Einstein's Reality

Space and Time in Modern Physics

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Outline

1. Invariance of c
2. Relativistic Kinematics
3. Spacetime Diagrams
4. Interesting Examples

Acoustic Doppler Effect

- We hear different frequencies depending on our relative velocity with a sound source

Galilean Invariance

- When you run after things, you catch up
- Physically intuitive, unquestioned for centuries

$$x \to x - vt$$

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- Solution: air provides a preferred frame of reference

Luminiferous Ether

- Is there a preferred frame for the entire universe?
- In principle: measure different Doppler shifts
- In practice: Michelson & Morley, 1887

Michelson-Morley Experiment

Postulates of Relativity

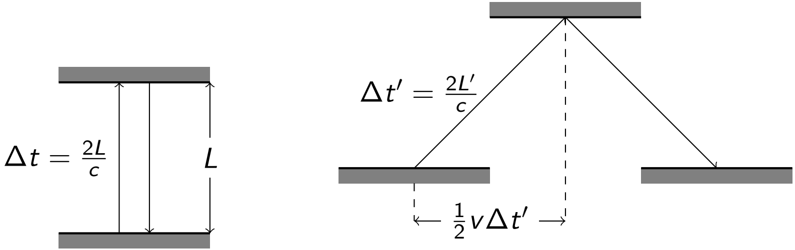
- All inertial frames of reference are equivalent.
- The speed of light is the same for all observers.
- Result: all of special relativity

Lorentz Transformation

- The Galilean transformation allows catching up to light
- We need a new way to change coordinates between observers
- This will incorporate three kinematic effects
    1. Time dilation
    2. Length contraction
    3. Relativity of simultaneity

Time Dilation

- How do we measure time in a relativistic setting?



(a) (b)

Figure: A moving light-clock will report a dilated time.

Time Dilation

- How do we measure time in a relativistic setting?
- Light is the best way to measure anything (theoretially)

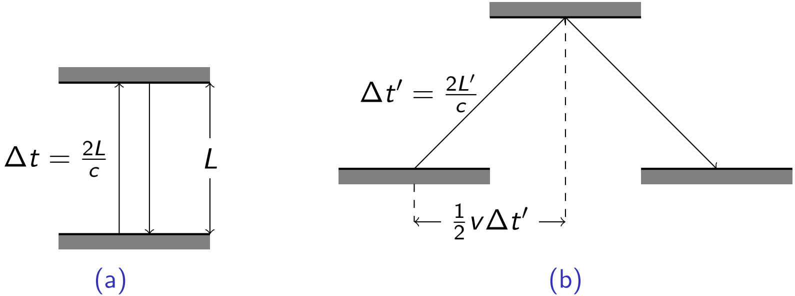


Figure: A moving light-clock will report a dilated time.

How much is time dilated?

- Distance = rate×time. Using the diagram from before:

$$2\sqrt{\left(\frac{1}{2}v\Delta t'\right)^2 + L^2} = c \cdot \Delta t'$$

- Square both sides:

$$\left(v\Delta t'\right)^2 + 4L^2 = \left(c\Delta t'\right)^2$$

- Isolate $\Delta t'$:

$$\Delta t'^2 = \frac{4L^2}{c^2 - v^2}$$

- Square root:

$$\Delta t' = \frac{2L/c}{\sqrt{1 - v^2/c^2}} = \frac{\Delta t}{\sqrt{1 - v^2/c^2}}.$$

Length Contraction

- We observe particles lasting longer and reaching Earth's surface
- Particles must observe Earth's atmosphere contracted by same factor

$$\Delta L' = \Delta L\sqrt{1 - v^2/c^2}$$

- Use light to check if events are simultaneous

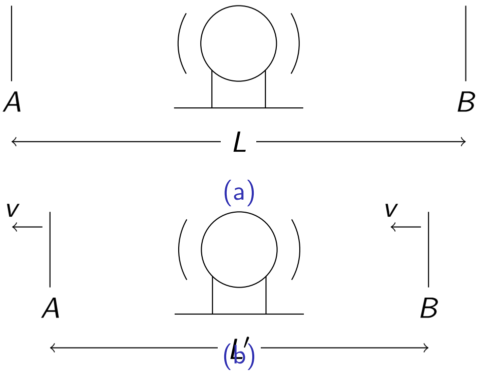
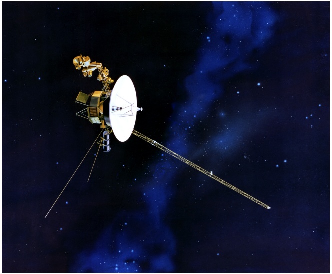


Figure: A synchronization device is only effective in its own rest frame.

