Computational Cognitive Modeling

Probabilistic graphical models

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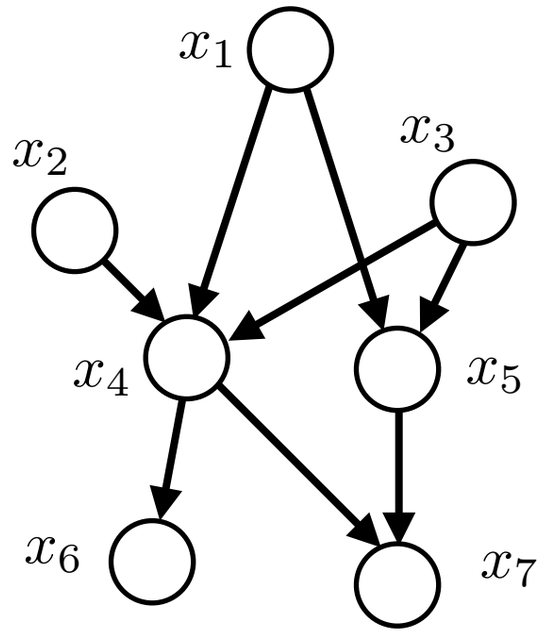
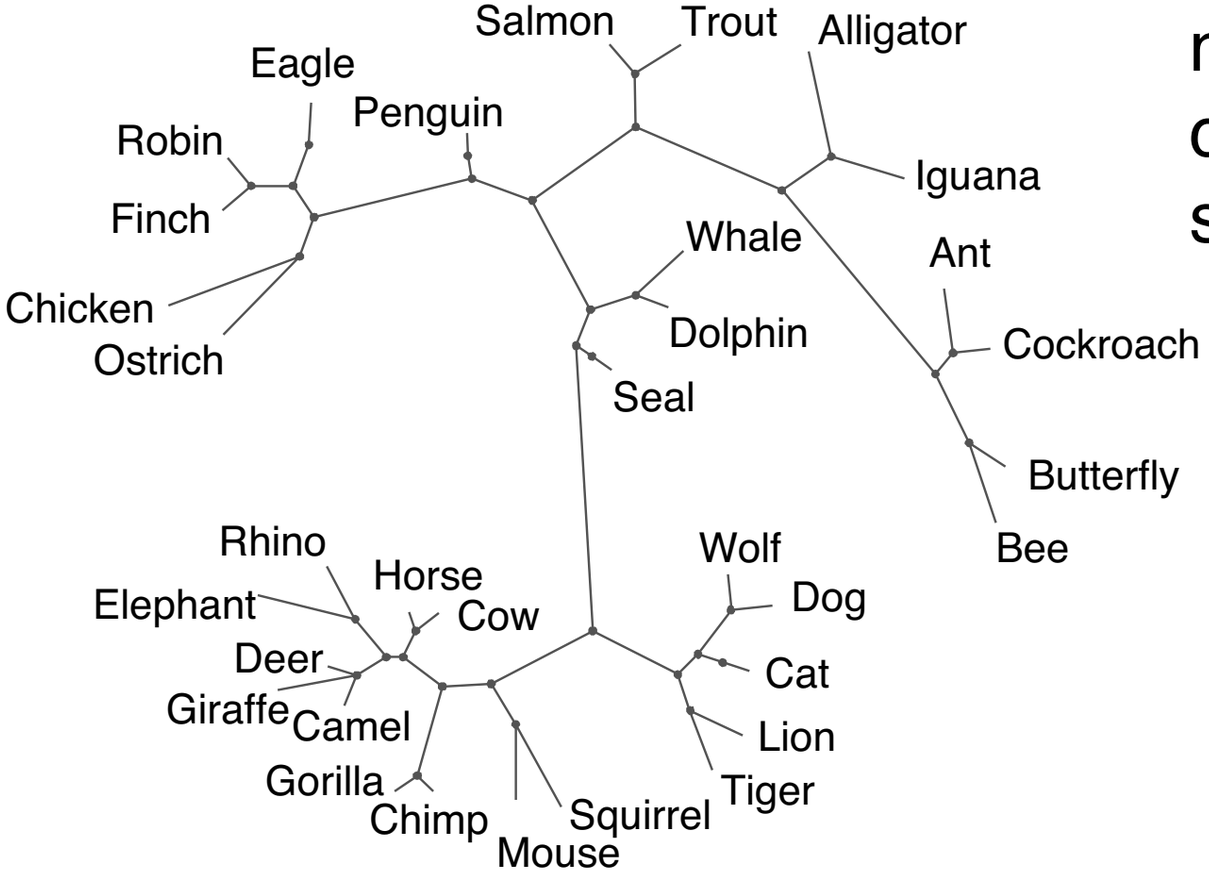
Probabilistic graphical models

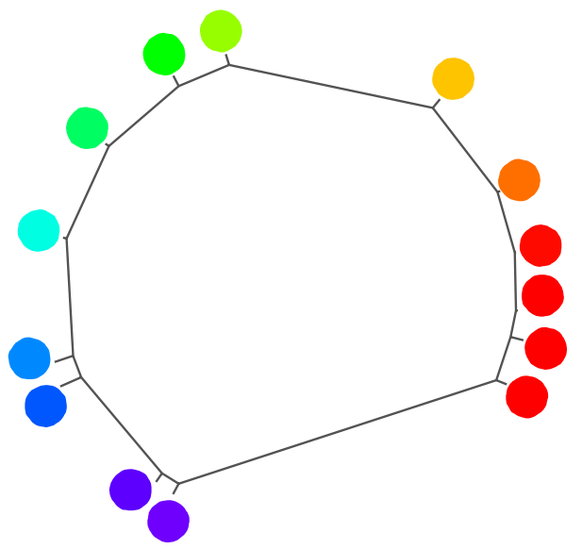




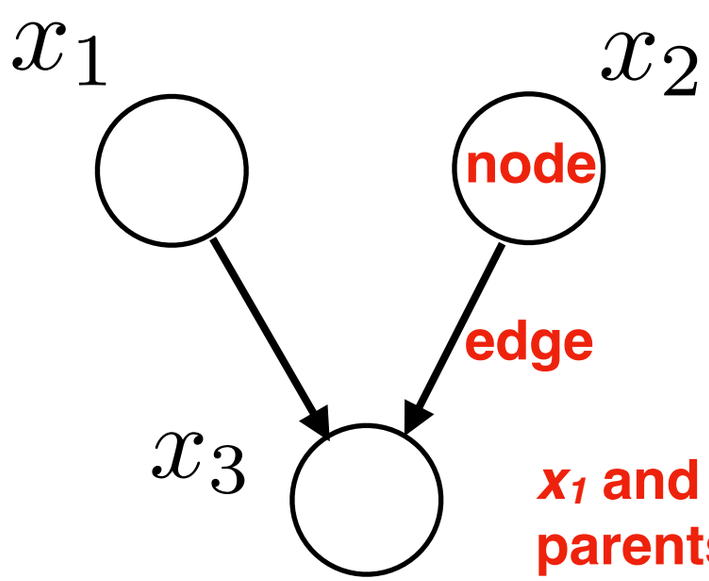
In this section, we will cover:

- The basic technical concepts behind probabilistic graphical models and how to work with them.

- Applications in computational cognitive modeling, including problems in classification, causal learning, and structure discovery.



Bayesian networks ("Bayes net")



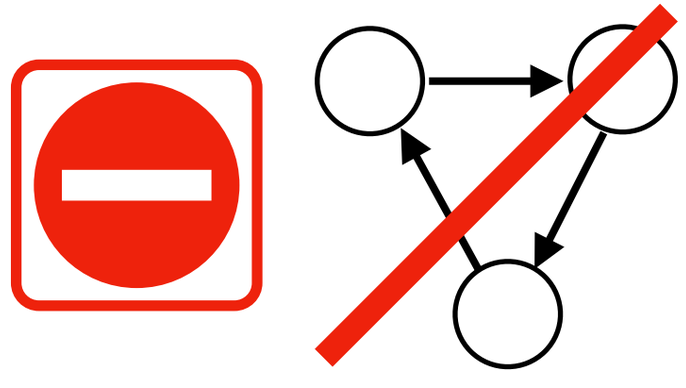
Factorization of the joint distribution:

$$P(x_1, x_2, x_3) = P(x_1)P(x_2)P(x_3|x_1, x_2)$$

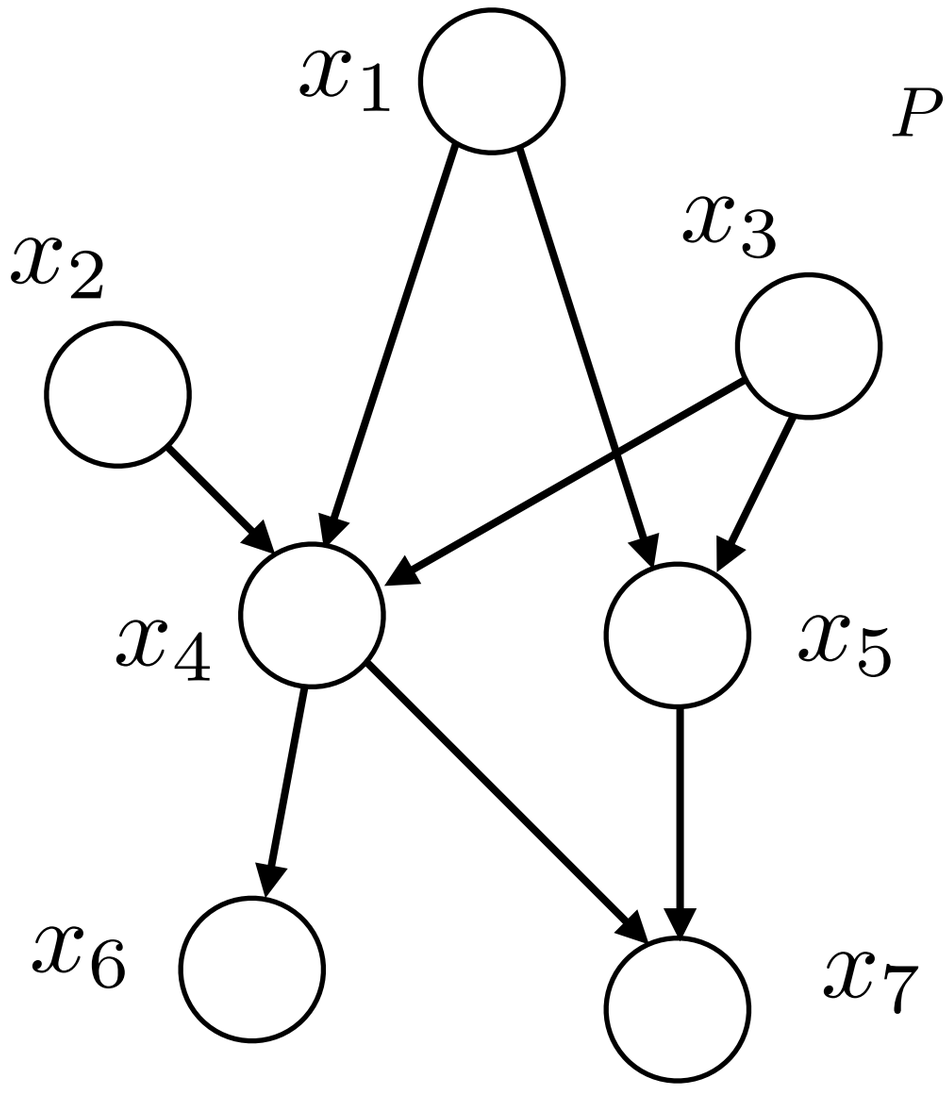
- *Bayesian network*: a directed graph that represents dependencies between random variables, giving a concise specification of a joint probability distribution.

- In a well-constructed network, an arrow indicates that two variables have a path of direct (causal) influence.

- Bayesian networks must be *directed, acyclic graphs (DAGs)*, meaning that they have no cycles.



Bayesian networks



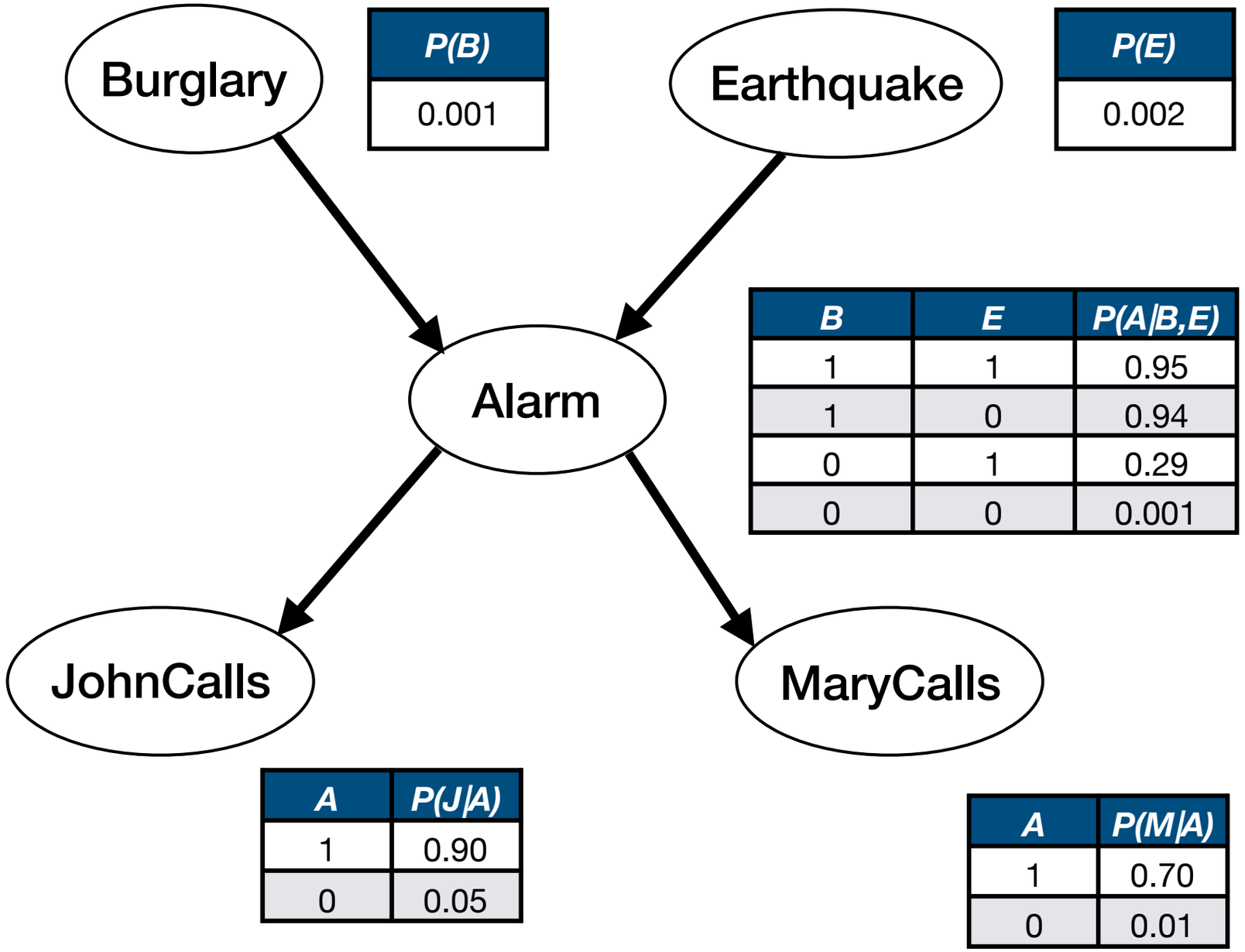
$$P(x_1, \ldots, x_7) = P(x_1)P(x_2)P(x_3)P(x_4|x_1, x_2, x_3)$$
$$P(x_5|x_1, x_3)P(x_6|x_4)P(x_7|x_4, x_5)$$

General formula for factorizing the joint distribution over a Bayes net:

$$P(X) = \prod_{i=1}^{K} P(x_i|\text{Parents}(x_i))$$

Slide credit: Christopher Bishop

An example: the alarm network



P(B)
0.001

P(E)
0.002

B	E	P(A|B,E)
1	1	0.95
1	0	0.94
0	1	0.29
0	0	0.001

A	P(J|A)
1	0.90
0	0.05

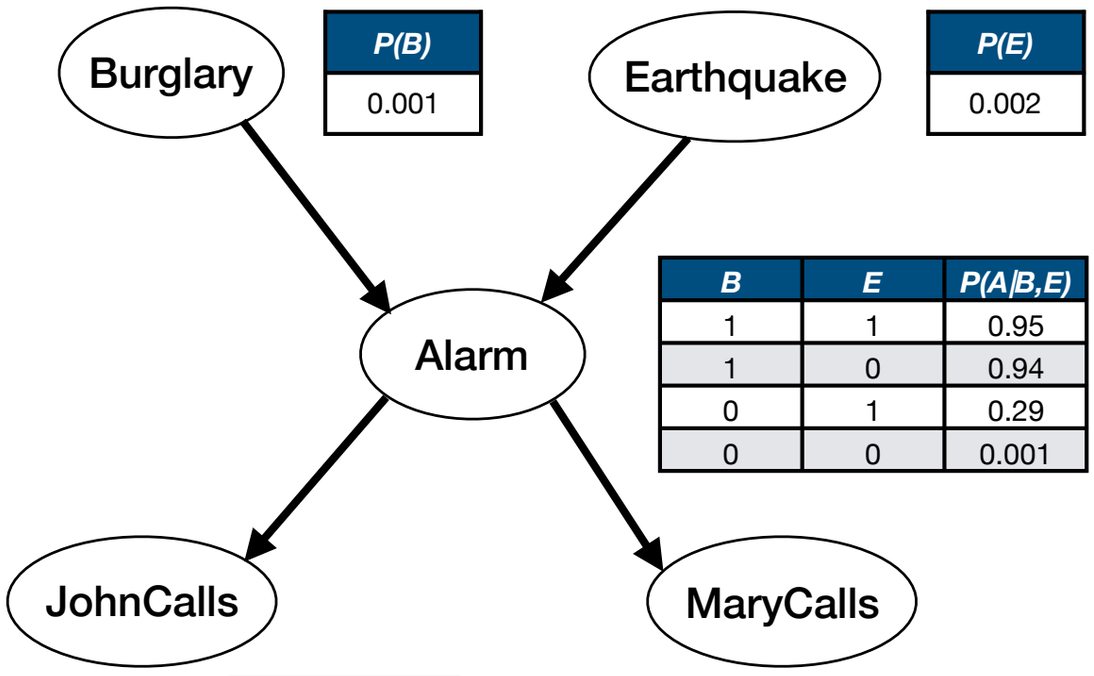
A	P(M|A)
1	0.70
0	0.01

Conditional Probability Table (CPT)

$$P(B, E, A, J, M) = P(B)P(E)P(A|B, E)P(J|A)P(M|A)$$

(particular version from Russell and Norvig)

Evaluating the joint probability of data



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1	0.90
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A	P(M|A)
1	0.70
0	0.01

We use the decomposed joint distribution to evaluate the probability of a setting of all of the variables.

$$P(B, E, A, J, M) = P(B)P(E)P(A|B, E)P(J|A)P(M|A)$$

What is the probability that there is no burglary or earthquake, and yet the alarm rings and both John and Mary call?

$$P(B = 0, E = 0, A = 1, J = 1, M = 1)$$
$$= P(B = 0)P(E = 0)P(A = 1|B = 0, E = 0)P(J = 1|A = 1)P(M = 1|A = 1)$$
$$= 0.999 * 0.998 * 0.001 * 0.9 * 0.7 = 0.00063$$

Example: Bayesian networks for understanding categorization

$$y = f(\mathbf{x})$$

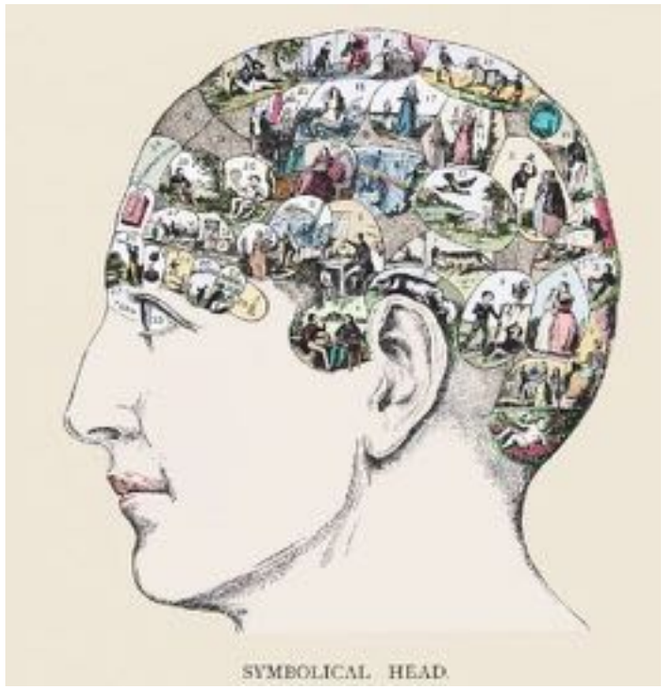
output — prediction function — Image feature

Causal mechanisms are important in everyday categorization and reasoning. **Is useful to think of the function *f* (the representation for a specific category) as a Bayesian network?**

data

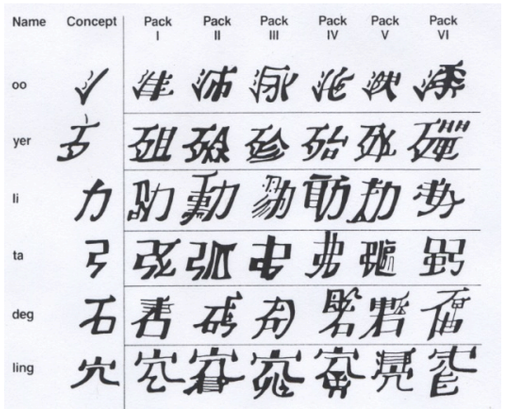
x_0	x_2	x_3	x_4	x_5	x_6	x_7
0	0	0	1	0	0	0
0	1	0	0	1	0	1
1	0	0	1	0	0	0
1	1	1	1	0	1	1

representation of category



predicted labels

y
0
0
1
1



Example of *Builder*

Example of *Digger*

Journal of Experimental Psychology:
Learning, Memory, and Cognition
2003, Vol. 29, No. 6, 1141–1159

0278-7393/03/$12.00 DOI: 10.1037/0278-7393.29.6.1141

A Causal-Model Theory of Conceptual Representation and Categorization

Bob Rehder
New York University

This article presents a theory of categorization that accounts for the effects of causal knowledge that relates the features of categories. According to causal-model theory, people explicitly represent the probabilistic causal mechanisms that link category features and classify objects by evaluating whether they were likely to have been generated by those mechanisms. In 3 experiments, participants were taught causal knowledge that related the features of a novel category. Causal-model theory provided a good quantitative account of the effect of this knowledge on the importance of both individual features and interfeature correlations to classification. By enabling precise model fits and interpretable parameter estimates, causal-model theory helps place the theory-based approach to conceptual representation on equal footing with the well-known similarity-based approaches.

For the last several decades, research on the topic of categorization has focused on the problem of learning new categories via examples of category members, that is, from empirical observations. The result has been a host of categorization models that are based on representational ideas such as central prototypes, stored exemplars, and variabilized rules, and on processing principles such as similarity, that have considerable explanatory power and experimental support. More recently, the influence of the prior "theoretical" knowledge that learners often contribute to their representations of categories has also been a topic of study (Carey, 1985; Keil, 1989; Murphy & Medin, 1985; Schank, Collins, & Hunter, 1986). For example, people not only know that birds have wings and that they can fly and build nests in trees, but also that birds build nests in trees *because* they can fly, and fly *because* they have wings. Many people even believe that morphological features of birds such as wings are ultimately caused by the kind of DNA that birds possess. However, in comparison with the development of models accounting for the effects of empirical observations, there has been relatively little development of formal models to account for the effects of such prior knowledge (although see Heit, 1994; Heit & Bott, 2000; Pazzani, 1991; Rehder & Murphy, in press; Sloman, Love, & Ahn, 1998).

The purpose of this article is to present a theory of categorization that accounts for the effects of causal knowledge that relates features (Rehder, 1999; Waldmann, Holyoak, & Fratianne, 1995). Further, according to this theory, people use causal models to determine a new object's category membership.

In this article, causal-model theory is applied to two outstanding problems in the domain of categorization research. The first problem concerns determining the importance, or *weight,* that individual features have on establishing category membership. Since the popularization of the notion of probabilistic categories in the 1970s, it has usually been assumed that features of a category vary regarding their influence on category membership (Hampton, 1979; Rosch, 1973; Rosch & Mervis, 1975; Smith & Medin, 1981). Indeed, formal models of categorization have formalized the manner in which a feature's weight is influenced by its perceptual saliency (Lamberts, 1995, 1998) and by the frequency with which it appears in category members and nonmembers (Nosofsky, 1986; Reed, 1972; Rosch & Mervis, 1975; Shepard, Hovland, & Jenkins, 1961). However, these models do not account for the fact that feature weights are also determined by a categorizer's domain theories. For instance, Medin and Shoben (1988) have found that *straight bananas* are rated as better members of the category *bananas* than *straight boomerangs* are of the category

Artificial categorization task

Task: Learn and make predictions about a new category, e.g., “Lake Victoria Shrimp”

Four binary features

F_1: High amounts of ACh neurotransmitter.

F_2: Long-lasting flight response.

F_3: Accelerated sleep cycle.

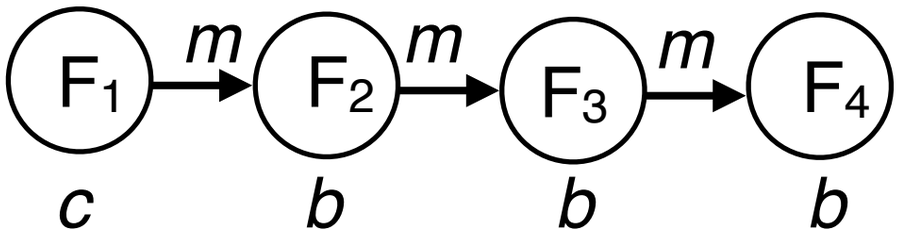
F_4: High body weight.

Base rate information: 75% of Lake Victoria Shrimp have each feature, e.g., 75% have feature F_4

Artificial categorization task

participants assigned to one of two conditions

chain (causal framing) condition



free parameters *c, b* (background mechanism) and *m* (causal strength)

$F_1 \rightarrow F_2$ A high quantity of the ACh neurotransmitter causes a long-lasting flight response. The duration of the electrical signal to the muscles is longer because of the excess amount of neurotransmitter.

