Exploring the Euclidean Algorithm

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Overview

The Euclidean Algorithm is used to find the Greatest Common Divisor between any pair of whole numbers p, q such that $\mathrm{p} > \mathrm{q}$. It follows that

$$p = n_1 * q + r_1$$

$$q = n_2 * r_1 + r_2$$

$$\vdots$$

$$r_{k-1} = n_{k+1} * r_k$$

Where

$$r_k = \gcd(\mathrm{p}, \mathrm{q}).$$

For example, here is the gcd(42, 36):

$$\gcd(42, 36) = 6 :$$

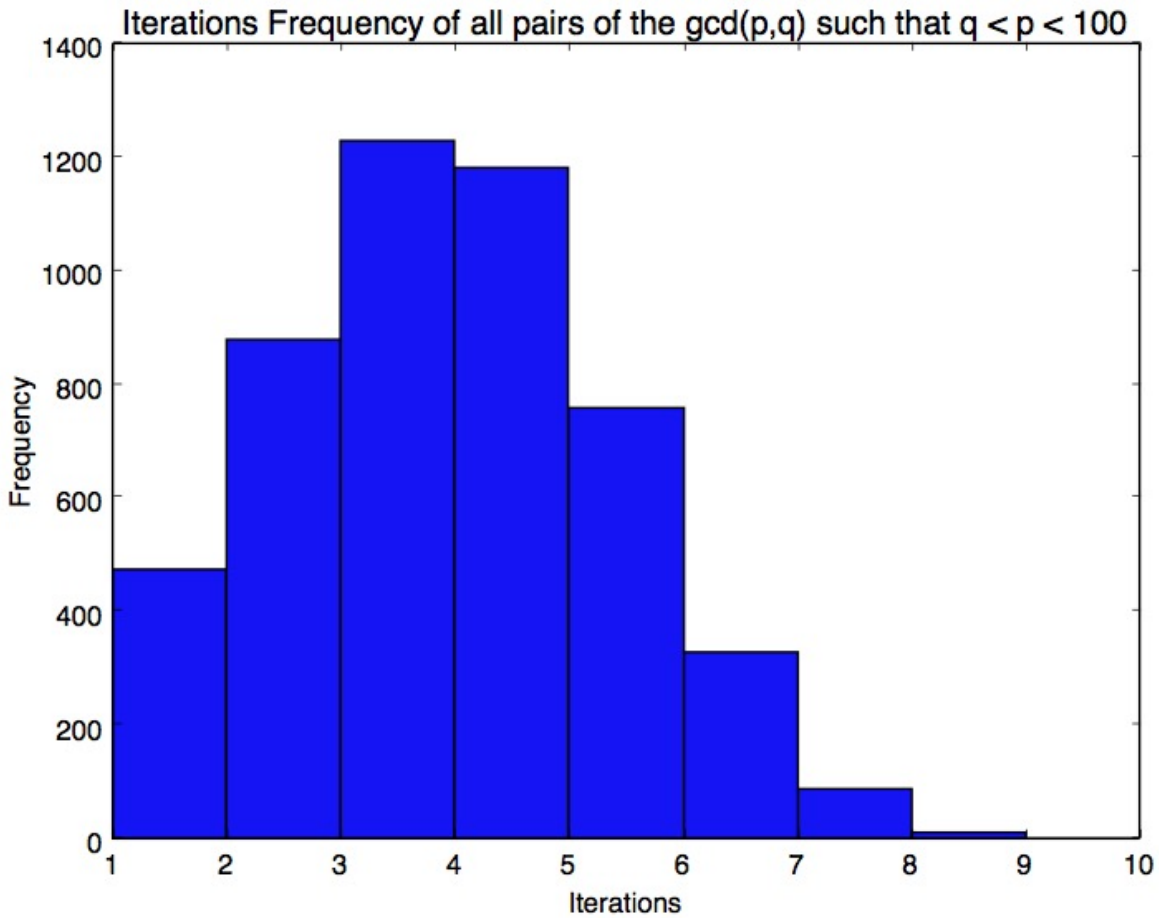
$$42 = 1 * 36 + 6 \quad (1)$$

$$36 = 6 * 6 + 0 \quad (2)$$

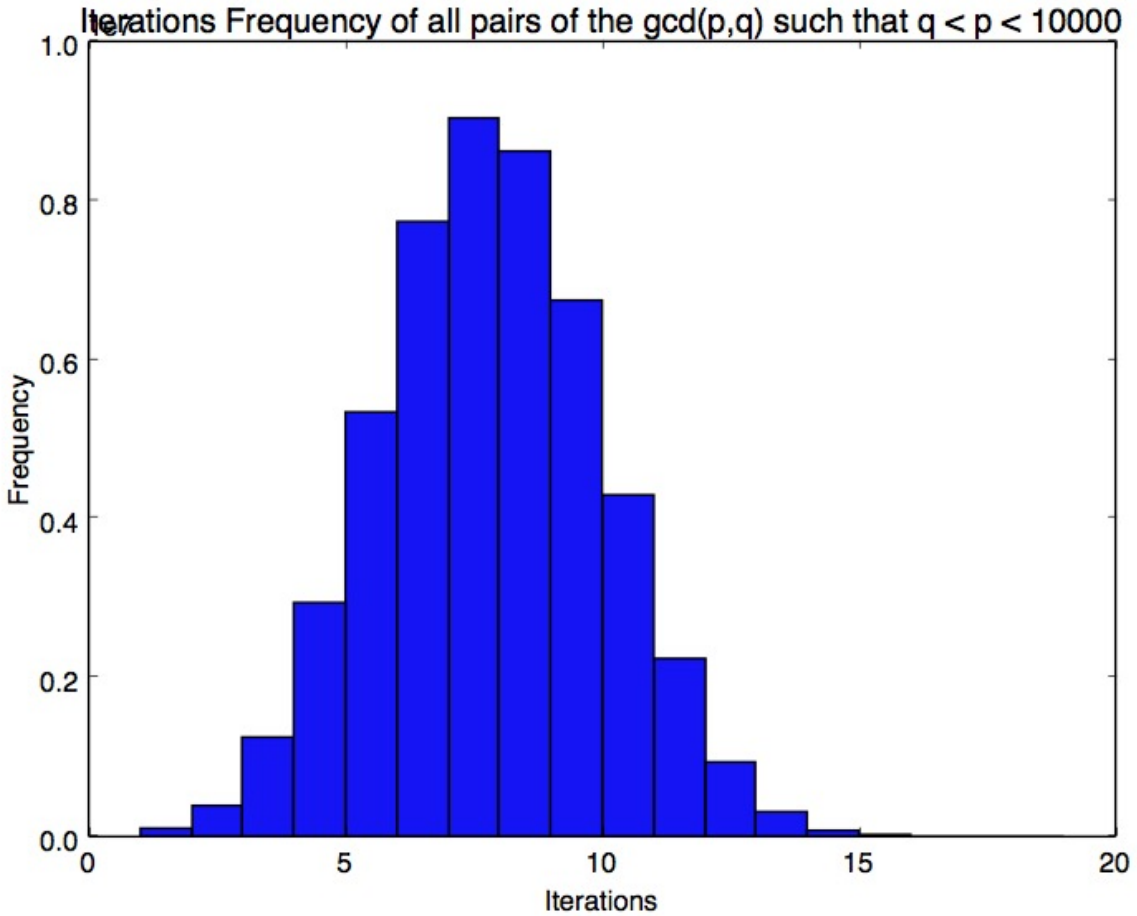
As you can see, it took 2 iterations to complete the algorithm. This is what we will explore. Here are some more gcds and their iterations:

$\gcd(p, q) = d$	Iterations
$\gcd(689, 456) = 1$	6
$\gcd(78, 45) = 3$	5
$\gcd(8394, 238) = 2$	7

