ECEN 322

Summary of Coordinate Transformations

Cartesian Cylindrical Cylindrical Cartesian Cartesian Spherical Spherical Cartesian

Divergence Theorum and Curl

Gradient - Scalar Field

Divergence - Vector Field

If $\nabla \cdot \vec{A} = 0$ then $\vec{A} =$ solenoidal vector fied (whatever goes in comes out)

for Circulation *** Δ is underlined in Nguyen's notation. Look into this.

Divergence Theorum

$$\int\limits_{V} \nabla \vec{A} \cdot dV = \oint\limits_{S} \vec{A} \vec{dS}$$

Circulation of \vec{A} around countour C

$$\Delta \oint\limits_C \vec{A} \vec{dl}$$

Curl

$$\nabla \times \vec{A} = \frac{1}{\Delta s} \lim_{\Delta s \to \emptyset} \oint\limits_{S} \vec{A} \vec{dl}$$

Stoke's Theorum

$$\int\limits_{S} \nabla \times \vec{A} \cdot \vec{ds} = \oint\limits_{C} \vec{A} \cdot \vec{dl}$$

Two Null Identitites

$$\nabla\times(\nabla V)\equiv 0$$

$$\nabla \cdot (\nabla \vec{A}) \equiv 0$$

Maxwell's Equations

$$\vec{E}(x,y,z,t), \vec{H}, \vec{D}, \vec{B}, \vec{J}, \varphi_v$$

Can be in differential or integral form - Differential form is most generally used

 $\int\limits_{S} \vec{J} \cdot \vec{dS} = I \leftarrow$ total current flow thru open surface

Contributor

Differential Form (Point Form)

Integral Form (Large Scale Form)

Faraday

$$\nabla imes ec{E} = rac{-\delta ec{B}}{\delta t}$$

$$\oint_C \vec{E} \cdot \vec{dl} = \frac{-\delta \vec{B}}{\delta t} \cdot \vec{dS}$$

Ampere

$$\nabla \times \vec{H} = \vec{J} + \frac{-\delta \vec{D}}{\delta t}$$

$$\int\limits_{S} \vec{J} \cdot \vec{dS} + \int\limits_{S} \frac{d\vec{D}}{dt} \cdot \vec{dS}$$

Gauss

$$\nabla \cdot \vec{D} = \varphi_v$$

$$\oint_C \vec{D} \cdot \vec{ds} = Q$$

No Magnetic Charges

$$\nabla \cdot \vec{B} = 0$$

$$\oint_C \vec{B} \cdot \vec{ds} = 0$$

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Modified Ampere's Law

$$\frac{\delta \vec{D}}{\delta t}$$

When a field is static, $\frac{d}{dt} = 0$

Special Cases of Maxwell's Equations: Static \vec{E} and \vec{H}

$$\vec{E}, \vec{H} = f(x, y, z) \neq f(t) \rightarrow \frac{\delta}{\delta t} =$$

Faraday
$$\nabla \times \vec{E} = 0 \quad \oint\limits_C \vec{E} \cdot \vec{dl} = 0$$

Ampere
$$\nabla \times \vec{H} = \vec{J} \quad \oint\limits_C \vec{H} \cdot \vec{dl} = I$$

Gaus
$$\nabla \cdot \vec{D} = \varphi_v \quad \oint\limits_C \vec{D} \cdot \vec{ds} = Q$$

Magnetic Charge
$$\nabla \cdot \vec{B} = 0$$
 $\oint_C \vec{B} \cdot \vec{ds} = 0$

$$\vec{E} = -\nabla V$$

Poisson
$$\nabla^2 V = \frac{\varphi_v}{\epsilon}$$

LaPlace
$$\nabla^2 V = 0$$

Charge Conservation
$$\nabla \cdot \vec{J} = -\frac{d\varphi_v}{dt}$$
, v - some arbitrary volume

Special Cases of Maxwell's Equations: Source-Free Regions ($\varphi=0,J=0$)

Faraday
$$\nabla \times \vec{E} = \frac{d\vec{B}}{dt}$$

Ampere
$$\nabla \times \vec{H} = \frac{d\vec{D}}{dt}$$

Gaus
$$\nabla \cdot \vec{D} = 0$$

No Magnetic Charges
$$\nabla \cdot \vec{B} = 0$$