to experiment with angle-of-arrival estimation.

Task: A radio signal of frequency $f_0 = 3$ GHz is detected by an array of $M = 16$ elements. The array elements are equally spaced by $d = 5$ cm. The signal arriving at the array element # 0 is $s_0(t) = A \cos(2\pi f_0 t + \varphi_0)$. This signal is observed in noise as $r_0(t) = s_0(t) + n_0(t)$. The remaining elements, numbered $m = 1, \dots, M - 1$, observe the waveforms $r_m(t) = s_0(t - m\Delta\tau) + n_m(t)$. The differential delay between the array elements is $\Delta\tau = \frac{d \sin \theta}{c}$, where θ is the angle of signal arrival, and $c = 3 \cdot 10^8$ m/s. The noise processes are zero-mean white Gaussian, and uncorrelated between the array elements. The M waveforms are processed over $T_0 = 1$ ns, yielding observations $y_m = \frac{1}{T_0} \int_0^{T_0} r_m(t) e^{-j2\pi f_0 t} dt$, $m = 0, \dots, M - 1$. These observations are stored in the Matlab file `hwk7.mat`. Your task is to download the file and use the observations to tell from which direction θ is the signal arriving. Support your answer by theory, and illustrate your approach by figures. Consider also a "sanity check," in which you generate a noiseless signal arriving from a certain angle, and apply your technique to show that it indeed provides a correct estimate.

Reporting: Your report should be typed, and not exceed one single-sided page. It should be written in a professional manner. Figures and mathematical expressions should be used whenever meaningful. Figures should always have axes labeled in appropriate units (e.g. time [s], time [ms], frequency [Hz], frequency [kHz], SNR or SNR [dB], etc.). Include any Matlab code as an appendix. Please put your name on top of the report.