EECE 5612 HW2
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1. The random normal signal present or absent in random gaussian noise hypothesis test can be written as follows:

$$H0: y = z_0, \qquad H1: y = A + z_1, \qquad z \sim N(0, \sigma_z^2), \qquad A \sim N(0, \sigma_A^2)$$

$$=> H0: y \sim N(0, \sigma_z^2\ ), \qquad H1: y \sim N(0, \sigma_A^2 + \sigma_z^2), \qquad \sigma_A^2 = 10\sigma_z^2$$

i. The ML decision rule can be simplified as the following equation:

$$y^2 \lessgtr \frac{\sigma_z^2(\sigma_A^2 + \sigma_z^2)}{\sigma_A^2} \ln\left(\frac{\sigma_A^2 + \sigma_z^2}{\sigma_z^2}\right)$$

ii. Given that $\frac{\sigma_A^2}{\sigma_z^2} = 10\ db, \sigma_A^2 = 10$ we can find the threshold $\gamma$ and the errors:

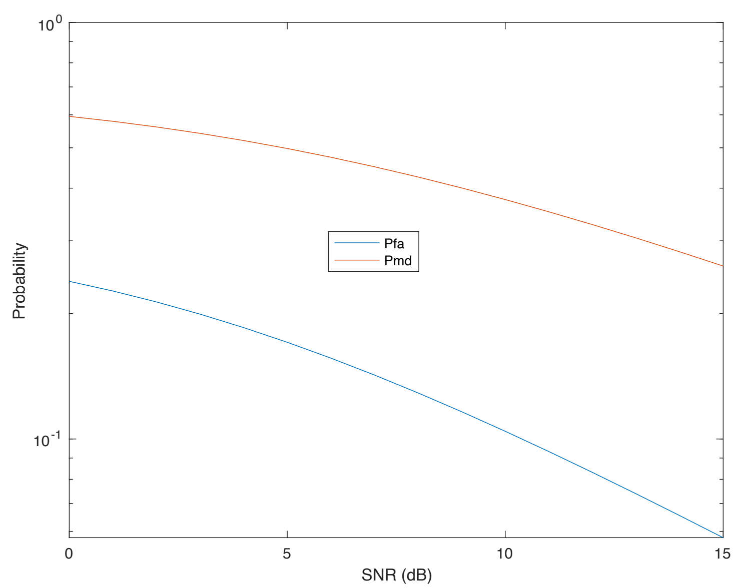
$$\gamma = \sqrt{\frac{11}{10} \ln{(11)}} = 1.624$$

$$P_{fa} = 2Q(\gamma) = 0.1044$$

$$P_{md} = 1 - 2Q\left(\frac{\gamma}{\sqrt{11}}\right) = 0.3756$$

$P_e$ cannot be determined without knowing priors.

iii. Figure 1. SNR vs $P_{fp}, P_{fn}$ on logarithmic y axis



2. The signal present or absent in random Laplacian noise hypothesis test can be written as follows:

$$H0: y = z, \qquad H1: y = A + z, \qquad z \sim Z(0, \lambda)$$

i. The ML decision rule simplifies to the following equation:

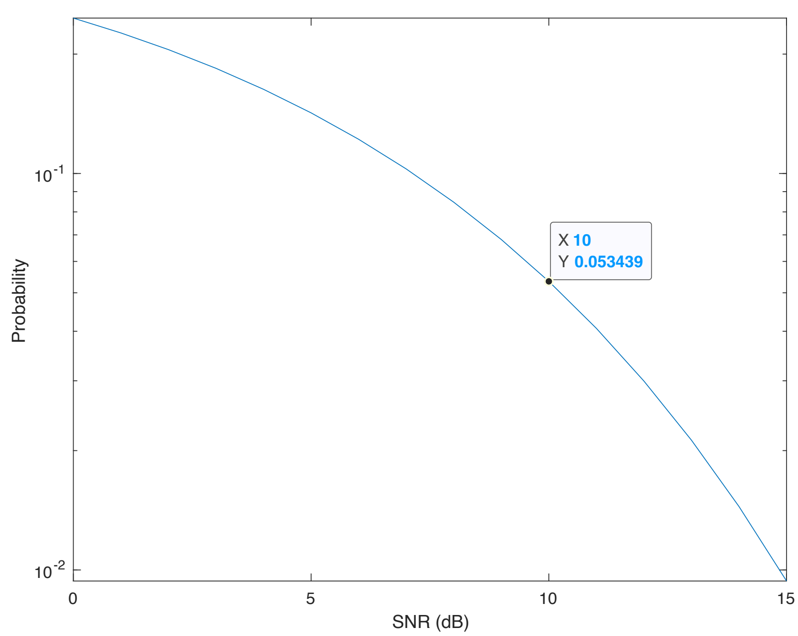
$$y \lesseqgtr \frac{A}{2}$$

ii.

$$\frac{A^2}{\sigma_z^2} = \sqrt{10^{SNR\_db/10}} \rightarrow A = \sqrt{10}, \lambda = \sqrt{2}$$

$$P_{fa} = P_{md} = P_e = \frac{1}{2}e^{-\frac{\lambda A}{2}} = \frac{1}{2}e^{-\frac{\sqrt{10}\sqrt{2}}{2}} = 0.0534$$

iii. Figure 2. SNR vs $P_{fp}, P_{fn}, P_e$




3. First, the signal amplitude is calculated from the SNR.

$$A = \sqrt{10^{SNR/10}} = 1.4125, \sigma_z = 1$$

For ML, $\gamma = \frac{A}{2}$

$$P_{fa} = P_{md} = P_e = Q\left(\frac{\sqrt{10^{0.3}}}{2}\right) = 0.24$$

For MAP, $\gamma = \frac{A}{2} + \frac{\sigma_Z^2}{A}\ln\left(\frac{0.3}{0.7}\right) = \frac{\sqrt{10^{0.3}}}{2} + \frac{\ln(0.3/0.7)}{\sqrt{10^{0.3}}} = 0.1064$

$$P_{fa} = Q(\gamma) = 0.4576$$

$$P_{md} = Q(A-\gamma) = 0.0958$$

$$P_e = (0.3)(P_{fa}) + (0.7)(P_{md}) = 0.2043$$

The computations show that for MAP, the $P_{fa}$ is higher than ML and the $P_{md}$ is lower than ML when $P_0 < P_1$, which agrees with theoretical charts.

4. $P_{fa} = Q\left(\frac{\sqrt{10^{0.3}}}{2} + \frac{\ln(0.1/0.9)}{\sqrt{10^{0.3}}}\right) = 0.8021$