NEW YORK UNIVERSITY

MATH-104 CALCULUS

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Calculus Practice 3: 5.3-5.9

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Instructions

Go through this practice paper under the guidance of the tutor.

5.1

1. (a) Frame the problem of finding the area of a circle by approximating the area of a circle with areas of inscribed regular regular polygons inside the circle.

   (b) How can we make this approximation exactly equal to the area of a circle.

   (c) State your answer in part b) in terms of limits.

2. The area problem is defined as such. Given a function f that is continuous and non-negative on an interval [a,b], find the area between the graph of f and the interval [a,b] on the x-axis.

   (a) How would you approximate the area A between the function f, which is non-negative and continuous, on an interval [a,b], and the x-axis? Provide a visual sketch.

   (b) Under what condition would your approximation yield the exact area A?

   (c) Use your answer in b, to compute the exact area, A. A is the area between the graph of the function $y = x^2$, which is nonnegative and continuous on $(-\infty, +\infty)$, and the x-axis on the interval [0,1].

   (d) State the anti-derivative method of solving the area problem.

   (e) The area $A(x)$ under the graph of f and over the interval $[a, x]$ is given. Find the function f and the value of a, when $A(x) = x^2 - 4$. (5.1, 27)

5.2

1. State the definition of an antiderivative.

2. Prove the following theorem. Suppose that $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ and $g(x)$, respectively, and that C is a constant. Then:

   (a) $\int cf(x)\,dx = cF(x) + C$

   (b) $\int [f(x) + g(x)]\,dx = F(x) + G(x) + C$

   (c) $\int [f(x) - g(x)]\,dx = F(x) - G(x) + C$

3. In each part, confirm that the formula is correct, and state a corresponding integration formula:

   (a) $\frac{d\left[\sqrt{1+x^2}\right]}{dx} = \frac{x}{\sqrt{1+x^2}}$

   (b) $\frac{d[xe^x]}{dx} = (1 + x)e^x$ (5.2, 1 a), b))

4. Evaluate the integral and check your answer by differentiating.
   (a) $\int x(1+x^3)\,dx$ (5,2, 15)
   (b) $\int (1+x^2)(2-x)\,dx$ (5,2, 18)
   (c) $\int \frac{sin(x)}{(cos(x))^2}\,dx$ (5.2, 29)
5. Use the double-angle formula $cos(2x) = 2(cos(x))^2 - 1$ to evaluate the integral: $\int \frac{1}{1+cos(2x)}\,dx$ (5.2,36)