$F_2 \rightarrow F_3$ A long-lasting flight response causes an accelerated sleep cycle. The long-lasting flight response causes the muscles to be fatigued, and this fatigue triggers the shrimp's sleep center.

$F_3 \rightarrow F_4$ An accelerated sleep cycle causes a high body weight. Shrimp habitually feed after waking, and shrimp on an accelerated sleep cycle wake three times a day instead of once.

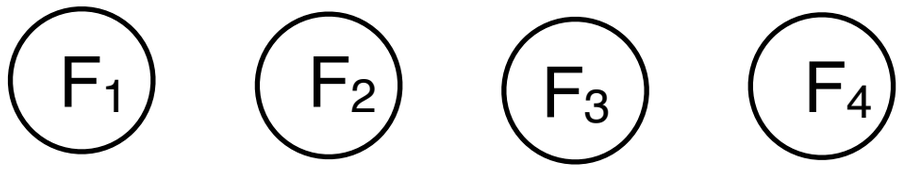
F_1: High amounts of ACh neurotransmitter.

F_2: Long-lasting flight response.

F_3: Accelerated sleep cycle.

F_4: High body weight.

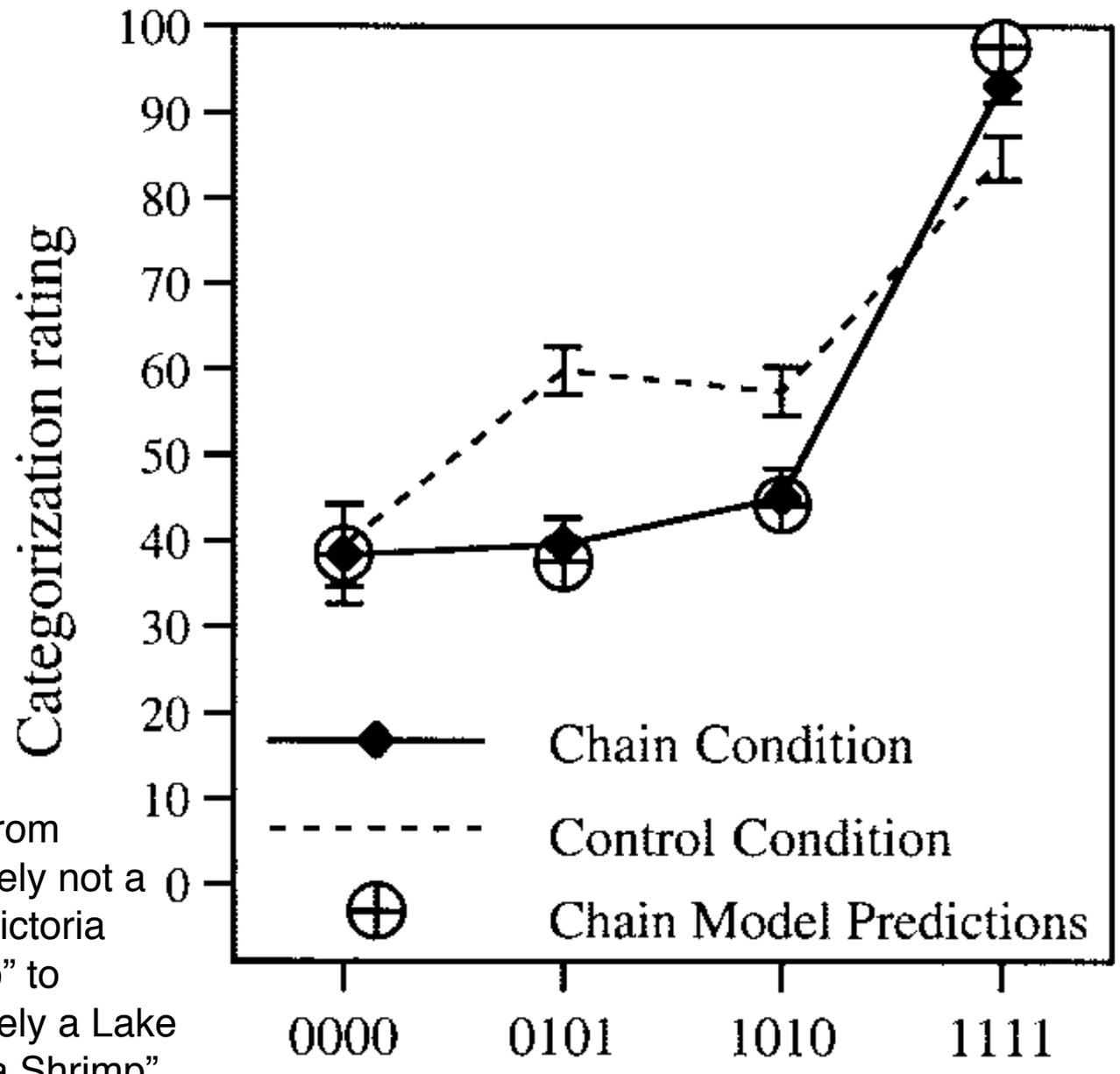
control condition (no causal framing)



exactly the same instructions, but without causal information between features

Artificial categorization task: Results

Conclusion: Causal/structural information influences people's categorization decisions, in a way predicted by a Bayesian network model.



scale from "definitely not a Lake Victoria Shrimp" to "definitely a Lake Victoria Shrimp"

new object for category judgment

(0101 means both F_2 and F_4 are present, and the others absent)

Test judgments: is *F* a Lake Victoria Shrimp?

In causal condition, compute judgement as:

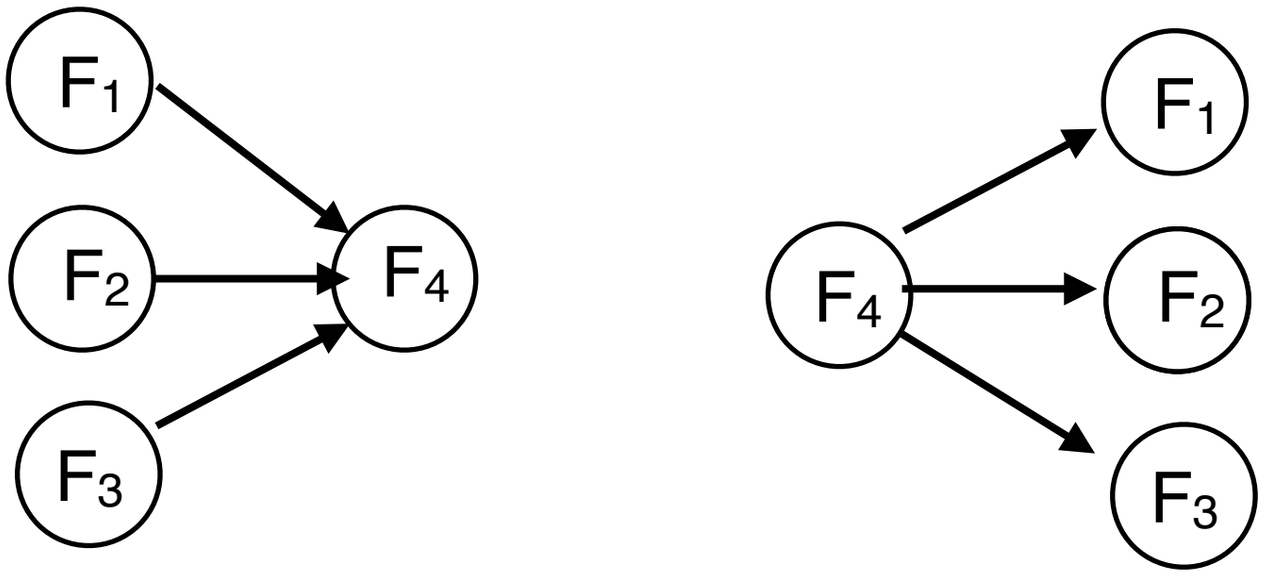
$$P(F_1, F_2, F_3, F_4)$$
$$= P(F_1)P(F_2|F_1)P(F_3|F_2)P(F_4|F_3)$$

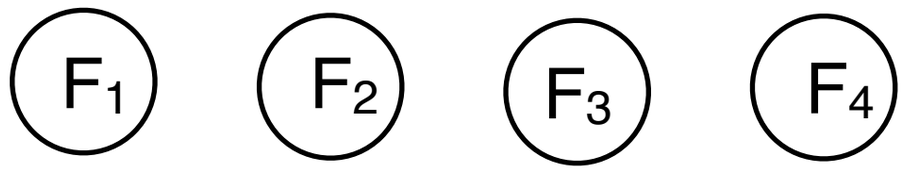
Key idea: categorization decision is computing joint probability under a Bayes net model of that category.

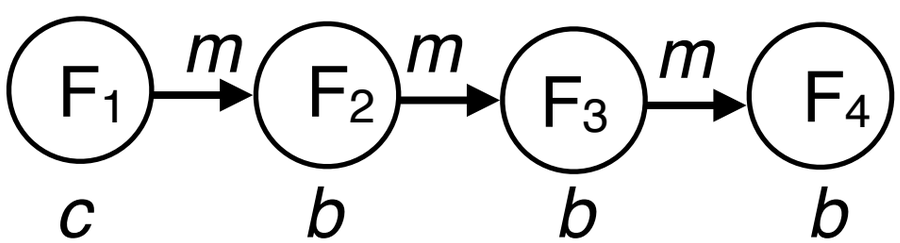
Causal structure matters in categorization judgments

Further work from Rehder and colleagues have studied categories with these alternative Bayes net structures…

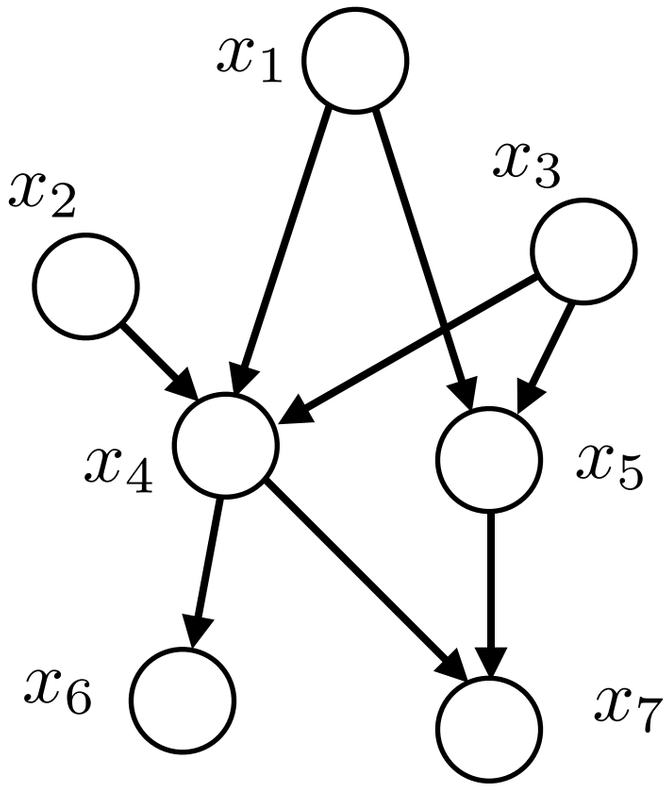
(e.g., Rehder and Hastie, 2001)







How do Bayesian networks relate to other Bayesian models used in cognitive modeling?



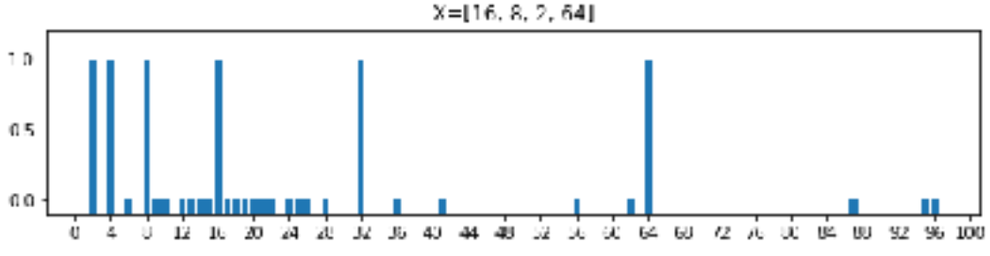
$$P(x_1, \ldots, x_7) = P(x_1)P(x_2)P(x_3)P(x_4|x_1, x_2, x_3)$$
$$P(x_5|x_1, x_3)P(x_6|x_4)P(x_7|x_4, x_5)$$

General formula for factorizing the joint distribution

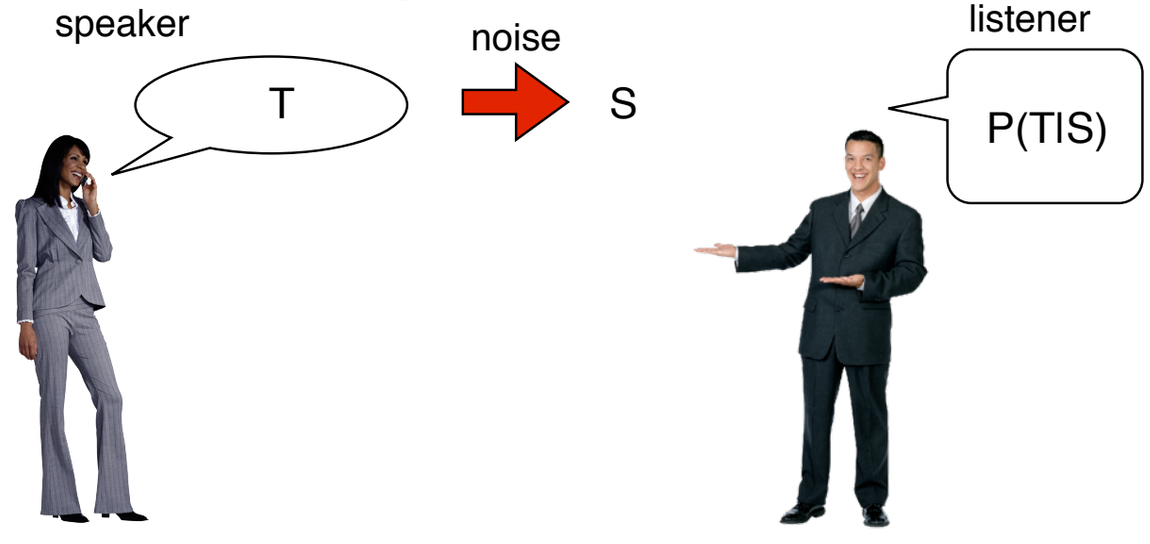
$$P(X) = \prod_{i=1}^{K} P(x_i|\text{Parents}(x_i))$$

The number game

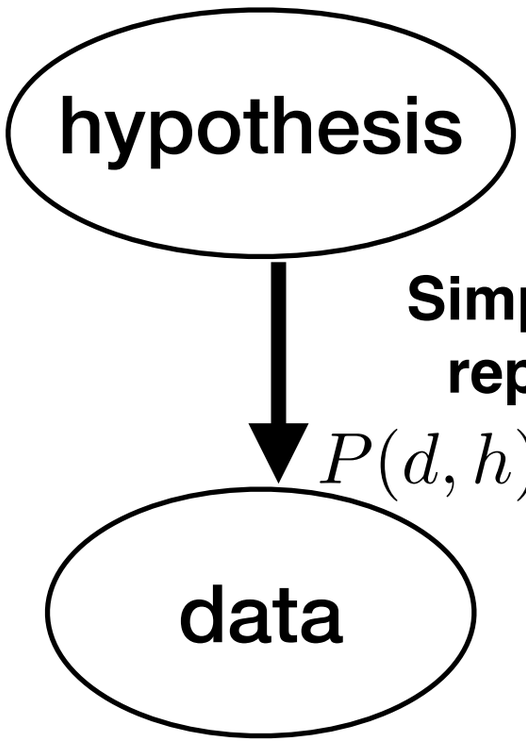
16 8 2 64



Perceptual magnet effect



Connection with simple Bayesian models



Simple Bayes net representation

$$P(d,h) = P(d|h)P(h)$$

Many of the Bayesian models developed for cognitive modeling can be interpreted as two node Bayesian networks, with a complex (potentially very complex) conditional probability table (aka likelihood function)

Diagnosis example

Data (D): John is coughing

Hypotheses:

h_1 = John has a cold
h_2 = John has emphysema
h_3 = John has a stomach flu

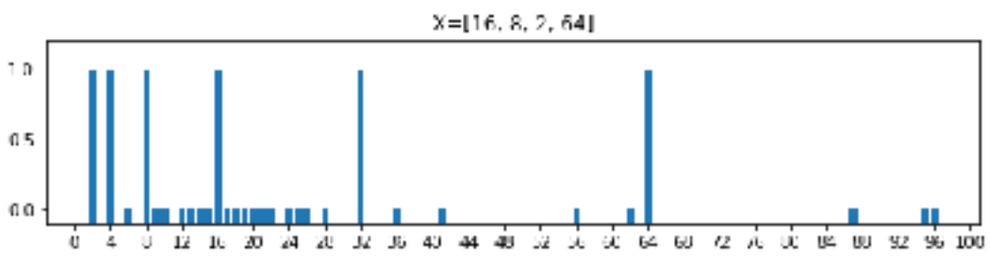
"Bayes' rule"

posterior — likelihood — prior

$$P(h_i|D) = \frac{P(D|h_i)P(h_i)}{\sum_j P(D|h_j)P(h_j)}$$

The number game

16 8 2 64



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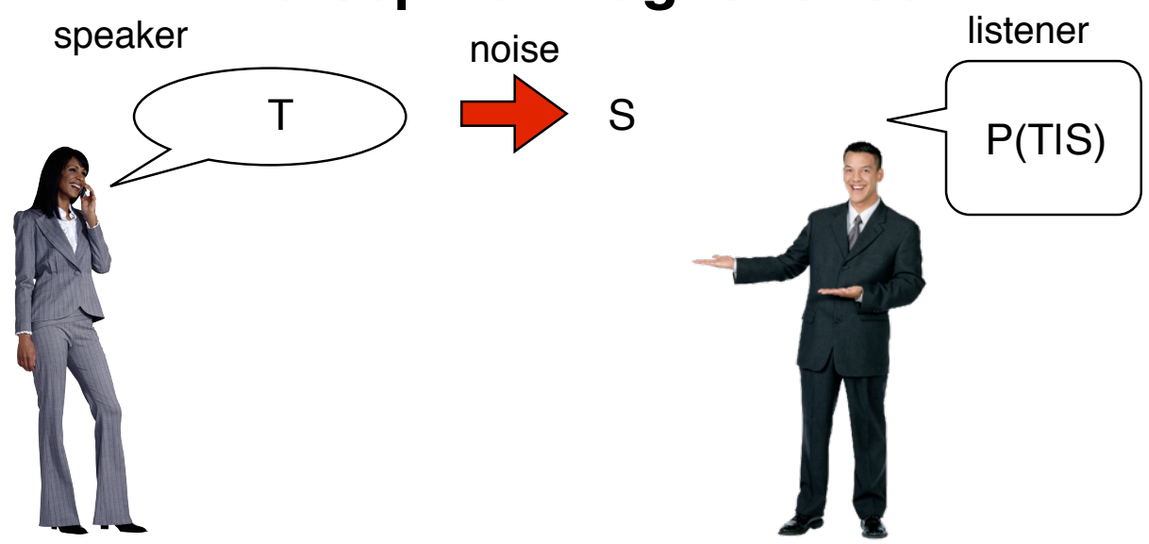


Review from the number game: Probabilistic inference is very flexible!

$$E[\phi(h)|D] \approx \frac{1}{M}\sum_{m}\phi(h^{(m)})$$

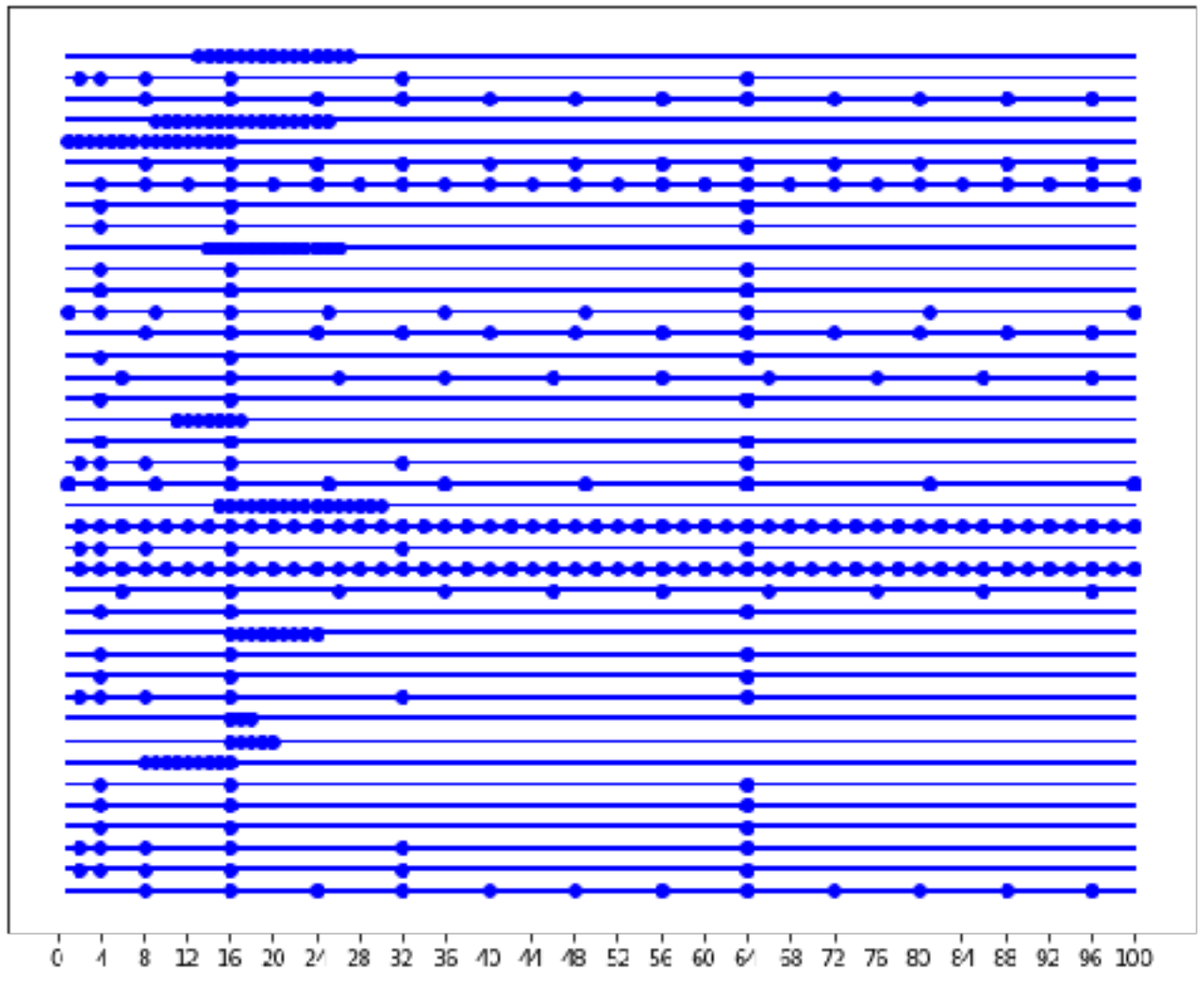
If we can compute the posterior, or draw samples from the posterior, we can automatically reason about a huge range of questions $\phi(\cdot)$

Examples of reusing the sample for new queries

- Is 64 a member of the set? **(probability is 0.73)**
- Are both 36 and 64 members of the set? **(0.36)**
- Is there a member of the set greater than or equal to 80? **(0.27)**
- If we sample a new number from the hypothesis, what is the chance it will be 64? (**0.16)**
- If we sample a new number from the hypothesis, what is the chance it will be 80? **(0.02)**

rows are samples $h^{(m)}$

→ 16



The type of flexibility in reasoning is natural in Bayesian models, but it is difficult to capture in neural networks trained with supervised learning, or model-free reinforcement learning.

Inference flexibility is not specific to rejection sampling, but to Bayesian models in general.

