Texas A&M University

Preliminary Investigation

Applications of the Abundancy Function in Primality Testing

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1 Introduction

There are several papers in which some numbers were proven to be solitary, such as 18. There are also papers in which possibilities for friends of 10 are narrowed down.

2 Theoretical Analysis: Neural Networks

2.1 Artificial Neural Networks and their Architectures

2.2 The Neuron

2.3 Network Architectures

2.3.1 Feed-Forward Networks

2.3.2 Recurrent Networks

2.4 Learning

3 Computational Approaches

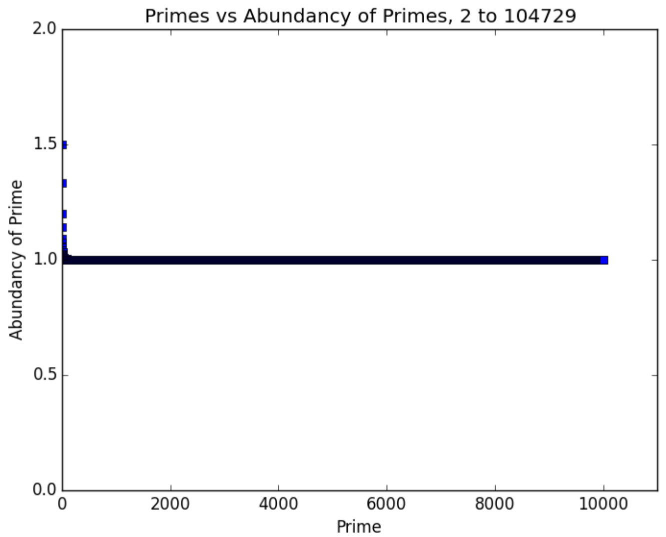
3.1 Implementation Detail and Process

All calculations were performed in Python 2.7, utilizing Matplotlib for graph renderings, excepting curve-fitting, which was performed in Microsoft Excel due to the limited scope of this investigation. Computations were performed on a Linux-based server hosted by Texas A& M University.

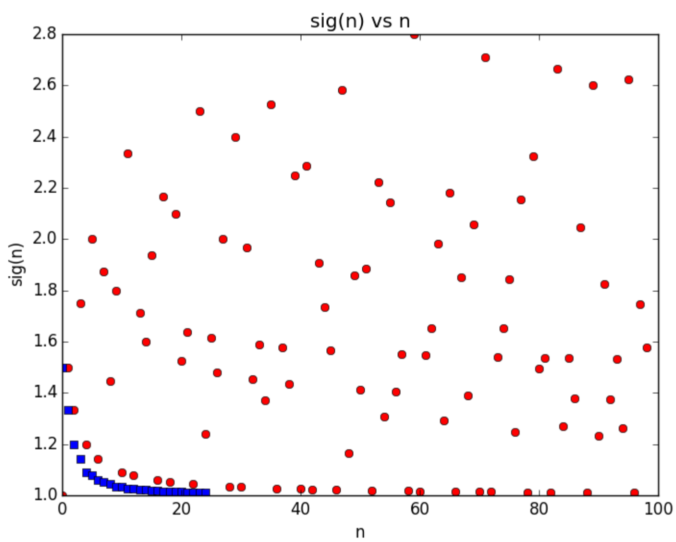
Random samples of the first 1000, 10,000, and 100,000 primes were taken to generate curves, with r^2 values recorded for each. A logarithmic curve was used to fit each data set.

4 Results

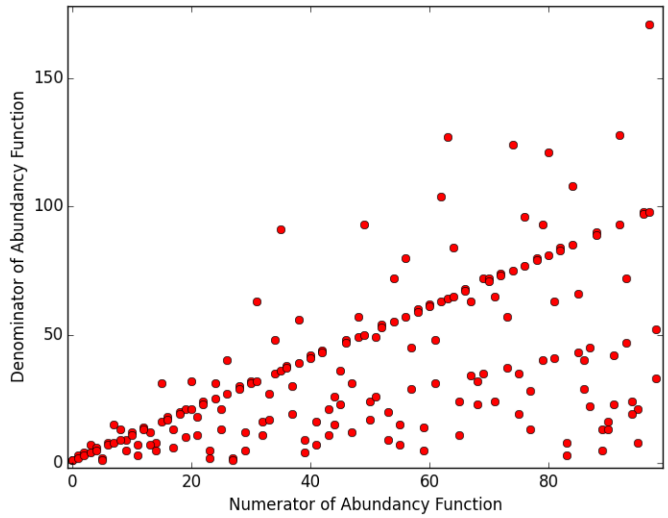
The abundancy function, when given prime inputs, has results as below:



For all integer inputs, it is as below

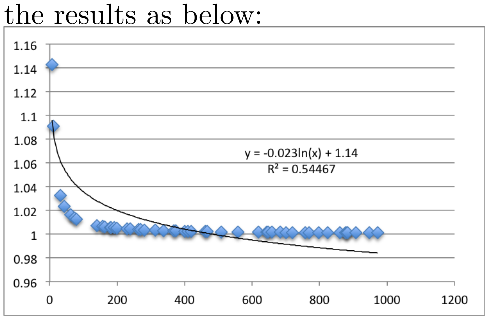


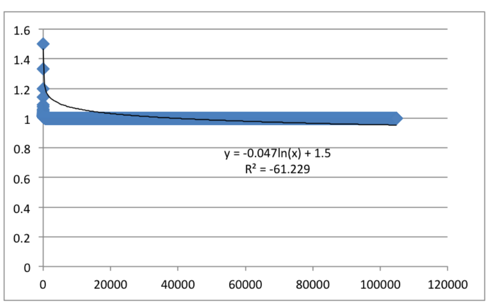
The ratio of $sig(n)$ to n was also graphed. It is relevant to note that due to the nature of the $sig(n)$ function for prime numbers, there is a strong linear

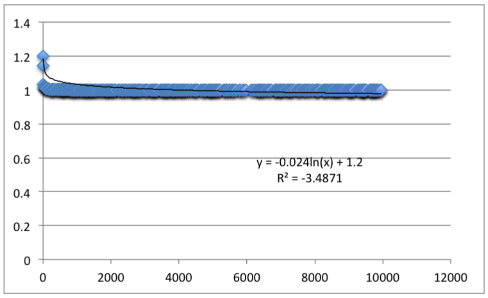


correlation between $sig(n)$ and n.

A random sample was taken from the aforementioned subssets of the first million primes using a random number generator from the Random library in Python. Logarithmic regression equations were performed, with the results as below:







5 Discussion

It is clear that the abundancy function cannot be fitted with a logarithmic curve. No numbers were correctly matched by any of the equations determined for each sample. It is not possible that the accuracy of each equation could be improved with a larger sample size of primes: none of the curves obtained will asymptotically approach 1. Future research should consider a better fit to the abundancy function and perhaps first analyze the rate at which it converges to 1.

6 Individual Contributions

6.1 Stephen Capps

6.2 Sarah Sahibzada

Performed computational investigations on the densities of the abundancy function, distribution of friendly numbers, and the convergence of the abundancy function for the prime numbers.

6.3 Taylor Wilson

7 References