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Date

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SUBJECT: CS 577

SEMESTER: FALL 22

Assignment - 0

A.

1.

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$\therefore 2a - b = 2 \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$\therefore 2a - b = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

2. $$|a| = \sqrt{\sum_i^n (a_{ij})^2} = \sqrt{(1)^2 + (2) + (3)^2} = \sqrt{14}$$

$$\therefore \hat{a} = \frac{a}{|a|} = \frac{1}{|a|} \cdot a = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \hat{a} = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix} \approx \begin{bmatrix} 0.2672 \\ 0.5345 \\ 0.8017 \end{bmatrix}$$

3.

for a, $\|a\| = \sqrt{14} \approx 3.7416$

for angle between a & x axis we use:

$$\cos\theta = \frac{a \cdot x}{|a| \cdot |x|}$$

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$\overrightarrow{x\,axis} = [1, 0, 0]$

$\therefore$ ~~a . b =~~

$$a \cdot xaxis = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1$$

$|a| = \sqrt{14}$ ... from previous question

$|x| = 1$

$$\therefore \cos\theta = \frac{a \cdot x}{|a| \cdot |x|} = \frac{1}{\sqrt{14} \cdot 1} = 1/\sqrt{14}$$

$$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{14}}\right) = 1.3002 \text{ radians}$$

$$\theta = \underline{74.4986^\circ} \approx \underline{1.3002} \text{ radians}$$

4.

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{a} = 1\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\therefore \cos\alpha = l = \frac{1}{\sqrt{14}}$$

$$\cos\beta = m = \frac{2}{\sqrt{14}}$$

$$\cos\gamma = n = \frac{3}{\sqrt{14}}$$

$$\therefore \text{directional cosines of } a = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$$

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5.

To calculate angle we use

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a|\cdot|b|}$$

$$\vec{a}\cdot\vec{b} = [1,2,3]\begin{bmatrix}4\\5\\6\end{bmatrix} = [32]$$

$$|a| = \sqrt{(1)^2+(2)^2+(3)^2} = \sqrt{14}$$

$$|b| = \sqrt{(4)^2+(5)^2+(6)^2} = \sqrt{77}$$

$$\therefore \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|} = \frac{32}{\sqrt{14}\times\sqrt{77}}$$

$$\therefore \cos\theta = \frac{32}{32.8329}$$

$$\therefore \theta = \cos^{-1}\left(\frac{32}{32.8329}\right)$$

$$\therefore \theta = \underline{\underline{12.93315^\circ}} \approx \underline{0.2257} \text{ radians}$$

6.

$$\vec{a}\cdot\vec{b} = \underline{\underline{32}} \quad \ldots \text{from previous question}$$

$$\vec{b}\cdot\vec{a} = [4,5,6]\cdot\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$= [4+10+18]$$

$$b\cdot a = \underline{\underline{[32]}}$$

(A side not, its not possible to multiple a.b or b.a as both are 3X1 orientation, I changed the orientation accordingly to multiply them considering both a & b as vectors.)

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7.

To find a·b using $\theta = 12.93315^\circ \ldots$ from previous question.

$$\therefore \cos\theta = \frac{a \cdot b}{|a| \cdot |b|}$$

$$\Rightarrow a \cdot b = |a| \cdot |b| \cdot \cos\theta$$
$$= \sqrt{14} \cdot \sqrt{77} \cdot \cos(12.93315)$$
$$= 32$$

$\therefore$ $a \cdot b = \underline{\underline{32}}$ using $\theta = 12.93315$

8.

scalar projection of b on a is calculated by using the formula :-

$$\frac{b \cdot a}{|a|} = \frac{32}{\sqrt{14}} = \underline{\underline{8.5523}}$$

9.

To find vector $\overrightarrow{p}$ (say), ⊥ar to $\vec{a}$

let $\vec{p} = [p, q, r]$

$\Rightarrow$ Angle b/t $\vec{p}$ & $\vec{a}$ = 90°

$$\therefore \cos\theta = \cos(90) = 0$$

$$\because \cos\theta = \frac{a \cdot p}{|a| \cdot |p|}$$

$$\Rightarrow a \cdot p = 0$$

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$$\Rightarrow [1, 2, 3]\begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

$$\Rightarrow p + 2q + 3r = 0$$

for the above eq^n, there are many possible sol^{n}s.

we consider, $p = q = 1$

$\Rightarrow 1 + 2(1) + 3r = 0$

$\Rightarrow 3 + 3r = 0$

$\Rightarrow r = -1$

$\therefore \vec{p} = [1, 1, -1]$ is $\perp$ar to $\vec{a}$

10.