Probabilistic inference

as a generalization of Bayes' rule to arbitrary queries in a probabilistic model

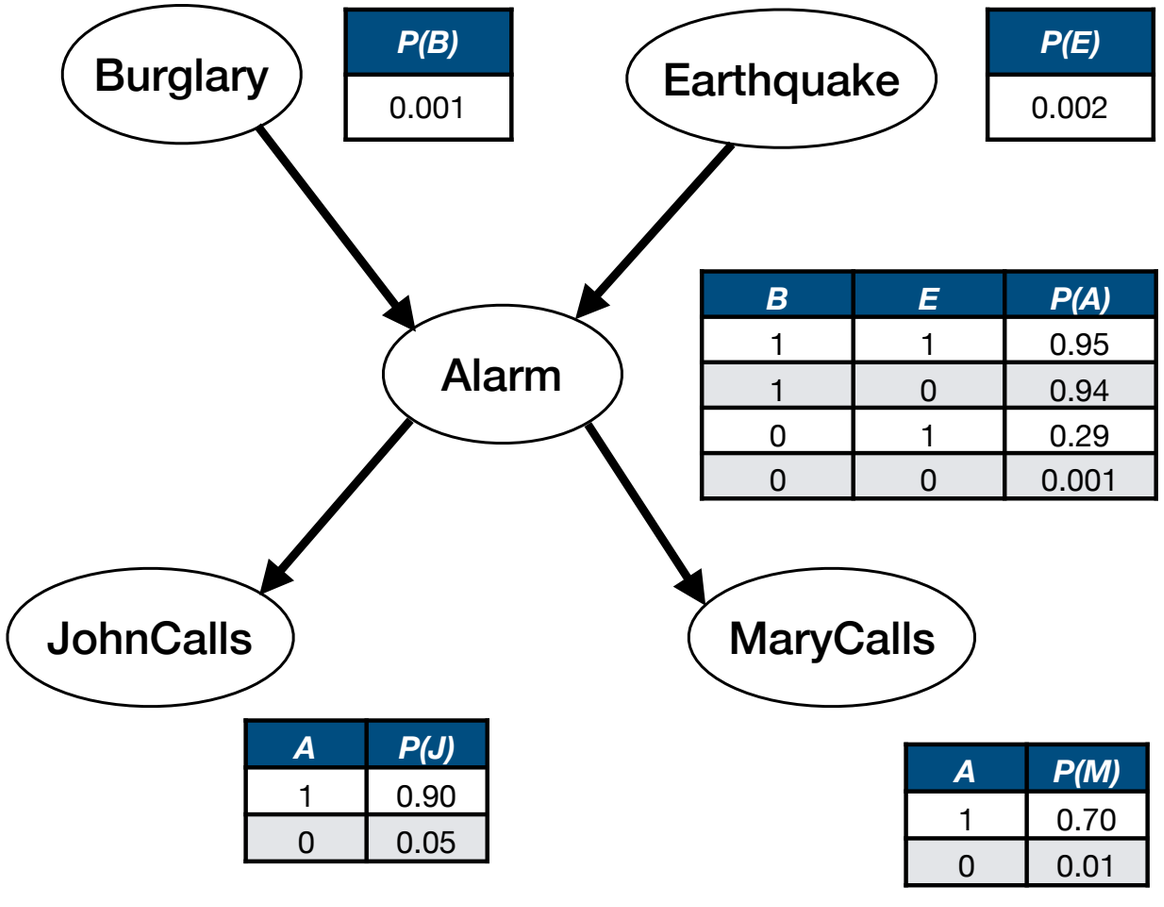
General formula for probabilistic inference

$$P(X|e) \propto \sum_y P(X, e, y) \qquad P(X|e) = \frac{\sum_y P(X, e, y)}{\sum_{y,X'} P(X', e, y)}$$

X = query variables

e = evidence variables

Y = hidden variables



Example with the alarm network:

Probability of a burglary given that Mary calls:

$$P(B = 1|M = 1) = 0.056$$

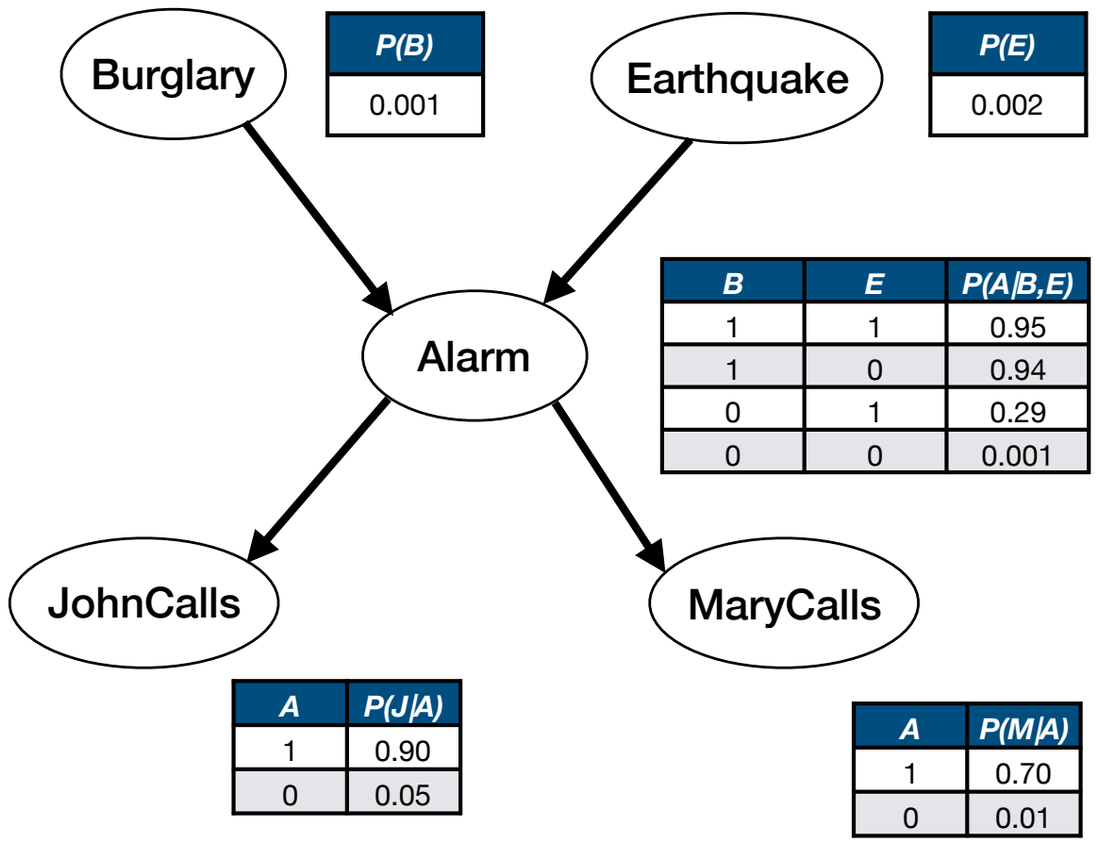
$X = \{B\}$

$e = \{M\}$

$Y = \{E, A, J\}$

Flexible reasoning through probabilistic inference

as a generalization of Bayes' rule to arbitrary queries in a probabilistic model



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0.001

P(E)
0.002

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1	1	0.95
1	0	0.94
0	1	0.29
0	0	0.001

A	P(J|A)
1	0.90
0	0.05

A	P(M|A)
1	0.70
0	0.01

Marginal distributions

$$P(B = 1) = 0.001$$
$$P(A = 1) = 0.003$$
$$P(M = 1) = 0.01$$

Is there a burglary, given John and Mary both call?

$$P(B = 1|J = 1, M = 1) = 0.284$$

Is there an earthquake, given John and Mary both call?

$$P(E = 1|J = 1, M = 1) = 0.176$$

Does Mary call, given a burglary?

$$P(M = 1|B = 1) = 0.66$$

Does Mary call, given a burglary and earthquake?

$$P(M = 1|B = 1, E = 1) = 0.67$$

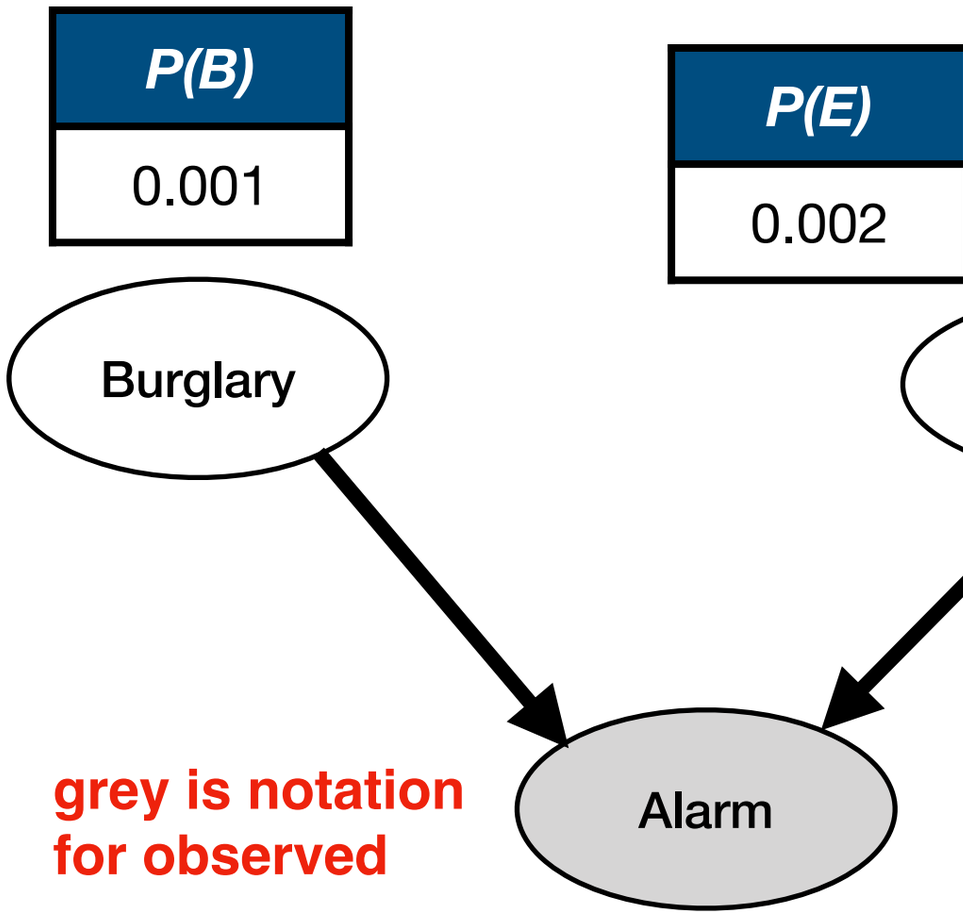
What is the chance there is a burglary with an alarm and Mary calls, assuming no earthquake?

$$P(A = 1, B = 1, M = 1|E = 0) = 0.0006$$

Algorithms for inference:

- exact enumeration (equation from previous slide)
- rejection sampling
- importance sampling
- MCMC
- etc.

“Explaining away” with Bayes nets



Where one cause can explain away the need for a second one…

B	E	P(A|B,E)
1	1	0.95
1	0	0.94
0	1	0.29
0	0	0.001

If alarm rings, then an earthquake becomes a real possibility:

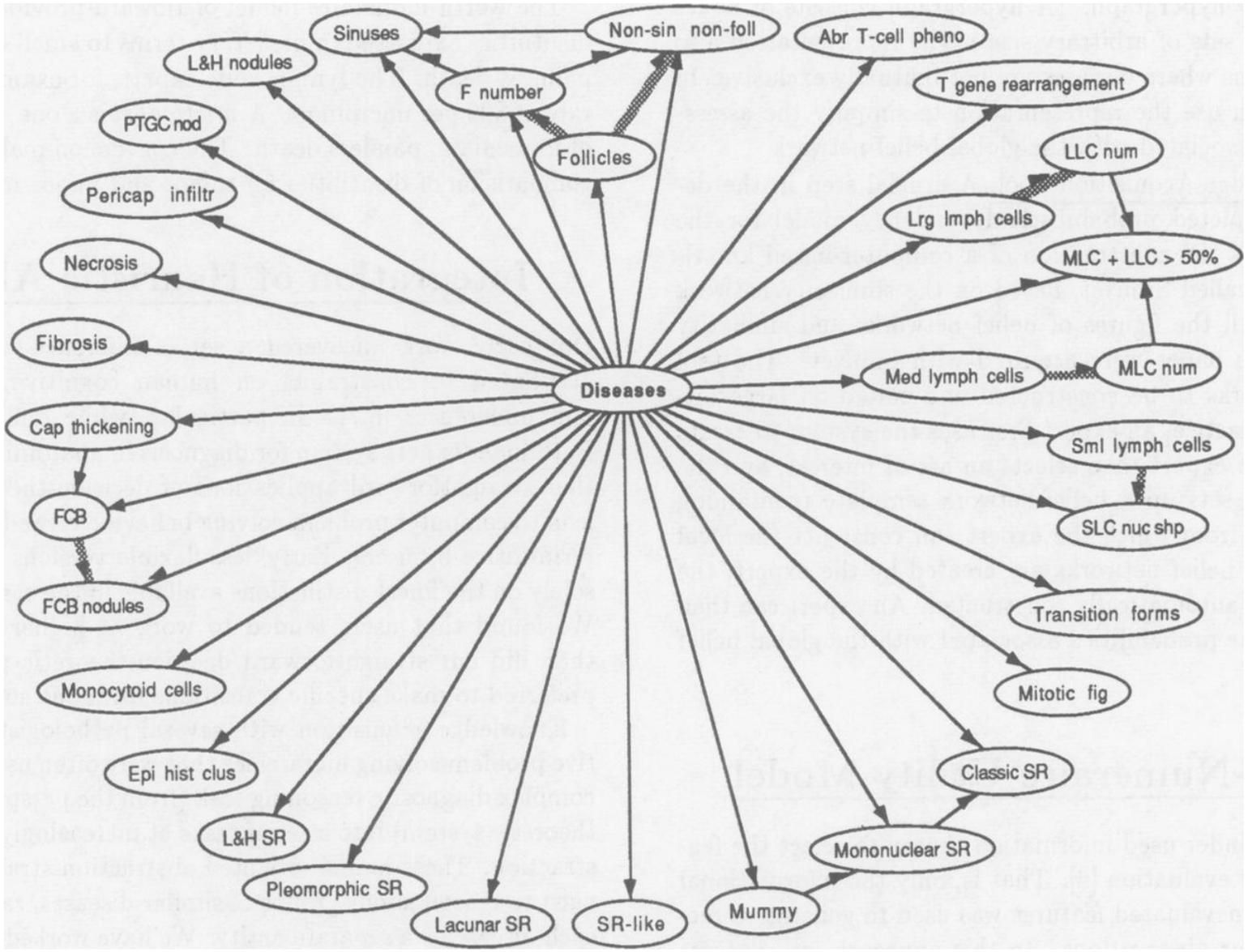
$$P(E = 1|A = 1) = 0.23$$

Unless we know *there was a burglary*, in which case we can “explain away” the indication for an earthquake.

$$P(E = 1|A = 1, B = 1) = 0.002$$

Probabilistic inference in practice

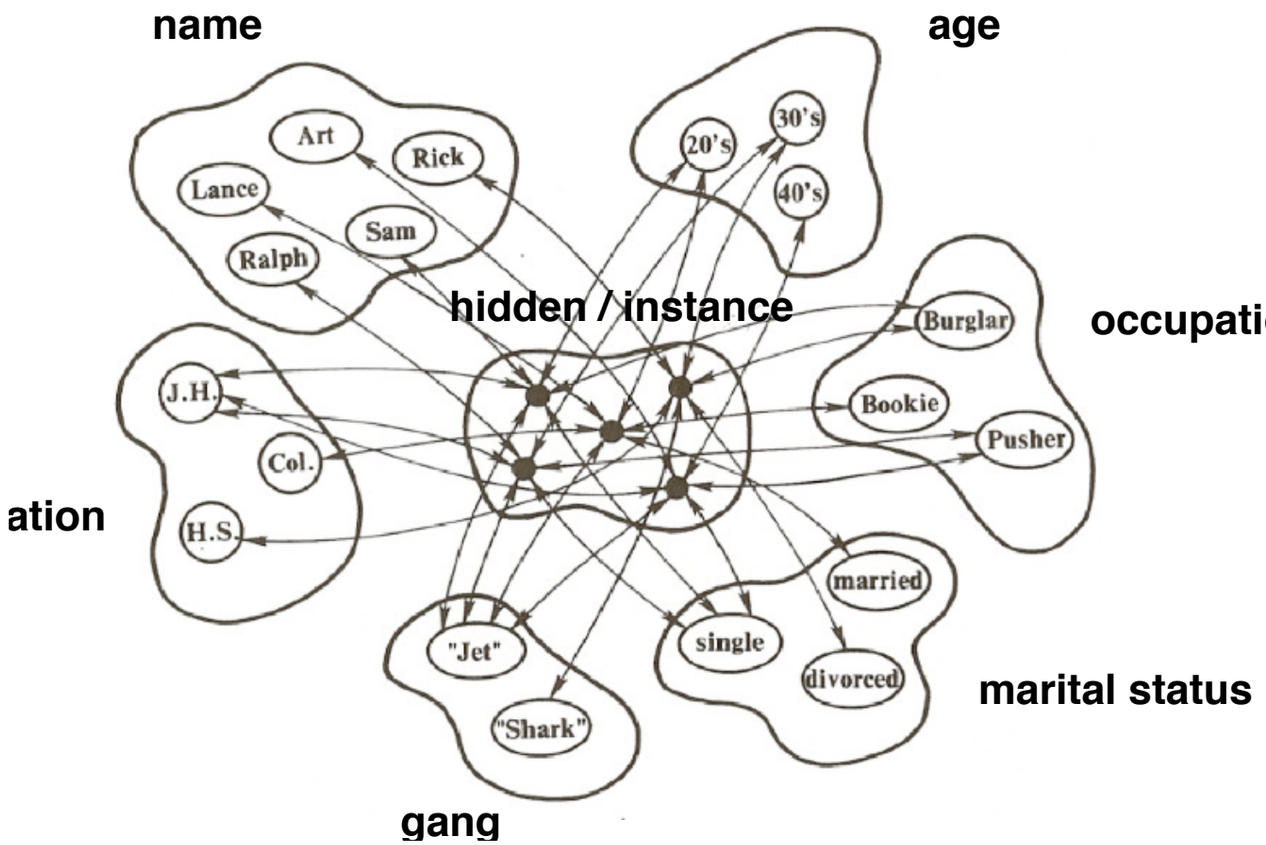
Pathfinder project for medical diagnosis (Heckerman, Horvitz, & colleagues; late 1980s)



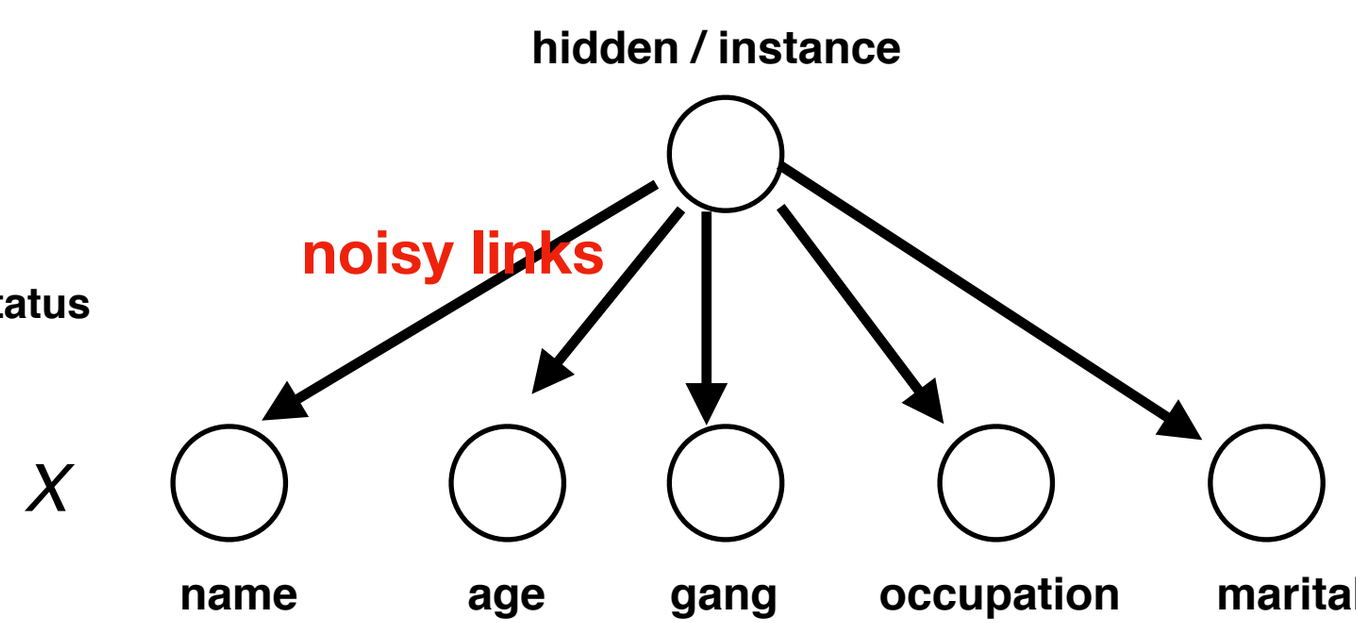
- Commercial system for diagnosing lymph-node pathology
- Probabilistic inference used to compute $P(\text{Disease}|\text{Symptoms})$
- CPTs determined by expert knowledge from pathologists

Example of probabilistic inference: Interactive activation model

recurrent neural network



Bayesian network alternative



- **Retrieval by name** $P(X|\text{name} = \text{Ken})$
- **Content addressability** $P(\text{name}|\text{age} = 30\text{s}, \text{gang} = \text{Sharks})$
- **Spontaneous generalization** $P(\text{age}|\text{gang} = \text{Sharks})$

Conditional independence

x_1 is independent of x_2 given x_3

$$P(x_1|x_2, x_3) = P(x_1|x_3)$$

Equivalently

$$P(x_1, x_2|x_3) = P(x_1|x_2, x_3)P(x_2|x_3) \quad \text{(product rule)}$$

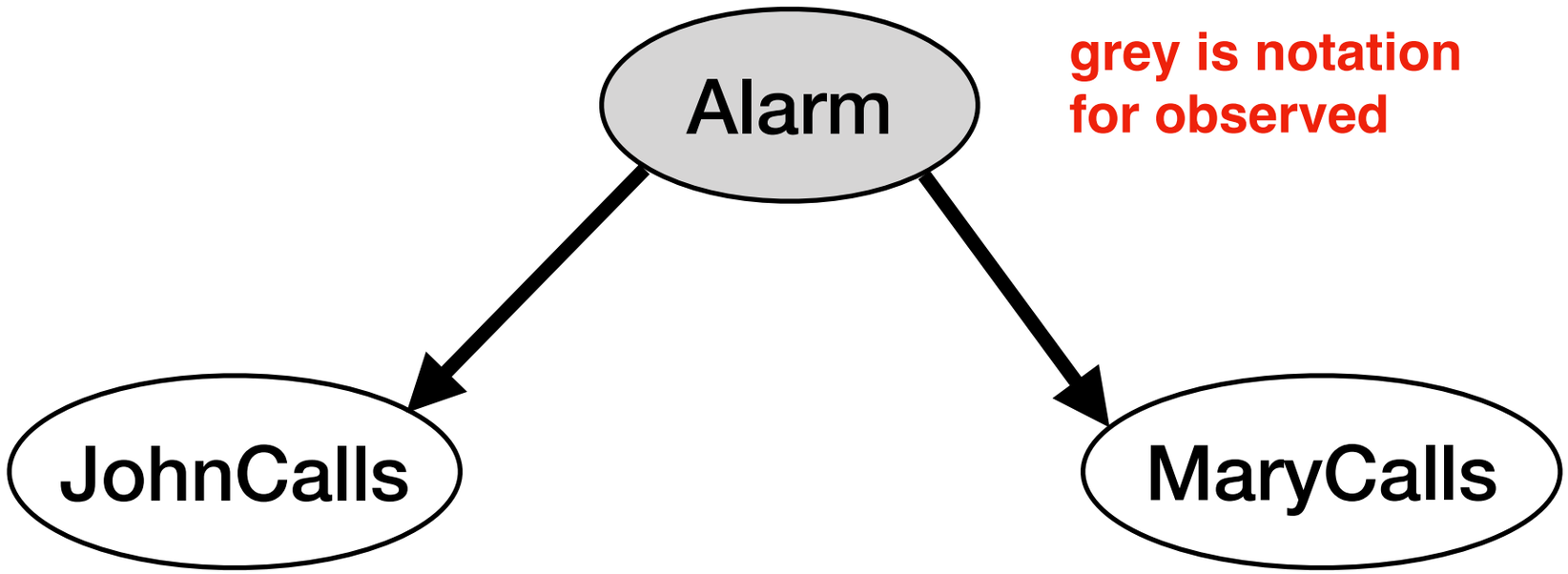
$$= P(x_1|x_3)P(x_2|x_3)$$

Written as

$$x_1 \perp\!\!\!\perp x_2 \mid x_3$$

Slide credit: Christopher Bishop

Conditional independence: Example



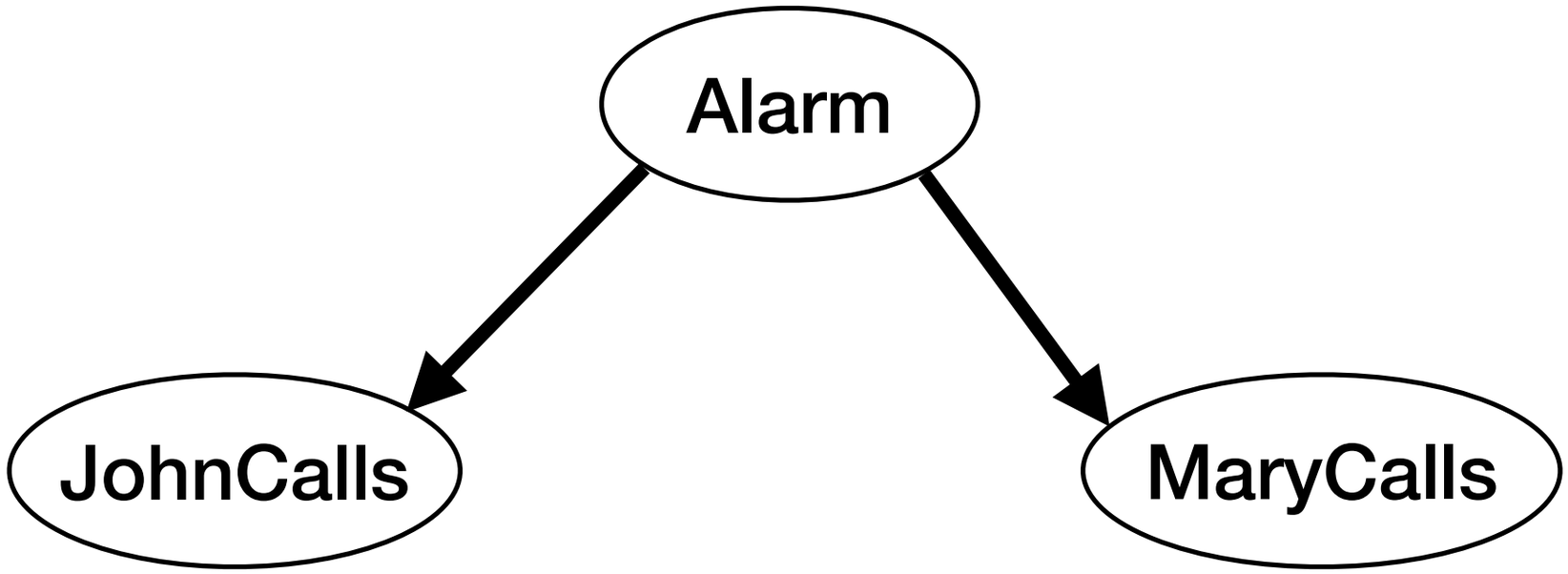
$$P(J, M|A) = \frac{P(J, A, M)}{P(A)} \quad \text{(def. condition prob)}$$