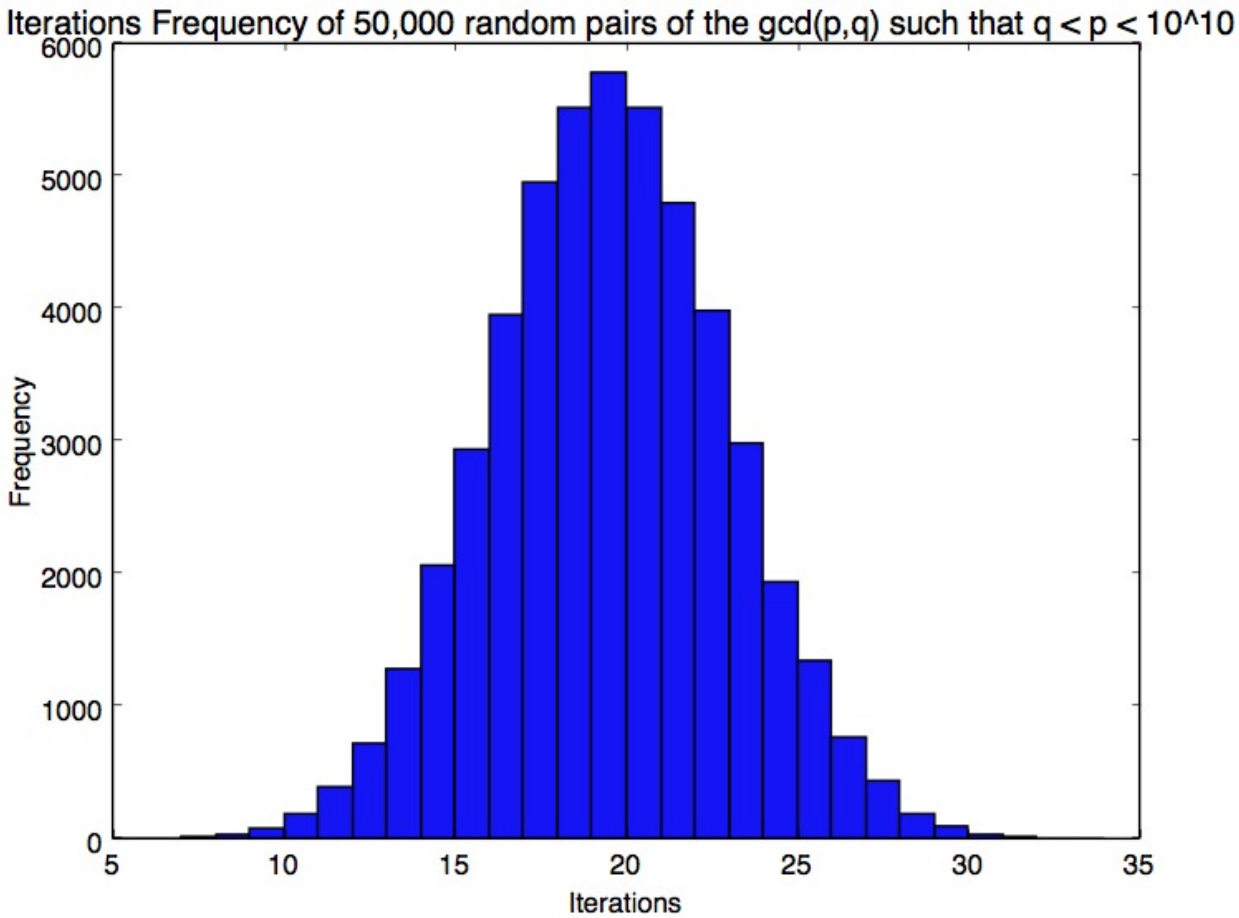
Next, we decided to explore the distributions of these iterations: Do most pairs take many iterations? What is the distribution? The following graphs are the answer to these questions.


Iterations Frequency of all pairs of the gcd(p,q) such that q < p < 100
Frequency
Iterations
0
200
400
600
800
1000
1200
1400
1
2
3
4
5
6
7
8
9
10

