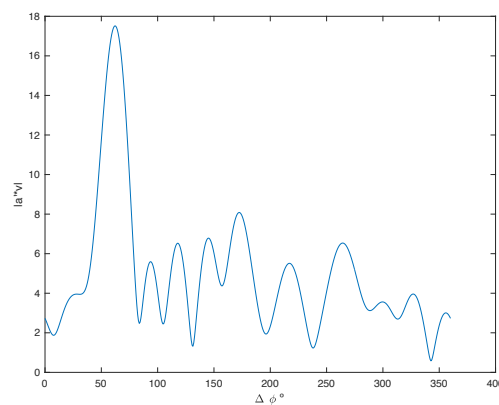
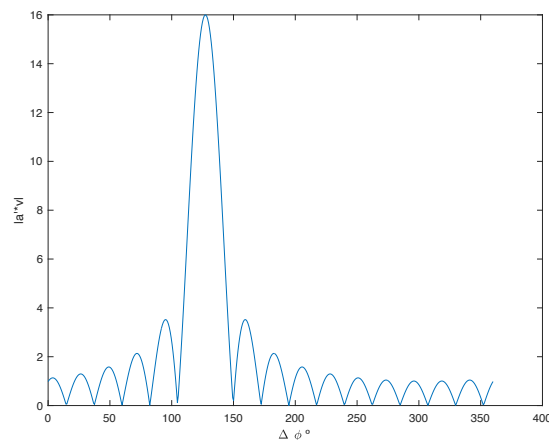


For every possible $\Delta\phi$ from 0 to 2π , the steering vector is calculated and the value for $|a' * v|$ is calculated. The $\Delta\phi$ that maximizes this function is found to be 62.04 degrees, and when plugging in values for frequency, speed of wave and the distance between array receivers, theta is found to be **20.16 degrees**. A plot of $\Delta\phi$ vs the GML function is shown below



To test this method, a noiseless signal was constructed as $e^{-jm\Delta\phi}$ for $m = 0:16$. When plotting the $\Delta\phi$ vs GML function, we see the beam pattern below. The value that maximizes the GML function is when $\Delta\phi$ is 126.97 degrees, or when theta is 44.86 degrees. This validates the method since theta was chosen to be 45 degrees.



APPENDIX

function hw7

```
clc;

% ----- Load data -----
v = load("hwk7-1.mat").Y;
m = length(v);

% ----- Find delta_phi GML -----
delta_phi = linspace(0, 2*pi, 500); % search space

GML = zeros(1,500);
for i = 1:length(delta_phi)

    % calculate steering vector
    a = exp((0:m-1)' * -1j * delta_phi(i));

    % calculate delta phi estimate
    GML(i) = abs(a'*v);
end

[~, i_max] = max(GML);
delta_phi_gml = delta_phi(i_max);

fprintf("Noise:\ndelta_phi_gml (deg): %6.2f\n", rad2deg(delta_phi_gml));

% ----- visualize GML function -----
plot(rad2deg(delta_phi), GML);
xlabel("\Delta \phi ^o");
ylabel("|a'*v|");

% ----- delta_phi -> Theta -----
f0 = 3E9;
c = 3E8;
d = 0.05;

theta_gml = asin(delta_phi_gml*c / (2*pi*f0*d));
fprintf("theta_gml (deg): %6.2f\n", rad2deg(theta_gml));

% -----
% ----- Noiseless -----
% -----

d = .05;           % distance between sensors
c = 3E8;           % wave speed
f0 = 3E9;          % 3 KHz frequency (what is this)?

theta_true = pi/4;
delta_phi_true = 2*pi*f0*d/c*sin(theta_true);

% ----- Generate observed array -----

V_t = zeros(16,1);
```

```

for m = 1:16
    V_t(m) = exp(-1j*(m-1)*delta_phi_true);
end

% ----- Find GML delta phi -----
delta_phi = linspace(0, 2*pi, 500); % search space

GML = zeros(1,500);
for i = 1:length(delta_phi)

    % calculate steering vector
    a = exp((0:15)' * -1j * delta_phi(i));

    % calculate delta phi estimate
    GML(i) = abs(a'* V_t);
end

[~, i_max] = max(GML);
delta_phi_gml = delta_phi(i_max);

fprintf("\nNoiseless:\ndelta_phi_gml (deg): %6.2f\n",
rad2deg(delta_phi_gml));

% ----- visualize GML function -----
plot(rad2deg(delta_phi), GML);
xlabel("\Delta \phi ^o");
ylabel("|a'*v|");

% ----- delta_phi -> Theta -----
f0 = 3E9;
c = 3E8;
d = 0.05;

theta_gml = asin(delta_phi_gml*c / (2*pi*f0*d));
fprintf("theta_gml (deg): %6.2f\n", rad2deg(theta_gml));

end

```