$$= \frac{P(J|A)P(M|A)P(A)}{P(A)}$$

$$= P(J|A)P(M|A)$$

$$J \perp\!\!\!\perp M \mid A$$

Conditional independence: Example

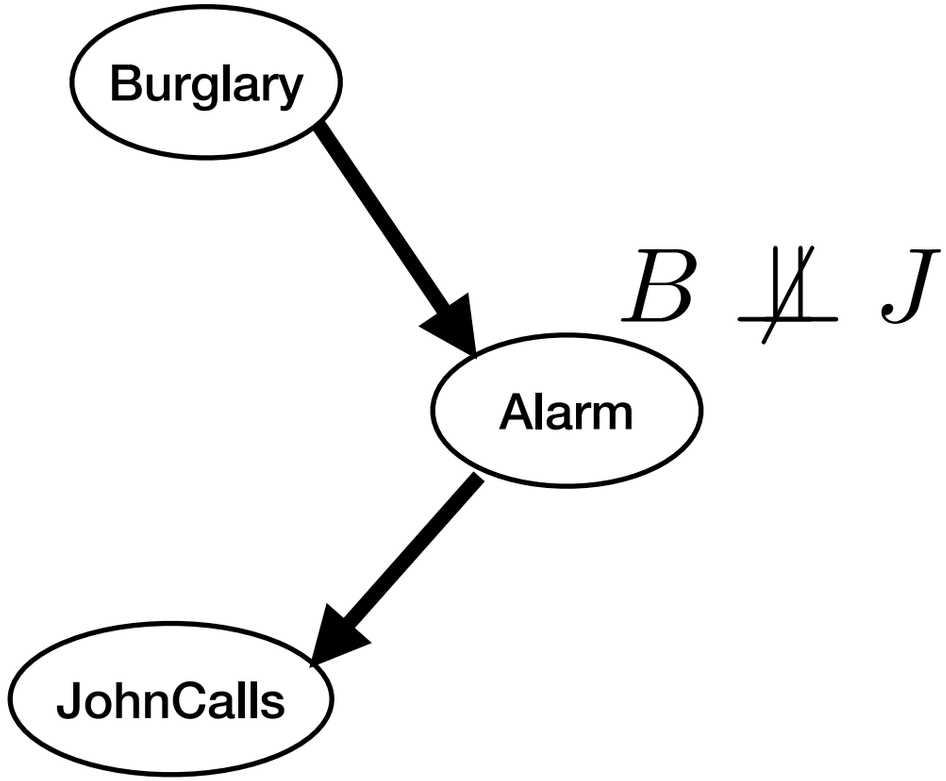


$$P(J, A, M) = P(J|A)P(M|A)P(A)$$

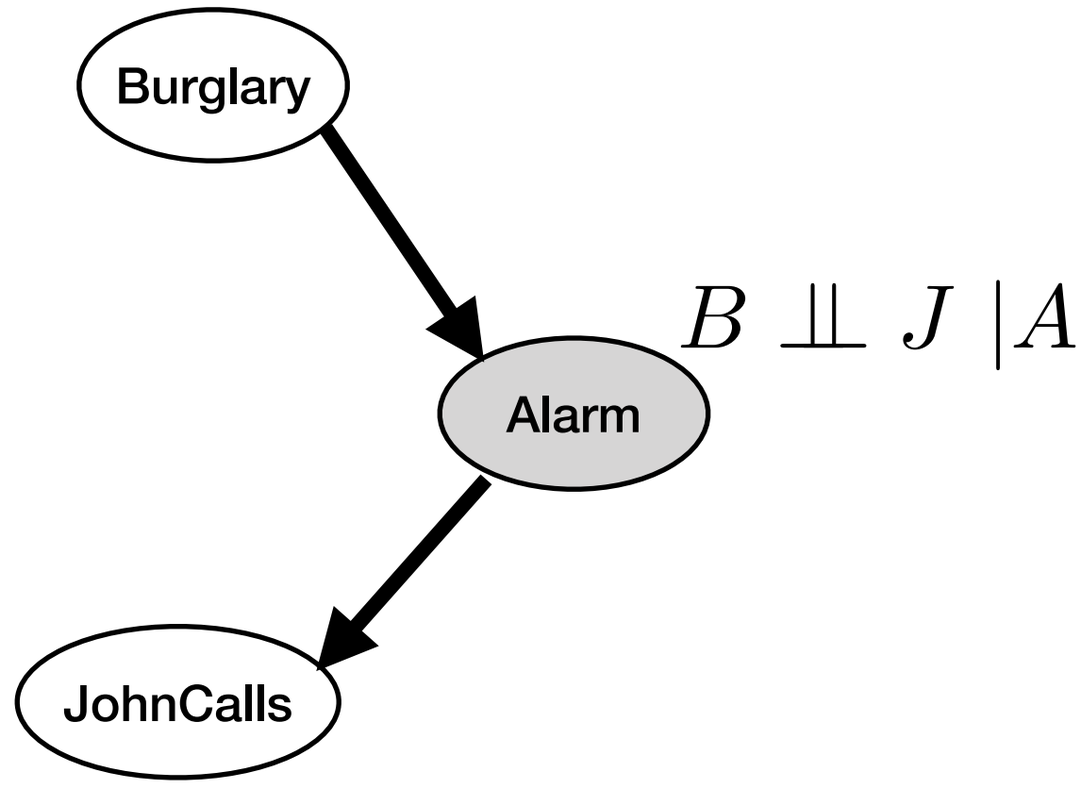
$$P(J, M) = \sum_A P(J|A)P(M|A)P(A)$$

$$J \not\perp\!\!\!\perp M$$

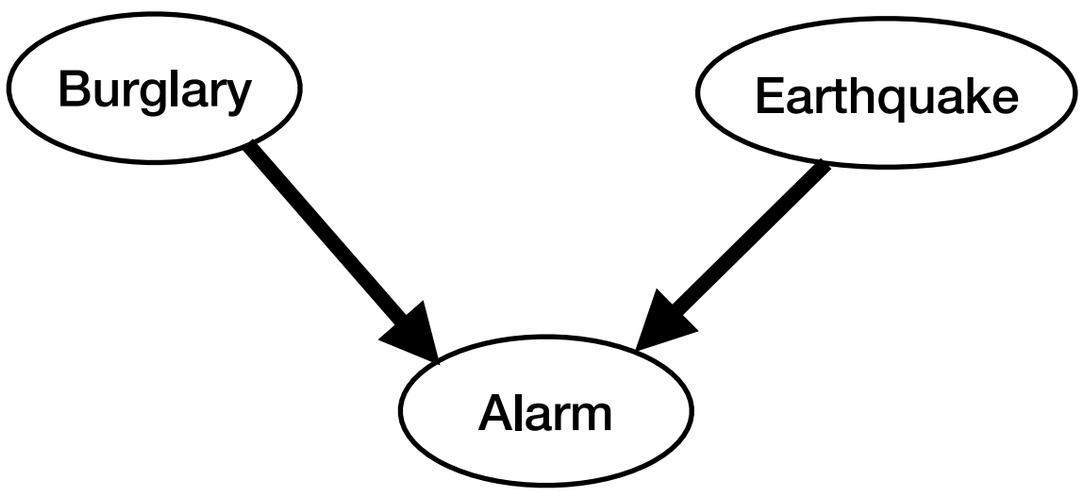
Conditional independence: More examples



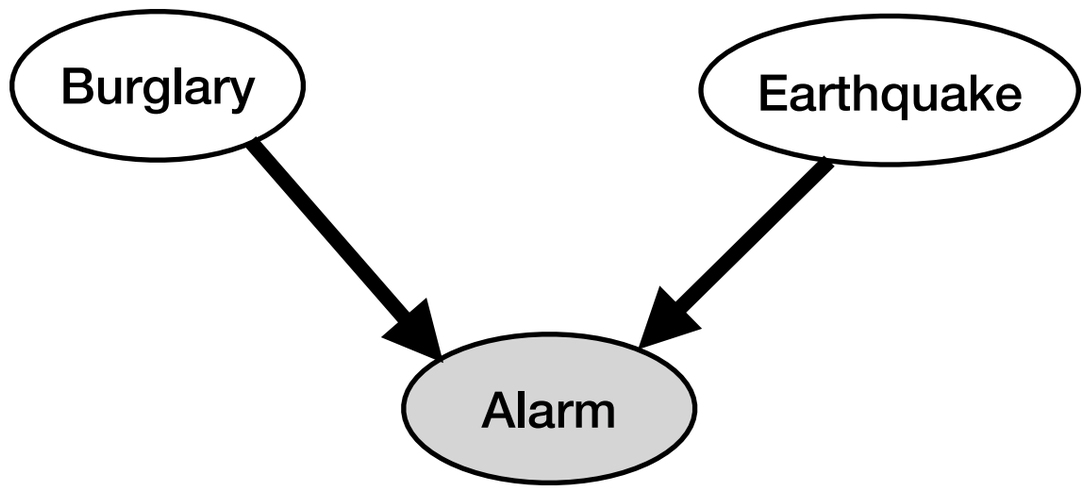
$$B \not\perp\!\!\!\perp J$$



$$B \perp\!\!\!\perp J \mid A$$



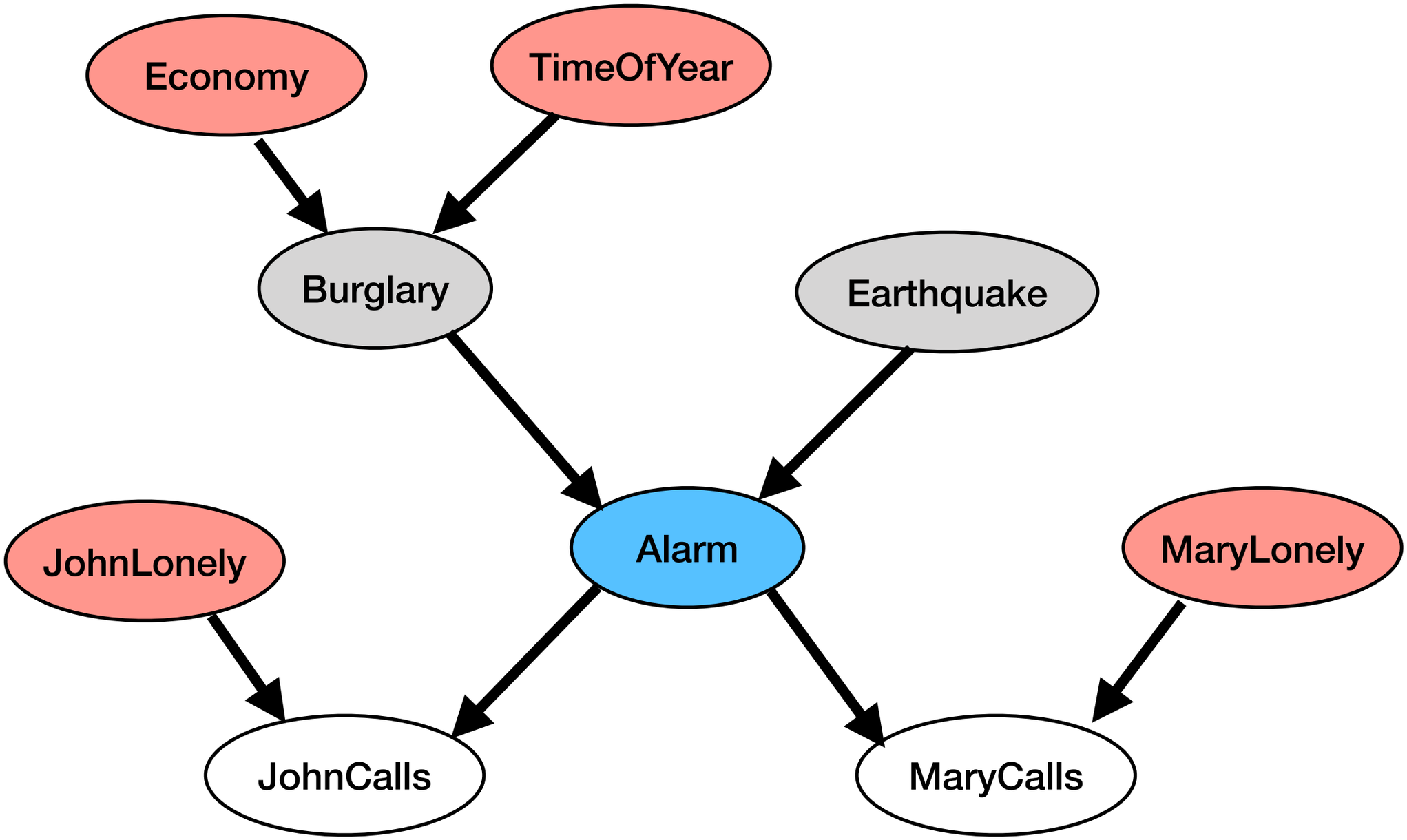
$$B \perp\!\!\!\perp E$$



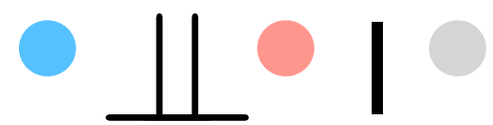
$$B \not\perp\!\!\!\perp E \mid A$$

"explaining away" case

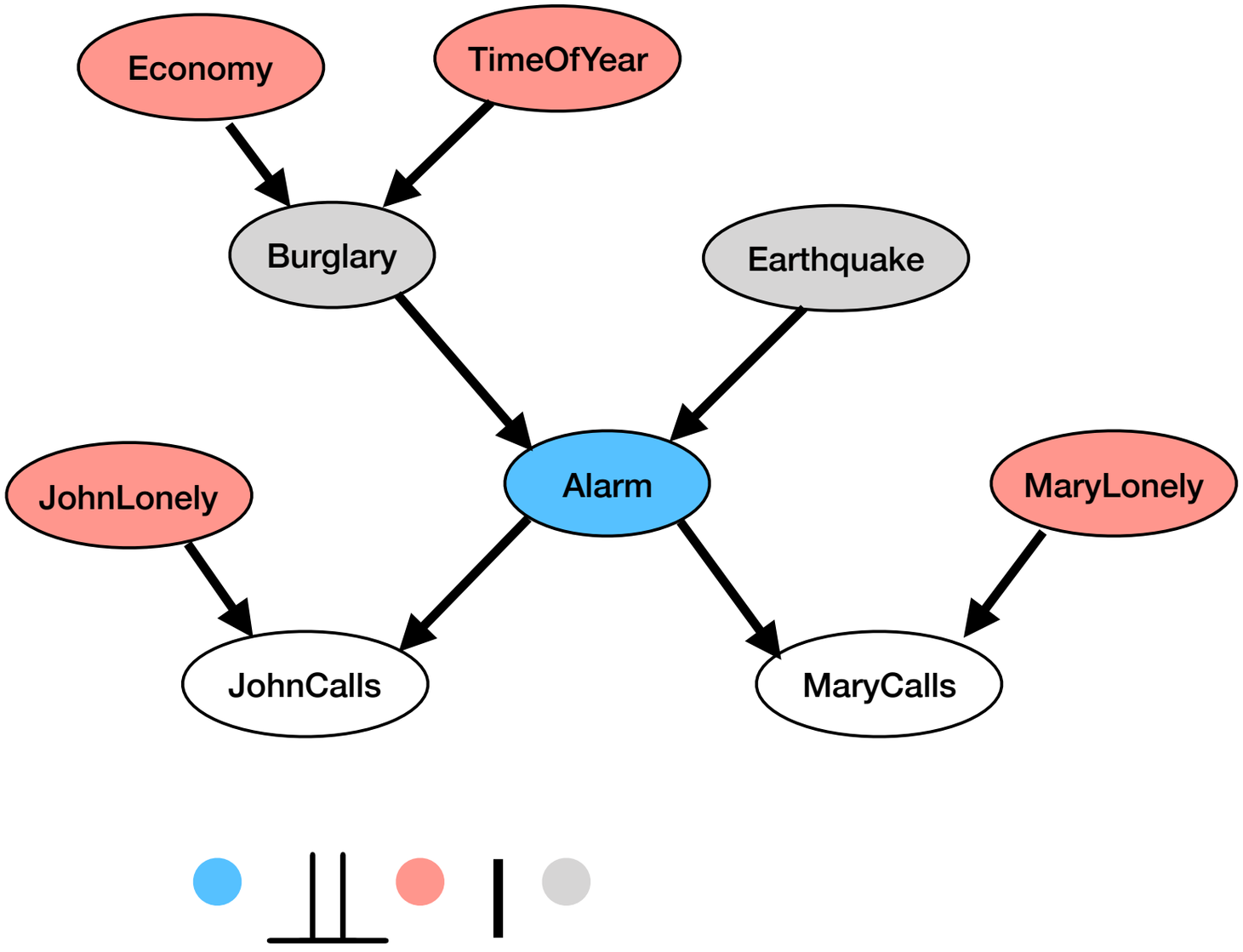
General statement on conditional independence



A variable is conditionally independent of its non-descendants **given** its parents



Significance of Bayes nets and conditional independence

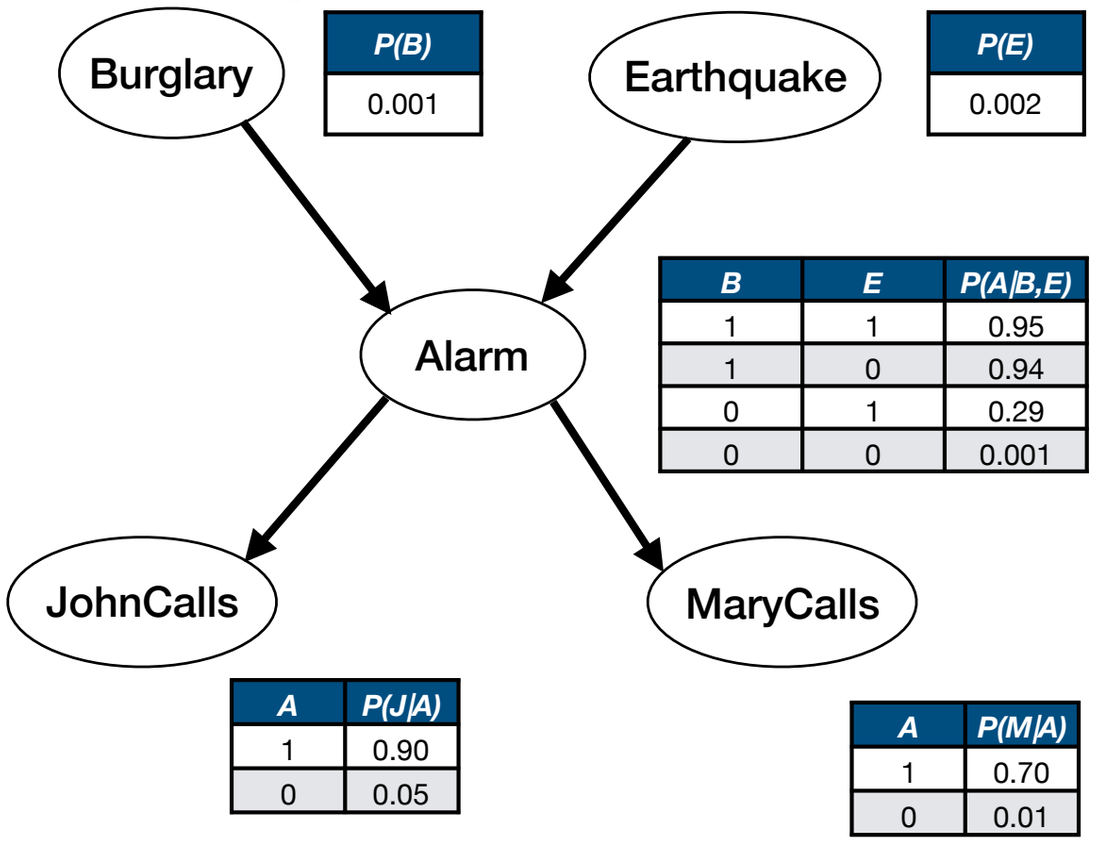


- We can read conditional independence properties directly off the graph structure, rather than having to derive them analytically (as we did with simple examples of conditional independence).

- We can exploit the conditional independence properties for efficient probabilistic inference / Bayesian reasoning (using exact inference, MCMC, etc.)

Learning Bayesian networks: Parameter learning

Known structure (e.g., consulting experts, prior knowledge), but unknown parameters



P(B)
0.001

P(E)
0.002

B	E	P(A|B,E)
1	1	0.95
1	0	0.94
0	1	0.29
0	0	0.001

A	P(J|A)
1	0.90
0	0.05

A	P(M|A)
1	0.70
0	0.01

S : graph structure

θ : parameters (numbers in CPTs)

D : data set

example empirical data set D

	B	E	A	J	M
$D^{(1)}$	1	0	1	0	1
$D^{(2)}$	0	0	0	0	0
…	0	0	0	0	1
$D^{(N)}$	more rows like this….				

$$P(B, E, A, J, M) = P(B)P(E)P(A|B, E)P(J|A)P(M|A)$$

maximum likelihood parameter learning:

$$\arg\max_{\theta} \sum_{i} \log P(D^{(i)}|\theta; S)$$

straightforward solution: we can fit CPTs independently, and each CPT is very intuitive (simply count the relevant occurrences of a variable given its parents)

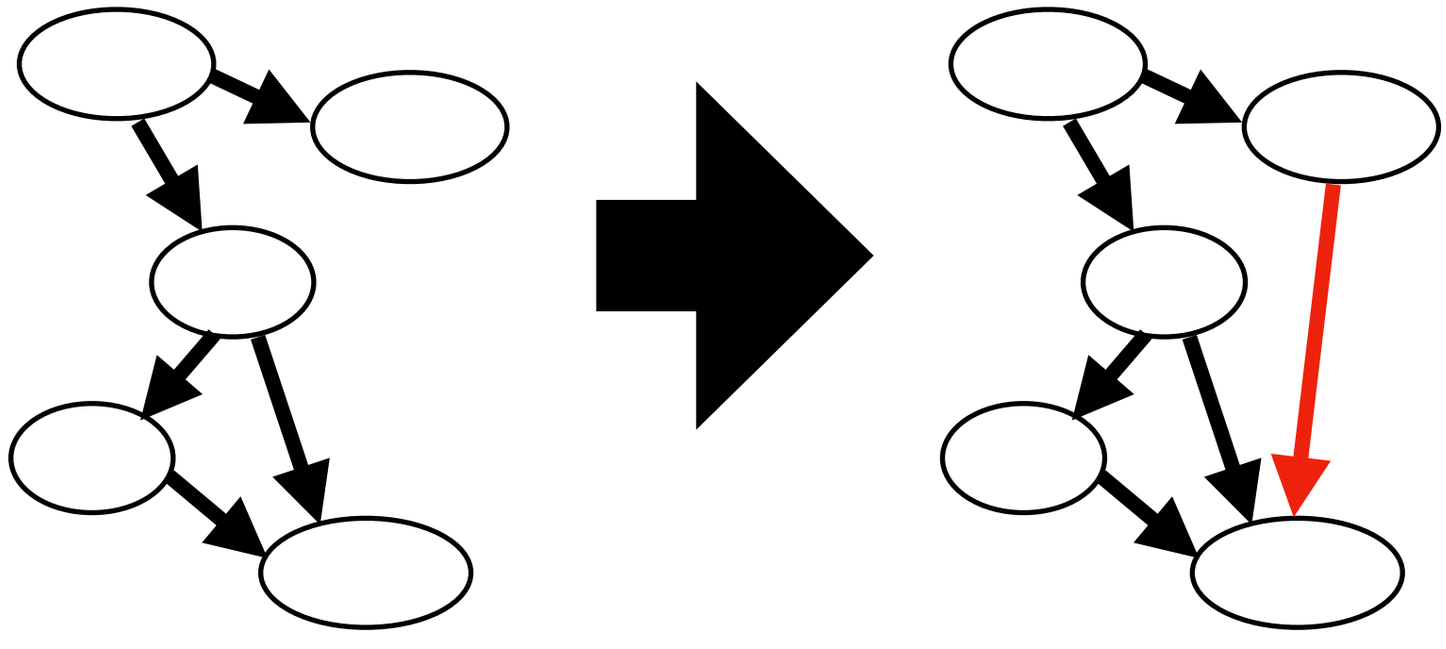
Learning Bayesian networks: Structure learning

Unknown structure, unknown parameters

$$\arg\max_{\theta,S} \sum_i \log P(D^{(i)}|\theta, S) - \text{cost}(S)$$

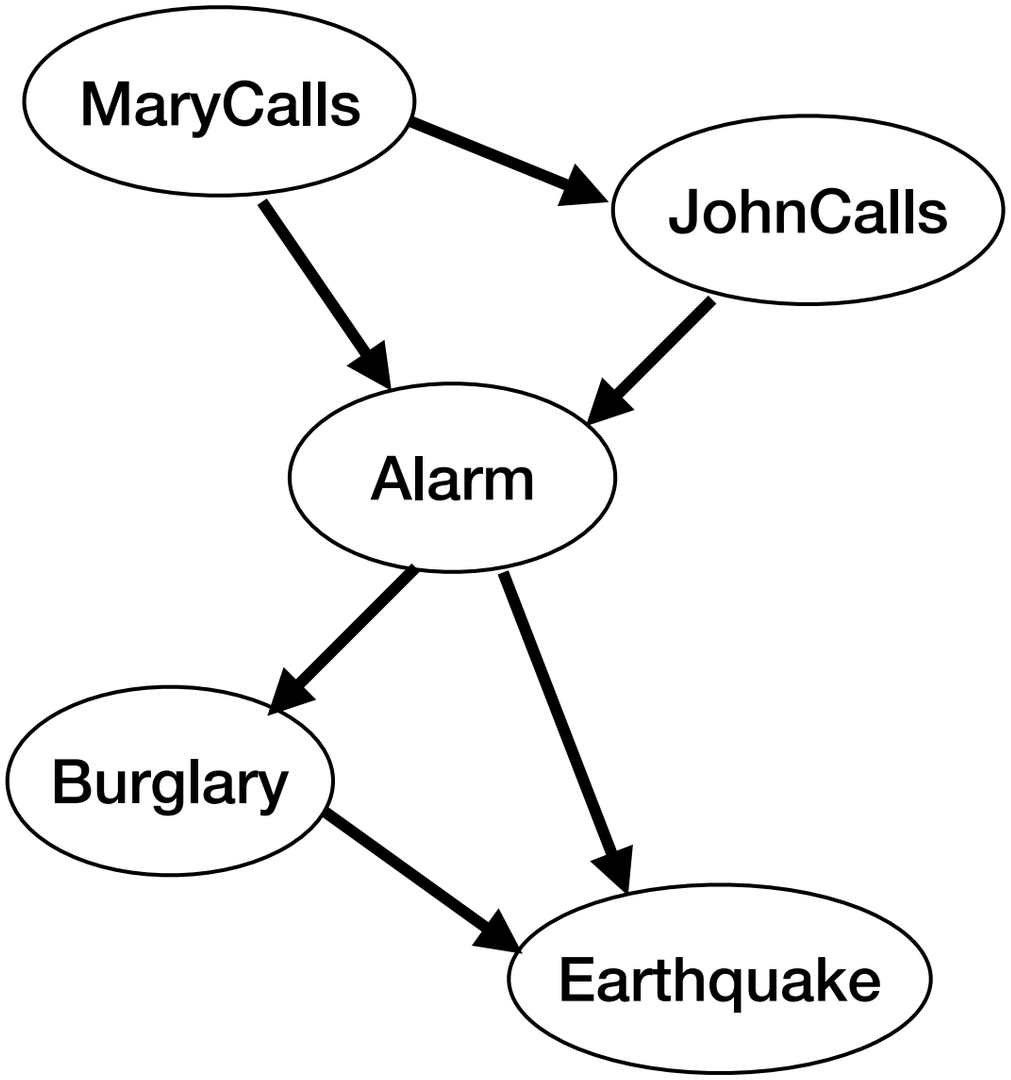
- Structure learning is much more difficult computationally than parameter learning
- The objective function includes some type of regularization to favor simple graphs (BIC, AIC, etc.)
- Finding the optimal graph structure *S* often involves a huge combinatorial search problem over structures, and we need to be careful not to introduce cycles.
- We usually have to resort to heuristic search methods (such as greedy proposal for adding, removing, or switching the direction of edges).
- Data can include both observations and (optionally) interventions.