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}\hat{i} - \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix}\hat{j} + \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}\hat{k}$$

$$= (12-15)\hat{i} - (6-12)\hat{j} + (5-8)\hat{k}$$

$$= -3\hat{i} + 6\hat{j} - 3\hat{k}$$

$\therefore a \times b = [-3, 6, -3]$

$$b \times a = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = (15-12)\hat{i} - (12-6)\hat{j} + (8-$$

$$= 3\hat{i} - 6\hat{j} + 3\hat{k}$$

$\therefore b \times a = [3, -6, 3]$

11.

The resultant vector of $a \times b$ will be perpandicular to to both a & b.

$$\therefore\ a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$$= (12-15)\hat{i} - (6-12)\hat{j} + (5-8)\hat{k}$$

$$a \times b = -3\hat{i} + 6\hat{j} - 3\hat{k}$$

$\therefore$ vector $\vec{p}$ $\perp$ar to both a & b $= -3\hat{i} + 6\hat{j} - 3\hat{k}$

12.

To check for linear dependency we calculate determinate of combine matrix of a, b & c.

~~a1~~

$$abc = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{bmatrix}$$

$$\therefore \det(abc) = 1 \cdot \det\begin{vmatrix} 5 & 1 \\ 6 & 3 \end{vmatrix} - 4 \cdot \det\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} + (-1)\det\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix}$$

$$= (15-6)(1) - (6-3)(4) + (-1)(12-15)$$

$$= 9 - 12 + 3$$

$$= 0$$

$\because \det(abc) = 0$

$\Rightarrow$ columns of a, b & c are linearly dependent.

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13.

$$a^T = [1, 2, 3]_{1\times 3}$$

$$\therefore a^T \cdot b = [1, 2, 3] \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \underline{\underline{32}}$$

$$b^T = [4, 5, 6]$$

$$\therefore a \cdot b^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1} \cdot [4, 5, 6]_{1\times 3}$$

$$= \begin{bmatrix} (4)(1) & (1)(5) & (1)(6) \\ (2)(4) & (2)(5) & (2)(6) \\ (3)(4) & (3)(5) & (3)(6) \end{bmatrix}$$

$$a \cdot b^T = \underline{\underline{\begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}}}$$

B.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & -1 \end{bmatrix}$$

$$\therefore 2A - B = 2\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}}}$$

2.

$$A \cdot B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

3.

$$(A \cdot B)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$B^T \cdot A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix} = \begin{bmatrix} (1+4+9) & (4+4+9) & (10-3) \\ (2+2-6) & (8-2-6) & (5+2) \\ (1-8+3) & (4+8+3) & (-20-1) \end{bmatrix}$$

$$B^T \cdot A^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

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4.

$\det A = 1 \det \begin{vmatrix} -2 & 3 \\ 5 & -1 \end{vmatrix} - (2) \det \begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} + (3) \det \begin{vmatrix} 4 & -2 \\ 0 & 5 \end{vmatrix}$

$= (1)(2-15) - (2)(-4) + 3(20)$

$= -13 + 8 + 60$

$\det(A) = 55$

$\det(C) = (1) \det \begin{vmatrix} 5 & 6 \\ 1 & 3 \end{vmatrix} - (2) \det \begin{vmatrix} 4 & 6 \\ -1 & 3 \end{vmatrix} + (3) \det \begin{vmatrix} 4 & 5 \\ -1 & 1 \end{vmatrix}$

$= (1)(15-6) - (2)(12+6) + (3)(4+5)$

$= \cancel{(9)} - \cancel{(18)} + \cancel{27}$

$= (9) - 36 + 27$

$= 0$

$\therefore \det(C) = 0$

5.

To check for matrix A if rows form orthogonal set.

$ar_1 = [1 \ 2 \ 3]$

$ar_2 = [4 \ -2 \ 3]$

if $ar_1 . ar_2 = 0$, they are linearly depende

$\Rightarrow ar_1 . ar_2 = [1 \ 2 \ 3] . \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$

$= 4 - 4 + 9 = 9 \neq 0$

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$\therefore$ ar_1 & ar_2 are not linearly dependent.

|||ly for $ar_1 \cdot ar_3 = [1\ 2\ 3]\begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix} = 7$

$$ar_1 \cdot ar_3 = 7 \neq 0$$

$\therefore$ ar_1 & ar_3 are not linearly dependent

|||ly for $ar_2 \cdot ar_3 = [4\ -2\ 3]\begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix} = [-13]$

$$ar_2 \cdot ar_3 = -13 \neq 0$$

$\Rightarrow$ $ar_2 \cdot ar_3$ are not linearly dependend.

