

**Statistical Inference**  
**(EECE5612)**  
**Spring 2022**

**Homework 1**  
**(Assigned Jan. 26, due Feb. 2, 2022 at 1pm)**

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**Objective:** The objective of this exercise is to experiment with detecting a signal in noise. Specifically, we are looking at the signal present/absent problem, zero-mean Gaussian noise, and the maximum likelihood (ML) detection principle.

**Task:**

- Review the class notes. Make sure you understand the concept of binary hypothesis testing, ML detection rule, and the notions of probability of false alarm, probability of correct detection (or missed detection), and the probability of error.
- Use Matlab (or a program of your choice) to do the following:
  - Generate a realization  $z$  of a zero-mean Gaussian random variable  $Z$  with variance  $\sigma_Z^2 = 1$ . You can do this using a built-in function `randn`.
  - Set the SNR to a desired value, e.g. 0 dB, and calculate the signal amplitude  $A$ .
  - Toss a random 0/1 coin with  $P_1 = 0.7$  ( $P_0 = 0.3$ ) and accordingly set the signal to be present or absent. If present, generate  $y = A + z$ ; if absent, generate  $y = z$ .
  - Apply the ML detection rule on the so-generated  $y$  to decide whether the signal is present or absent.
  - Repeat the process for  $N = 10,000$  realizations and count the instances of false alarm and missed detection. From the count, estimate the probability of false alarm  $P_{fa}$  and the probability of missed detection  $P_{md}$ .
  - Store the result, and repeat everything for different values of the SNR, ranging from 0 dB to 15 dB in steps of 1 dB.
  - Plot the results: Show the graph of the estimated  $P_{fa}$  vs. SNR. On the same graph, plot the theoretical expression for  $P_{fa}$  as a function of the SNR. (For best viewing, use circles for the simulation results, and solid line for the theoretical curve.)
  - Comment on the agreement: Does your simulation agree with theory? Do the same for the probability of missed detection and the probability of error.

**Reporting:** Your report should be typed, and not exceed two single-sided pages. It should be written in a professional manner. Figures and mathematical expressions should be used whenever meaningful. Figures should always have axes labeled in appropriate units (e.g. time [s], time [ms], frequency [Hz], frequency [kHz], SNR or SNR [dB], etc.). Include your Matlab code as an appendix. Please put your name on top of the report.