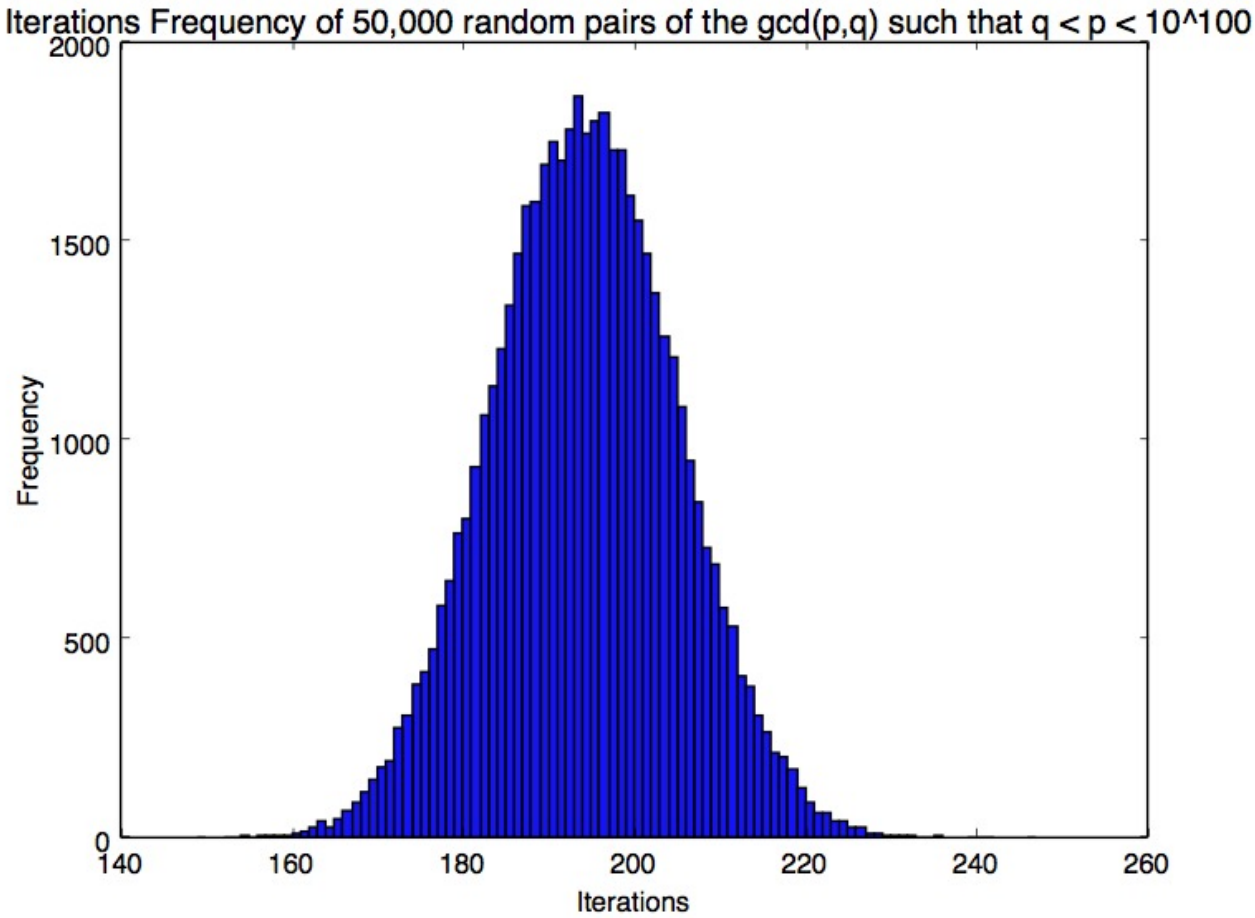
(Figure 1)


Iterations Frequency of all pairs of the gcd(p,q) such that q < p < 10000
Frequency
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Iterations