$\therefore$ In matrix A all rows are linearly independent.

for matrix B

$br_1 \cdot br_2 = [1\ 2\ 1]\begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} = 0$

$\because$ $br_1 \cdot br_2 = 0$

$\therefore$ they are linearly dependent

$br_1 \cdot br_3 = [1\ 2\ 1]\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = 0$

$\because$ $br_1 \cdot br_3 = 0$

$\therefore$ they are linearly dependent

&

$br_2 \cdot br_3 = [2\ 1\ -4]\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = 0$

$\because$ $br_2 \cdot br_3 = 0$

$\therefore$ they are linearly dependent

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$\therefore$ In matrix B all rows are linearly dependent.

|||ly for matrix C.

$$Cr_1 . Cr_2 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 32$$

$\because Cr_1 . Cr_2 \neq 0$

$\therefore$ they are not linearly dependent

$$Cr_2 . Cr_3 = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} = 19$$

$\because Cr_2 . Cr_3 \neq 0$

$\therefore$ they are not linearly dependent

$$Cr_1 . Cr_3 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} = 10$$

$\because Cr_1 . Cr_3 \neq 0$

$\therefore$ they are not linearly dependent

$\therefore$ In matric C all rows are linearly independent.

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6.

By formula

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$$

where $|A| \neq 0$

$\therefore \quad |A| = 55$ from previous question

$$\text{adj } A = (\text{co factor } (A))^T$$

cofactor (A), we find all cofactor acc^n. to the formula

$$C_{ij} = (-1)^{ij} M_{ij}, \quad M_{ij} \text{ is a minor}$$

$$\therefore C_{11} = (-1)^{1+1} \begin{vmatrix} -2 & 3 \\ 5 & -1 \end{vmatrix} = -13$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} = 4$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 4 & -2 \\ 0 & 5 \end{vmatrix} = 20$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 5 & -1 \end{vmatrix} = 17$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} = -1$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix} = -5$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 3 \\ -2 & 3 \end{vmatrix} = 12$$

$$C_{32} = (-1) \begin{vmatrix} 1 & 3 \\ 4 & 3 \end{vmatrix} = 9$$

$$C_{33} = (1) \begin{vmatrix} 1 & 2 \\ 4 & -2 \end{vmatrix} = -10$$

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$$\text{Cofactors } \cancel{(A)} \text{ of}(A) = \begin{bmatrix} -13 & 4 & 20 \\ 17 & -1 & -5 \\ 12 & 9 & -10 \end{bmatrix}$$

Co factor matrix

$$Co(A)^T = \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$\therefore \operatorname{adj}(A) = \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \operatorname{adj}(A)$$

$$= \frac{1}{55} \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -0.2363 & 0.3090 & 0.2181 \\ 0.0727 & -0.0181 & 0.1636 \\ 0.3636 & -0.0909 & -0.1818 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \cdot \operatorname{adj}(B)$$

$$|B| = 1 \cdot \det\begin{vmatrix} 1 & -4 \\ -2 & 1 \end{vmatrix} - (2) \cdot \det\begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix} + (1) \det\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix}$$

$$= -42$$

co factors of B

$$C_{11} = (1)\begin{vmatrix} 1 & -4 \\ -2 & 1 \end{vmatrix} = -7$$

$$C_{12} = (-1)\begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix} = -14$$

$$C_{13} = (1)\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} = -7$$

$$C_{21} = (-1)\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} = -4$$

$$C_{22} = (1)\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -2$$

$$C_{23} = (-1)\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = 8$$

$$C_{31} = (1)\begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix} = -9$$

$$C_{32} = (-1)\begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} = 6$$

$$C_{33} = (1)\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$$

$$\therefore \text{cofactor Martrix}(B) = \begin{bmatrix} -7 & -14 & -7 \\ -4 & -2 & 8 \\ -9 & 6 & -3 \end{bmatrix} = Co(B)$$

$$\therefore \text{adj } B = Co(B)^T = \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{1}{-42}\begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix} = \begin{bmatrix} 1/6 & 2/21 & 3/14 \\ 1/3 & 1/21 & -1/7 \\ 1/6 & -4/21 & 1/14 \end{bmatrix}$$

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7.

$\because |C| = 0$

$\Rightarrow$ Inverse of C does not exsist.

