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by

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THESIS

Submitted as partial fulfillment of the requirements
for the degree of Master's in Computer Science
in the Graduate College of the
University of Illinois at Chicago, 2023

Chicago, Illinois

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ACKNOWLEDGMENTS

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SUMMARY

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CHAPTER 1

INTRODUCTION

In this work, we study two mirror-descent based reinforcement learning algorithms and propose novel improvements to them.

1.1 Outline

The rest of the thesis is organized as follows. We begin by providing some background and definitions in section 2 that are useful for the understanding of the algorithms and methods described in section 3 and 4. Section 3 introduces Mirror Decsent and expands on Mirror Descent based methods for solving Reinforcement learning problems. Section 4 discusses combining novel improvements on top of these methods and discusses their structure and the expected effects. In Section 5, we dive into some experimental results and discuss the performance of these algorithms in different settings. We then close the thesis with some discussion.

CHAPTER 2

BACKGROUND

We begin by providing some necessary background in Reinforcement learning , Game Theory, and Online Learning to make the reader familiar with the concepts required to follow the ideas discussed in the following sections.

2.1 Reinforcement Learning

2.1.1 Policy gradient methods

- One way of doing RL is to directly manipulate the policy to maximize some reward. - Then use the reward and compute a gradient with respect to policy. - Reward might not be typically differentiable, so some workaround is needed to arrive at a gradient-based update rule.

2.1.1.1 Reinforce

- Widely popular PG algorithm, many insantiations possible including variants with baselines to reduce variance. - Convergence is proven using PG theorem.

2.1.1.2 Softmax Policy Gradients

- Parametrizing policies - One way of projecting parametrized policy to a probability simplex is Softmax

PG with softmax parameterization is referred to as Softmax Policy gradients.

2.1.2 PPO

- Trust region methods - PPO an approximation of TRPO with hueristic objective

2.2 Game Theory

Game theory is the mathematical study of interaction between agents to produce outcomes while trying to uphold certain individual or group preferences. It has a wide range of applications including economics, biology, and computer science.

In this work, we mainly focus on a branch of game theory called non-cooperative game theory in which each agent has their own indivudual preference.

2.2.1 Problem Representations

In discussing agent interactions, and preferences, we need a formal notion of how agents act, and how agent preferences can be defined. In game theory, agent preferences are formalized using Utility theory, where each agent has a utility function that maps the agents preferences over outcomes to a real value.

The primary way of modeling problems in game theory is through a *game* that encodes information about the agents, possible actions agents can take in different situations, their preferences, and the outcome of an interaction. There are many types of such representations, a few relevant of which we introduce below. Before we introduce such representations, we first cover some preliminary concepts that will be helpful in formally defining those representations and agent preferences.

- what are utilities, and strategies?

Normal-Form Games

Normal-Form games are a popular way of representing situations in which all agents act simultaneously and the outcome is revealed after each agent has taken their action. A few

popular games that can be represented in this form are rock-paper-scissors, matching pennies, prisoner's dilemma etc. A more formal definition of a normal-form game is as follows:

Definition 1 (Normal-form games) *A* (N, A, u) *tuple is a n-player normal form game, where* N *is the set of players,* $A = A_1 \times A_2 \ldots \times A_n$*, with* A_i *being the set of actions available to player* i*, and* $u = (u_i \forall i \in N)$ *is the set of utility functions that map an action profile to a real utility value for each agent,* $u_i : A \mapsto \mathbb{R}$*.*

Normal-form games are typically represented using a n-dimensional tensor, where each dimension represents the possible actions available to each agent, and every entry represents an outcome. The actual entries of the

Sequential Games

Although normal-form games provide a neat representation, many real-world scenarios necessitate agents act sequentially which is difficult to represent as a matrix. These problems require a tree-like representation where each node is an agent's turn to make a choice, and each edge is a possible action. There are a few ways to represent such scenarios, one being normal-form games themselves. A downside is that the size of the normal-form representation for sequential games explode exponentially in the size of the game tree. Other possible representations include the Extensive-form, and Sequence-form.

