

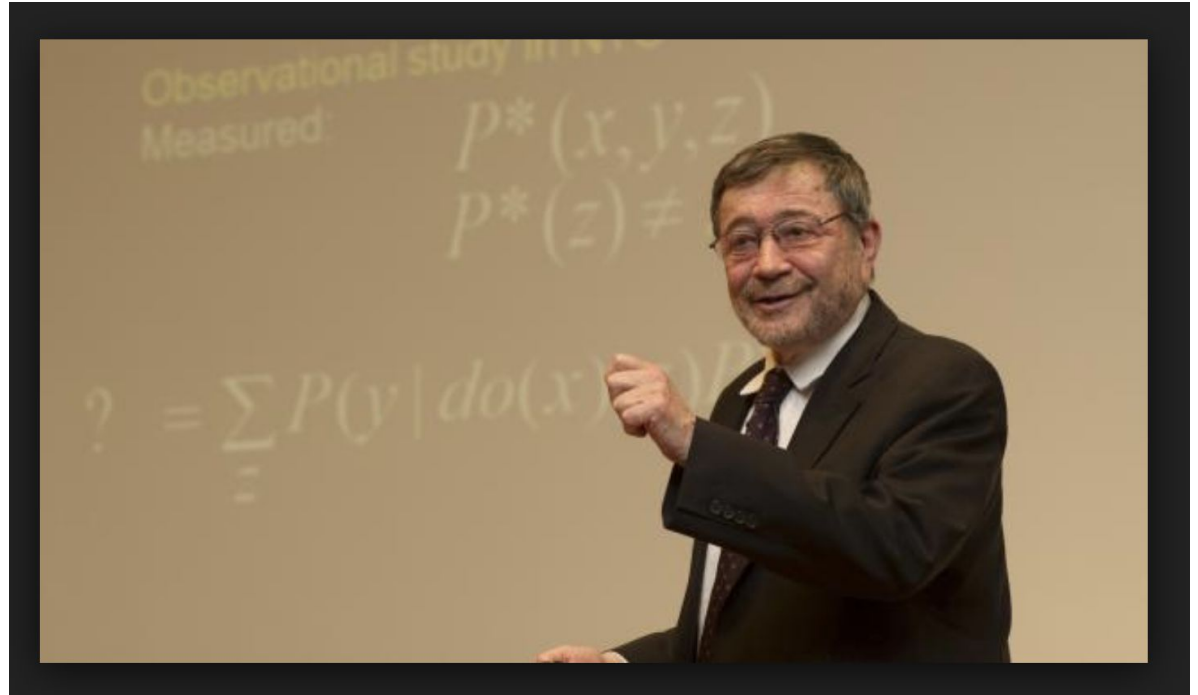
# Seeing Doing and Imagining

Dr. Judea Pearl's view on Causal Inference in Statistics

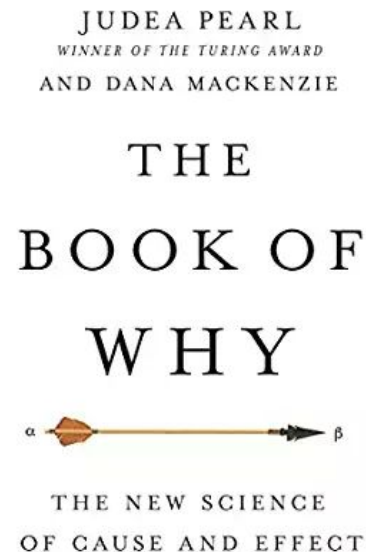
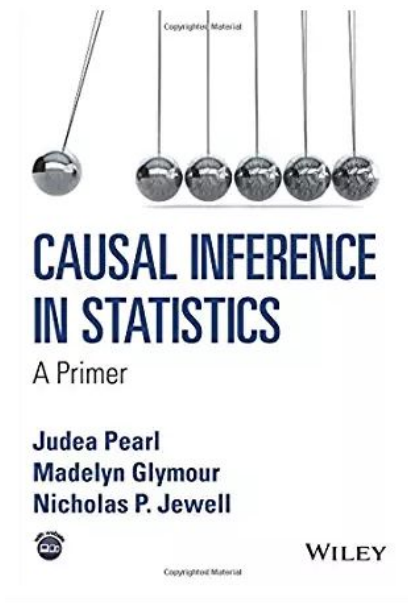
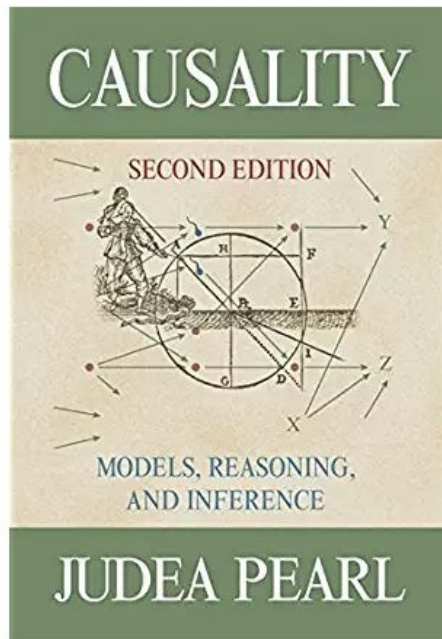


