Exploring the Euclidean Algorithm

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Overview

The Euclidean Algorithm is used to find the Greatest Common Divisor between any pair of whole numbers p, q such that $\mathrm{p} > \mathrm{q}$. It follows that

$$p = n_1 * q + r_1$$

$$q = n_2 * r_1 + r_2$$

$$.$$

$$.$$

$$.$$

$$r_{k-1} = n_{k+1} * r_k$$

Where

$$r_k = \gcd(\mathrm{p}, \mathrm{q}).$$

For example, here is the gcd(42, 36):

$$\gcd(42, 36) = 6:$$

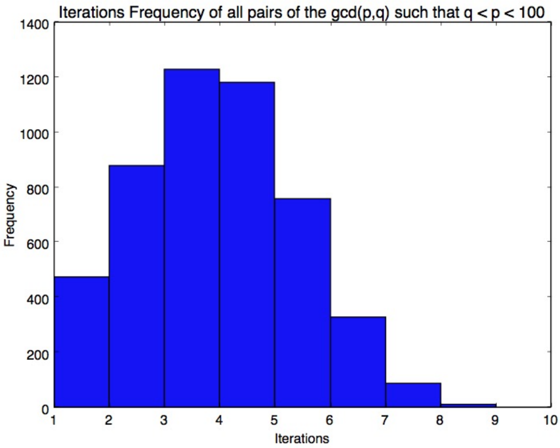
$$42 = 1 * 36 + 6 \tag{1}$$

$$36 = 6 * 6 + 0 \tag{2}$$

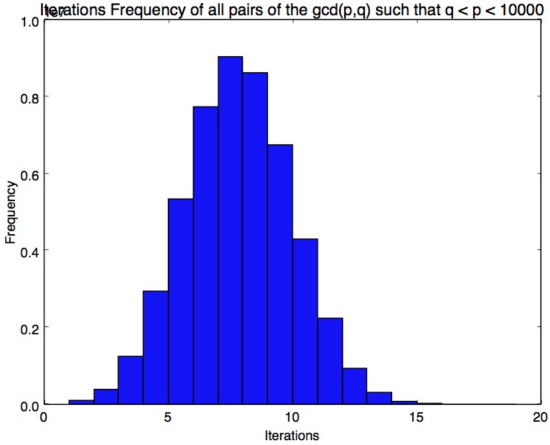
As you can see, it took 2 iterations to complete the algorithm. This is what we will explore. Here are some more gcds and their iterations:

$\gcd(p, q) = d$	Iterations
$\gcd(689, 456) = 1$	6
$\gcd(78, 45) = 3$	5
$\gcd(8394, 238) = 2$	7

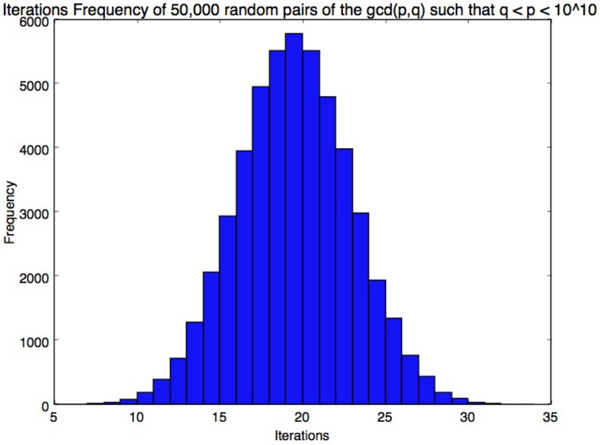
Next, we decided to explore the distributions of these iterations: Do most pairs take many iterations? What is the distribution? The following graphs are the answer to these questions.


Iterations Frequency of all pairs of the gcd(p,q) such that q < p < 100
Frequency
Iterations
0
200
400
600
800
1000
1200
1400
1
2
3
4
5
6
7
8
9
10

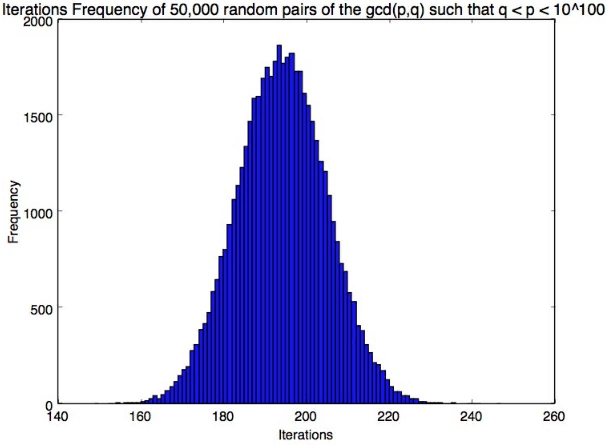
(Figure 1)


Iterations Frequency of all pairs of the gcd(p,q) such that q < p < 10000
Frequency
Iterations
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20

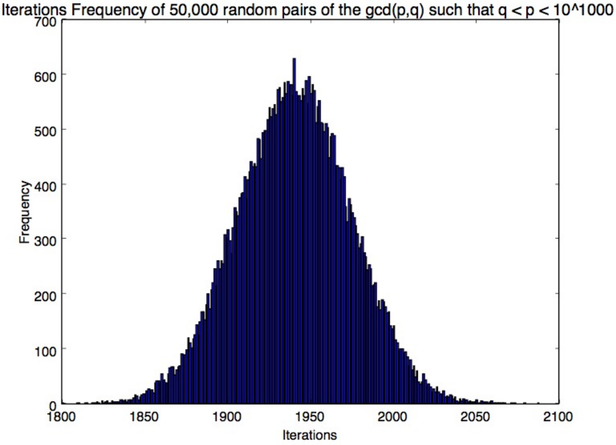
(Figure 2)


Iterations Frequency of 50,000 random pairs of the gcd(p,q) such that q < p < 10^10
Frequency
Iterations
0
1000
2000
3000
4000
5000
6000
5
10
15
20
25
30
35

(Figure 3)


Iterations Frequency of 50,000 random pairs of the gcd(p,q) such that q < p < 10^100
Frequency
2000
1500
1000
500
0
140
160
180
200
220
240
260
Iterations

(Figure 4)


Iterations Frequency of 50,000 random pairs of the gcd(p,q) such that q < p < 10^1000
Frequency
0
100
200
300
400
500
600
700
1800
1850
1900
1950
2000
2050
2100
Iterations

(Figure 5)

- in Figure 1, it must be noted that the lack of normality here is due to the lack of available bins. The size of this data set was no more than 4950, and spanned across merely 9 bins. As the number of bins needed increases, the more normal the graph becomes.
- in Figure 5, the inconsistencies in the normal distribution can be attributed to a too small a sample size. If given the computing power and time, one could compute all pairs less than 10^{1000}. Note the increasing mean iterations as we climb the upper bound.

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End