8.

$$A \cdot d = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1) + (2)(2) + (3)(3) \\ (4)(1) + (-2)(2) + (3)(3) \\ (0)(1) + (5)(2) + (-1)(3) \end{bmatrix}$$

$$A \cdot d = \begin{bmatrix} 14 \\ 9 \\ 7 \end{bmatrix}$$

8.

normalizing d: $\dfrac{d}{|d|}$

$$= \frac{1}{|d|} \cdot d$$

$$= \frac{1}{\sqrt{14}} \begin{bmatrix} \cancel{14} \\ \cancel{9} \\ \cancel{7} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore d_{norm} = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix} \approx \begin{bmatrix} 0.2672 \\ 0.5345 \\ 0.8017 \end{bmatrix}$$

9.

Scalar projection of rows on normalized d

$$Ar_1 \text{ onto } d_{norm} = \frac{Ar_1 \cdot d_{norm}}{|d_{norm}|}$$

$$Ar_1 \cdot d_{norm} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$$

$$Ar \cdot d_{norm} = \sqrt{14}$$

$$|d_{norm}| = \sqrt{(1/\sqrt{14})^2 + (2/\sqrt{14})^2 + (3/\sqrt{14})^2}$$

$$= \sqrt{1/14 + 4/14 + 9/14}$$

$$|d_{norm}| = \cancel{0}\ 1$$

$$\therefore Ar_1 \text{ noto } d_{norm} = \frac{\sqrt{14}}{1} = \sqrt{14} \approx 3.7416$$

for Ar_2 ont d_{norm}

$$Ar_2 \cdot d_{norm} = \begin{bmatrix} 4, & -2, & 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$$

$$= \frac{9\sqrt{14}}{14} \approx 2.4053$$

$$\therefore Ar_2 \text{ onto } d_{norm} = \frac{Ar_2 \cdot d_{norm}}{|d_{norm}|} = \frac{9\sqrt{14}}{14} \approx 2.4053$$

for Ar_3 onto d_{norm}

$$Ar_3 \cdot d_{norm} = \begin{bmatrix} 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix} = 7/\sqrt{14} = \sqrt{14}/2$$

$$\therefore Ar_3 \text{ onto } d_{norm} = \frac{Ar_3 \cdot d_{norm}}{|d|} = \frac{\sqrt{14}}{2} \approx \cancel{1.8078}\ 1.8708$$

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10.

Vector projection formula for Ar_i onto d_{norm}:

$$= \frac{Ar_i \cdot d_{norm}}{||d||^2} \cdot d_{norm}$$

$\because |d| = 1$

$\Rightarrow ||d||^2 = 1$

To $\cancel{\text{prove } ||d||^2}$

$$Ar_1 \cdot d_{norm} = \sqrt{14}$$

$$\therefore Ar_1 \text{ on } d_{norm} = \frac{\sqrt{14}}{(1)} \cdot \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$$

Vector projections of Ar_1 onto d_{norm} $= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Similarly, $Ar_2 \cdot d_{norm} = \dfrac{9\sqrt{14}}{14}$

$$\therefore Ar_2 \text{ onto } d_{norm} = \frac{9\sqrt{14}}{14} \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$$

Vector projections of Ar_2 onto d_{norm} $= \begin{bmatrix} 9/14 \\ 9/7 \\ 27/14 \end{bmatrix}$

$$Ar_3 \cdot d_{norm} = \frac{\sqrt{14}}{2}$$

$\therefore$ Vector projections of Ar_3 onto d_{norm} $= \dfrac{\sqrt{14}}{2} \cdot \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 3/2 \end{bmatrix}$

11.

let x, y & z be linear combination of 1^{ST} columns of A using elements of D.

$\Rightarrow$ $\cancel{\begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \cancel{[1, 2, 3]}}$

$x[1] + y[2] + 3[z]$

$$\vec{Ac_1} = \cancel{[x, y, z] \cdot [1, 2, 3]}$$
$$= x, 2y, 3z$$

$\Rightarrow \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ 2y \\ 3z \end{bmatrix}$

$\therefore$ $x = 1$, $y = 2$ & $z = 0$

111^{ly} linear combinations for 2^{nd} column.

$$\vec{Ac_2} = x[1] + y[2] + z[3]$$

$\Rightarrow \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} x \\ 2y \\ 3z \end{bmatrix}$

$\therefore$ $x = 2$, $y = -1$ and $z = 3/5 \approx 1.66$

1×3 3×1 3×1 1×3

Ill^{ly} linear combination of 3rd column of A using d.