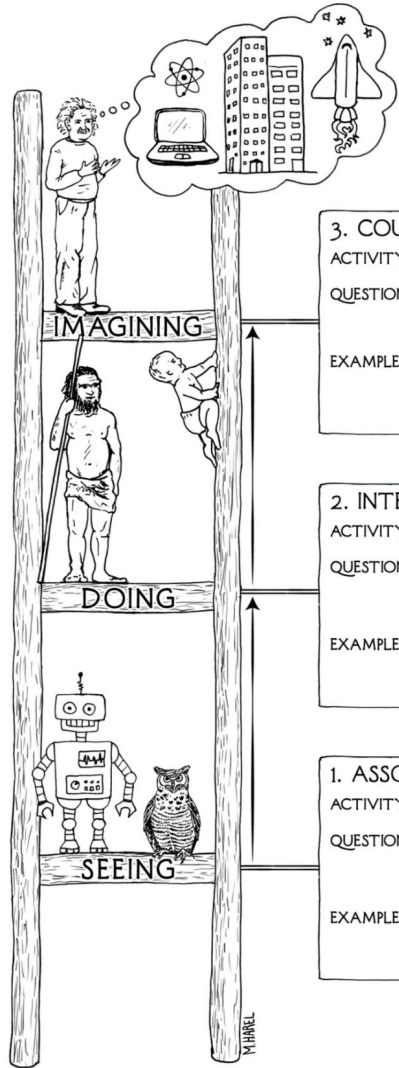
# Biography

- Computer Scientist
- Philosopher
- One of the developers of  
Bayesian Networks
- 2011 Winner of ACM Turing  
Award



# Books About Causality (By Pearl)





### 3. COUNTERFACTUALS

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**QUESTIONS:** *What if I had done ...? Why?*  
(Was it X that caused Y? What if X had not occurred? What if I had acted differently?)

**EXAMPLES:** Was it the aspirin that stopped my headache?  
Would Kennedy be alive if Oswald had not killed him? What if I had not smoked for the last 2 years?

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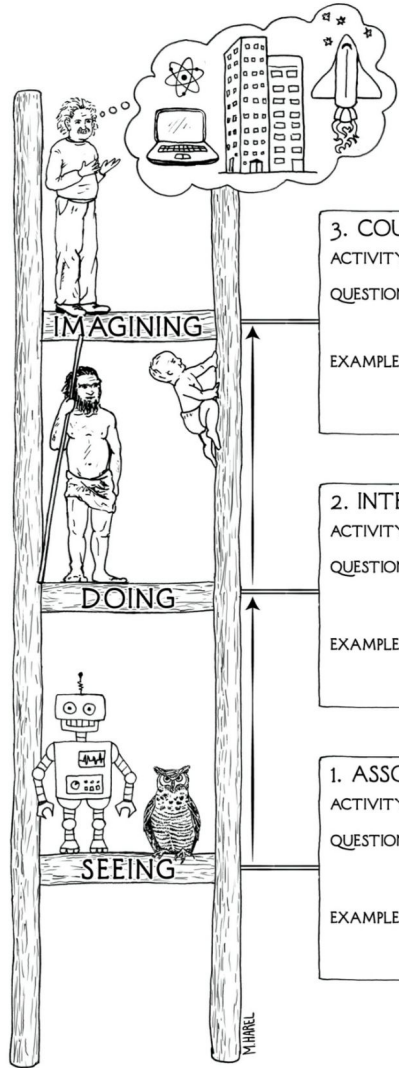
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# Seeing



# Seeing

History and Milestones:

--David Hume

--Frederick Galton

--Karl Pearson

Models:

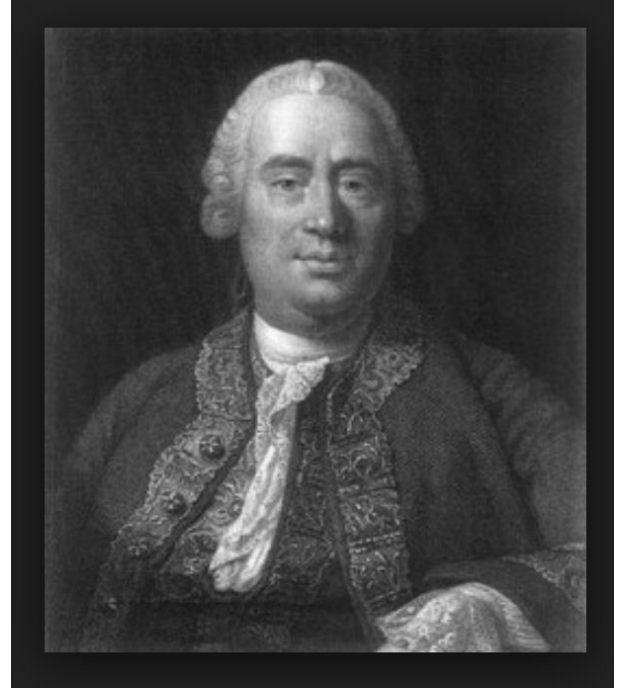
--Graphical Models

--Confounder

--Collider

# David Hume

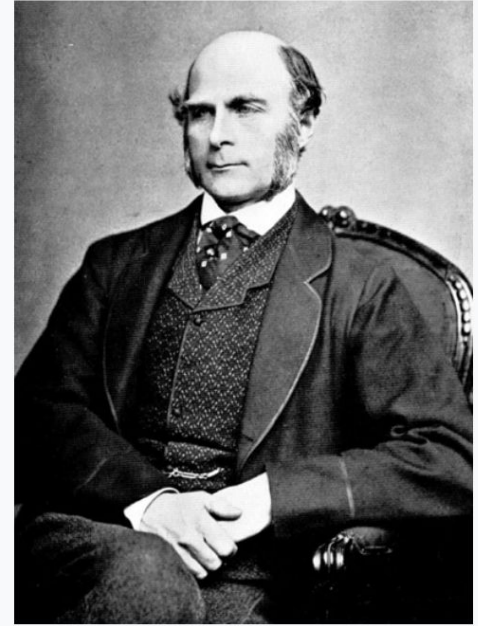
- 18 Century philosopher
- Causality cannot be justified rationally
- Causality is result of mental or custom habit
- Attributable only to the experience of  
“Constant Conjunction”



# Galton

- 19 Century statistician
- Toward causality but found correlation
- Popularized the word regression

Sir Francis Galton





# Pearson

--Galton's student

--Define causation as a special case of correlation

1. when correlation coefficient is 1 and

2.  $x$  and  $y$  are deterministic(which can never be proven)

--Completely ignored intervention and counterfactual

**Karl Pearson**

FRS



# Notation

V: Endogenous variables  $\{X, Y, Z\}$

U: Exogenous variables  $\{U_x, U_y, U_z\}$

F: Functions  $\{F_x, F_y, F_z\}$

Every endogenous variable in a model is a descendant of at least one exogenous variable.

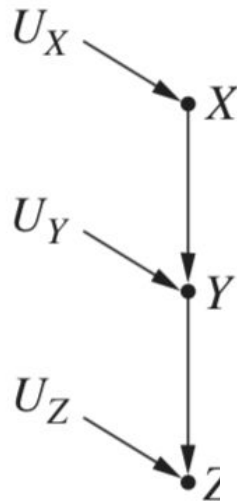
Exogenous variables cannot be descendants of any other variables, and in particular, cannot be a descendant of an endogenous variable; they have no ancestors and are represented as *root* nodes in graphs.

If we know the value of every exogenous variable, then using the functions in  $f$ , we can determine with perfect certainty the value of every endogenous variable.

# Chain and Forks

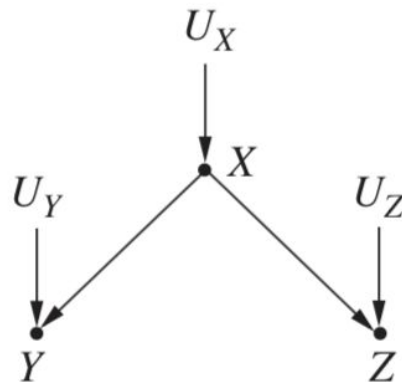
## Rule 1 (Conditional Independence in Chains)

Two variables,  $X$  and  $Z$  are conditionally independent given  $Y$ , if there is only one unidirectional path between  $X$  and  $Y$  and  $Z$  is any set of variables that intercepts that path.



## Rule 2 Conditional Independence in Forks)

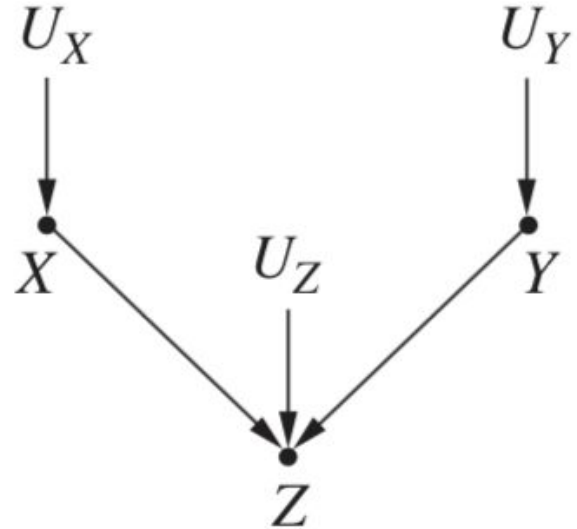
If a variable  $X$  is a common cause of variables  $Y$  and  $Z$ , and there is only one path between  $Y$  and  $Z$ , then  $Y$  and  $Z$  are independent conditional on  $X$ .



