Statistical Inference
(EECE5612)
Spring 2022

Homework 9
(Assigned April 11, due April 18, 2022.)

Objective: The objective of this exercise is to experiment with AR-1 process modeling and estimation.

Task: A small company comes on the market with an initial value $y(0)$. The company's growth is monitored over the first $N = 24$ months. The value of the company over the N months, $y(n), n = 0, \ldots N-1$, shows a linearly increasing trend with a random deviation around it. It is thus established that the value of the company can be modeled as $Y(n) = m(n) + \Delta Y(n)$, where $m(n) = a_0 + a_1 n, n = 0, 1, \ldots$, represents the linear trend, and $\Delta Y(n)$ the zero-mean deviation. The deviation itself is modeled as an auto-regressive process of first order (AR-1) with the one-step correlation coefficient a. In other words, the statistical model for the random deviation is $\Delta Y(n+1) = a\Delta Y(n) + Z(n)$, where $Z(n)$ is a zero-mean white noise sequence, and $Z(n)$ is uncorrelated with $\Delta Y(n)$.

The Matlab file `hwk9.mat` contains the vector $\mathbf{y}$ whose first 24 elements represent the observed values over the first 2 months. The remaining elements represent the observations made during the next 12 months. Your tasks are the following:

(a) Using the first 24 values, estimate the model parameters a_0, a_1 and a. With the estimated values $\hat{a}_0, \hat{a}_1$ and $\hat{a}$, build a model, and use it to make month-to-month predictions for the next 12 months. Plot your predictions along with the true values and comment on the agreement.

(b) Ignore the model, and make predictions as $\hat{y}(n+1) = c(n)y(n)$, determining the coefficient $c(n)$ on the fly using the least mean squares (LMS) adaptive algorithm. You can set the value of the step-size μ to be on the order of 0.01. Plot this result on top of the previous and comment on the agreement.

Reporting: Your report should be typed, and not exceed one single-sided page. It should be written in a professional manner. Figures and mathematical expressions should be used whenever meaningful. Figures should always have axes labeled in appropriate units (e.g. time [s], time [ms], frequency [Hz], frequency [kHz], SNR or SNR [dB], etc.). Include any Matlab code as an appendix. Please put your name on top of the report.