APPENDIX

When we apply a power source equal to $P_{dyn}\ Watts$, the temperature rise $\mathcal{T}$ according to Equation 3 is given by Equation 19:

$$\mathcal{T} = f_{sp} \star (P_{dyn} + \Delta P_{leak}) \tag{19}$$

Let P_{dyn} be a delta function applied in layer k. the heat spreading function in layers 1 to l would be denoted by fsp_{1k}, fsp_{2k}, ... fsp_{lk}. Thus, fsp_{ik} denotes the effect in layer i, when a point source is applied in layer k. Let the temperature rise in layer i , when a point source is applied in layer k be denoted by $\mathcal{T}_{ik}$. As discussed in the previous Section, $\mathcal{T}_{ik}$ is affected by the dynamic power dissipated by layer k, as well as the leakage sources created in all the layers. From Equation 2, $\Delta Pleak_{ik} = \beta\mathcal{T}_{ik}$, which denotes the new leakage sources created in layer i, due to the source in layer k. Using these terms in Equation 19, and the fact that convolution of any function with a delta function is the function itself, we arrive at the following Equations:

$$\begin{aligned}\mathcal{T}_{1k} =& fsp_{1k} + \beta(fsp_{11} \star \mathcal{T}_{1k} + fsp_{12} \star \mathcal{T}_{2k} + fsp_{13} \star \mathcal{T}_{3k} + \\ &... + fsp_{1l} \star \mathcal{T}_{lk}) \\ &... \\ \mathcal{T}_{lk} =& fsp_{lk} + \beta(fsp_{41} \star \mathcal{T}_{1k} + fsp_{42} \star \mathcal{T}_{2k} + fsp_{43} \star \mathcal{T}_{3k} + \\ &... + fsp_{4l} \star \mathcal{T}_{lk})\end{aligned} \tag{20}$$

To transform convolution to multiplication, we compute the 2-D Fourier Transform of the above Equations. This allows us to solve for $\mathcal{F}(\mathcal{T}_{ik})$, where $\mathcal{F}$ denotes the Fourier transform operator:

$$\begin{aligned}\mathcal{F}(\mathcal{T}_{1k}) &= \frac{\begin{array}{r}\mathcal{F}(fsp_{1k}) + \beta(\mathcal{F}(fsp_{12})\mathcal{F}(\mathcal{T}_{2k}) + \mathcal{F}(fsp_{13})\mathcal{F}(\mathcal{T}_{3k}) \\ + ... + \mathcal{F}(fsp_{1l})\mathcal{F}(\mathcal{T}_{lk}))\end{array}}{1 - \beta\mathcal{F}(fsp_{11})} \\ &... \\ \mathcal{F}(\mathcal{T}_{lk}) &= \frac{\begin{array}{r}\mathcal{F}(fsp_{lk}) + \beta(\mathcal{F}(fsp_{l1})\mathcal{F}(\mathcal{T}_{1k}) + \mathcal{F}(fsp_{l2})\mathcal{F}(\mathcal{T}_{2k}) \\ + ... + \mathcal{F}(fsp_{l(l-1)})\mathcal{F}(\mathcal{T}_{(l-1)k}))\end{array}}{1 - \beta\mathcal{F}(fsp_{ll})}\end{aligned} \tag{21}$$

This gives us a set of simultaneous linear equations in $\mathcal{F}(\mathcal{T}_{1k})$, $\mathcal{F}(\mathcal{T}_{2k})$, ... $\mathcal{F}(\mathcal{T}_{lk})$. To solve this system of equations, we used Mathematica [7].

Now, β is generally a very small number. For most modern day chips, it is of the order of 10^{-3}. Hence we neglect all terms containing powers of β greater than one. Assume we have the source in layer 2. We then arrive at Equation 22.

$$\mathcal{F}(\mathcal{T}_{12}) = \frac{\begin{array}{r}\mathcal{F}(fsp_{12}) + \beta(\mathcal{F}(fsp_{13})\mathcal{F}(fsp_{32}) - \mathcal{F}(fsp_{33})\mathcal{F}(fsp_{12}) \\ + ... + \mathcal{F}(fsp_{1l})\mathcal{F}(fsp_{l2}) - \mathcal{F}(fsp_{ll})\mathcal{F}(fsp_{12}))\end{array}}{1 - \beta\left(\mathcal{F}(fsp_{11}) + \mathcal{F}(fsp_{22}) + ... + \mathcal{F}(fsp_{ll})\right)} \tag{22}$$

In Equation 22, the first term fsp_{12} denotes the temperature rise in layer 1 due to the dynamic power present in layer 2. The second term, $\beta\mathcal{F}(fsp_{13})\mathcal{F}(fsp_{32})$, accounts for secondary effects of the dynamic power source. The dynamic power source creates leakage sources in the adjoining layer (layer 3), which in turn act as sources themselves and heat the other layers. The effect of these sources in layer 1 is captured by the second term.