# Collider

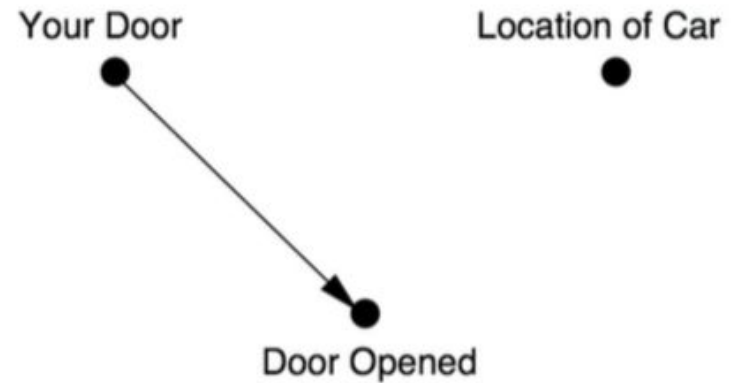
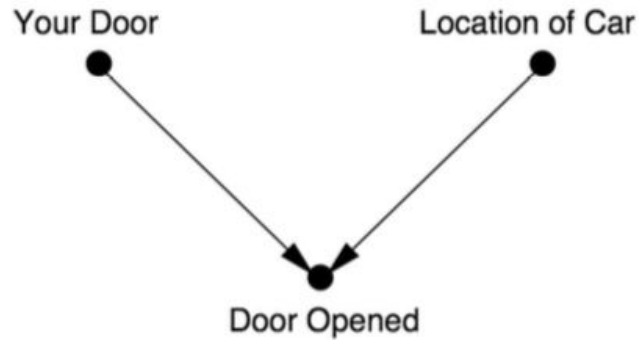
**Rule 3 (Conditional Independence in Colliders)** *If a variable  $Z$  is the collision node between two variables  $X$  and  $Y$ , and there is only one path between  $X$  and  $Y$ , then  $X$  and  $Y$  are unconditionally independent but are dependent conditional on  $Z$  and any descendants of  $Z$ .*

(The grass rain, sprinkler example)



# Examples of Collider

## Monty Hall Problem

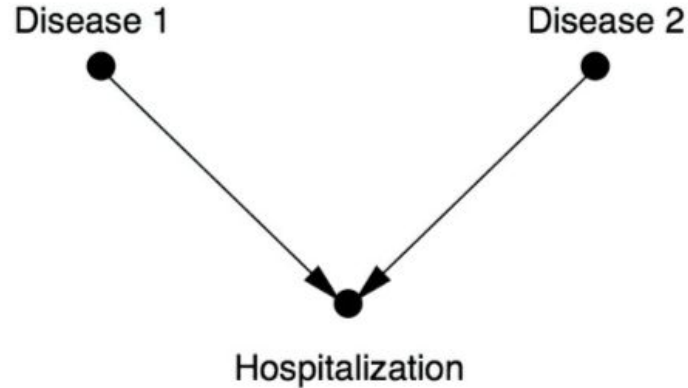


# Collider Bias

## Berkson's Paradox

Two traits are independent from each other (or negative related) will appear to be positive related once conditioned on a collider (or a descendent of a collider)