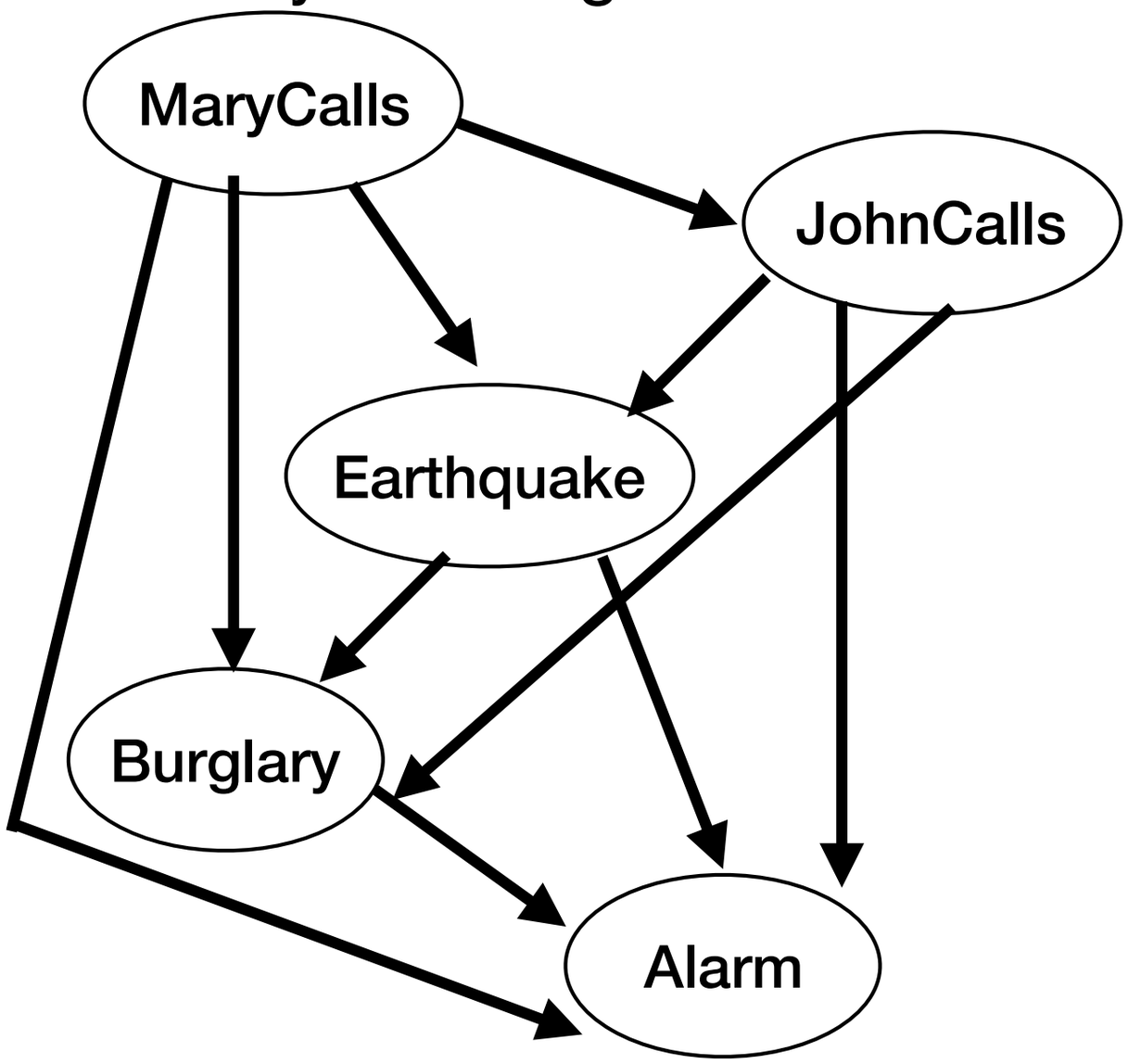
example proposal to add an edge



Learning Bayesian networks: Structure learning

We can also search over “node orders”, where network structure is determined by ordering





- These networks can represent the same probability distribution as the original alarm network, but are much clumsier and require more parameters (node order is indicated by height on the slide)
- If **we get the causal structure wrong**, we can easily overfit when learning the parameters and make bad inferences.

A Theory of Causal Learning in Children: Causal Maps and Bayes Nets

Alison Gopnik
University of California, Berkeley

Clark Glymour
Carnegie Mellon University and
Institute for Human and Machine Cognition

David M. Sobel
Brown University

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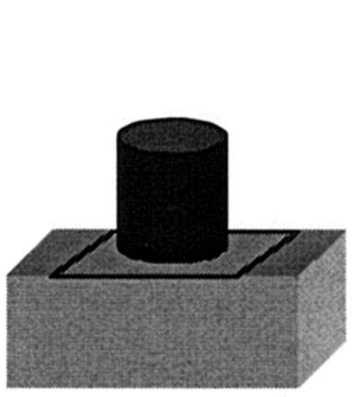
The authors outline a cognitive and computational account of causal learning in children. They propose that children use specialized cognitive systems that allow them to recover an accurate "causal map" of the world: an abstract, coherent, learned representation of the causal relations among events. This kind of knowledge can be perspicuously understood in terms of the formalism of directed graphical causal models, or Bayes nets. Children's causal learning and inference may involve computations similar to those for learning causal Bayes nets and for predicting with them. Experimental results suggest that 2- to 4-year-old children construct new causal maps and that their learning is consistent with the Bayes net formalism.

The input that reaches children from the world is concrete, particular, and limited. Yet, adults have abstract, coherent, and largely veridical representations of the world. The great epistemological question of cognitive development is how human beings get from one place to the other: How do children learn so much about the world so quickly and effortlessly? In the past 30 years, cognitive developmentalists have demonstrated that there are systematic changes in children's knowledge of the world. However, psychologists know much less about the representations that underlie that knowledge and the learning mechanisms that underlie changes in that knowledge.

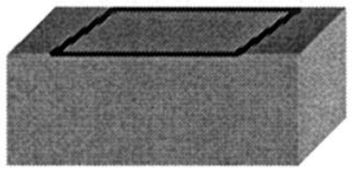
In this article, we outline one type of representation and several related types of learning mechanisms that may play a particularly important role in cognitive development. The representations are of the causal structure of the world, and the learning mechanisms involve a particularly powerful type of causal inference. Causal knowledge is important for several reasons. Knowing about causal structure permits us to make wide-ranging predictions about future events. Even more important, knowing about causal structure allows us to intervene in the world to bring about new events—often events that are far removed from the interventions themselves.

Children can make novel interventions based on causal hypotheses

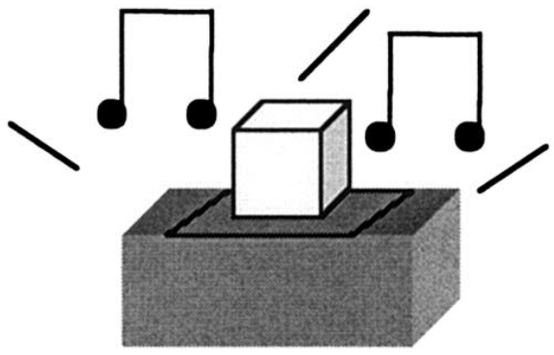
"Blicket detector" studies by Gopnik and colleagues



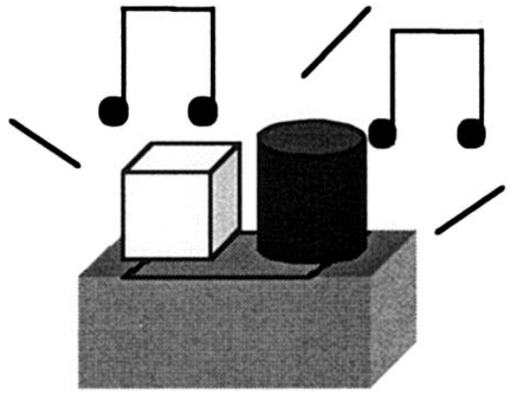
Object B is placed on the detector and nothing happens



Object B is removed



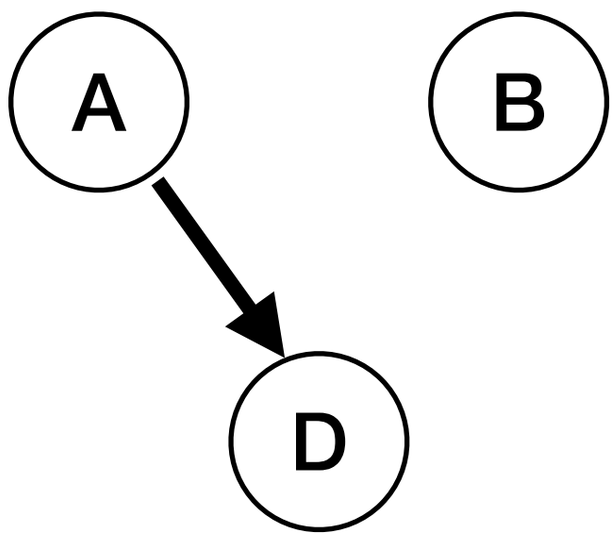
Object A is placed on the detector by itself and the detector activates



Object B is added to the detector with Object A. The detector continues to activate. Children are asked to make it stop

Results: 75% of 3-4 year olds remove Object A from the detector (Gopnik et al., 2001)

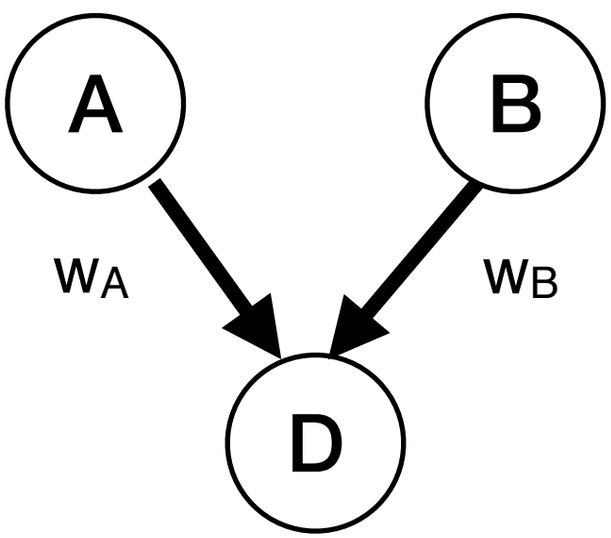
Bayesian network representation



Neural network representation

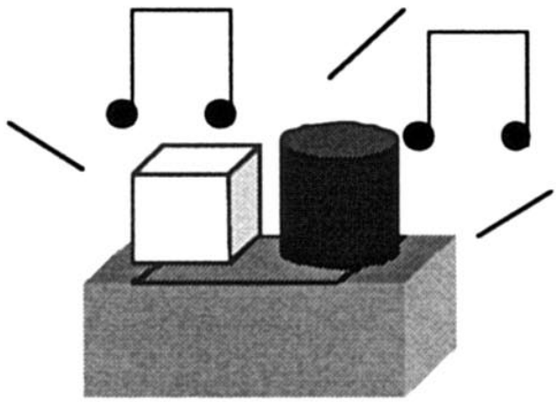
(Rescorla-Wagner model of classical conditioning)

R-W model can learn association between *A* and *D,* but it doesn't account for interventions as naturally as Bayesian networks

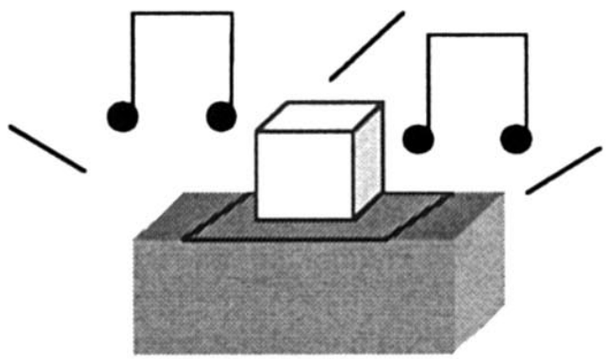


Backward blocking as evidence of causal learning

Backward blocking condition



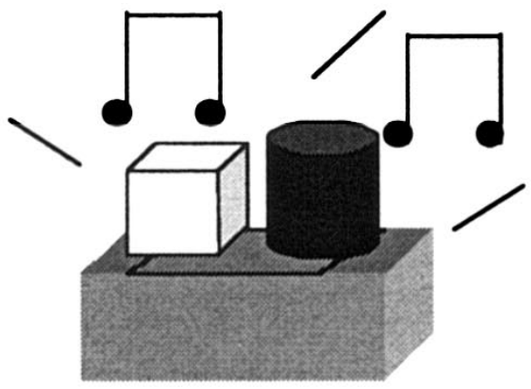
Both objects activate the detector



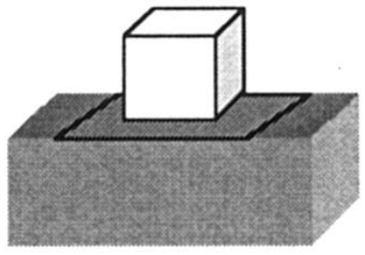
Object A activates the detector by itself

Children are asked if each is a blicket, then they are asked to make the machine go

Control condition



Both objects activate the detector

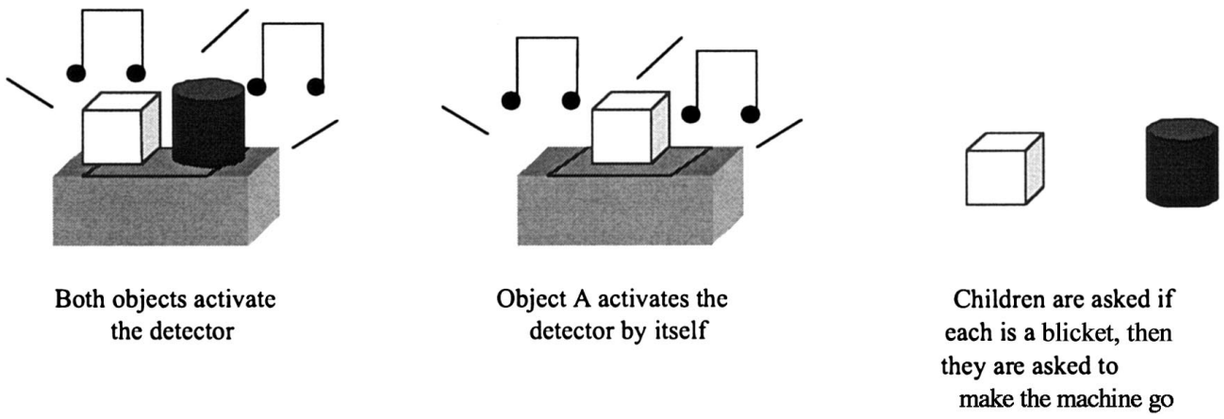


Object A does not activate the detector by itself

Children are asked if each is a blicket, then they are asked to make the machine go

3-4 year old children categorized **Object B as a blicket only 31% of the time** in backward blocking condition, but 100% of the time in control (Sobel et al., 2004)

Backward blocking as Bayesian structure learning



Both objects activate the detector

Object A activates the detector by itself

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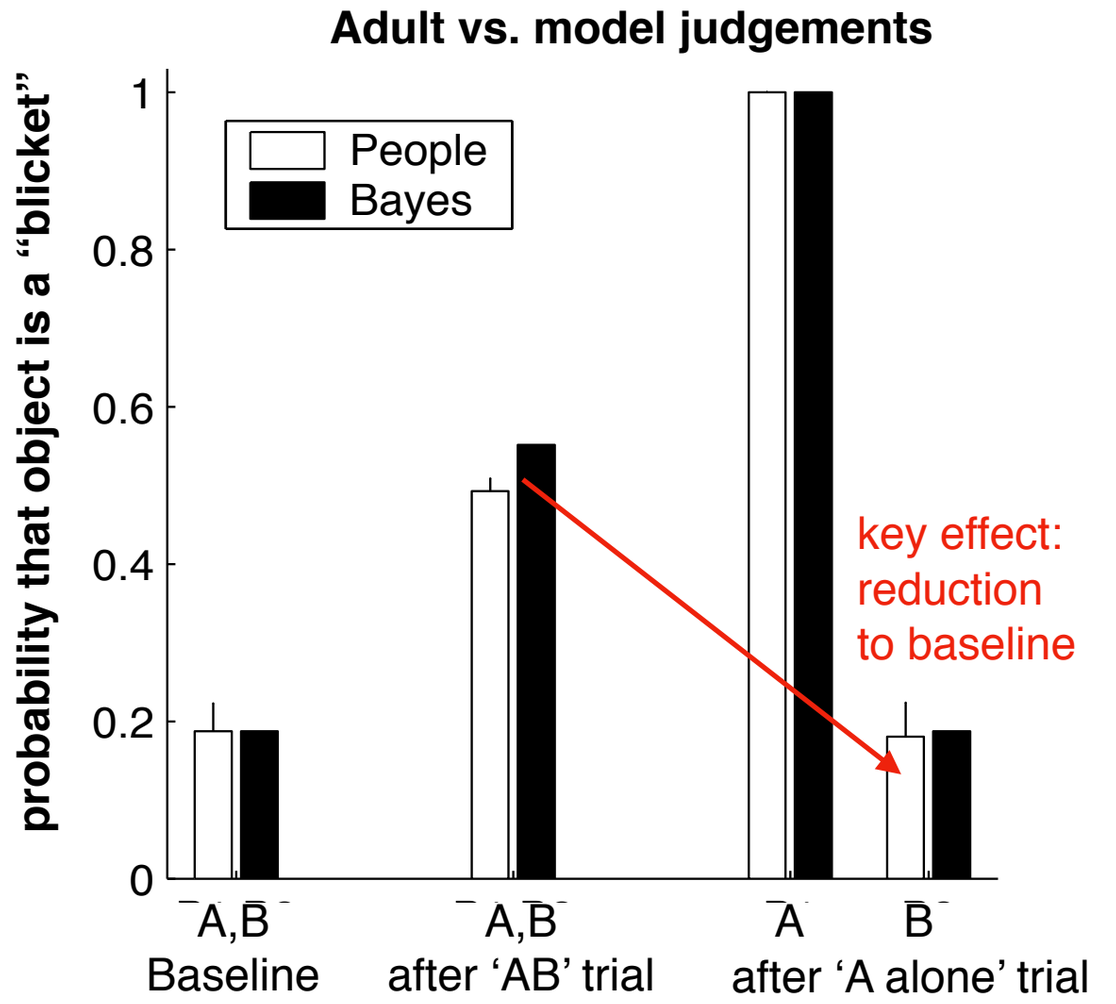
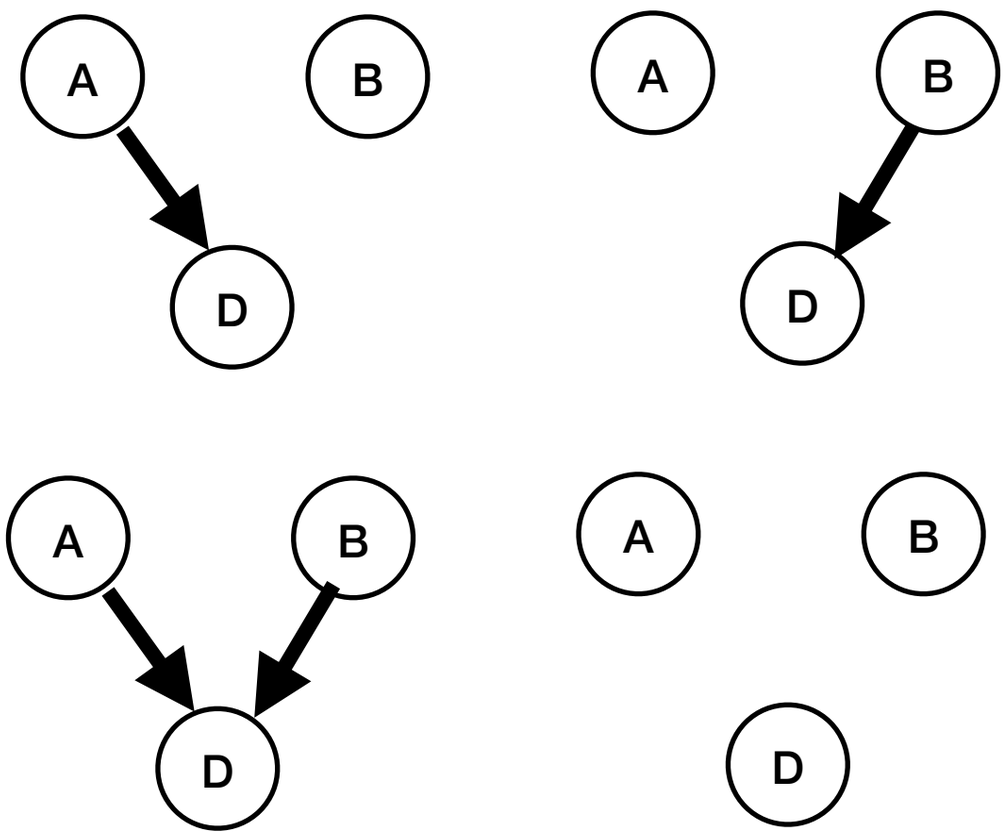


(Tenenbaum & Griffiths, 2003)

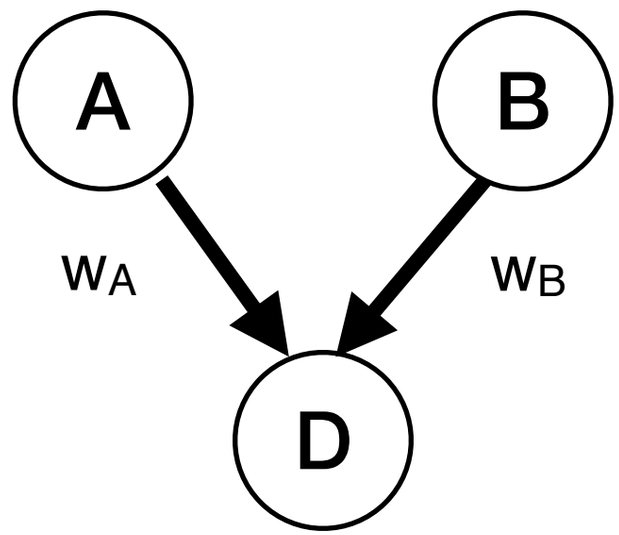
Four hypotheses for Bayesian structure learning

(prior favors fewer edges)



Neural network representation

Neural nets only account for decrease to baseline with particular input encoding schemes, where it comes naturally from structure learning.



Structure learning and semantic cognition

PNAS

The discovery of structural form

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Edited by Richard M. Shiffrin, Indiana University, Bloomington, IN, and approved May 30, 2008 (received for review March 17, 2008)

Algorithms for finding structure in data have become increasingly important both as tools for scientific data analysis and as models of human learning, yet they suffer from a critical limitation. Scientists discover qualitatively new forms of structure in observed data: For instance, Linnaeus recognized the hierarchical organization of biological species, and Mendeleev recognized the periodic structure of the chemical elements. Analogous insights play a pivotal role in cognitive development: Children discover that object category labels can be organized into hierarchies, friendship networks are organized into cliques, and comparative relations (e.g., ''bigger than'' or ''better than'') respect a transitive order. Standard algorithms, however, can only learn structures of a single form that must be specified in advance: For instance, algorithms for hierarchical clustering create tree structures, whereas algorithms for dimensionality-reduction create low-dimensional spaces. Here, we present a computational model that learns structures of many different forms and that discovers which form is best for a given dataset. The model makes probabilistic inferences over a space of graph grammars representing trees, linear orders, multidimensional spaces, rings, dominance hierarchies, cliques, and other forms and successfully discovers the underlying structure of a variety of physical, biological, and social domains. Our approach brings structure learning methods closer to human abilities and may lead to a deeper computational understanding of cognitive development.

cognitive development | structure discovery | unsupervised learning

Discovering the underlying structure of a set of entities is a fundamental challenge for scientists and children alike. Higher-level discoveries about structural form are rarer but more fundamental, and often occur at pivotal moments in the development of a scientific field or a child's understanding (1, 2, 4). For centuries, the natural representation for biological

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Structured Statistical Models of Inductive Reasoning

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Everyday inductive inferences are often guided by rich background knowledge. Formal models of induction should aim to incorporate this knowledge and should explain how different kinds of knowledge lead to the distinctive patterns of reasoning found in different inductive contexts. This article presents a Bayesian framework that attempts to meet both goals and describe 4 applications of the framework: a taxonomic model, a spatial model, a threshold model, and a causal model. Each model makes probabilistic inferences about the extensions of novel properties, but the priors for the 4 models are defined over different kinds of structures that capture different relationships between the categories in a domain. The framework therefore shows how statistical inference can operate over structured background knowledge, and the authors argue that this interaction between structure and statistics is critical for explaining the power and flexibility of human reasoning.

Keywords: inductive reasoning, property induction, knowledge representation, Bayesian inference

Humans are adept at making inferences that take them beyond the limits of their direct experience. Even young children can learn the meaning of a novel word from a single labeled example (Heibeck & Markman, 1987), predict the trajectory of a moving object when it passes behind an occluder (Spelke, 1990), and choose a gait that allows them to walk over terrain they have never before encountered. Inferences like these may differ in many respects, but common to them all is the need to go beyond the information given (Bruner, 1973).