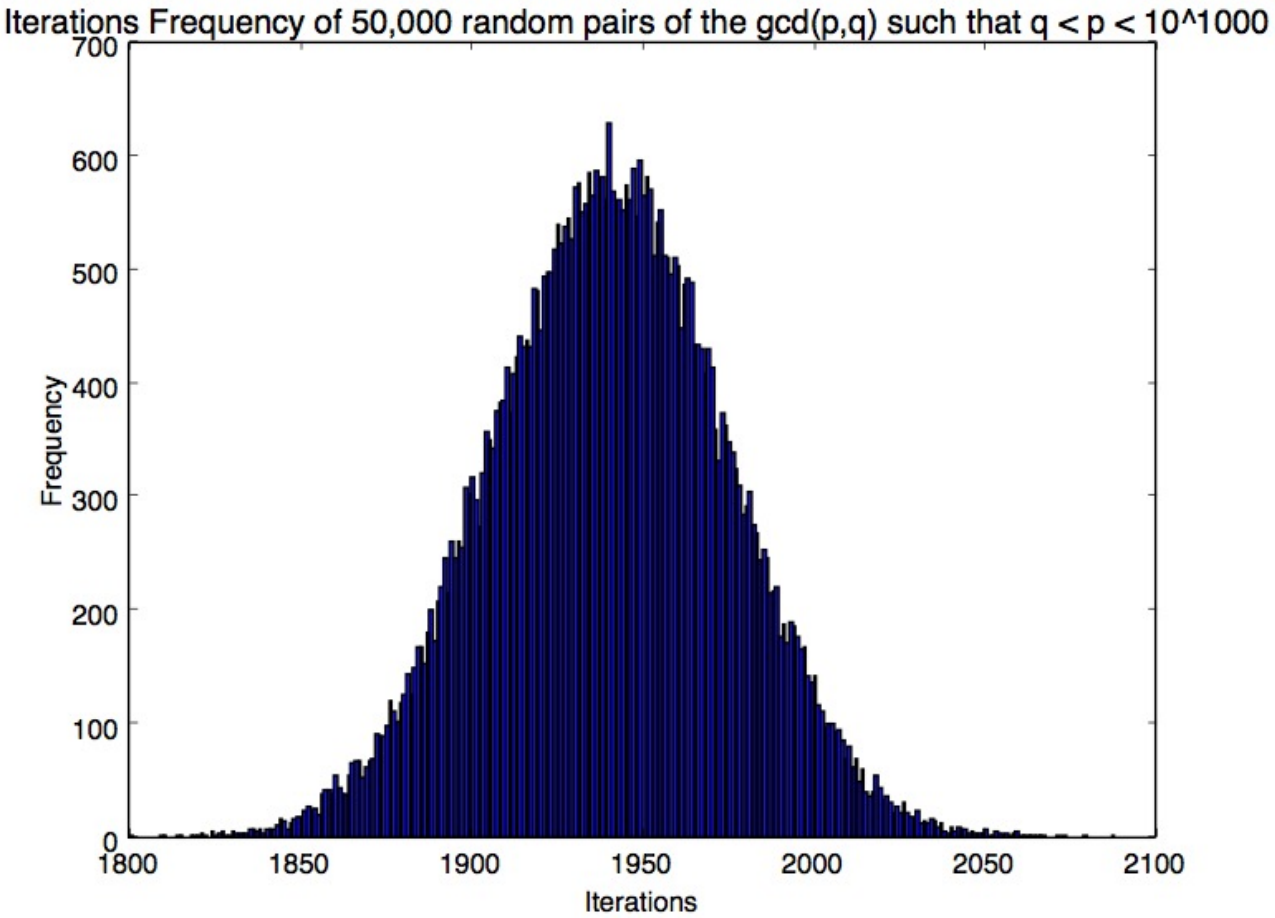
(Figure 2)


Iterations Frequency of 50,000 random pairs of the gcd(p,q) such that q < p < 10^10
Frequency
Iterations
0
1000
2000
3000
4000
5000
6000
5
10
15
20
25
30
35

(Figure 3)


Iterations Frequency of 50,000 random pairs of the gcd(p,q) such that q < p < 10^100
Frequency
2000
1500
1000
500
0
140
160
180
200
220
240
260
Iterations

(Figure 4)


Iterations Frequency of 50,000 random pairs of the gcd(p,q) such that q < p < 10^1000
Frequency
0
100
200
300
400
500
600
700
1800
1850
1900
1950
2000
2050
2100
Iterations

(Figure 5)

Bullet Points

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block 1

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Block 2

Pellentesque sed tellus purus. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos. Vestibulum quis magna at risus dictum tempor eu vitae velit.

Block 3

Suspendisse tincidunt sagittis gravida. Curabitur condimentum, enim sed venenatis rutrum, ipsum neque consectetur orci, sed blandit justo nisi ac lacus.

Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)

Title of the publication

Journal Name 12(3), 45 – 678.

The End