Sometimes as selection bias

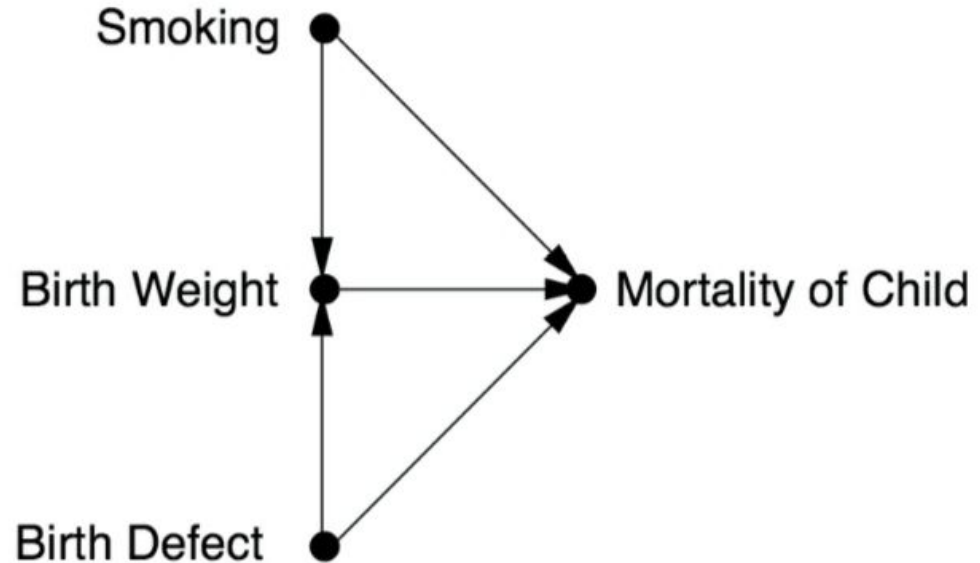


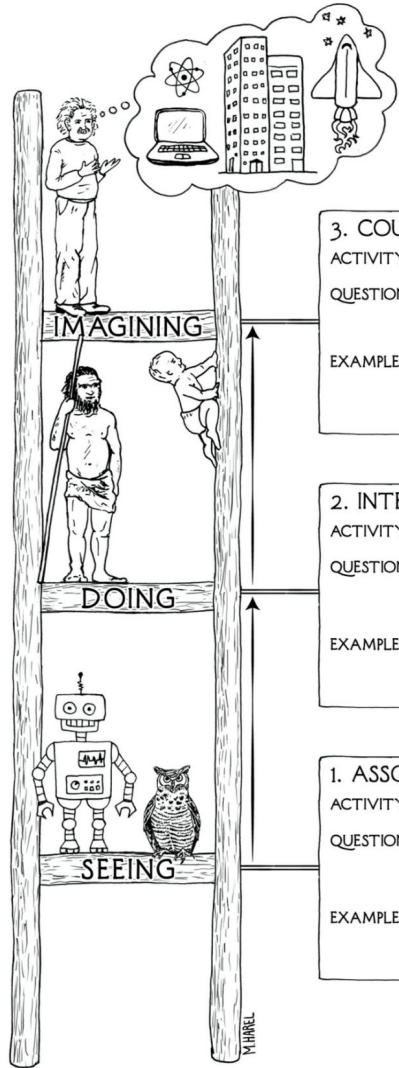
# Newborn Smoking Study

Mid 1960, researcher pointed out that mother's smoking during pregnancy seemed to benefit the health of her newborn baby.

Reason:

Newborn baby from smoking mother had a better survival rate than non-smoking mother.





