Cruel and Unusual DC Circuits: Solutions

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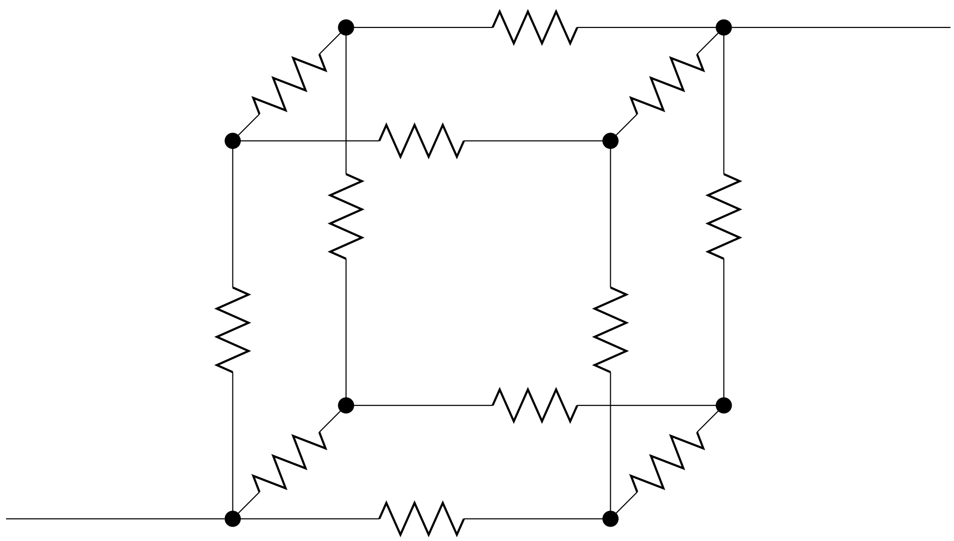
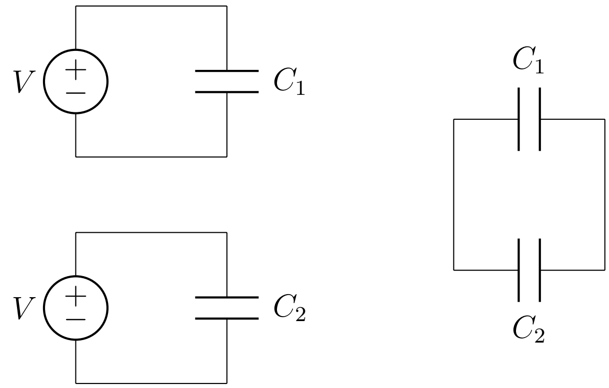
1. Find the equivalent resistance of the following resistor network, given that each resistor has a resistance R.



Look at the network beginning at the starting point. At first, the current splits evenly across 3 resistors. Then, irrespective of which path the current followed, it will have to split yet again into two equal parts (verify that this occurs along $3 \cdot 2 = 6$ distinct resistors). Once the current has followed one of these paths, it will follow one more resistor to the final node. So, we can segment this whole path into three parts. First, three resistors act essentially in parallel (even though they arrive at different nodes, they arrive at entirely equivalent nodes, so we can collapse them). This gives a resistance of $\frac{R}{3}$. Then, it flows along one of 6 resistors, which again act essentially in parallel. This adds a resistance of $\frac{R}{6}$. Finally, it flows along one of three resistors, which add a resistance of $\frac{R}{3}$. The total is $\frac{5R}{6}$.

2. Two capacitors, with capacitances C_1 and C_2, are charged separately by a voltage source V. They are then connected in series with no voltage source. Find the ratio of the energies stored originally to the energy stored once the capacitors are connected.



The initial energy is trivial: $\frac{1}{2}(C_1 + C_2)V^2$. To find the final energy, we first have to solve for the charge that ends up on each capacitor after they reach equilibrium. To do this, we write equations for the conservation of charge (letting Q_1 be the charge on the left of C_1 and Q_2 be the charge on the right of C_2) and for Kirchoff's voltage law:

$$Q_1 + Q_2 = (C_1 - C_2)V$$
$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

Solving for Q_1 and Q_2 yields:

$$Q_1 = \frac{C_1 - C_2}{C_1 + C_2} C_1 V$$
$$Q_1 = \frac{C_1 - C_2}{C_1 + C_2} C_2 V$$

So the final energy is:

$$\frac{1}{2}\left(\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2}\right) = \frac{1}{2}(C_1 + C_2)V^2 \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$$

The ratio is then:

$$\left(\frac{C_1 + C_2}{C_1 - C_2}\right)^2$$

3. Find currents everywhere.