This article describes a formal approach to inductive inference that should apply to many different problems, but we focus on the problem of property induction (Sloman & Lagnado, 2005). In particular, we consider cases where one or more categories in a domain are observed to have a novel property and the inductive task is to predict how the property is distributed over the remaining categories in the domain. For instance, given that bears have sesamoid bones, which species is more likely to share this property: moose or salmon (Osherson, Smith, Wilkie, Lopez, & Shafir,

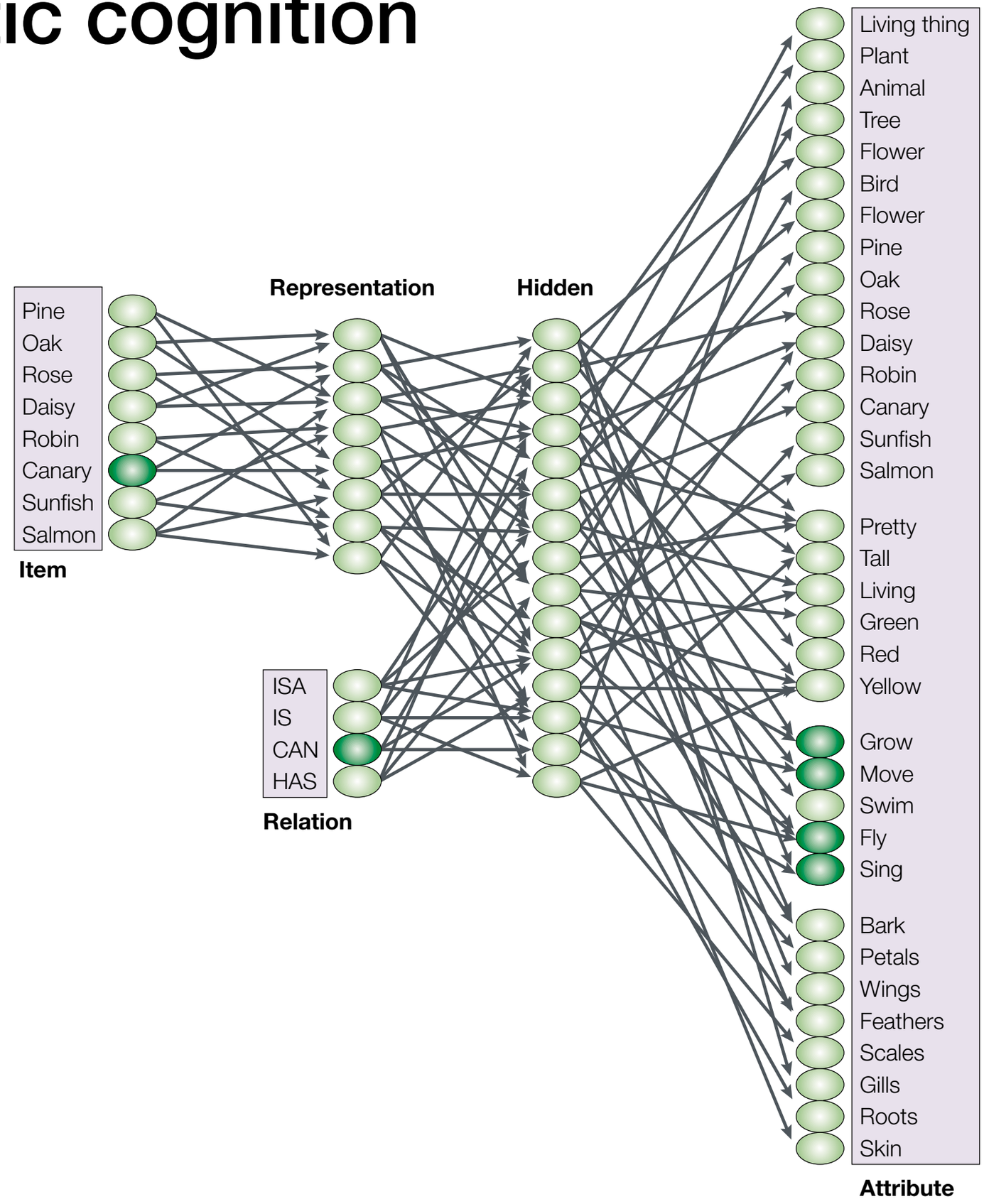
Review: A neural network model of semantic cognition

- Network is trained to answer queries involving an **item** (e.g., “Canary”) and a **relation** (e.g., “CAN”), outputting all **attributes** that are true of the item/ relation pair (e.g., “grow, move, fly, sing”)

- Trained with stochastic gradient descent, as we learned about in this lecture

- The model helps us to understand the broad-to-specific pattern of differentiation in children’s cognitive development

- It also helps us to understand the specific-to-general deterioration in semantic dementia



Alternative: Property induction as probabilistic inference in a probabilistic graphical model

Question: "Given that cows and seals have T9 hormones, how likely is it that horses do?"

property induction as probabilistic inference:

$$P(f_Y = 1 | f_X = 1)$$

f : T9 hormones

$Y = \{\text{horses}\}$

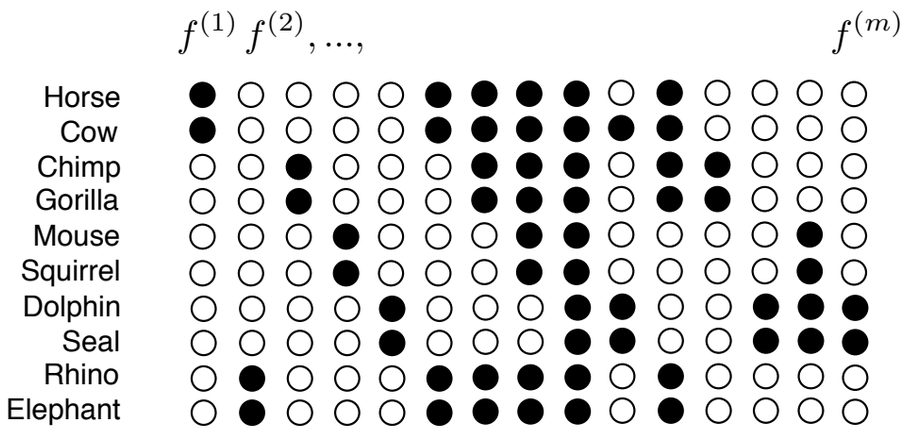
$X = \{\text{cows, seals}\}$

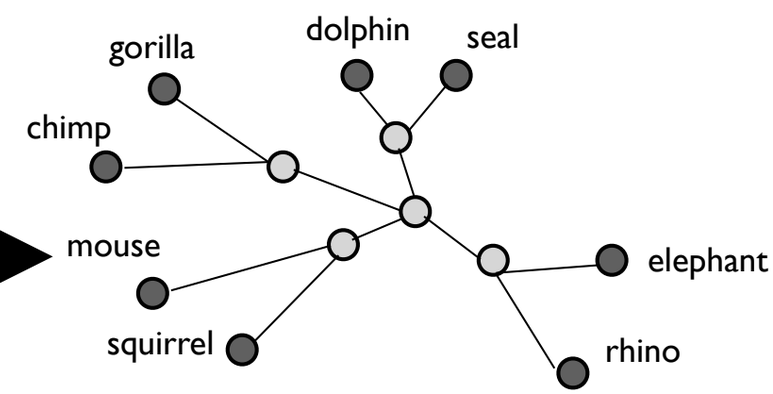
Bayesian modeling roadmap

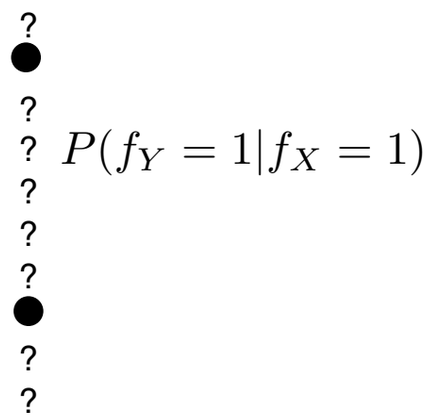
background data

graphical model structure learning

property induction as prob. inference



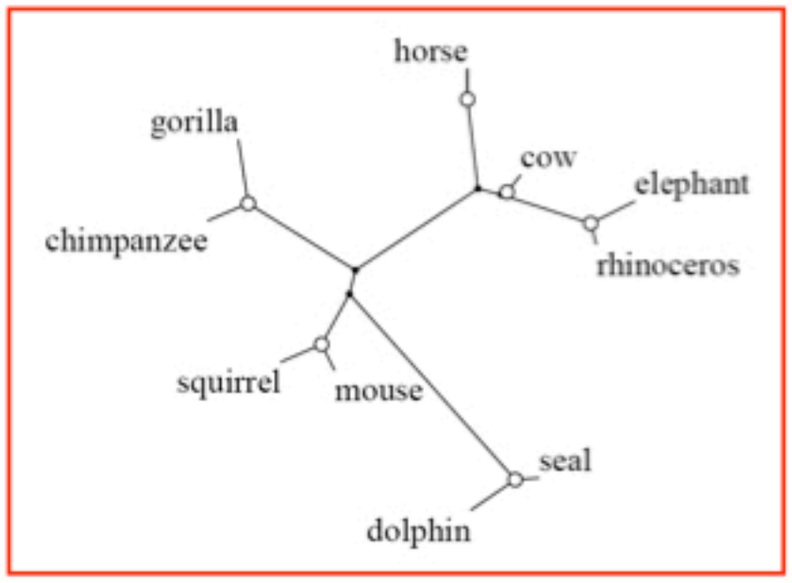




Features for Elephant: 'gray', 'hairless', 'toughskin', 'big', 'bulbous', 'longleg', 'tail', 'chewteeth', 'tusks', 'smelly', 'walks', 'slow', 'strong', 'muscle', 'fourlegs',…

Biological reasoning about animals

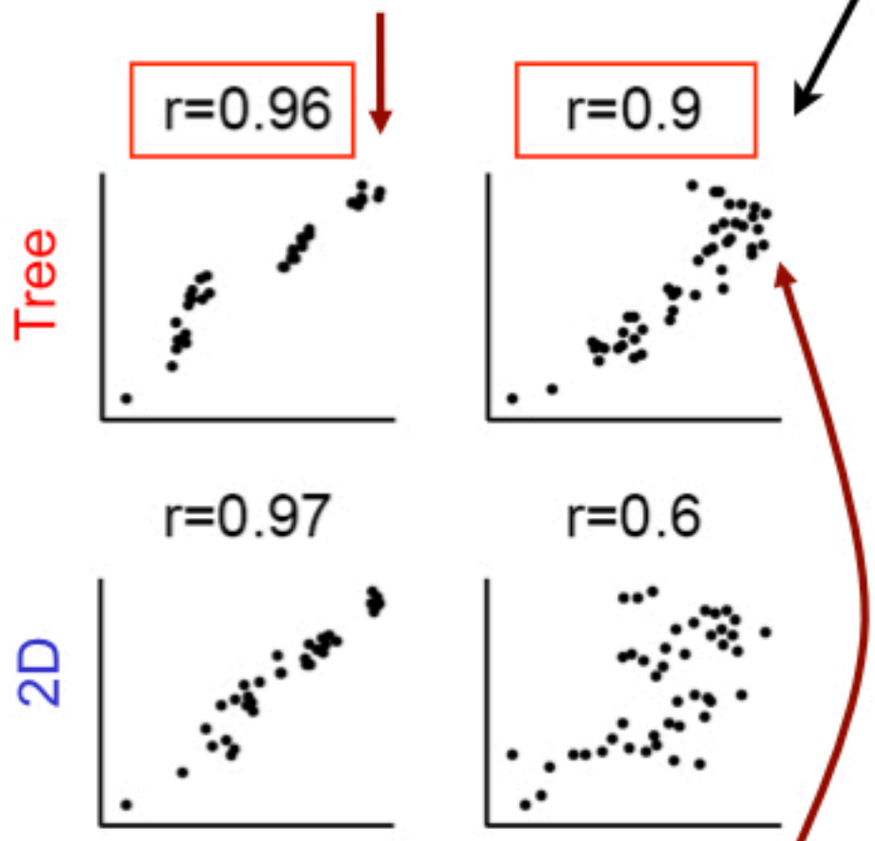
A tree fits better than a 2D space



Cows have property P.
Elephants have property P.

Horses have property P.

Correlation of human participant judgments with model judgements



Evaluated across a range of difference premise/conclusion combinations .

Gorillas have property P.
Mice have property P.
Seals have property P.

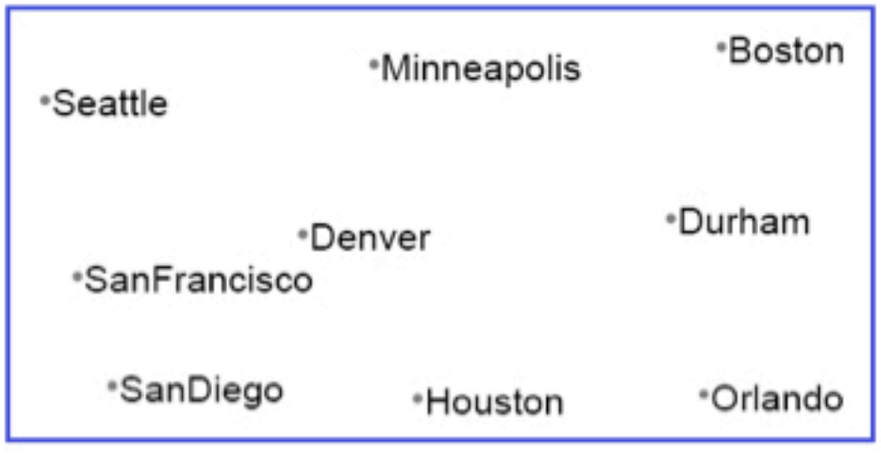
All mammals have property P.

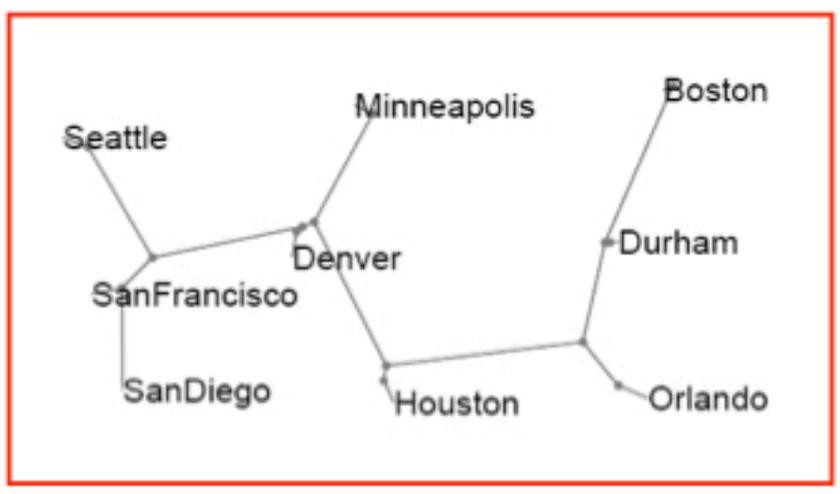
slide credit: Josh Tenenbaum

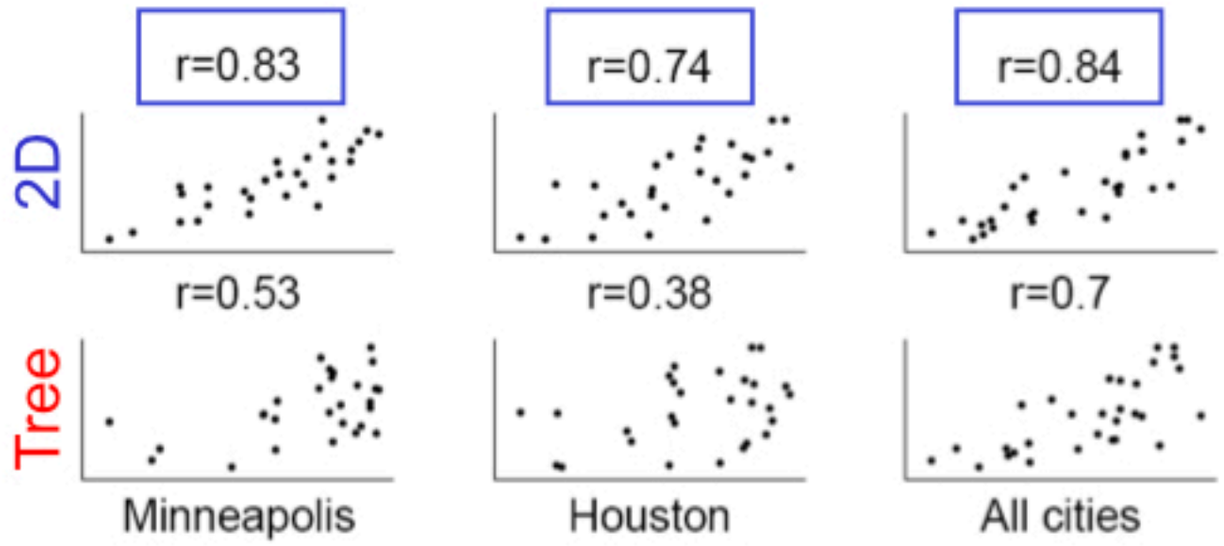
Spatial reasoning about cities

A 2D space fits better than a tree

“Given that a certain kind of native American artifact has been found in sites near city X, how likely is the same artifact to be found near city Y?"







slide credit: Josh Tenenbaum

Learning structural forms

How do we know what the right form is?

People can discover intuitive structural forms:

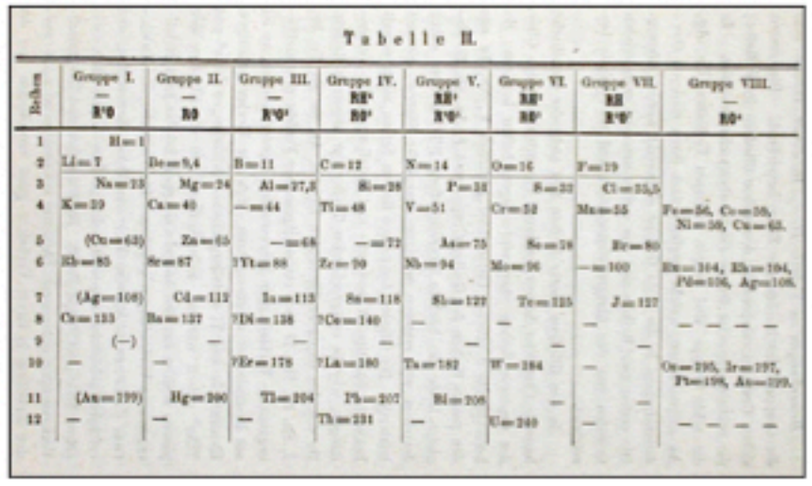
Famous examples in Science

Linnaeus

Kingdom Animalia
Phylum Chordata
Class Mammalia
Order Primates
Family Hominidae
Genus Homo
Species *Homo sapiens*

Darwin

Mendeleev



Examples from childhood

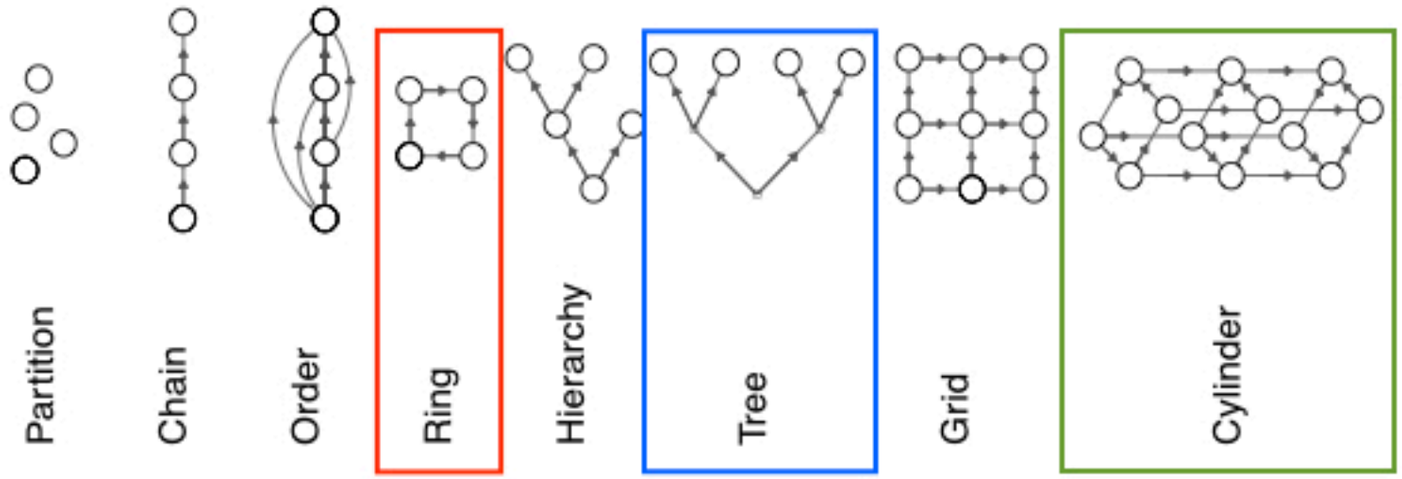
- e.g., days of the week form a cycle, social networks are cliques, comparative relations are transitive, names can be organized in taxonomies

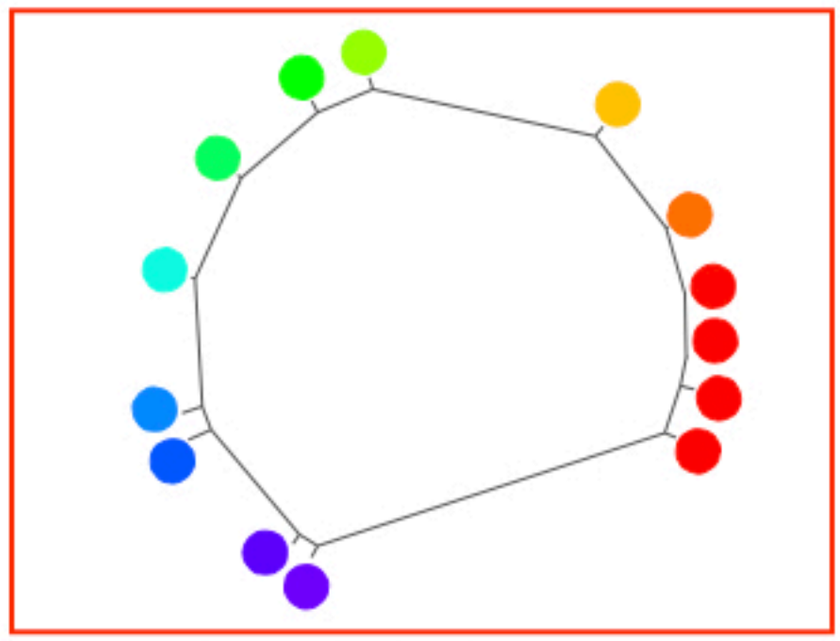
slide credit: Josh Tenenbaum

Learning structural forms

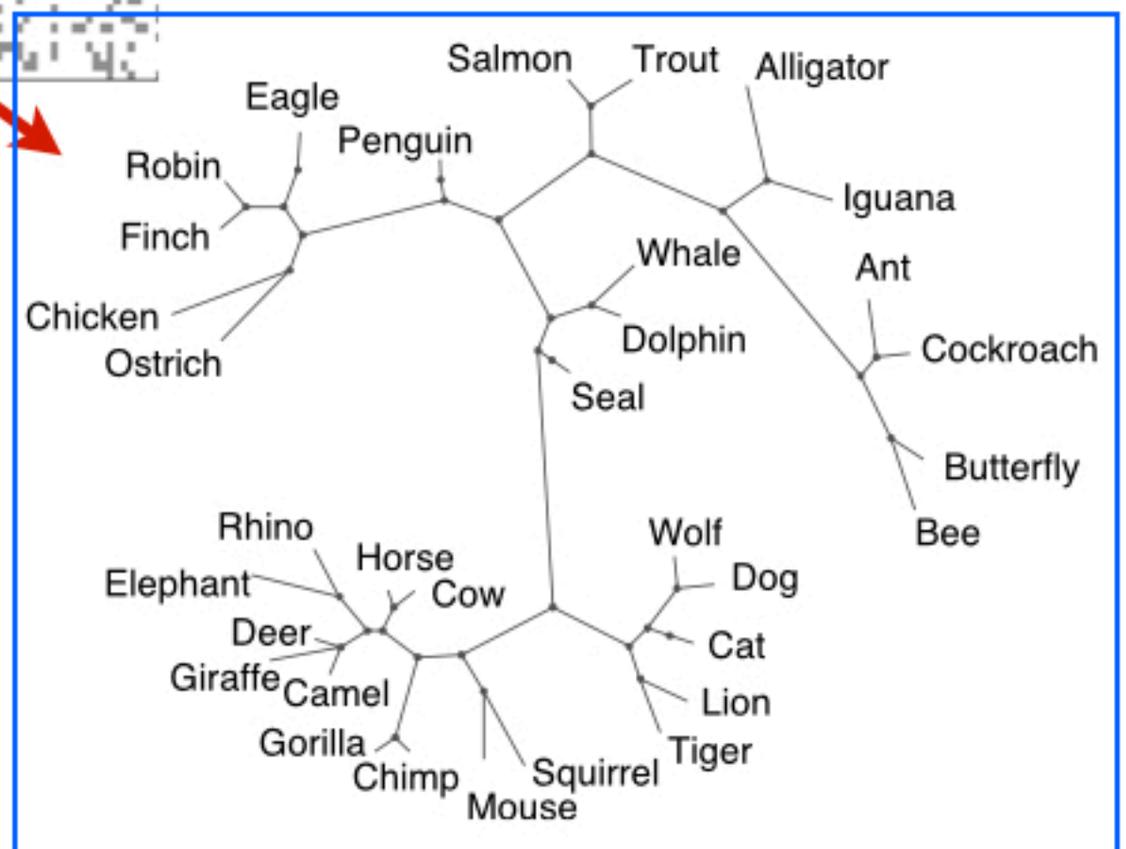
Kemp & Tenenbaum (2008). The discovery of structural form. *PNAS.*