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# Doing





# R. A. Fisher

--Statistician

--Geneticist

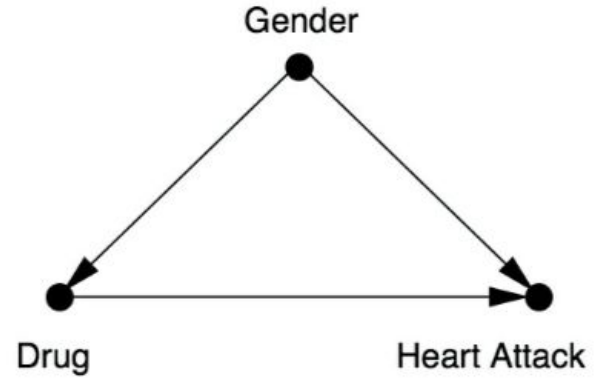
--Expert in experiment design

**Sir Ronald Fisher**

FRS

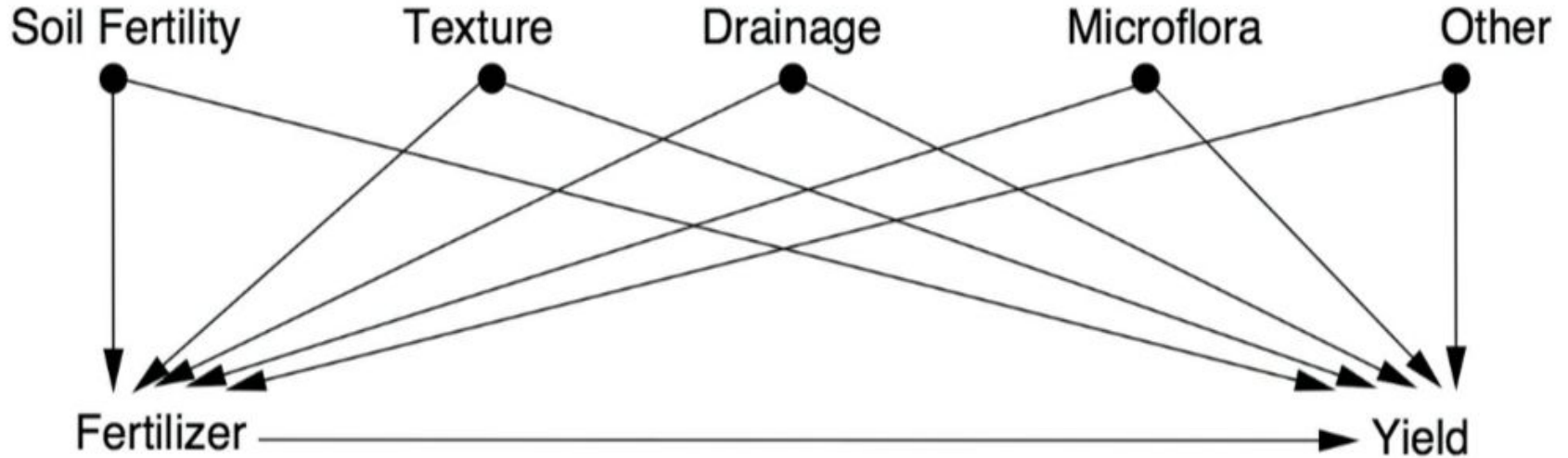


# Confounder

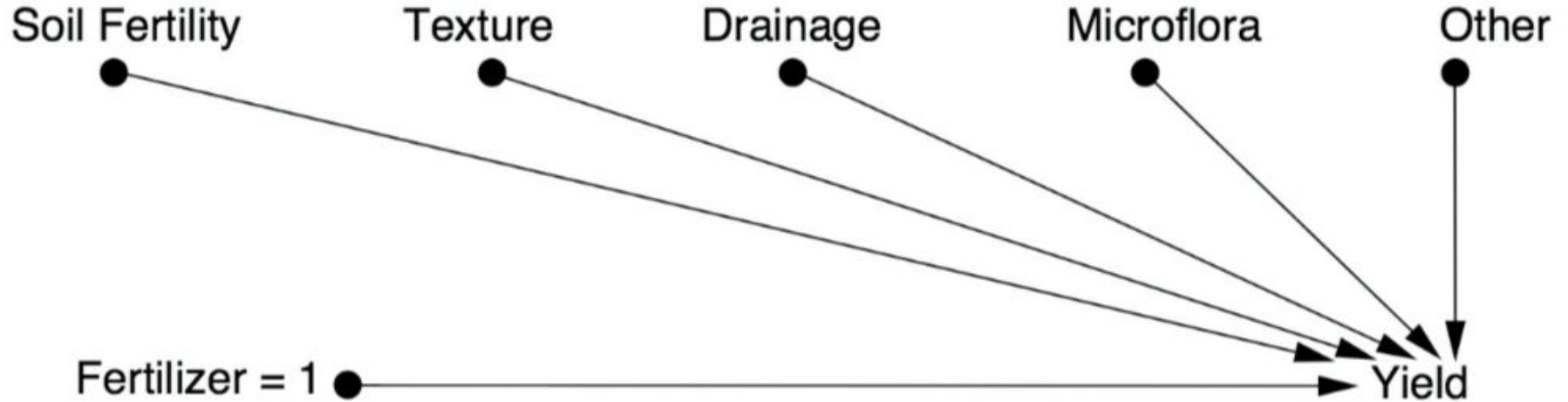


	Control Group (No Drug)		Treatment Group (Took Drug)	
	<i>Heart attack</i>	<i>No heart attack</i>	<i>Heart attack</i>	<i>No heart attack</i>
Female	1	19	3	37
Male	12	28	8	12
Total	13	47	11	49