$$A C_3 = x[1] + y[2] + 3[2]$$

$$\Rightarrow \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ 2y \\ 3z \end{bmatrix}$$

$$\therefore \quad x = 3, \quad y = 3/2 = 1.5 \quad \& \quad z = -1/3$$

12.

considering $x = [x_1, x_2, x_3]$

$$\therefore \quad Bx = d$$

$$\Rightarrow \quad x = B^{-1} . d$$

$$x = \begin{bmatrix} 1/6 & 2/21 & 3/14 \\ 1/3 & 1/21 & -1/14 \\ 1/6 & -4/21 & 1/14 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \quad x = \begin{bmatrix} (1/6) + (4/21) + (9/14) \\ (1/3) + (2/21) + -(3/14) \\ (1/6) + (-8/21) + (3/14) \end{bmatrix}$$

$$\therefore \quad x = \begin{bmatrix} 1 \\ 0.014 \\ 0 \end{bmatrix}$$

13.

for $C \cdot x = d$

$\Rightarrow x = C^{-1} \cdot d$

but $|C| = 0$

$\Rightarrow C^{-1}$ does not exsist

$\therefore x$ has no solution.

C.

1.

To find eigen vectors & eigen value of D.

$$D = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

we use the formula

$$D - \lambda I = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = 0$$

now we calculate determinant of this matrix

$$\therefore \det \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = (1-\lambda)(2-\lambda) - 6$$

$$= 2 - \lambda - 2\lambda + \lambda^2 - 6$$

$$= \lambda^2 - 3\lambda - 4$$

$$= \cancel{\lambda}\{\lambda^2 + \lambda\} - 4\lambda - 4$$

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$$= (\lambda^2 + \lambda) + (-4\lambda - 4)$$
$$= \lambda(\lambda+1) - 4(\lambda+1)$$
$$= (\lambda+1)(\lambda-4)$$

$\therefore \quad (\lambda+1)(\lambda-4) = 0$

$\therefore \quad \lambda = -1, \quad \lambda = 4$

$\therefore$ eigen values of D = $\underline{\underline{-1, 4}}$

now for
$\lambda = 4$

$$\therefore \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = \begin{bmatrix} 1-4 & 2 \\ 3 & 2-4 \end{bmatrix}$$

let $A = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}$

solve $A\bar{x} = \bar{O}$

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

To find row echelon form of A

$R_1 = R_1 \div -3$

$$\begin{bmatrix} 1 & -2/3 \\ 3 & -2 \end{bmatrix}$$

$R_2 = R_2 - 3R_1$

$$\begin{bmatrix} 1 & -2/3 \\ 0 & 0 \end{bmatrix}$$

now we solve

$$\begin{bmatrix} 1 & -2/3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x_1 - \frac{2x_2}{3} = 0$

$\Rightarrow \quad x_1 = \frac{2}{3} x_2$

let $x_2 = 1$

$\Rightarrow \quad x_1 = 2/3$

$\therefore$ eigen vector $= \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$ of D for $\lambda = 4$.

for $\lambda = -1$

$\therefore \quad B = \begin{bmatrix} 1-(-1) & 2 \\ 3 & 2-(-1) \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$

now we find row echolon matrix

$R_1 = R_1 \div 2$

$$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

$R_2 = R_2 - 3R_1$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

now we solve : $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \quad x_2 + x_1 = 0$

$\Rightarrow \quad x_1 = -x_2$

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let $x_2 = 1$

$\Rightarrow x_1 = -1$

$\therefore \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ are eigen vectors of D for $\lambda = -1$.

2.

Dot product of eigenvectors D

$= \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$

$= -2/3 + 1$

$= \begin{bmatrix} 1/3 \end{bmatrix} \approx 0.3333$

3.

To find dot product of eigen vectors of E.

