# 2. MLE

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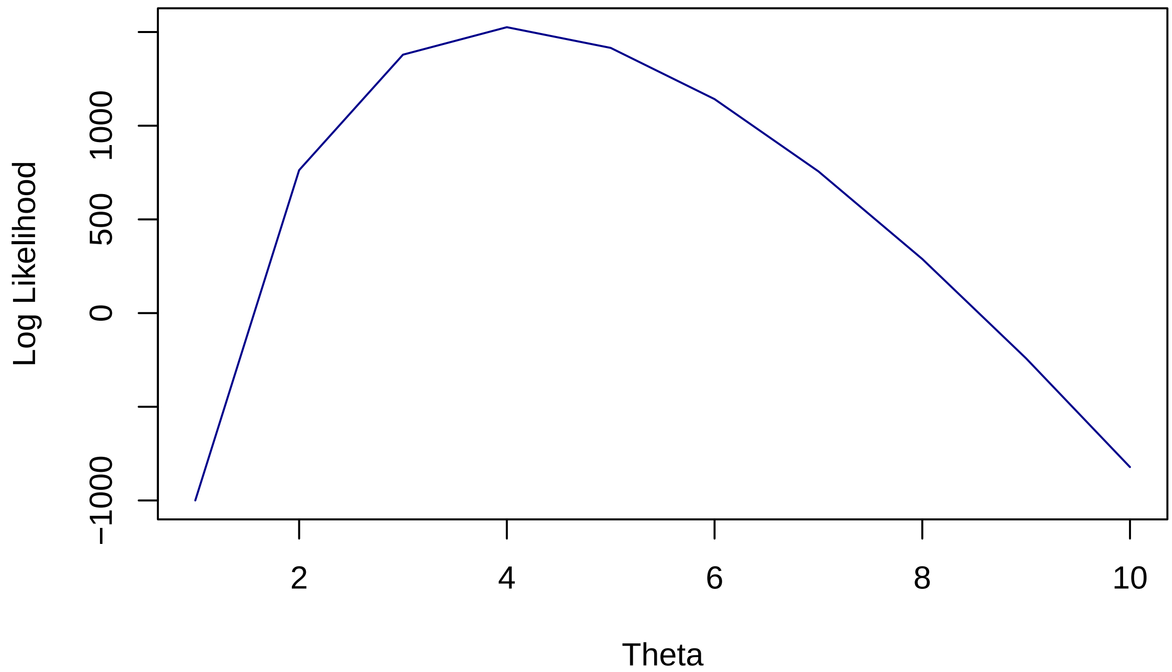
## Example 1: Poisson Distribution

```r
set.seed(123)
## Generate random numbers from Poisson
Data1 <- c(rpois(1000, 4))

## Write out the log-likelihood function
"logL.pois" <- function(theta, x) {

  n <- length(x)
  logL <- sum(x)*log(theta) - n*theta
  return(-logL)

}

## Find MLE
mle_mu1 <- optim(par = 1, fn = logL.pois, method = "BFGS",
      hessian = T, x = Data1)$par

## Find theoretical mean
mu1 <- mean(Data1)

print(cbind(mu1, mle_mu1))
```

```
##        mu1 mle_mu1
## [1,] 3.986   3.986
```

```r
## Plot
plot(-logL.pois(seq(1,10), Data1), type = "l", col = "darkblue",
     xlab = "Theta", ylab = "Log Likelihood")
```




### Example 2: BHP Trading Data (Poisson Distribution)

```
library(hexView)
BHP <- readEViews("etc3400_bhp_trade_counts.wf1", time.stamp=TRUE, as.data.frame = TRUE)
head(BHP)
```

```
##      Date BHP_1_MIN_TRADE_COUNTS
## 1 1-01-01                     26
## 2 2-01-01                      6
## 3 3-01-01                      6
## 4 4-01-01                     18
## 5 5-01-01                      0
## 6 6-01-01                     20
```

```
Data2 <- BHP$BHP_1_MIN_TRADE_COUNTS
length(Data2)
```