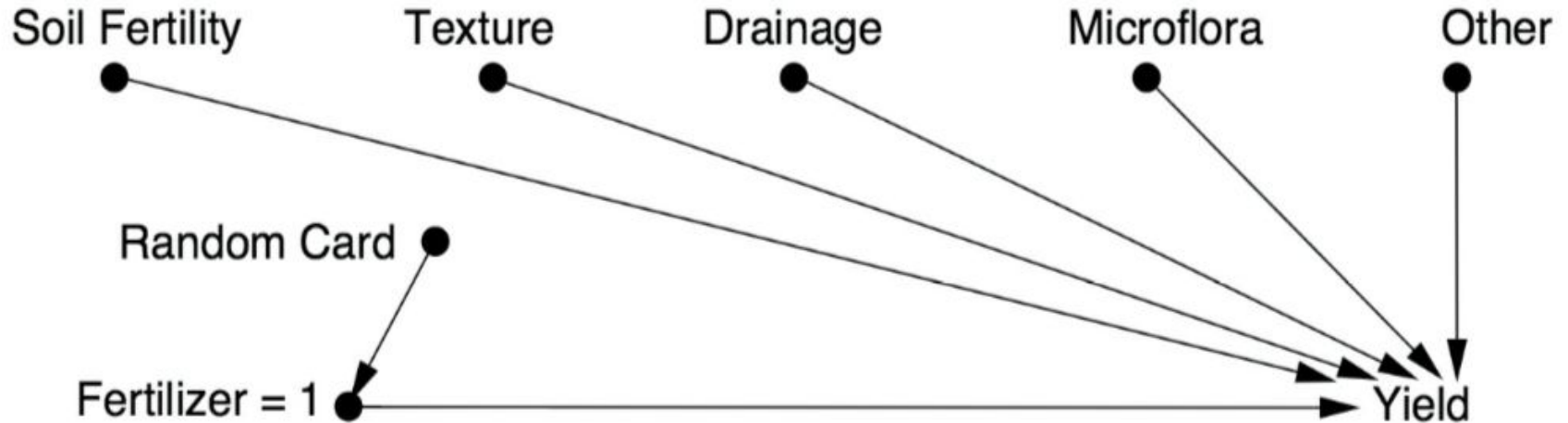
# Many Confounders



# Adjust for confounders



# RCT by Fisher



# Intervention

--Randomized Controlled Trials, the golden standard of intervention

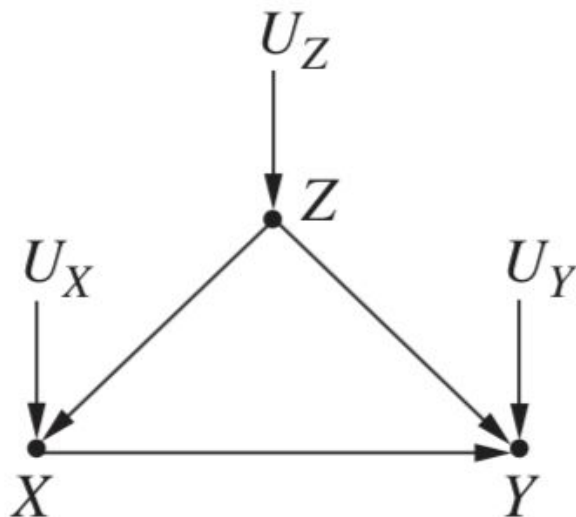
1. Solved potential confounding bias
2. Quantified uncertainty

--But always check if the design actually solved the problem

(Story of working condition and production in a factory)

# New Notation

$$P(Y = 1|do(X = 1)) - P(Y = 1|do(X = 0))$$



# Adjustment

**Rule 1 (The Causal Effect Rule)** *Given a graph  $G$  in which a set of variables  $PA$  are designated as the parents of  $X$ , the causal effect of  $X$  on  $Y$  is given by*

$$P(Y = y|do(X = x)) = \sum_z P(Y = y|X = x, PA = z)P(PA = z) \quad (3.6)$$



# Back Door

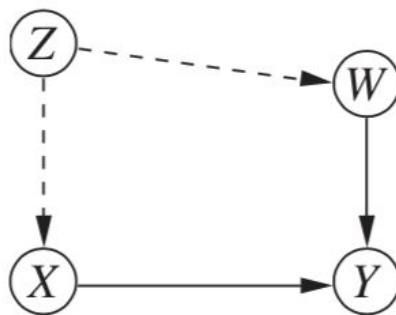
**Definition 3.3.1 (The Backdoor Criterion)** *Given an ordered pair of variables  $(X, Y)$  in a directed acyclic graph  $G$ , a set of variables  $Z$  satisfies the backdoor criterion relative to  $(X, Y)$  if no node in  $Z$  is a descendant of  $X$ , and  $Z$  blocks every path between  $X$  and  $Y$  that contains an arrow into  $X$ .*

If a set of variables  $Z$  satisfies the backdoor criterion for  $X$  and  $Y$ , then the causal effect of  $X$  on  $Y$  is given by the formula

$$P(Y = y|do(X = x)) = \sum_z P(Y = y|X = x, Z = z)P(Z = z)$$

1. We block all spurious paths between  $X$  and  $Y$ .
2. We leave all directed paths from  $X$  to  $Y$  untouched.
3. We create no newpaths.