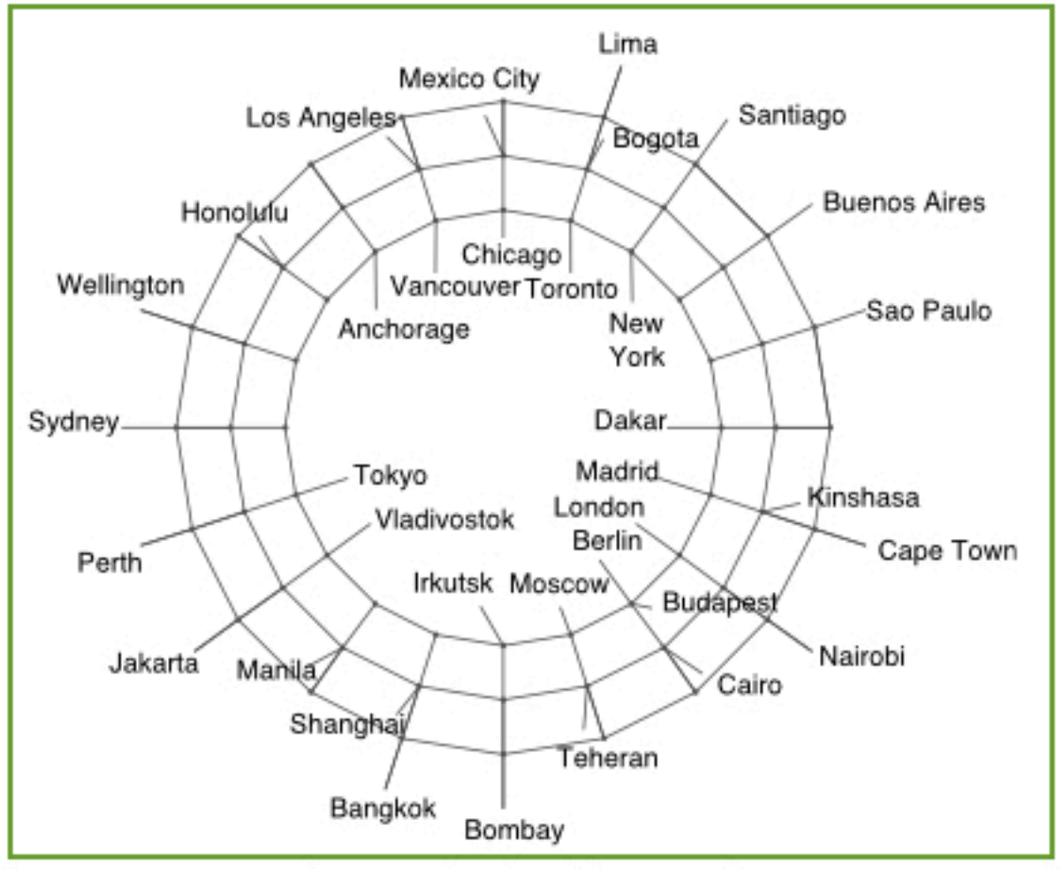
Structures available for selection





given pairwise similarity data



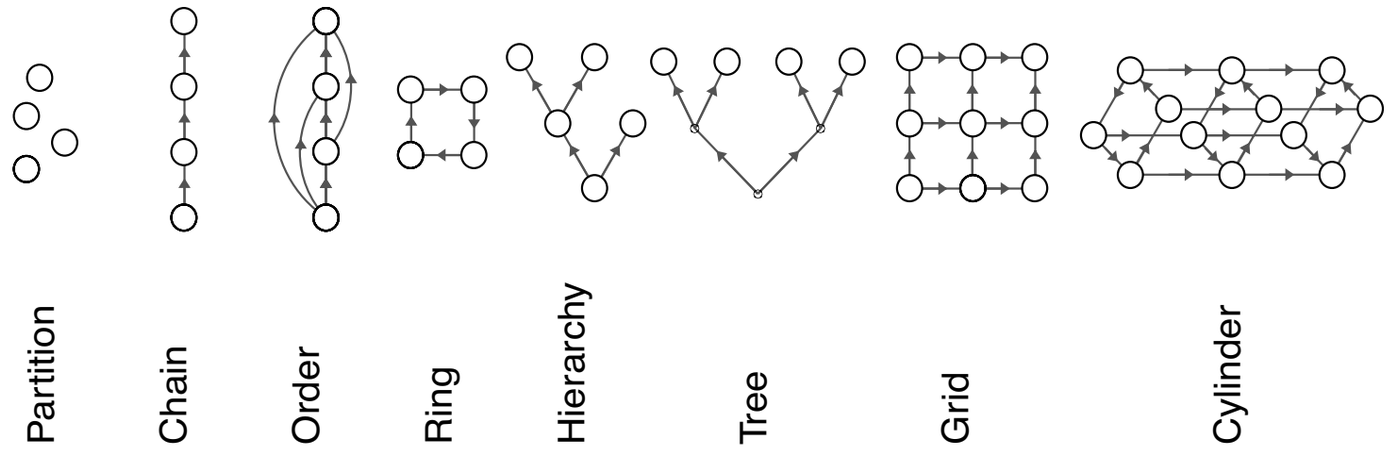


given pairwise distance data

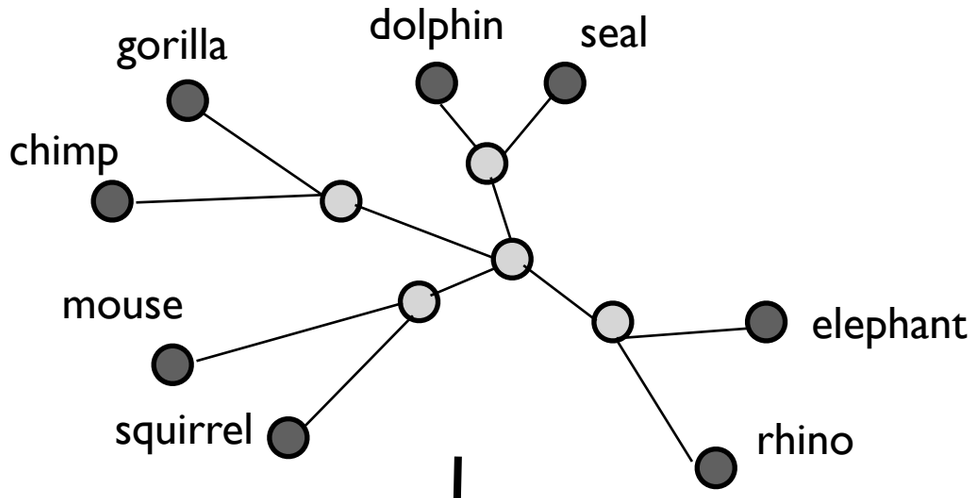
Bayesian structural forms model

Kemp & Tenenbaum (2008)

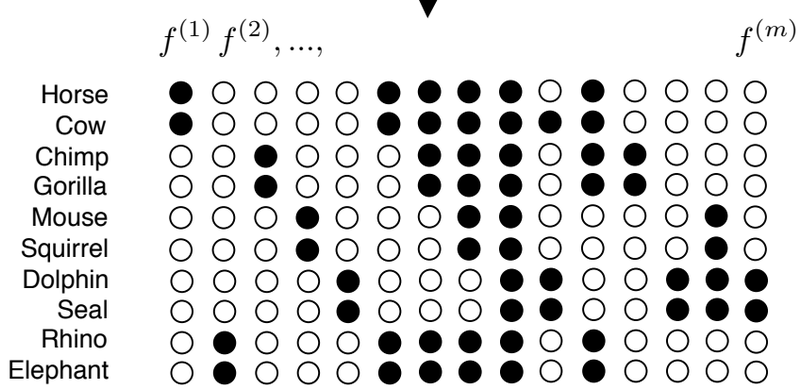
Form F

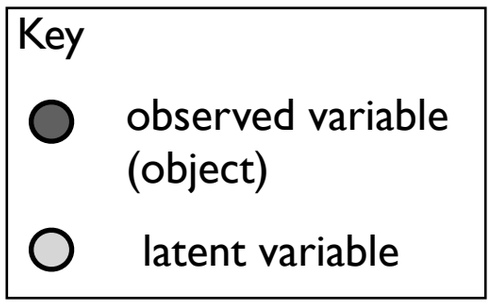


Structure S



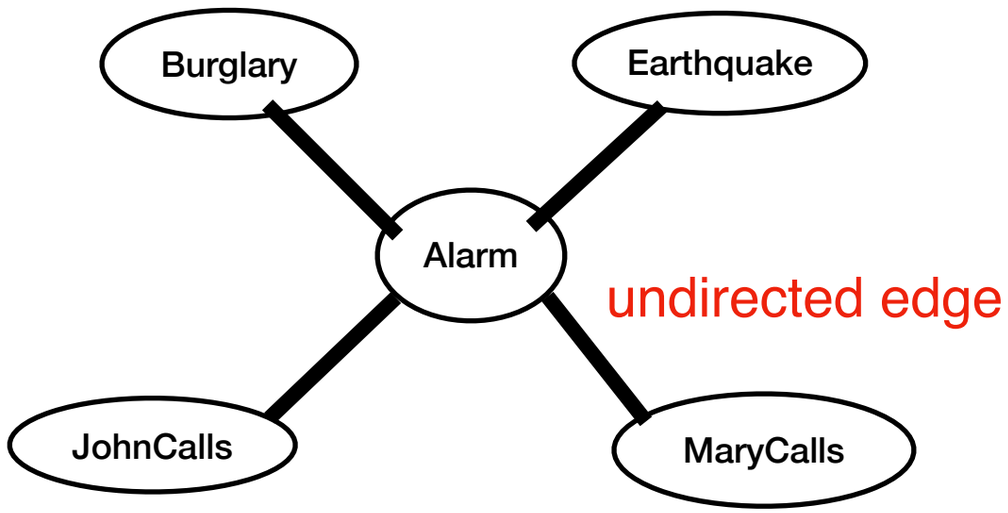
Data $f^{(k)}$





(Markov random field)

Very brief intro. to undirected graphical models



- Also known as Markov Random Fields or Markov Networks
- Bayesian networks are better suited for representing causal processes, while Markov networks are better suited for capturing soft constraints between variables

---

In the structural forms model, structure is operationalized as a Gaussian Markov Random Field, which enforces smoothness over the graph:

$$P(f^{(k)}|S) \quad \propto \quad \exp\left(-\frac{1}{4} \sum_{i,j} s_{ij} (f_i^{(k)} - f_j^{(k)})^2 \right)$$

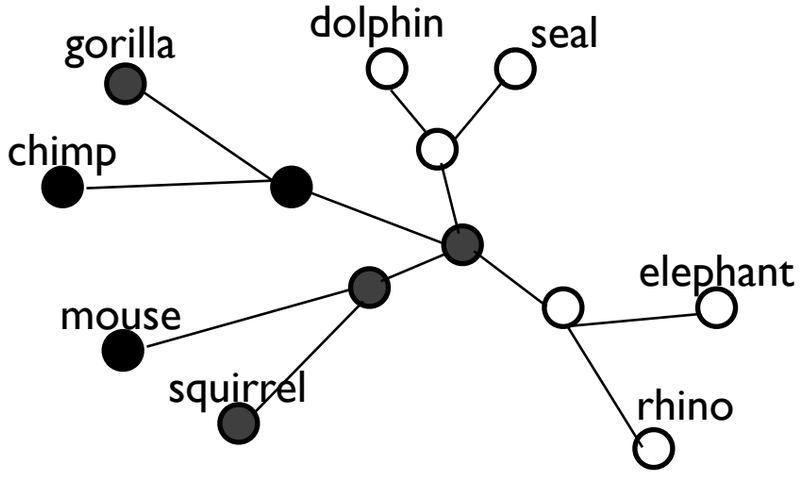
Feature $f^{(k)}$

- on
- off

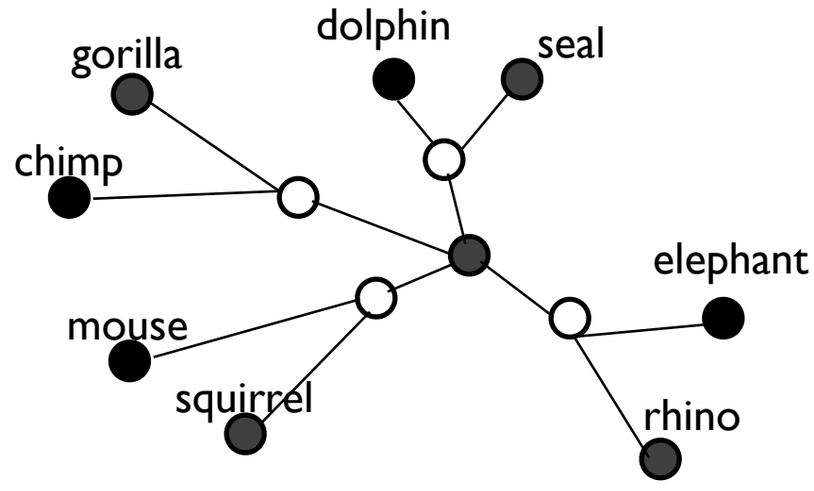
s_{ij} (weight) is non-zero for each edge in graph

*essential regularization term not shown, see paper for details

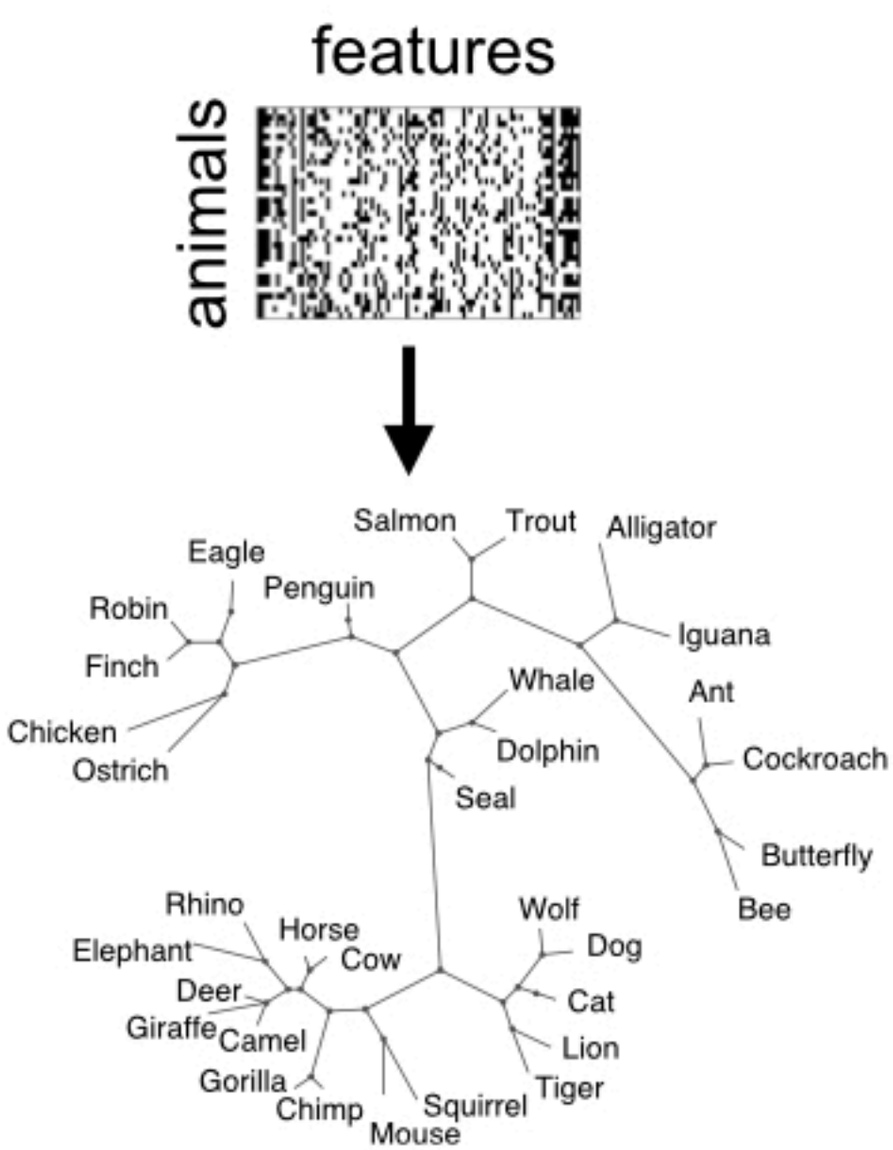
High probability

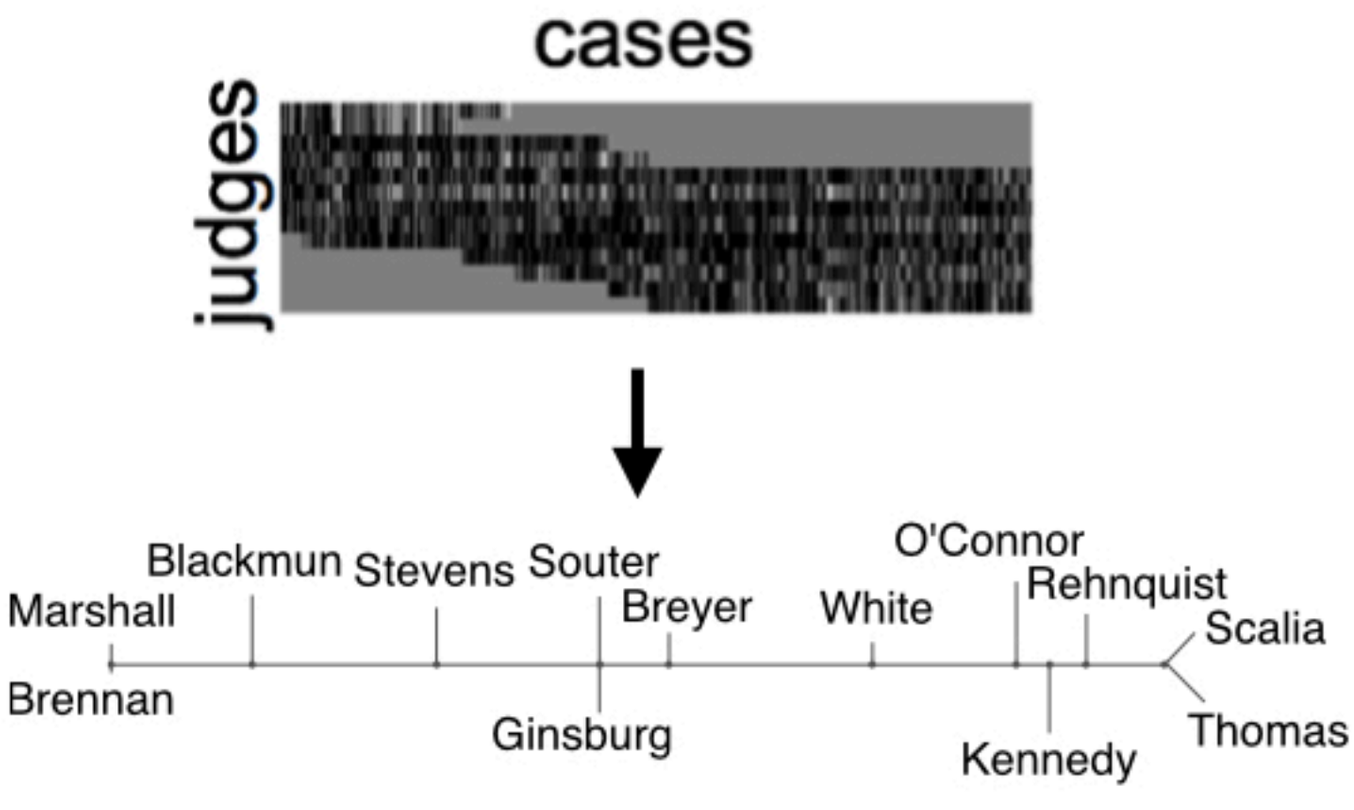


Low probability

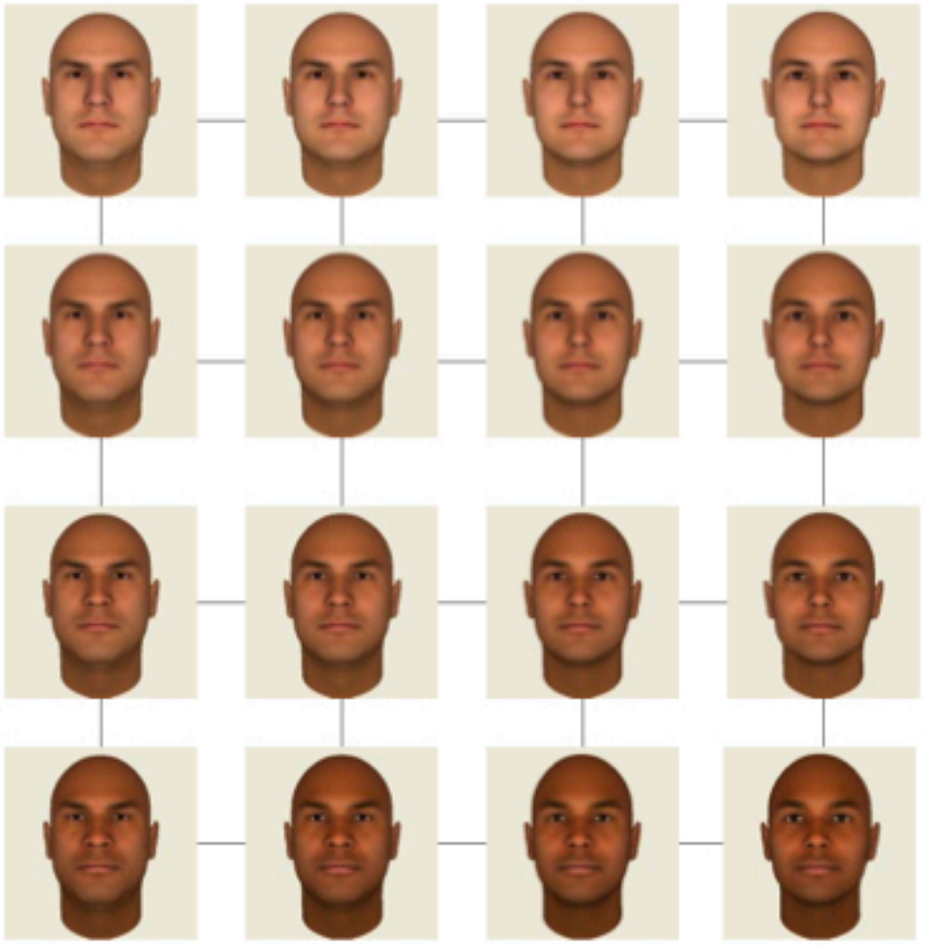


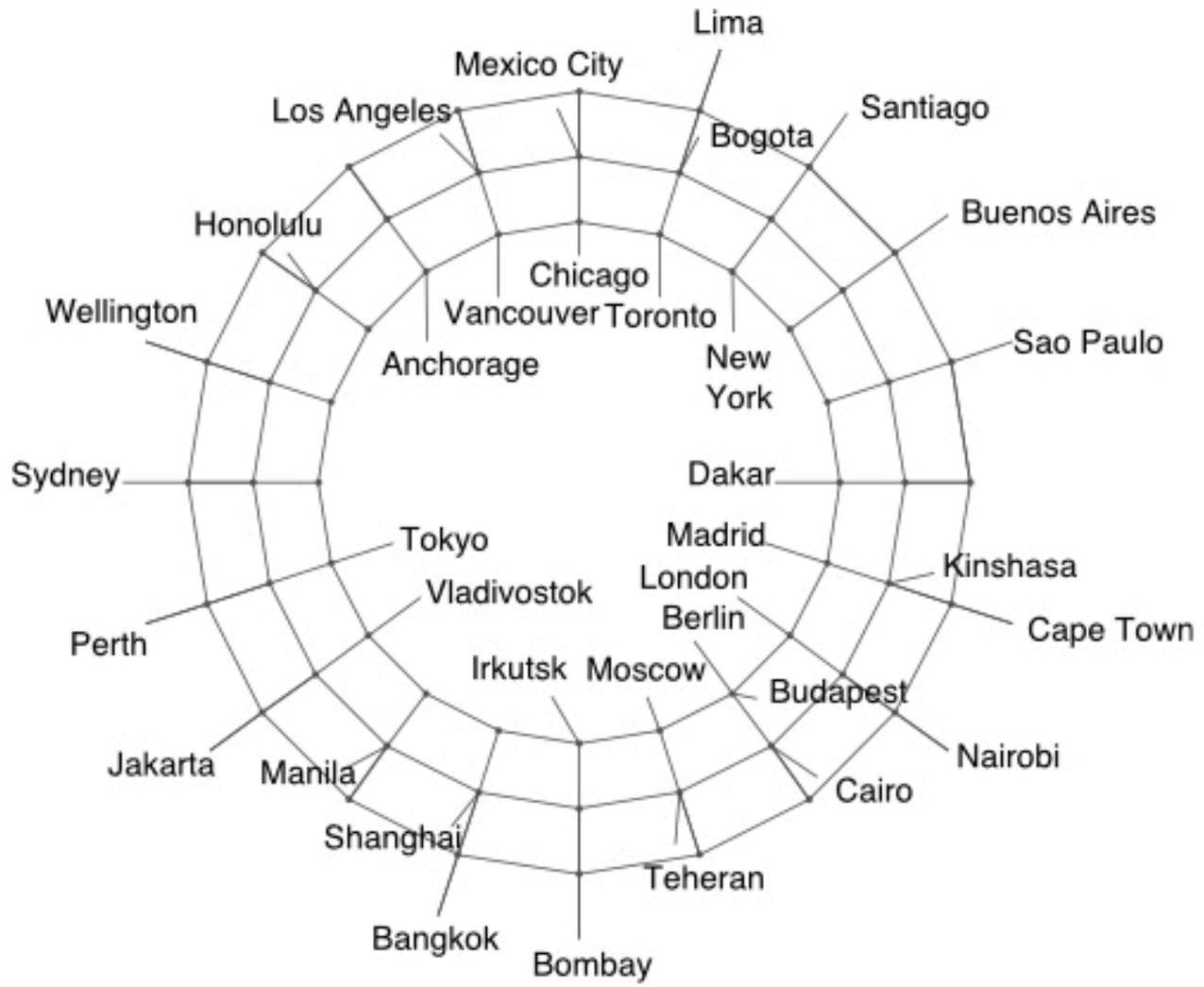
Results: Bayesian structural forms

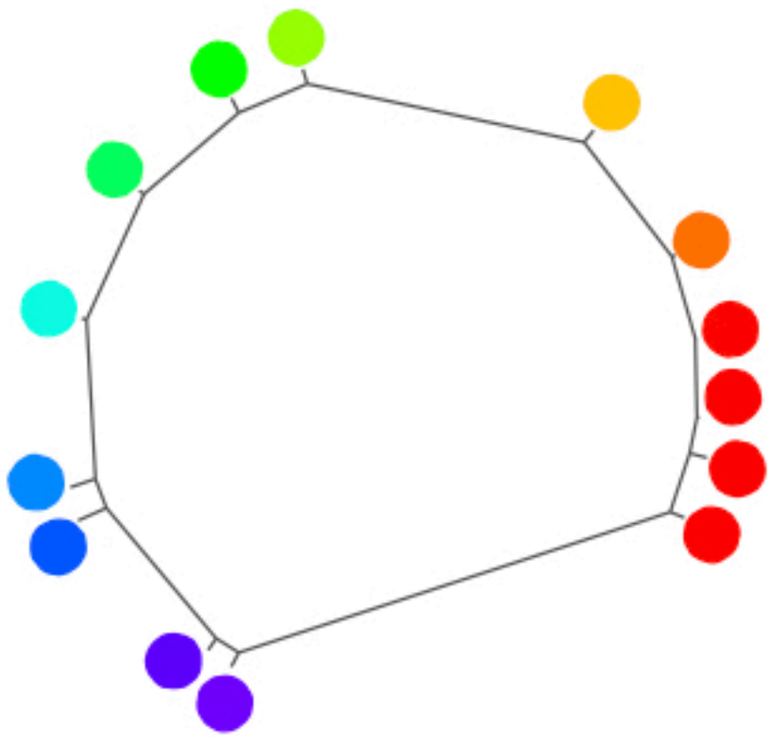


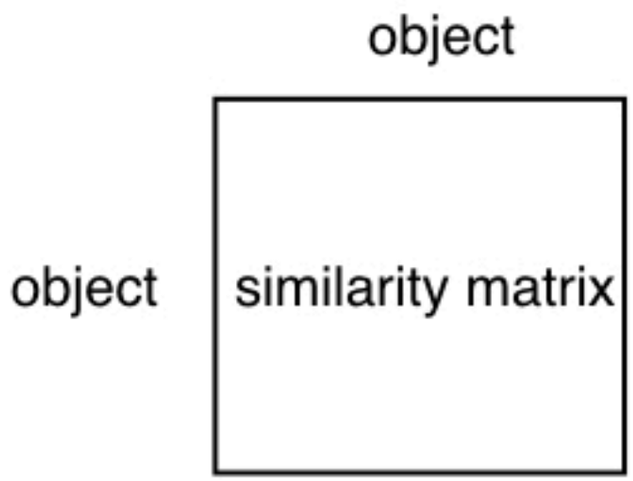


Results: Bayesian structural forms









Do we need pre-defined structural forms to make discoveries?

