Equivalent Capacitance

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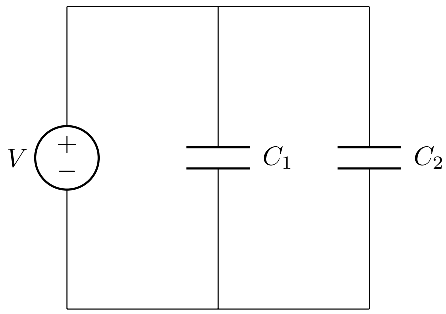
1 Parallel Capacitors

Consider the following general example of two capacitors linked in parallel:



We know that the charge developed on each capacitor is in proportion to the capacitance:

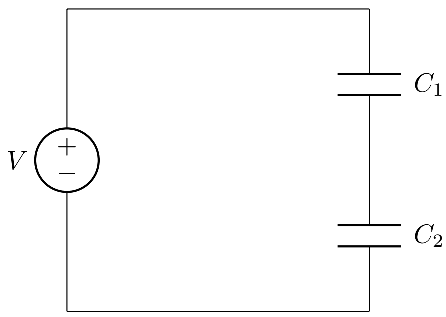
$$Q_1 = C_1 V$$
$$Q_2 = C_2 V$$

The equivalent capacitance is then simply the total charge divided by the voltage difference:

$$C_{eq} = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = C_1 + C_2$$

2 Series Capacitors

Similarly, consider the following example of two capacitors linked in series:



The charge on both capacitors is equal, because no charge can leave the wire between the two capacitors (and so the negative charge on the first capacitor must be equal in magnitude to the positive charge on the second capacitor). Thus, the voltage drop is inversely proportional to the capacitance in each:

$$V_1 = \frac{Q}{C_1}$$
$$V_2 = \frac{Q}{C_2}$$

The sum of these voltages must be equal to the total potential difference. From there, we derive the equivalent capacitance:

$$V = \frac{Q}{C_1} + \frac{Q}{C_2}$$
$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$