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by

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THESIS

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The thesis has been completed... (INSERT YOUR TEXTS)

YOUR INITIAL

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CHAPTER 3

MAGNETIC MIRROR DESCENT

3.0.1 Mirror Descent

3.0.2 Magnetic Mirror Descent

Magnetic mirror descent (MMD) [1] is a last-iterate equilibrium approximation algorithm for two-player zero-sum games that is an extension of mirror descent with entropy regularization.

The idea behind MMD begins with the observation that solving for QRE in two-player zero-sum games can be reformulated as the solution to a negative entropy regularized saddle point problem as follows,

$$\min_{x \in \mathcal{X}} \max_{y \in \mathcal{Y}} \alpha g_1(x) + f(x, y) + \alpha g_2(y), \tag{3.1}$$

where $\mathcal{X} \subset \mathbb{R}^n$, $\mathcal{Y} \subset \mathbb{R}^m$ are closed and convex, and $g_1 : \mathbb{R}^n \to \mathbb{R}$, $g_2 : \mathbb{R}^m \to \mathbb{R}$, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$.

The solution (x_*, y_*) to Equation 3.1, is a Nash equilibrium in the reguarlized game with the following best response conditions,

$$x_* \in \arg\min_{x \in \mathcal{X}} \alpha g_1(x) + f(x, y_*) \tag{3.2}$$

$$y_* \in \arg\min_{y \in \mathcal{Y}} \alpha g_2(y) + f(x_*, y) \tag{3.3}$$

Zero-sum games and Variational Inequalities

MMD reframes the solution to QRE as a variational inequality problem.

Definition 1 *Given $\mathcal{Z} \subseteq \mathbb{R}^n$ and mapping $G : \mathcal{Z} \to \mathbb{R}^n$, the variational inequality problem VI($\mathcal{Z}, G$) is to find $z_* \in \mathcal{Z}$ such that,*

$$\langle G(z_*, z - z_*) \rangle \geq 0 \quad \forall z \in \mathcal{Z}.$$

The optimality conditions are equivalent to VI($\mathcal{Z}, G$), where $G = F + \alpha \nabla g$, $\mathcal{Z} = \mathcal{X} \times \mathcal{Y}$, and $g : \mathcal{Z} \to \mathbb{R}$.

Now, the solution the the VI problem ($z_* = (x_*, y_*)$), corresponds to the solution of the saddle point problem stated in Equation 3.1, and satisfies the best response conditions Equation 3.2 and Equation 3.3.

The algorithm that the authors propose is a non-Euclidean proximal gradient method to solve the VI($\mathcal{Z}, F + \alpha \nabla g$) problem.

We now restate the main algorithm as stated in [1](3.1),

With the following assumptions, z_{t+1} is well defined:

- ψ is 1-strongly convex with respect to $\|.\|$ over $\mathcal{Z}$, and for any l, stepsize $\eta > 0$, $\alpha > 0$, $z_{t+1} = \arg\min_{z \in \mathcal{Z}} \eta(\langle l, z \rangle + \alpha g(z)) + B_\psi(z; z_t) \in \text{int dom } \psi$.

AlgorithmMMD Starting with $z_1 \in \text{int dom } \psi \cap \mathcal{Z}$, at each iteration t do

$z_{t+1} = \arg\min_{z \in \mathcal{Z}} \eta(\langle F(z_t), z\rangle + \alpha g(z)) + B_\psi(z, z_t)$.

- F is monotone and L-smooth with respect to $\|.\|$ and g is 1-strongly convex relative to ψ over $\mathcal{Z}$ with g differentiable over int dom ψ.

Algorithm 1 provides the following convergence guarantees,

Assuming that the solution z_* to the problem $\text{VI}(\mathcal{Z}, F + \alpha\nabla g)$ lies in the int dom ψ, then the algorithm 1 guarantees

$$B_\psi(z_*; z_{t+1}) \leq \left(\frac{1}{1+\eta\alpha}\right)^t B_\psi(z_*; z_1),$$

if $\alpha > 0$, and $\eta \leq \frac{\alpha}{L^2}$.

MMD is also a first attempt in developing algorithms that are performant in both Single-agent and Multi-agent settings.

In single-agent RL MMD's performance is competitive with that of PPO in Atari and MuJoCo environments (though not evaluated extensively). And, in the multi-agent setting the performance of tabular MMD is on par with that of CFR, but worse than that of CFR+.

CHAPTER 4

EXTENDING MMD WITH NEURD FIX, EXTRAGRADIENT, AND OPTIMISM

CHAPTER 5

EXPERIMENTS

5.1 Experimental Domains

Tabular Experiments: Normal Form games: Perturbed RPS

Neural Experiments: Kuhn Poker Abrupt Dark Hex (3x3) Phantom Tic-tac-toe

5.2 Evaluation Methods

Exact exploitability Apporximate exploitability

5.3 Tabular MMD Experiments

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5.4 Neural MMD Experiments

Outline:

- Implementation details (RLLib, PPO modifications, GAE)
- Neural network architecture, hyperparameters

5.4.1 Results

APPENDICES

Appendix A

SOME ANCILLARY STUFF

Ancillary material should be put in appendices.

Appendix B

SOME MORE ANCILLARY STUFF

[2]

CITED LITERATURE

1. Sokota, S., D'Orazio, R., Kolter, J. Z., Loizou, N., Lanctot, M., Mitliagkas, I., Brown, N., and Kroer, C.: A Unified Approach to Reinforcement Learning, Quantal Response Equilibria, and Two-Player Zero-Sum Games. In *The Eleventh International Conference on Learning Representations* , February 2023.

2. Farine, D. R., Strandburg-Peshkin, A., Couzin, I. D., Berger-Wolf, T. Y., and Crofoot, M. C.: Individual variation in local interaction rules can explain emergent patterns of spatial organization in wild baboons. *Proceedings of the Royal Society of London B: Biological Sciences* , 284(1853), 2017.