Equation 22 can be used directly to obtain the final temperature profile. We call this method *3DSim*$\mathcal{F}$. However, it requires taking a 2-D transform, which is computationally expensive ($O(n^2)$). To further speed-up the computation, we observe that for large die sizes, the temperature profile is radially symmetric, and has information only in one direction. Hence we can convert Cartesian co-ordinates to polar co-ordinates and use the Hankel Transform. A 1-D zero order Hankel transform is equivalent to a 2-D Fourier Transform for a radially symmetric function. Thus the 2-D problem ($O(n^2)$) is reduced to a single dimension ($O(n)$). Now, according to the definition of the Hankel transform that we use (Equation 4), we require an additional factor of 2π when calculating the transform of a convolution of two functions.

$$\begin{aligned}\mathcal{H}(\mathcal{T}_{12}) &= \frac{\begin{array}{r}\mathcal{H}(fsp_{12}) + 2\pi\beta(\mathcal{H}(fsp_{13})\mathcal{H}(fsp_{32}) - \mathcal{H}(fsp_{33})\mathcal{H}(fsp_{12}) \\ + ... + \mathcal{H}(fsp_{1l})\mathcal{H}(fsp_{l2}) - \mathcal{H}(fsp_{ll})\mathcal{H}(fsp_{12}))\end{array}}{1 - 2\pi\beta\left(\mathcal{H}(fsp_{11}) + \mathcal{H}(fsp_{22}) + ... + \mathcal{H}(fsp_{ll})\right)} \\ &...\end{aligned} \tag{23}$$

After computing the inverse Hankel transform of Equation 23, we can obtain the leakage aware Green's function. However, to calculate the final temperature profile, we would again need to take its transform. Hence we store it in the transform domain itself after converting the radial function to Cartesian coordinates. Let us refer to this avatar of the simulator as *3DSim*.

Thus we divide step 2 into two sub-stages. In the first sub-stage, we compute the transform of the heat spreading functions (Fourier transform in the case of *3DSimF*, and Hankel transform in the case of *3DSim*). In the second sub-stage, we use Equations 22 for *3DSimF* and 23 for *3DSim*, to calculate the final Green's functions in the transform domain. The advantage of this approach is that whenever β changes (as a result of varying supply voltage or the threshold voltage, or voltage-frequency scaling), we will have to re-run only the second sub-stage, that is, the calculation of the final Green's functions.

Figure 1 shows one of the calculated steady state Green's functions (for layer 2) along with the one obtained from Icepak. The temperature profile rapidly decays down to less than 0.1°C within four grid points (0.25 cm), thereby indicating that the boundary effects should be minimal. Also the temperature profile is radially symmetric, as shown by the contour map of the Green's function in Figure 3. The temperature profile obtained when multiple power sources are applied is shown in Figure 2.

