Statistical Inference
(EECE5612)
Spring 2022

Homework 3
(Assigned Feb. 9, due Feb 16, 2022.)

Objective: The objective of this exercise is to study the Neyman-Pearson detection principle.

Task:

- Consider the following detection problem: The observation y of a random variable Y is known to contain a signal in noise. The signal either equals A_0, or it equals $A_1 > A_0$. The noise is zero-mean Gaussian of known variance σ_Z^2.
  (i) Develop the Neyman-Pearson detection rule.
  (ii) Determine the probability of correct detection as a function of the probability of false alarm.
- Use Matlab to generate 10,000 detection trials, with A_0 occurring with probability $P_0 = 0.3$ (and A_1 occurring with probability $P_1 = 0.7$). Set the noise variance to $\sigma_Z^2 = 1$, and the signal values such that $SNR_0 = \frac{A_0^2}{\sigma_Z^2}$ is 10 dB and $SNR_1 = \frac{A_1^2}{\sigma_Z^2}$ is 15 dB.
  (i) Set the desired probability of false alarm to $P_{fa}^* = 10\%$. Count the instances of correct detection and estimate the probability of correct detection.
  (ii) Repeat for different values of P_{fa}^*, ranging from 0 to 100 % in steps of 10%. Plot the estimated probability of correct detection vs. the probability of false alarm. Plot the theoretical ROC on the same graph. Comment on the result.
  (iii) Estimate the actual probability of false alarm, and plot it against the design value P_{fa}^*. Comment on the result.

Reporting: Your report should be typed, and not exceed two single-sided pages. It should be written in a professional manner. Figures and mathematical expressions should be used whenever meaningful. Figures should always have axes labeled in appropriate units (e.g. time [s], time [ms], frequency [Hz], frequency [kHz], SNR or SNR [dB], etc.). Include any Matlab code as an appendix. Please put your name on top of the report.