4. Chapter 4 Solutions

4E1. The first line is the likelihood. The second line is very similar, but is instead the prior for the parameter μ. The third line is the prior for the parameter σ. Likelihoods and priors can look very similar, because a likelihood is effectively a prior for the residuals.

4E2. Two parameters in the posterior: μ and σ.

4E3. There are boxes in the chapter that provide examples. Here's the right form in this case, ignoring the specific distributions for the moment:

$$\Pr(\mu, \sigma | y) = \frac{\Pr(y|\mu, \sigma) \Pr(\mu) \Pr(\sigma)}{\int \int \Pr(y|\mu, \sigma) \Pr(\mu) \Pr(\sigma) d\mu d\sigma}$$

Now inserting the distributional assumptions:

$$\Pr(\mu, \sigma | y) = \frac{\text{Normal}(y|\mu, \sigma)\text{Normal}(\mu|0, 10)\text{Exponential}(\sigma|1)}{\int \int \text{Normal}(y|\mu, \sigma)\text{Normal}(\mu|0, 10)\text{Exponential}(\sigma|1) d\mu d\sigma}$$

4E4. The second line is the linear model.

4E5. There are 3 parameters in the posterior: α, β, and σ. The symbol μ is no longer a parameter in the posterior, because it is entirely determined by α, β, and x.

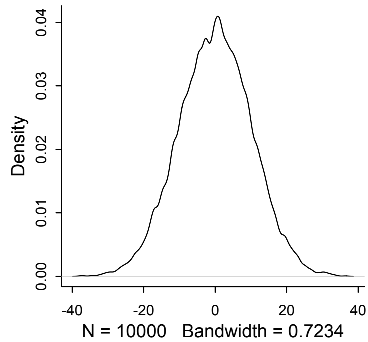
4M1. To sample from the prior distribution, we use `rnorm` to simulate, while averaging over the prior distributions of μ and σ. The easiest way to do this is to sample from the priors and then pass those samples to `rnorm` to simulate observations. This code will sample from the priors:

R code 4.1
```
mu_prior <- rnorm( 1e4 , 0 , 10 )
sigma_prior <- rexp( 1e4 , 1 )
```

You may want to visualize these samples with `dens`, just to help school your intuition for the priors.

Now to simulate observations that average over these prior distributions of parameters:

R code 4.2
```
h_sim <- rnorm( 1e4 , mu_prior , sigma_prior )
dens( h_sim )
```



4M2. As a quap formula, the model in **4M1** is:

R code 4.3

```
f <- alist(
    y ~ dnorm( mu , sigma ),
    mu ~ dnorm( 0 , 10 ),
    sigma ~ dexp( 1 )
)
```

4M3. This is straightforward, but remember that mathematical notation makes use of index variables like i, while the R code is instead implicitly vectorized over observations.

$$\begin{aligned}
y_i &\sim \text{Normal}(\mu, \sigma) \\
\mu &= \alpha + \beta x_i \\
\alpha &\sim \text{Normal}(0, 10) \\
\beta &\sim \text{Normal}(0, 1) \\
\sigma &\sim \text{Exponential}(1)
\end{aligned}$$

4M4. This is an more open-ended problem than the others. But perhaps the simplest model structure that addresses the prompt would be:

$$\begin{aligned}
h_{ij} &\sim \text{Normal}(\mu_{ij}, \sigma) \\
\mu_{ij} &= \alpha + \beta(y_j - \bar{y}) \\
\alpha &\sim \text{Normal}(100, 10) \\
\beta &\sim \text{Normal}(0, 10) \\
\sigma &\sim \text{Exponential}(1)
\end{aligned}$$

where h is height and y is year and $\bar{y}$ the average year in the sample. The index i indicates the student and the index j indicates the year.

The problem didn't say how old the students are, so you'll have to decide for yourself. The priors above assume the students are still growing, so the mean height α is set around 100 cm. The slope with year β is vague here—we'll do better in the next problem. For σ, this needs to express how variable students are in the same year