Definition 2 (Extensive-form games)

Definition 3 (Sequence form games)

2.2.2 Solution Concepts

Now that we have - what are solution concepts? - what are the common solution concepts? Nash equilibrium, Quantal response equilibrium. - what are the relevant information related to solution conepts for this work? Existence of a nash equilibrium Uniqueness of QRE

2.3 Online Learning

- what is online learning?

Online learning is the study of designing algorithms that use historical knowledge in predicting actions for future rounds while trying to minimize some loss function in an adaptive (possibly adversarial) setting.

- why is it useful? - why is it relevant here?

2.3.1 FoReL

- what is forel? - relevant info?

2.3.2 Hedge

- what is hedge?

2.3.3 Gradient Descent

Gradient descent as a FTRL variant

CHAPTER 3

MAGNETIC MIRROR DESCENT

3.1 Mirror Descent

In this section we discuss about Mirror Descent, and two mirror descent-based reinforcement learning algorithms (MDPO, and MMD).

A disadvantage of FoReL 2.3.1 in solving online learning problems is that, there is a minimization at every step. Mirror descent overcomes this by using a recursive update rule that does not required us to perform a minimization at every step.

There are different views of arriving at Mirror Descent as an optimization algorithm, some of which include the FTRL view, the mirror map view, ...

- Mirror descent algorithm

- Mirror descent convergence guarantee for OCO

3.1.1 Mirror Descent Policy Optimization

3.2 Magnetic Mirror Descent

Magnetic Mirror Descent [1] is an approach that is applicable both as an equilibrium solving algorithm and a reinforcement learning algorithm. The main results of Sokota et.al [1] are as follows. First, they establish an equivalence between solving for normal-form reduced QRE of a two-player zero-sum EFG and solving a variational inequality problem with some specific properties. Second, based on this connection they propose a non-euclidean proximal gradient

method as an equilibrium finding algorithm in two-player zero-sum games. They also prove a linear last-iterate convergence guarantee for the algorithm as a QRE solver.

3.2.1 Connection between Variational Inequalities and QREs

Variational inequalities are a general class of problems that have a wide range of applications. A Variational Inequality problem is generally of the following form (we use the same notation and symbols as in [1]):

Definition 4 *Given $\mathcal{Z} \subseteq \mathbb{R}^n$ and mapping $G : \mathcal{Z} \to \mathbb{R}^n$, the variational inequality problem VI $(\mathcal{Z}, G)$ is to find $z_* \in \mathcal{Z}$ such that,*

$$\langle G(z_*), z - z_* \rangle \geq 0 \quad \forall z \in \mathcal{Z}.$$

Solving for QREs in two-player zero-sum games can be represented as the following bilinear saddle point problem:

$$\min_{x \in \mathcal{X}} \max_{y \in \mathcal{Y}} \alpha g_1(x) + f(x, y) + \alpha g_2(y), \tag{3.1}$$

where $\mathcal{X} \subset \mathbb{R}^n$, $\mathcal{Y} \subset \mathbb{R}^m$ are closed and convex, and $g_1 : \mathbb{R}^n \to \mathbb{R}$, $g_2 : \mathbb{R}^m \to \mathbb{R}$, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$.

The solution (x_*, y_*) to the saddle point problem Equation 3.1, corresponds to the Nash equilibrium of the reguarlized game with the following first-order optimality conditions:

$$\langle \nabla g_1(x_*) + \nabla_{x_*} f(x_*, y_*), x - x_* \rangle \geq 0, \forall x \in \mathcal{X}. \tag{3.2}$$

$$\langle \nabla g_2(y_*) + \nabla_{y_*} f(x_*, y_*), y - y_* \rangle \geq 0, \forall y \in \mathcal{Y}. \tag{3.3}$$

These optimality conditions are equivalent to VI($\mathcal{Z}, G$), where $G = F + \alpha \nabla g$, $\mathcal{Z} = \mathcal{X} \times \mathcal{Y}$, and $g : \mathcal{Z} \to \mathbb{R}$. Hence, the solution the the VI problem ($z_* = (x_*, y_*)$), corresponds to the solution of the saddle point problem stated in Equation 3.1.

