

Algebra notes , problems and solutions

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1 System of real numbers

1. Natural numbers 1,2,3,4,..... (three dots mean "and so on") used in counting and is also known as the positive integers.If two such numbers added or multiplied,the result is always a natural numbers.
2. Positive rational numbers or positive fractions are the quotients of two positive integers, such as $\frac{2}{3}$, $\frac{8}{5}$, $\frac{121}{17}$.The positive rational numbers include the get of natural numbers. Thus the rational number $\frac{3}{1}$ is the natural number 3.
3. Positive irrational numbers are numbers which are not, such as $\sqrt{2}$, π .
4. Zero, written 0, which in order to enlarge the number system so at to permit such operations as 6-6 on 10 - 10 . Zero has the property that any number multiplied by zero is zero. Zero divided by any number $\neq 0$.
5. Negative integers, negative rational numbers and negative irrational numbers such as -3, $-\frac{2}{3}$,and $-\sqrt{2}$, arose in the to enlarge the number system so as to permit such operations 2-8, $\pi - 3\pi$ or $2 - 2\sqrt{2}$.

2 Exponent laws

1.

$$a^p a^q = a^{p+q}$$

2.

$$\frac{a^p}{a^q} = a^{p-q} = \frac{1}{a^{q-p}}$$

3.

$$(a^p)^q = a^{pq}$$

4.

$$(ab)^p = a^p b^p, \left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

3 Some laws pertaining fractions

1.

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$$

Polynomial is a monomial or multinomial in which every term is integral and rational.

4 Algebraic Expressions

An algebraic expression is a combination of ordinary numbers and letters which represent numbers

$$3x^3 - 5xy + 2y^4, 2a^3b^5, \frac{5xy + 3z}{2a^3 - c^2}$$

are algebraic expressions. A term consists of products and quotients of ordinary numbers and letters which represents numbers. Thus $6x^2y^3$, $\frac{5x}{3y^4}$, $-3x^7$ are terms. However, $6x^2 - 7xy$ is an algebraic expression consisting of two terms.

A monomial is an algebraic expression consisting of only one term. This $7x^3y^4$, $3xyz^2$, $\frac{4x^2}{y}$ are monomials. Because of this definition, monomials are sometimes simply called terms.

4.1 Grouping laws

1. If a + sign precedes a symbol of grouping may be removed without affecting the terms contained.

$$(3x + 7y) + (4xy - 3x^3) = 3x + 7y + 4xy - 3x^3$$

2. if a -sign precedes symbol of grouping, this symbol of grouping may be removed if each sign of the term contained is changed.

$$(3x + 7y) - (4xy - 3x^3) = 3x + 7y - 4xy + 3x^3$$

3. If more than one symbol of grouping is present, the inner ones are to be removed first

$$2x - 4x^3 - (3x^2 - 5y) = 2x - 4x^3 - 3x^2 + 5y = 2x - 4x^3 + 3x^2 - 5y$$

4.2 Computation with Algebraic expression

A polynomial is a monomial or multimomial in which every term is integral and rational. For example , $3x^2y^3 - 5x^4y + 2$, $2x^4 - 7x^3 + 3x^2 - 5x + 2$, $4xy + 2$ and $3x^2$ are polynomials. However, $3x^2 - \frac{-4}{x}$ and $4\sqrt{y} + 3$ are not polynomials.

4.2.1 Polynomial to polynomial multiplication

Polynomial by polynomial, multiply each of the terms of the terms of the other polynomial and combine results. It is very often useful to arrange the polynomials according to ascending (or descending) powers of one of the letters involved.

Multiply $-3x + 9 + x^2$ by $3 - x$

- 1.

$$x^2 - 3x + 9$$

$$-x - 3$$

2. Multiplying (1) by $-x$

$$-x^3 + 3x^2 + 9x$$

3. Multiplying (1) by 3

$$3x^2 - 9x + 27$$

4. (2) and (3) together

$$-x^3 + 6x^2 - 18x + 27$$

5 Word problems for practice

- (a) Two more than five times a certain number.
- (b) Six less than twice a certain number.
- (c) Each of two numbers whose difference is 25.
- (d) The squares of there consecutive integers.
- (e) The amount by which five times a certain number exceeds 40.