TEST CURS 2 PS

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$$B_a = \{ \underset{e_0}{1}, \underset{e_{s1}}{\sin t}, \underset{e_{c1}}{\cos t}, \dots \underset{e_{sn}}{\sin nt}, \underset{e_{cn}}{\cos nt}, \dots \}$$

1) $\|e_0\|^2 = 2\pi$

$e_0 = 1$

$$\|e_0\|^2 = \|1\|^2 = \int_{-\pi}^{\pi} 1\, dt = t \Big|_{-\pi}^{\pi} = \pi + \pi = 2\pi$$

$\Rightarrow \|e_0\|^2 = 2\pi$

2) $\langle e_0, e_{sn} \rangle = 0$

$$\langle e_0, e_{sn} \rangle = \int_{-\pi}^{\pi} e_0 \cdot \overline{e_{sn}}\, dt = \int_{-\pi}^{\pi} 1 \cdot (-\sin nt)\, dt = \int_{-\pi}^{\pi} -\sin(nt)\, dt$$

$$= \frac{\cos(nt)}{n} \Big|_{-\pi}^{\pi} = 0.$$

$\Rightarrow \langle e_0, e_{sn} \rangle = 0$

3) $\langle e_0, e_{cn} \rangle = 0$

$$\langle e_0, e_{cn} \rangle = \int_{-\pi}^{\pi} e_0 \cdot \overline{e_{cn}}\, dt = \int_{-\pi}^{\pi} 1 \cdot (-\cos nt)\, dt = \int_{-\pi}^{\pi} -\cos(nt)\, dt$$

$$= -\frac{\sin(nt)}{n} \Big|_{-\pi}^{\pi} = -\frac{2\sin(\pi n)}{n} \left. \begin{matrix} \\ n \in \mathbb{N}^* \end{matrix} \right\} \Rightarrow \langle e_0, e_{cn} \rangle = 0$$

4) $\langle e_{sn}, e_{sm} \rangle = \pi\, \delta_0[n-m]$

$$\langle e_{sn}, e_{sm} \rangle = \int_{-\pi}^{\pi} e_{sn} \cdot \overline{e_{sm}}\, dt = \int_{-\pi}^{\pi} \sin(nt)(-\sin(mt))\, dt =$$

$$= -\int_{-\pi}^{\pi} \sin(nt) \cdot \sin(mt)\, dt =$$

$$\sin x \cdot \sin y = \frac{1}{2}(\cos(y-x) - \cos(y+x))$$

$$= \int_{-\pi}^{\pi} -\frac{\cos((n+m)t) - \cos((n-m)t)}{2}\, dt =$$

$$= \underbrace{\frac{1}{2}\int_{-\pi}^{\pi} \cos((n-m)t)\, dt}_{X} - \underbrace{\frac{1}{2}\int_{-\pi}^{\pi} \cos((n+m)t)\, dt}_{Y}$$

$$x = \frac{1}{2}\int_{-\pi}^{\pi} \cos((n-m)t)\,dt$$

$$u = (n-m)t \Rightarrow \frac{du}{dt} = n-m \Rightarrow dt = \frac{1}{n-m}\,du$$

$$\Rightarrow x = \frac{1}{2(n-m)}\int_{-\pi}^{\pi} \cos u\,du = \frac{\sin u}{2(n-m)}$$

$$\Rightarrow x = \frac{\sin((n-m)t)}{2(n-m)}\Bigg|_{-\pi}^{\pi} = \frac{\sin((n-m)\pi)}{n-m}$$

$$n, m \in \mathbb{N}^*$$

$$Y = \frac{1}{2}\int_{-\pi}^{\pi} \cos((n+m)t)\,dt$$

$$u = (n+m)t \Rightarrow \frac{du}{dt} = n+m \Rightarrow dt - \frac{1}{n+m}\,du$$

$$\Rightarrow Y = \frac{1}{2(n+m)}\int \cos u\,du = \frac{\sin u}{2(n+m)}$$

$$\Rightarrow Y = \frac{\sin((n+m)t)}{2(n-m)}\Bigg|_{-\pi}^{\pi} = \frac{\sin((n+m)\pi)}{n+m}$$

