Introduction to Statistical Investigations

Second Edition

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Chapter 1.3

Significance: How Strong is the Evidence?

Review: Interpreting p-values

- P-values are conditional probabilities:

The probability of observing a statistic as extreme or more extreme than $\hat{p}$, assuming the null hypothesis is true.

- We know how to interpret probability in context:

If <random process> is repeated a very large number of times, then, in the long run, the proportion of repetitions in which <event of interest> occurs will approach <probability>.

Interpreting p-values

We can put these together.

- **Repeated random process**: the null-hypothesis version of the study activity
  - Harley guesses 10 times at random
  - 20 students choose R-P-S at random
- **Event of interest**: an observed statistic as extreme or more extreme than $\hat{p}$
  - Harley gets at least 9/10 correct
  - 2/20 or fewer students choose scissors
- **Probability**: p-value

Interpreting p-values

What does it mean to say "the p-value for the Harley study was 0.01"?

If *Harley guesses 10 times at random*, and this process is repeated a very large number of times, then, in the long run, the proportion of repetitions in which *Harley gets at least 9/10 correct* will approach *0.01*.

Now you try for "the p-value for the R-P-S study was 0.149."

Alternative Measure of Strength of Evidence

Section 1.3

Strength of Evidence

- P-values are one measure for the strength of evidence.
- Another approach is to *standardize* our observed statistic.
- *Standardizing* gives a quick, informal way to evaluate strength of evidence.

Describing a Distribution

- Some aspects to look for in a distribution:
  - ***Shape***
  - ***Center***
  - ***Variability*** (how spread out is the data?)
  - ***Unusual observations***

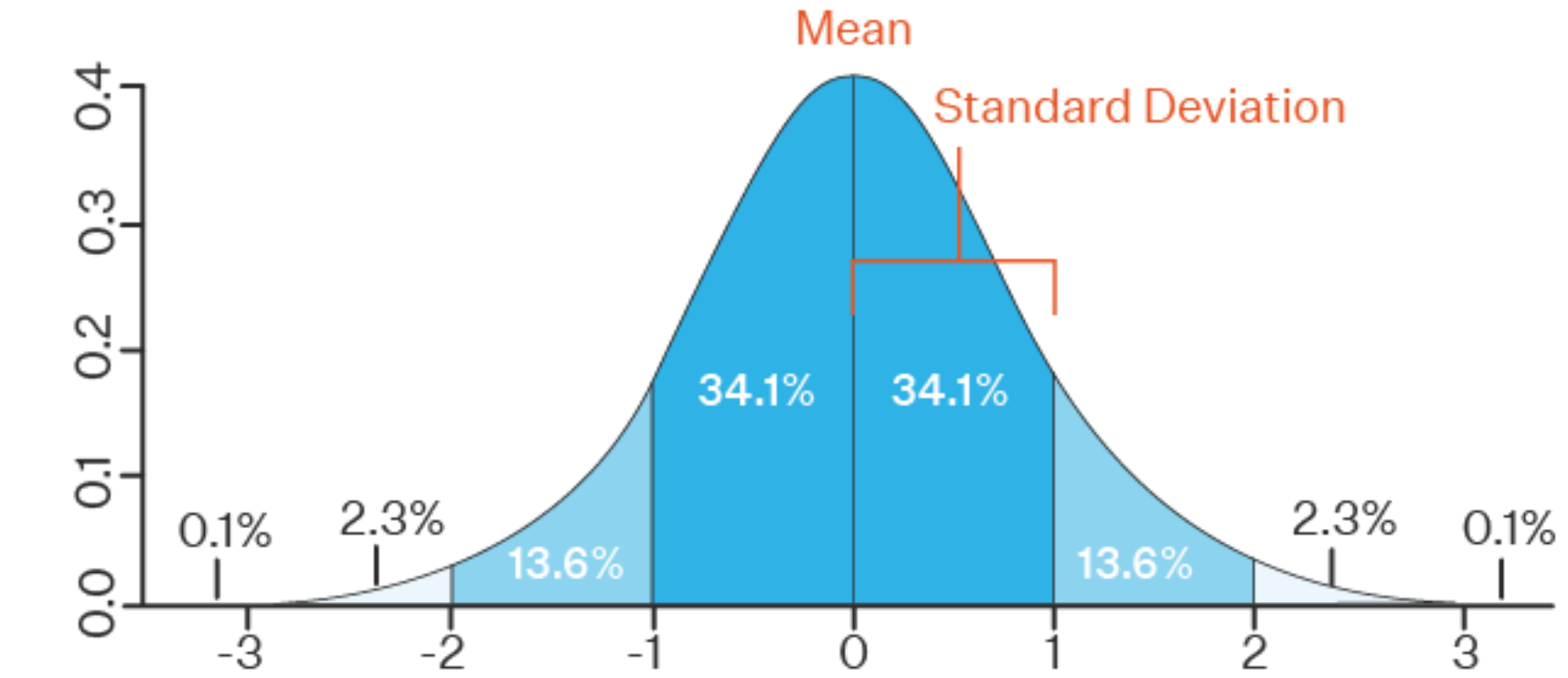
Describing a Distribution

- For our null distribution:
  - ***Shape:*** bell-curve (“normal”)
  - ***Center:*** mean (average)
  - ***Variability:*** standard deviation (SD)
  - ***Unusual observations:*** none

Standard Deviation

- We can think of the standard deviation as the *distance* a typical value in the distribution is away from the mean of the distribution.