# Example

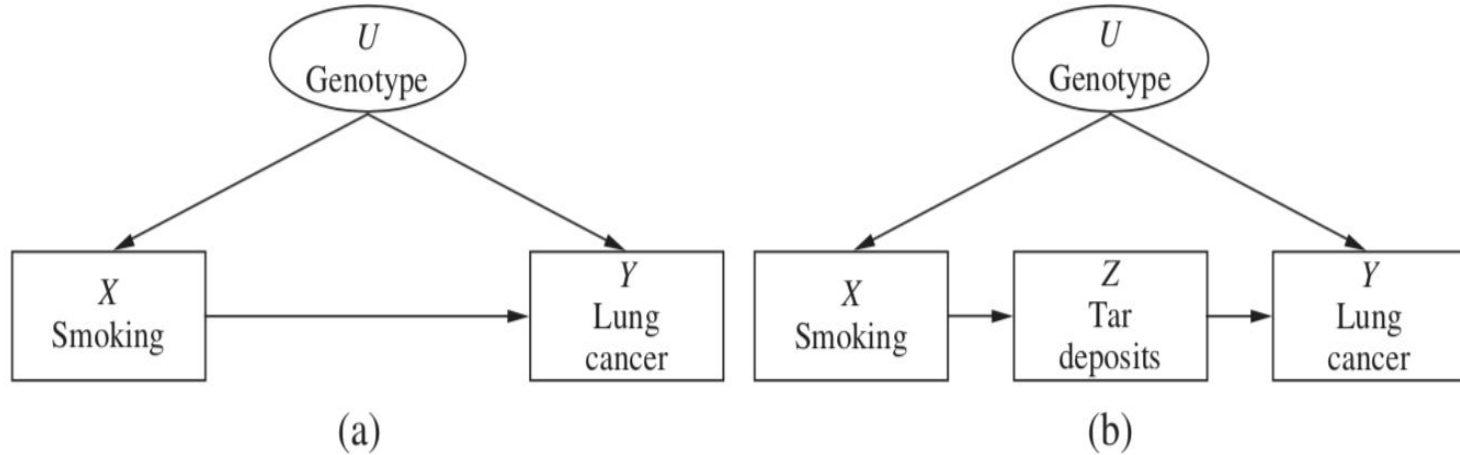


**Figure 3.6** A graphical model representing the relationship between a new drug ( $X$ ), recovery ( $Y$ ), weight ( $W$ ), and an unmeasured variable  $Z$  (socioeconomic status)

$$P(Y = y | do(X = x)) = \sum_{\dots} P(Y = y | X = x, W = w) P(W = w)$$

$$P(y | do(x)) = P(y | x)$$

# Front Door



**Figure 3.10** A graphical model representing the relationships between smoking ( $X$ ) and lung cancer ( $Y$ ), with unobserved confounder ( $U$ ) and a mediating variable  $Z$

# Front Door

**Definition 3.4.1 (Front-Door)** *A set of variables  $Z$  is said to satisfy the front-door criterion relative to an ordered pair of variables  $(X, Y)$  if*

- 1.  $Z$  intercepts all directed paths from  $X$  to  $Y$ .*
- 2. There is no unblocked path from  $X$  to  $Z$ .*
- 3. All backdoor paths from  $Z$  to  $Y$  are blocked by  $X$ .*

# Front Door

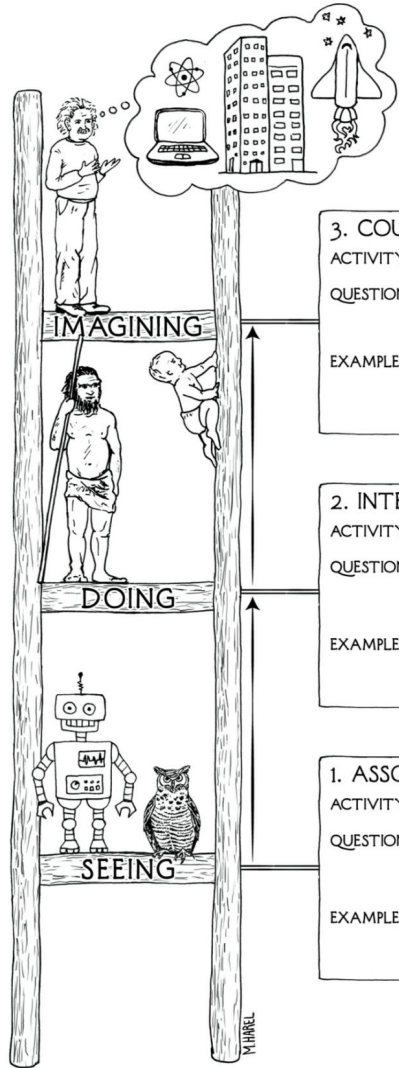
$$P(Z = z|do(X = x)) = P(Z = z|X = x)$$

$$P(Y = y|do(Z = z)) = \sum_x P(Y = y|Z = z, X = x)$$

$$P(Y = y|do(X = x)) = \sum_z P(Y = y|do(Z = z))P(Z = z|do(X = x))$$

$$P(Y = y|do(X = x)) =$$

$$\sum_z \sum_{x'} P(Y = y|Z = z, X = x')P(X = x')P(Z = z|X = x)$$



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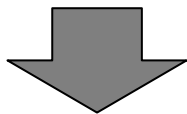
# Imagining

# Counterfactuals

Notation:

“Do” operator

$$E(Y_{X=1} | X = 0, Y = Y_0 = 1)$$



$$E[Y | do(X = x)]$$

# Three Steps

- (i) Abduction: Use evidence  $E = e$  to determine the value of  $U$ .
- (ii) Action: Modify the model,  $M$ , by removing the structural equations for the variables in  $X$  and replacing them with the appropriate functions  $X = x$ , to obtain the modified model,  $M_x$ .
- (iii) Prediction: Use the modified model,  $M_x$ , and the value of  $U$  to compute the value of  $Y$ , the consequence of the counterfactual.



# Topic skipped

IP weighting

Mediation

TE, DE, NDE, NID

Causal Inference in Linear Systems(Partial regression)