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The Emergence of Organizing Structure in Conceptual Representation

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Abstract

Both scientists and children make important structural discoveries, yet their computational underpinnings are not well understood. Structure discovery has previously been formalized as probabilistic inference about the right structural form—where form could be a tree, ring, chain, grid, etc. (Kemp & Tenenbaum, 2008). Although this approach can learn intuitive organizations, including a tree for animals and a ring for the color circle, it assumes a strong inductive bias that considers only these particular forms, and each form is explicitly provided as initial knowledge. Here we introduce a new computational model of how organizing structure can be discovered, utilizing a broad hypothesis space with a preference for sparse connectivity. Given that the inductive bias is more general, the model's initial knowledge shows little qualitative resemblance to some of the discoveries it supports. As a consequence, the model can also learn complex structures for domains that lack intuitive description, as well as predict human property induction judgments without explicit structural forms. By allowing form to emerge from sparsity, our approach clarifies how both the richness and flexibility of human conceptual organization can coexist.

Keywords: Structure discovery; Unsupervised learning; Bayesian modeling; Sparsity

1. Introduction

Structural discoveries play an important role in science and cognitive development (Carey, 2009; Kuhn, 1962). In biology, Linnaeus realized that living things were best organized as a tree, displacing the "great chain of being" used for centuries before.

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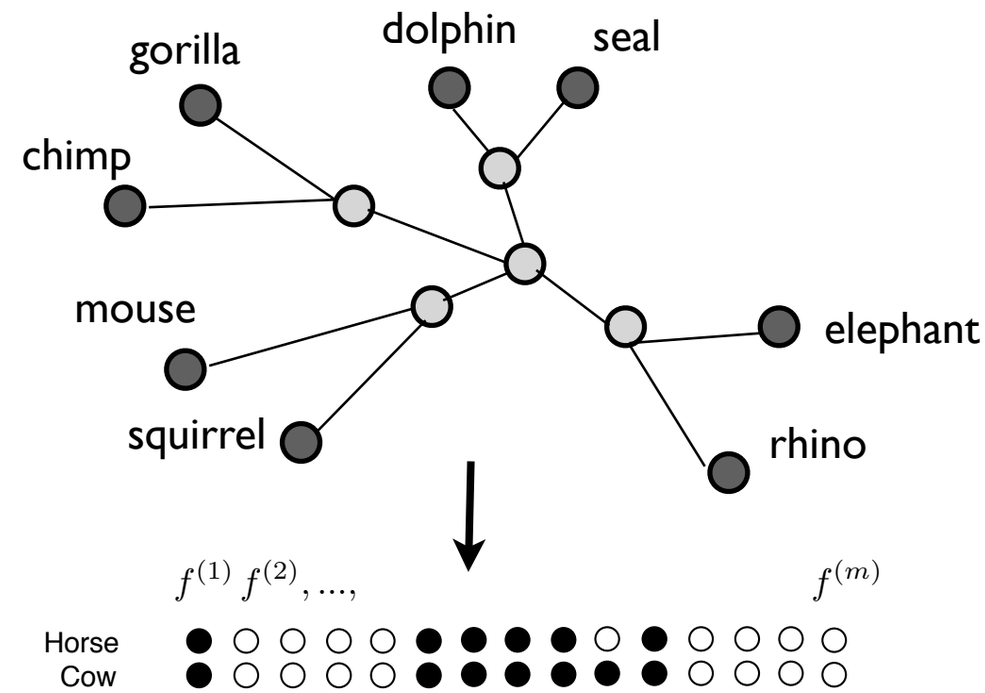
[†]This work was completed before N. Lawrence joined Amazon Research Cambridge.

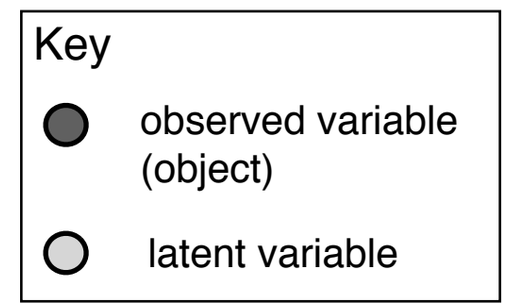
Structural sparsity model (Lake et al., 2018)

(more closely akin to traditional graphical model structure learning in machine learning)

Structure S

Data $f^{(k)}$





(Gaussian Markov Random Field with latent variables)

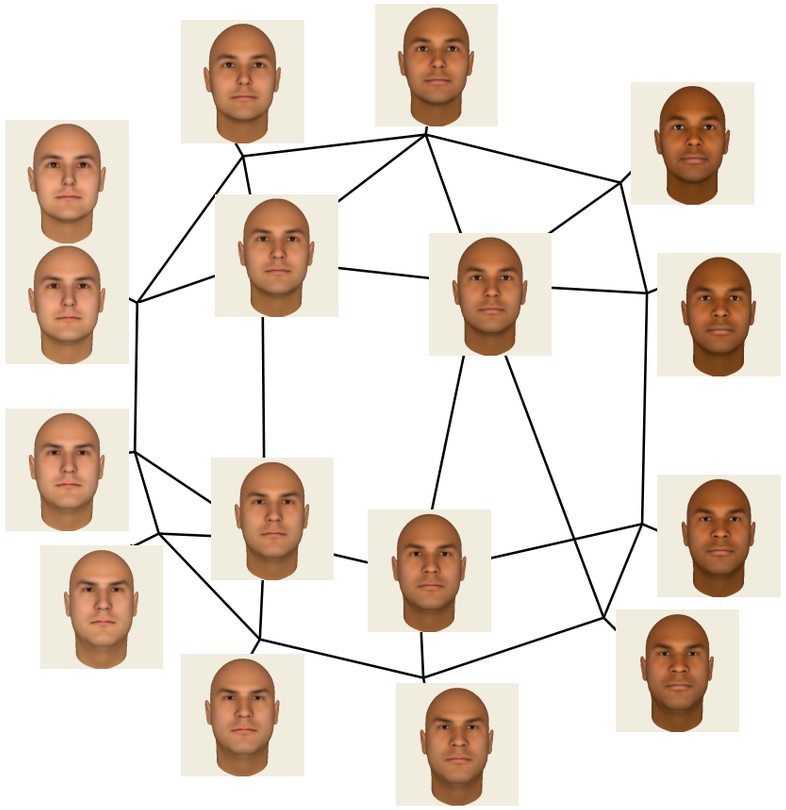
Find structure S that maximizes the objective function:

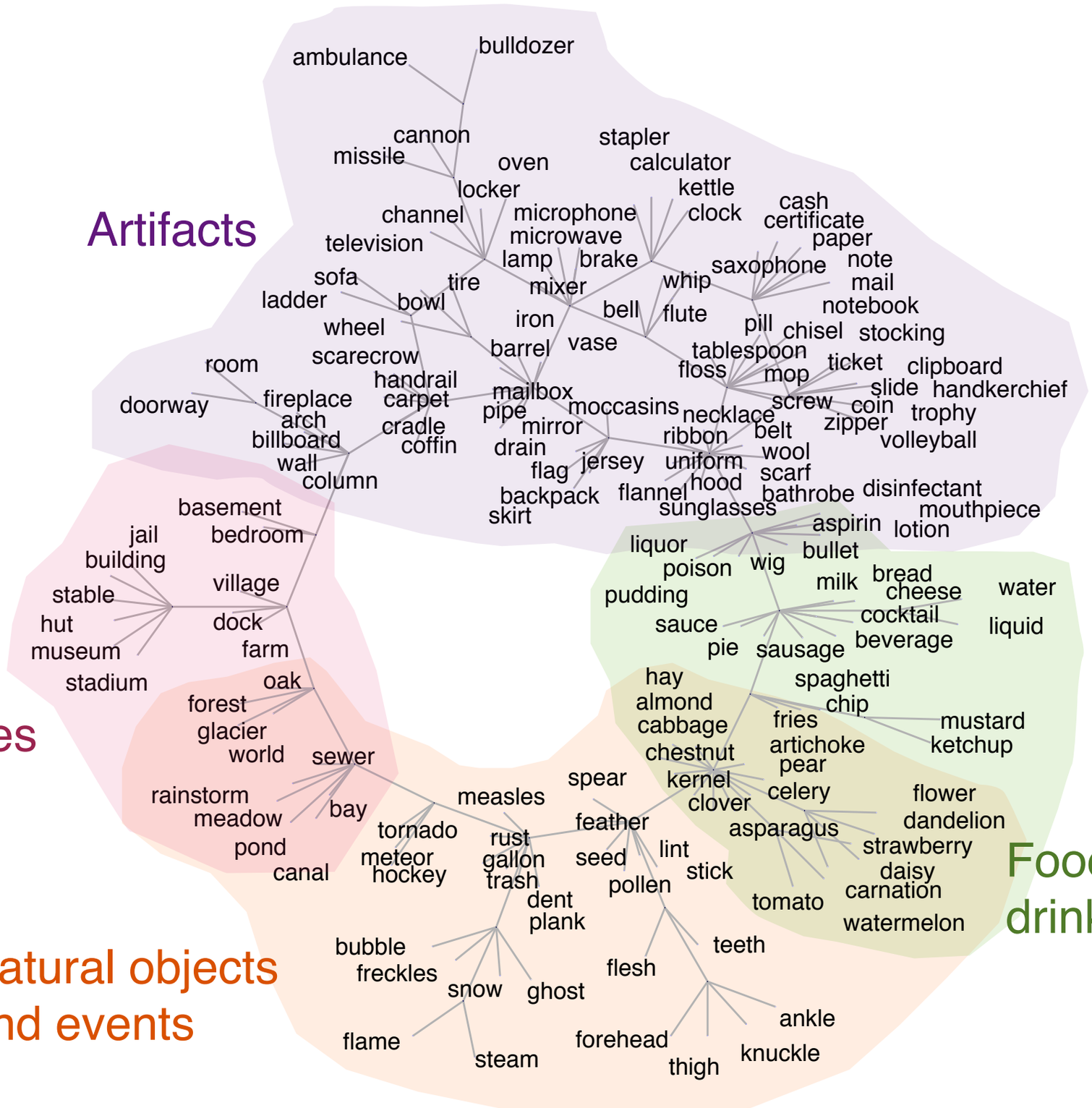
$$\underset{S}{\operatorname{argmax}}\ p(S|f^{(1)},...,f^{(k)}) \propto \prod_{i=1}^{m} p(f^{(i)}|S)p(S)$$

Likelihood favors fit to the data

Prior favors sparse graphs (fewest possible edges)

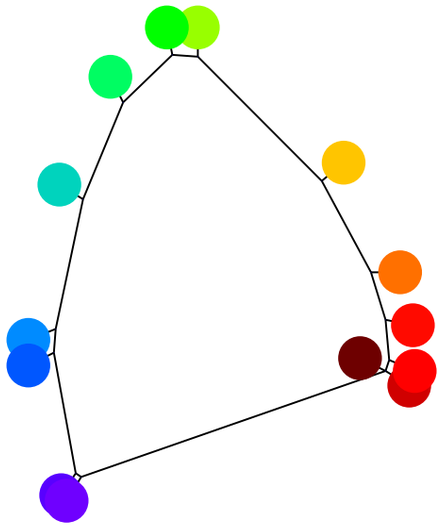
Learning complex structural organizations



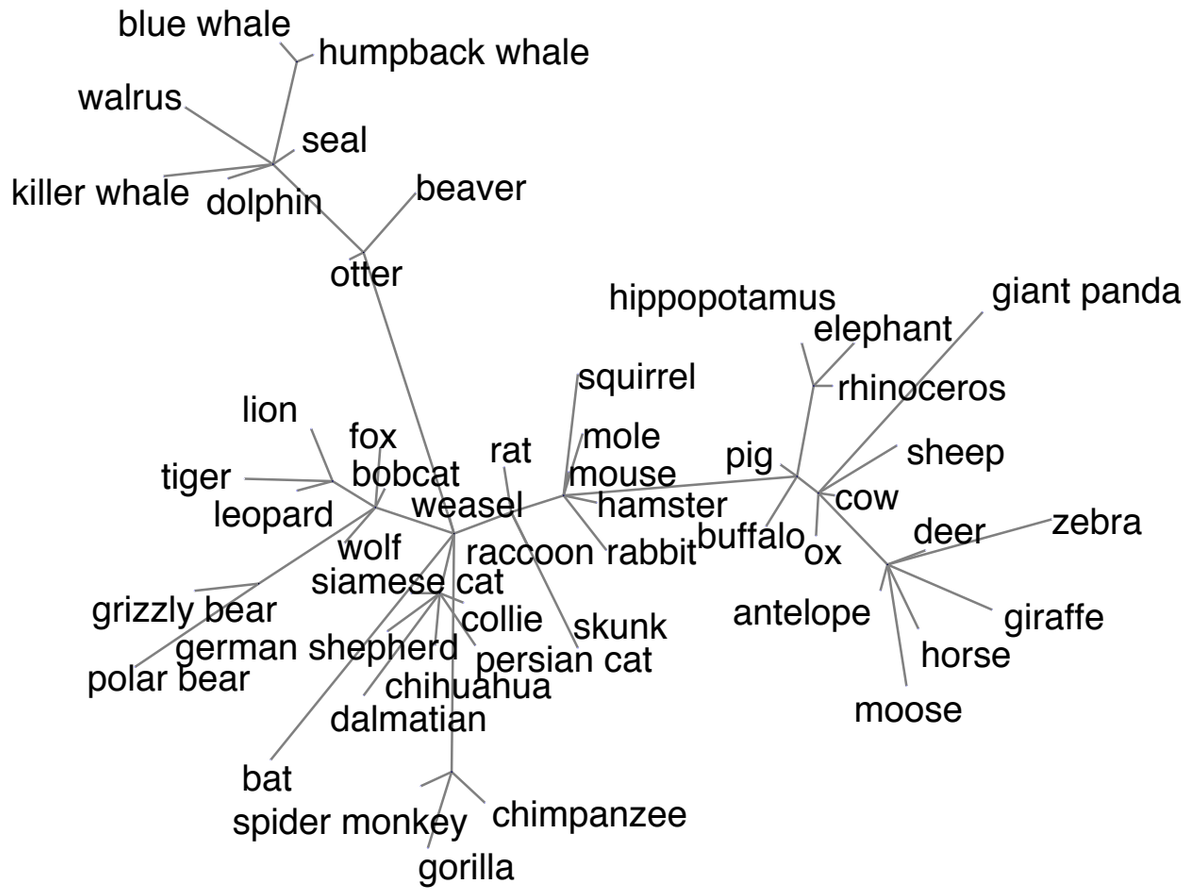


Crisp structural discoveries

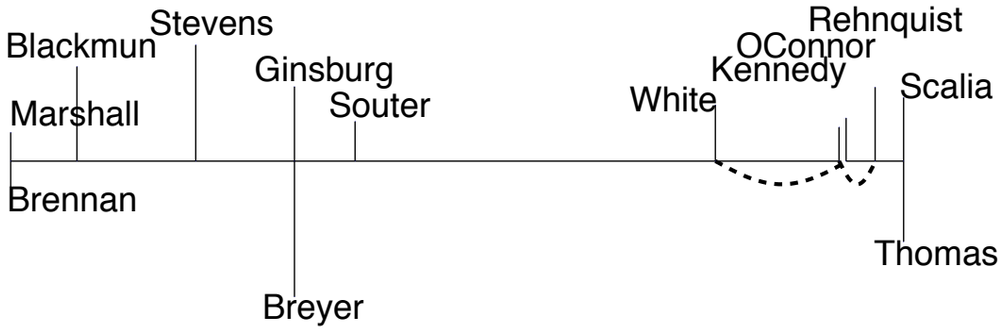
Circle for colors



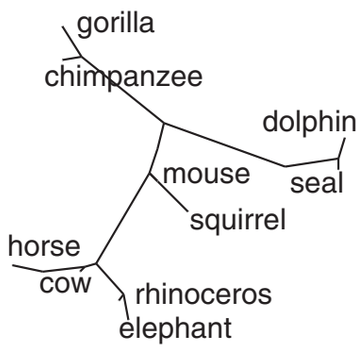
Tree for mammals

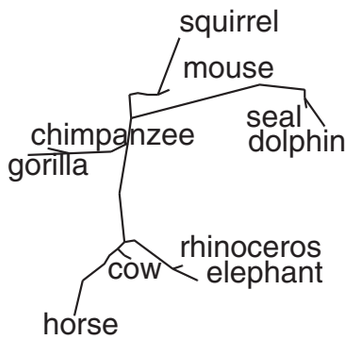


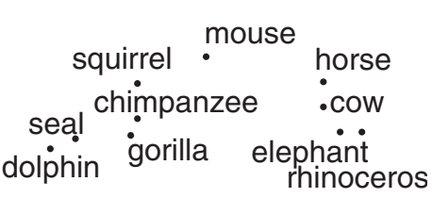
Linear order for Supreme Court judges

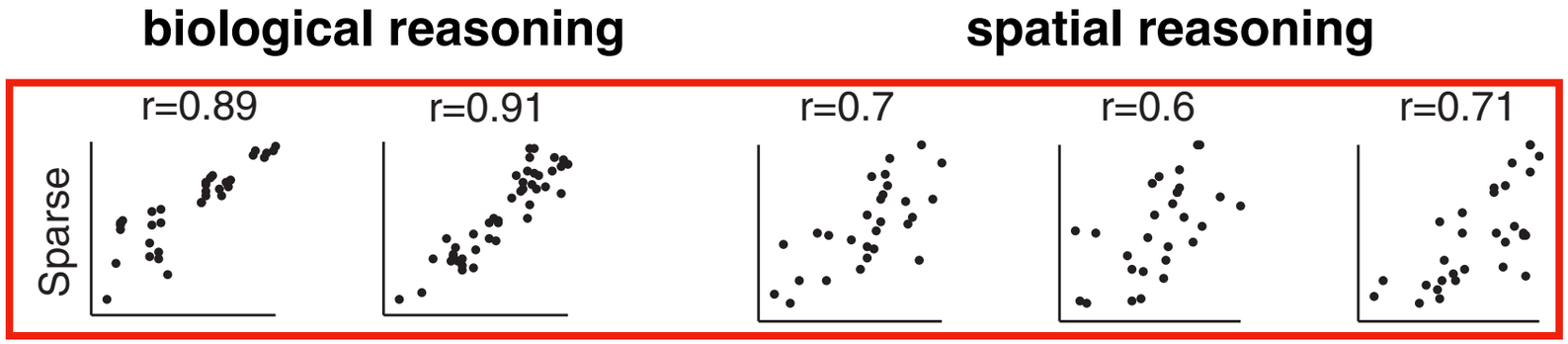


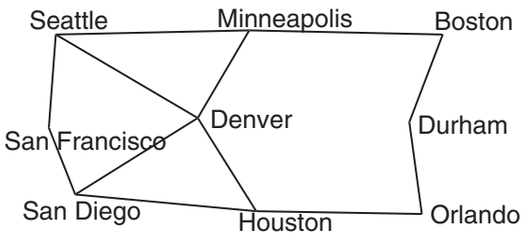
Accounting for inductive judgments without special purpose structural forms

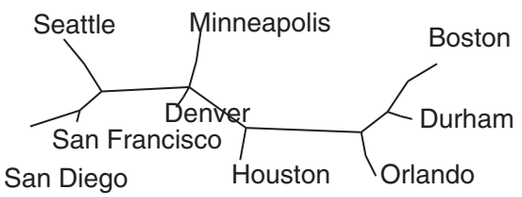










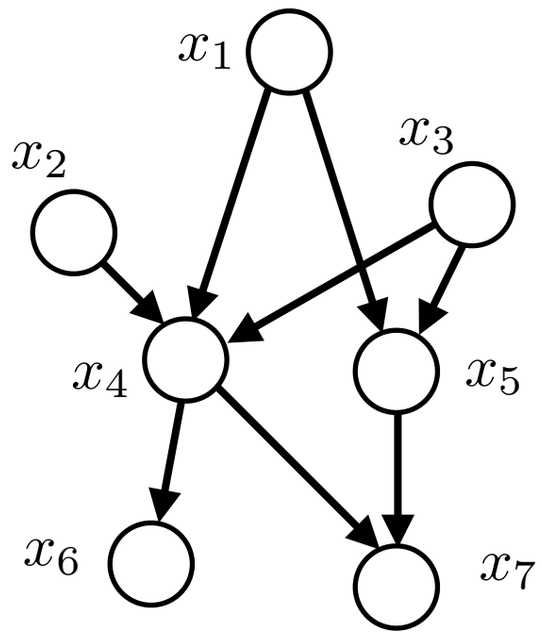


model vs. human judgments

Cows have property P.
Elephants have property P.

Horses have property P.

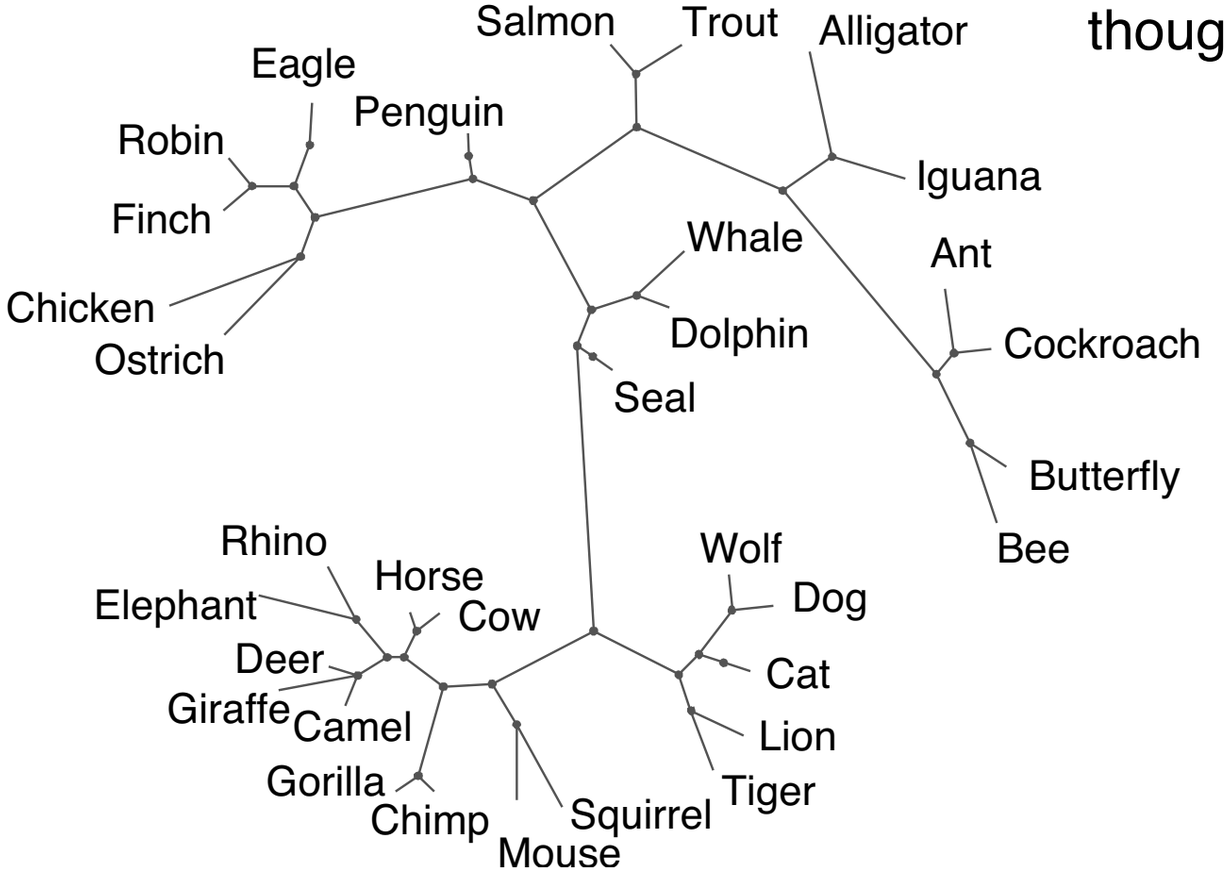
Conclusions: probabilistic graphical models

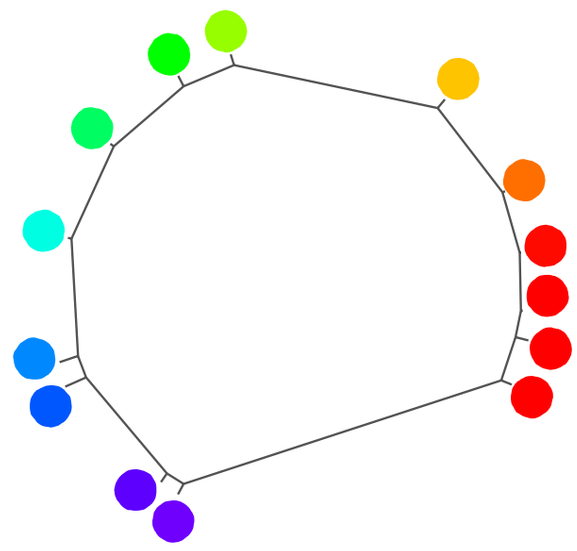


- Probabilistic graphical models are a powerful paradigm in machine learning, and they have been applied in computational cognitive modeling to problems in classification, causal learning, and structure discovery.

- Especially well-suited for modeling data where the causal process is transparent.

- Probabilistic inference, using a model of the world, helps us to understand the productivity of human thought and reasoning.





Excellent textbook for deeper reading