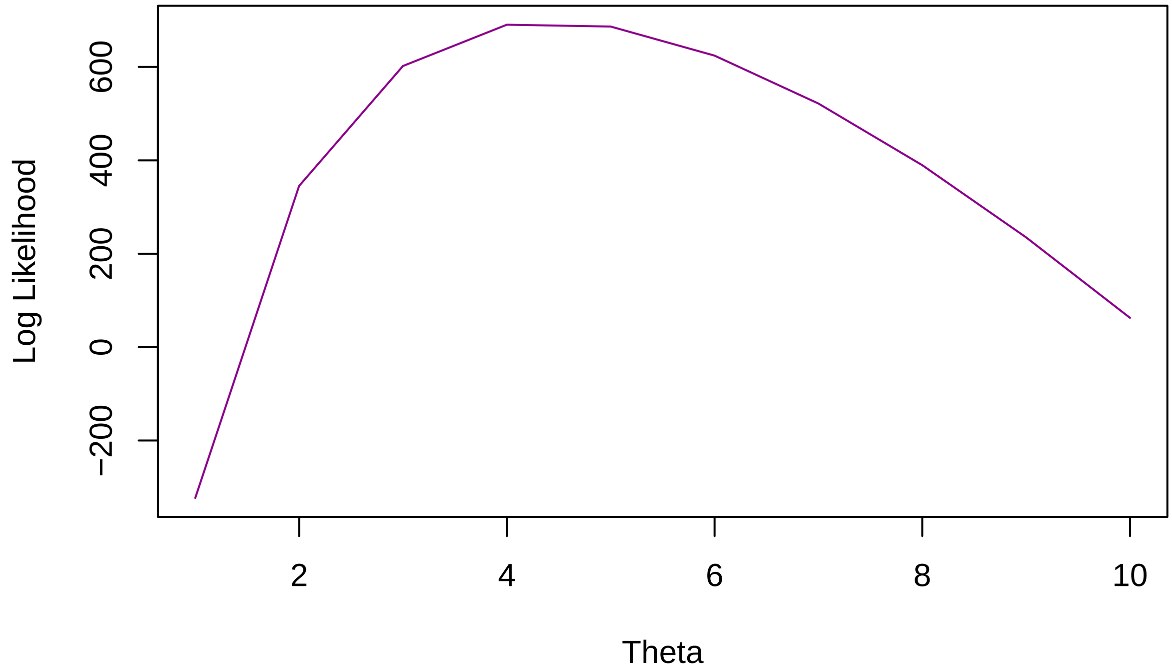
```
## [1] 323
```

```
## Calculate theoretical mean:
mu <- mean(Data2)

## Find MLE
mle_mu <- optim(par = 1, fn = logL.pois, method = "BFGS",
            hessian = T, x = Data2)$par

print(cbind(mu, mle_mu))
```

```
##            mu   mle_mu
## [1,] 4.427245 4.427308
```

```
## Plot
plot(-logL.pois(seq(1:10), Data2), type = "l", col = "darkmagenta",
```

```
    xlab = "Theta", ylab = "Log Likelihood")
```




## Example 3: Normal Distribution

```
Data3 <- rnorm(1000, 3, 2)
head(Data3)
```

```
## [1]  1.79621431  1.01260282  5.05357011  4.50212261 -0.01833307  2.80970510
```

```
## Theoretical sample mean and variance
mu3 <- mean(Data3)
sigma2 <- sd(Data3)

## Log Likelihood function for Normal
ll.normal <- function(theta, x) {

  n <- length(x)
  mu <- theta[1]
  sigma2 <- theta[2]
  logL <- -0.5*n*log(2*pi) - 0.5*n*log(sigma2) - (1/(2*sigma2))*sum(x-mu)^2

  return(-logL)

}

## Find MLE
mle_theta_vect <- optim(c(0,1), fn = ll.normal, x = Data3)$par


## MLE mean and sample variance
mle_mu3 <- mle_theta_vect[1]
mle_sigma2 <- mle_theta_vect[2]
```

```r
## Calculate biases
mle_mu3 - mu3
```

```
## [1] 0.01192669
```

```r
# MLE of normal mean is close to true population mean
mle_sigma2 - sigma2
```

```
## [1] -1.841254
```

```r
# MLE of sample variance is heavily biased down
```

Example 3 (Normal Distribution) validates our theoretical result that the MLE of our sample variance, with an underlying Normal Distribution, is biased downwards.