$$\Rightarrow \langle e_{sn}, e_{sm}\rangle = \frac{\sin((n-m)\pi)}{n-m} - \frac{\sin((n+m)\pi)}{n+m} = \pi\delta_0[n-m]$$

$$= \cancel{\frac{\sin((n-m)\pi)}{m}} - \cancel{\frac{\sin(n-m)\pi}{m}} - \cancel{\sin(n+m)}$$

b) $\langle e_{cn}, e_{cm}\rangle = \pi\delta_0[n-m]$

$$\langle e_{cn}, e_{cm}\rangle = \int_{-\pi}^{\pi} e_{cn}\cdot\overline{e_{cm}}\,dt - \int_{-\pi}^{\pi}\cos nt\cdot(-\cos mt)\,dt =$$

$$= -\int_{-\pi}^{\pi}\cos nt\cos mt\,dt =$$

$$\cos x\cos y = \frac{1}{2}(\cos(y+x) + \cos(y-x))$$

$$= -\int_{-\pi}^{\pi}\frac{\cos((n+m)t) + \cos((n-m)t)}{2}\,dt =$$

$$= \frac{1}{2}\int_{-\pi}^{\pi}\cos((n+m)t)\,dt + \frac{1}{2}\int\cos((n-m)t)\,dt$$

Se observă că această integrală este egală cu cea de la 4) $\Rightarrow$

$$\Rightarrow \quad \langle e_{sn}, e_{sm} \rangle = \langle e_{cn}, e_{cm} \rangle = \pi \delta_0[n-m]$$

6) $\langle e_{sn}, e_{cm} \rangle = 0$

$$\langle e_{sn}, e_{cm} \rangle = \int_{-\pi}^{\pi} e_{sn} \cdot \overline{e_{cm}}\, dt = \int_{-\pi}^{\pi} \sin nt\,(-\cos mt)\, dt =$$

$$= -\int_{-\pi}^{\pi} \sin nt \cos mt\, dt =$$

$$\sin x \cos y = \frac{1}{2}\left(\sin(y+x) - \sin(y-x)\right)$$

$$= \int_{-\pi}^{\pi} \frac{\sin((n+m)t) + \sin((n-m)t)}{2}\, dt =$$

$$= \underbrace{\frac{1}{2}\int_{-\pi}^{\pi} \sin((n+m)t)\, dt}_{X} + \underbrace{\frac{1}{2}\int_{-\pi}^{\pi} \sin((n-m)t)\, dt}_{Y}$$

$$X = \frac{1}{2}\int_{-\pi}^{\pi} \sin(n+m)t\, dt$$

$$u = (n+m)t \Rightarrow \frac{du}{dt} = n+m \Rightarrow dt = \frac{1}{n+m}\, du$$

$$\Rightarrow X = \frac{1}{2(n+m)} \int \sin u\, du = \frac{-\cos u}{2(n+m)}$$

$$\left.\begin{aligned} &\Rightarrow X = -\frac{\cos(n+m)t}{2(n+m)}\Big|_{-\pi}^{\pi} = -\frac{\cos(n+m)\pi}{2(n+m)} - \frac{\cos(n+m)\pi}{2(n+m)} \\ &\qquad n, m \in \mathbb{N}^* \end{aligned}\right\} \Rightarrow X = 0$$

$$Y = \frac{1}{2}\int_{-\pi}^{\pi} \sin(n-m)t\, dt$$

$$u = (n-m)t \Rightarrow \frac{du}{dt} = n-m \Rightarrow dt = \frac{1}{n-m}\, du$$

$$\Rightarrow Y = \frac{1}{2(n-m)} \int \sin u\, du = \frac{-\cos u}{2(n-m)}$$

$$\left.\begin{aligned} &\Rightarrow Y = -\frac{\cos(n-m)t}{2(n-m)}\Big|_{-\pi}^{\pi} = -\frac{\cancel{2}\cos(n-m)\pi}{\cancel{2}(n-m)} = -\frac{\cos(n-m)\pi}{(n-m)} \\ &\qquad n, m \in \mathbb{N}^* \end{aligned}\right\}$$

$$\Rightarrow Y = 0$$

$$\Rightarrow \langle e_{sn}, e_{cm} \rangle = 0$$

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