$E = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

$E - \lambda I = 0$

$\Rightarrow \begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix} = 0$

$\Rightarrow (2-\lambda)(5-\lambda) - (-2)(-2) = 0$

$\Rightarrow 10 - 2\lambda - 5\lambda + \lambda^2 - 4 = 0$

$\Rightarrow (\lambda - 6)(\lambda - 1) = 0$

$\therefore \ \lambda = 6, \ \lambda = 1$

for $\lambda = 1$

$$A = \begin{bmatrix} 2-1 & -2 \\ -2 & 5-1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

for rowechelon form

$R_2 = R_2 + 2R_1$

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

now $\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \ x_1 - 2x_2 = 0$

$\Rightarrow \ \Rightarrow x_1 = 2x_2$

say $x_2 = 1$

$\therefore \ x_1 = 2$

$\therefore$ vector $= \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

for $\lambda = 6$

$$A = \begin{bmatrix} 2-6 & -2 \\ -2 & \cancel{6}5-6 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix}$$

for row echelon form

$R_1 = -R_1/4$

$$\begin{bmatrix} 1 & 1/2 \\ -2 & -1 \end{bmatrix}$$

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$R_2 = R_2 + 2R_1$

$$\begin{bmatrix} 1 & 1/2 \\ 0 & 0 \end{bmatrix}$$

$\therefore \begin{bmatrix} 1 & 1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow x_1 + 1/2\, x_2 = 0$

$\Rightarrow x_1 = \frac{-1}{2} x_2$

say $x_2 = 1$

$\therefore x_1 = -1/2$

$\therefore$ vector $= \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$

$\therefore$ Dot product of eigen vectors of

$E = [2 \;\; 1] \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$

$= -1 + 1$

$= 0$

4.

Since dot product of eigen vectors of $E = 0$.

$\Rightarrow$ the eigenvectors are orthagonal and they are $\perp$ar to each other.

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5. To find trivial sol^n for $fx = 0$

if $|A| = 0$, where A is a matrix

$\Rightarrow$ matrix has trivial sol^n.

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$x_1 + 2x_2 = 0$$
$$2x_1 + 4x_2 = 0$$

for matrix (f), row 2 is a linear combination of row1 i.e $R_2 = 2R_1$

$\Rightarrow$ they are linearly dependent.

And the reduced row echelong matrix of (f):

by $R_2 = R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 = -2x_2$$

$\Rightarrow$ Rank of $(f) = 1$

and $[f|0] = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \end{bmatrix}$

$\Rightarrow$ Rank $[f|0] = 1$

$\therefore$ Rank of (f) = Rank $[f|0] < n$ (no of unknowns)

$\Rightarrow f$ has infinite $sol^n s$.

for f to have trivial sol^n, att

$x_1 = 0$, $x_2 = 0$

$\Rightarrow x_1 = 2x_2$

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6.

the value for non trivial sol^n. for $f x = 0$

where

$x_1 = -2x_2$ are

$(-2, 1)$ and $(-4, 2)$.

7.

The only sol^n x for $D\bar{x} = \bar{0}$

we find rank of D.

reduced echelon form of D

$R_1 = 3R_1$

$$\begin{bmatrix} 3 & 6 \\ 3 & 2 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$\therefore$ Rank $(D) = 2$

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now $[D|0] = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & 0 \end{bmatrix}$

Rank $[D|0] = 2$

$\Rightarrow$ Rank (D) = Rank $[D|0]$ = n....
no. of unknowns

$\therefore$ D has unique sol^n for $Dx = 0$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\Rightarrow$ $x + 2x_2 = 0 \rightarrow x_1 = -2x_2$

$3x_1 + 2x_2 = 0 \Rightarrow 3(-2x_2) + 2x_2 = 0$

$\Rightarrow -6x_2 + 2x_2 = 0$

$\therefore$ $\cancel{2x_1 = 0}$ $\Rightarrow -4x_2 = 0$

$\Rightarrow$ $\cancel{x = 0}$ $\therefore x_2 = 0$

$\Rightarrow$ $x_1 = -2(0)$

$x_1 = 0$

$\therefore$ Trivial sol^n for $Dx = 0$ is $(0, 0)$.

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D 1.

$f(x) = x^2 + 3$

$\therefore f'(x) = 2x$

$f''(x) = 2$

2.

∂p

$\frac{\partial q}{\partial x} = 2x$

$\therefore \frac{\partial q}{\partial y} = 2y$

3.

$\nabla q(x, y) = \langle \frac{\partial q}{\partial x}, \frac{\partial q}{\partial y} \rangle$

$= \sqrt{(2x)^2 + (2y)^2}$

$= 2\sqrt{x^2 + y^2}$

4. without chain rule

$\frac{\partial}{\partial x} f(g(x)) = \cancel{\partial} \frac{d}{dx} f(x^4 + 3)$

$= 4x^3$

with chain rule

let $u = x^2$

so $\frac{d}{dx}(u^2 + 3) \frac{d}{dx}(x^2)$

$\Rightarrow 2u \frac{d}{dx}(x^2)$

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$\Rightarrow 2x^2 \cdot 2x$

$\Rightarrow 4x^3$