Voyager I

- $v = 17\,\mathrm{km/s}$, $c = 3 \times 10^5\,\mathrm{km/s}$
- Length contraction and time dilation: one part in a billion
- Simultaneity difference: $3.7\,\mathrm{s}$



Voyager Golden Record

Spacetime

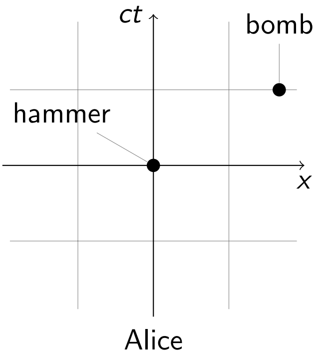
- Normally we think about the 3 dimensions of space
- In relativity, time and space are connected by the Lorentz transformation
- We move from Euclidean geometery in 3 dimensions to Minkowskian geometry in 3+1 dimensions

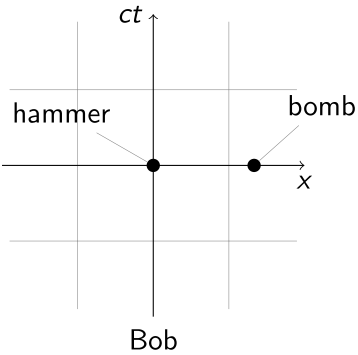
Symmetry

- Pre-relativity:
  - Allowed transformations are rotations and velocity boosts
  - Distance $(\Delta x^2 + \Delta y^2 + \Delta z^2)$ is preserved
- Relativity
  - Allowed transformations are Lorentz transformations (rotations and modified velocity boosts)
  - Spacetime interval $(\Delta x^2 + \Delta y^2 + \Delta z^2) - c^2 \Delta t^2$ is preserved

Plotting Spacetime

- If we choose an observer, we can plot events in terms of the space and time coordinates she observes





Lorentz Transformations

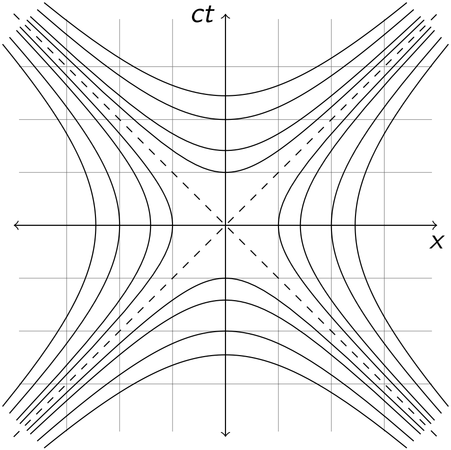


Figure: A point moves along a branch of a hyperbola when a Lorentz transformation is carried out.

Past, Present, and Future

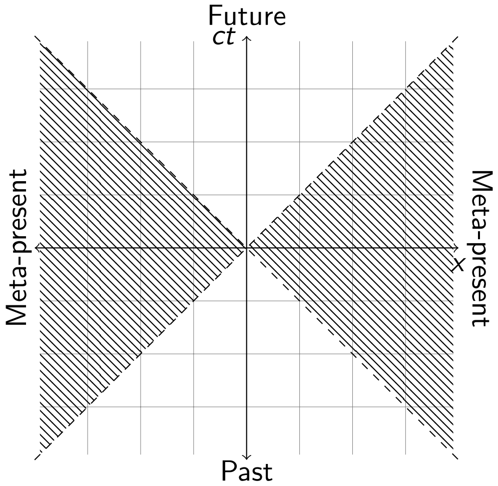
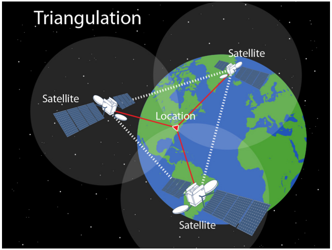


Figure: The future includes events which can be caused by events at the origin, and the past includes events which can cause events at the origin.

GPS Timing

- GPS satellites need distance estimates to triangulate position
- $r = ct$, but what is t?
- GPS onboard computers adjust for special relativity

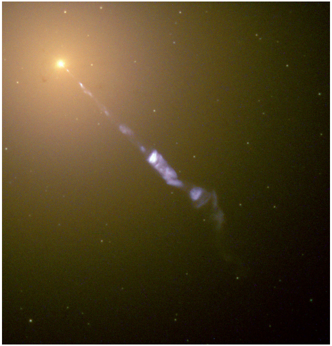


Relativistic Space Travel

- We see things as they were, not as they are
- Example: length contraction appears as rotation
- Problems with rigid bodies
- Doppler shift along spacecraft

Relativistic Beaming

- Direction of motion affects perceived brightness of light
- Most intensity emitted in direction of motion



Dodging a Phone Call

- Impossible to go faster than light
- Barely possible to stay ahead of light

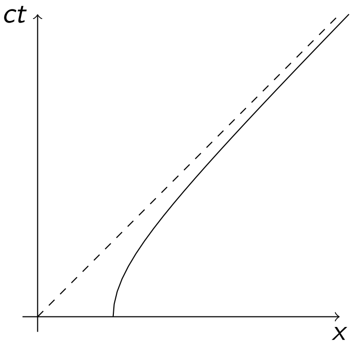


Figure: With constant proper acceleration, Bob can maintain an asymptotically small lead on a beam of light.