The transient temperature profile obtained is shown in Figure 4

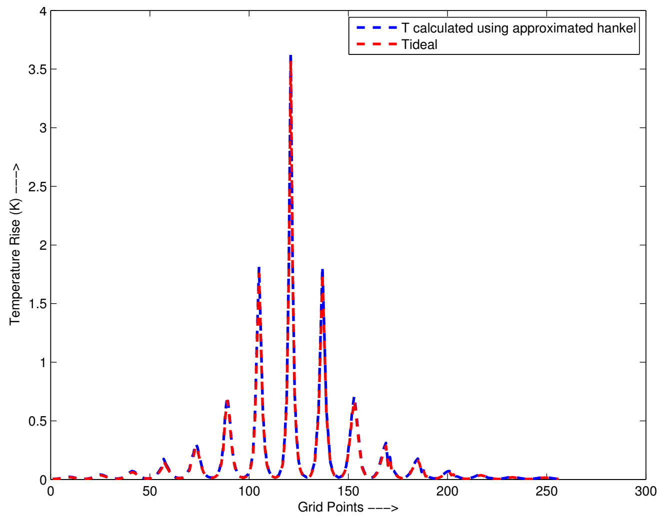

Fig. 1. Leakage Aware Green's function for Layer 2 with Source in Layer 3

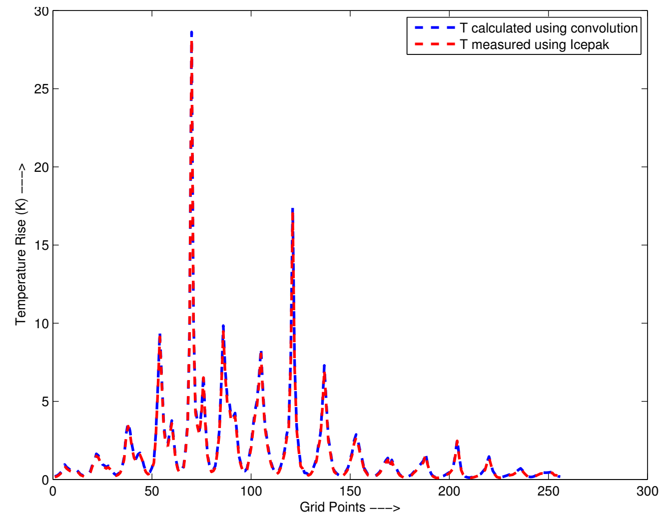

Fig. 2. Final temperature profile for Layer 2 for the power profile in Table III

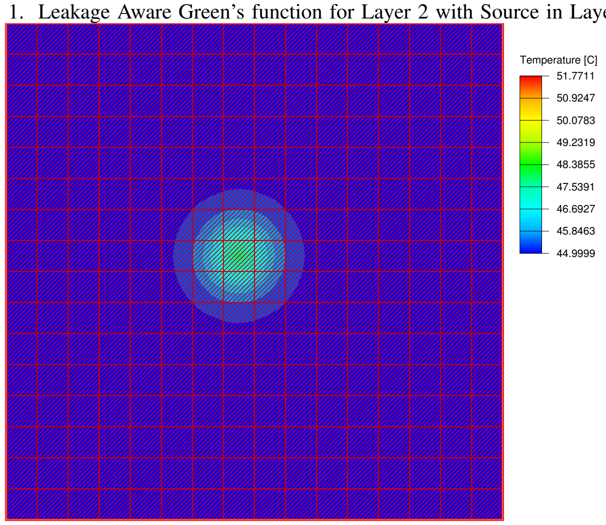

Fig. 3. Contour map for the Green's function in Figure 1

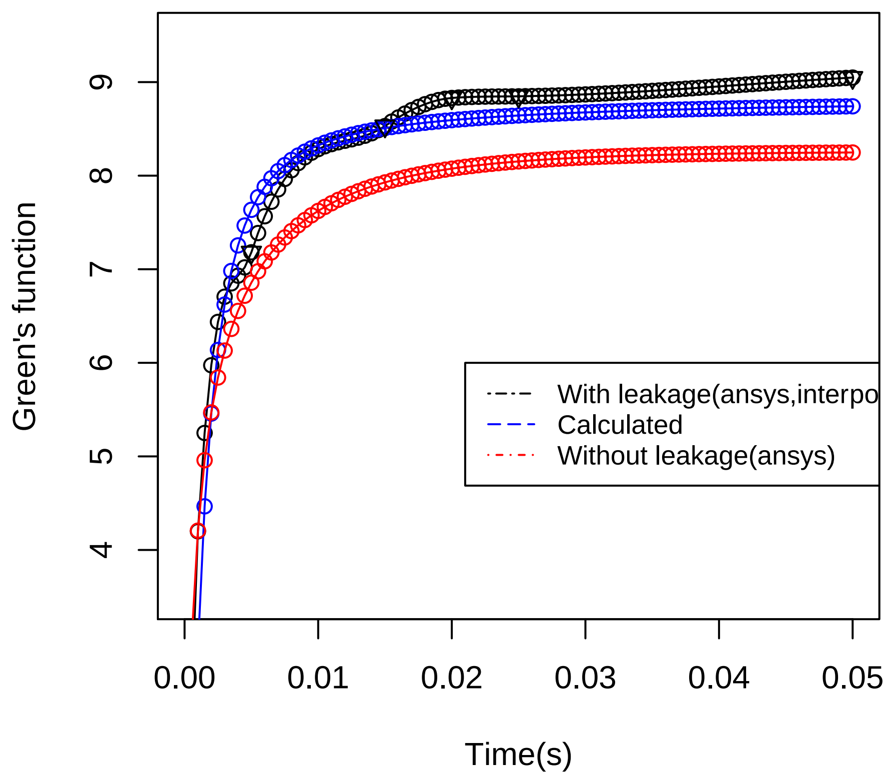

Fig. 4. Transient temmperature profile for layer 2