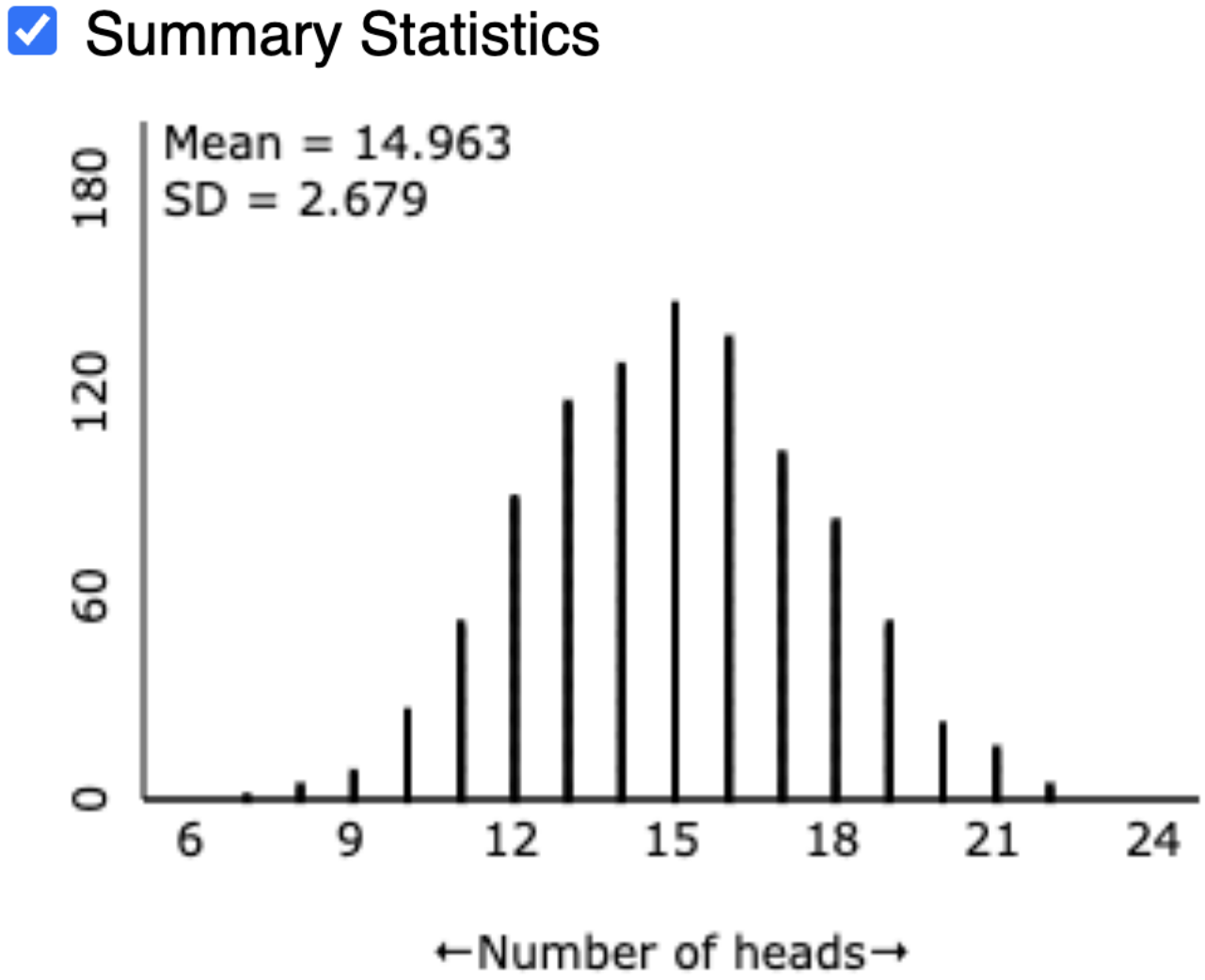
What's a summary statistic?



Describing a Distribution

- There are two main ways to find the mean and SD of a null distribution:
  - Have the applet compute them. (Click "Summary Stats" box in **One Proportion** applet.)
  - Use an appropriate theory-based approach (Section 1.5).

Describing a Distribution



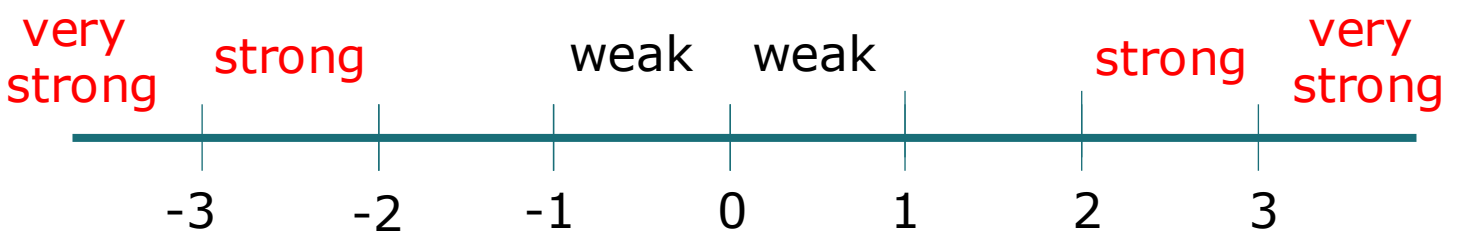
The Standardized Statistic

- The ***standardized statistic*** (denoted ***z***) is the number of standard deviations an observed statistic falls above or below the mean of a null distribution.
- Measure how far the observed value is from the mean of the distribution, using standard deviation units.

$$z = \frac{(\text{observed statistic}) - (\text{mean of null distribution})}{(\text{SD of null distribution})}$$

The Standardized Statistic

Standardized Statistic	Evidence Against Null
between −1.5 and 1.5	not much
below −1.5 or above 1.5	moderate
below −2 or above 2	strong
below −3 or above 3	very strong



Strength of evidence against H_0 for various values of z