3.2.2 MMD Algorithm

The authors first propose a non-Euclidean proximal gradient method to solve the VI ($\mathcal{Z}, F + \alpha \nabla g$) with the following update at each iteration:

$$z_{t+1} = \arg\min_{z \in \mathcal{Z}} \eta(\langle F(z_t), z \rangle + \alpha g(z)) + B_\psi(z; z_t). \tag{3.4}$$

With the following assumptions, z_{t+1} is well defined:

- ψ is 1-strongly convex with respect to $\|.\|$ over $\mathcal{Z}$, and for any l, stepsize $\eta > 0$, $\alpha > 0$, $z_{t+1} = \arg\min_{z \in \mathcal{Z}} \eta(\langle l, z \rangle + \alpha g(z)) + B_\psi(z; z_t) \in \text{int dom } \psi$.
- F is monotone and L-smooth with respect to $\|.\|$ and g is 1-strongly convex relative to ψ over $\mathcal{Z}$ with g differentiable over int dom ψ.

The algorithm that is termed MMD uses the same update as Equation 3.4, with g taken to be ψ or $B_\psi(.; z')$ for some z'.

We now restate the main algorithm as stated in Sokota et.al, [1],

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AlgorithmMMD [1, (Algorithm 3.6)] Starting with $z_1 \in \text{int dom } \psi \cap \mathcal{Z}$, at each iteration t do

$$z_{t+1} = \arg\min_{z \in \mathcal{Z}} \eta(\langle F(z_t), z\rangle + \alpha\psi(z)) + B_\psi(z; z_t).$$

or, Given some z', do

$$z_{t+1} = \arg\min_{z \in \mathcal{Z}} \eta(\langle F(z_t), z\rangle + \alpha B_\psi(z; z')) + B_\psi(z, z_t).$$

Algorithm 1 provides the following convergence guarantees.

Theorem 1 *[1, Theorem 3.4] Assuming that the solution z_* to the problem VI $(\mathcal{Z}, F + \alpha\nabla g)$ lies in the int dom ψ, then*

$$B_\psi(z_*; z_{t+1}) \leq \left(\frac{1}{1+\eta\alpha}\right)^t B_\psi(z_*; z_1),$$

if $\alpha > 0$, and $\eta \leq \frac{\alpha}{L^2}$.

3.2.3 Behavioral form MMD

Algorithm 1 can be either implemented in closed form for a given set of parameters or by performing stochastic gradient descent at each step to approximate the closed form. This approximation is especially useful when it is not possible to compute the closed form such as in function approximation settings.

In the two form of updates specified in Algorithm 1, the second form involves a z', term that is referred to as the magnet. This can either be a reference policy such as the ones used in Imitation learning, or one that is trailing the current policy to stabilize learning. Alternatively, one can also use a uniform policy in which case the term reduces to entropy and acts as a regularization term and encourages exploration. For the rest of our discussion on MMD, we assume the magnet to always be a uniform policy.

With ψ taken to be negative entropy, the behavioral form of MMD is to perform the following update at each information state,

$$\pi_{t+1} = \arg\max_{\pi} \mathbb{E}_{A\sim\pi} q_t(A) + \alpha H(\pi) - \frac{1}{\eta} KL(\pi, \pi_t), \tag{3.5}$$

where π_t is the current policy, q_t is a vector containing the q-values following the policy π_t, and $H(\pi)$ is the entropy of the policy being optimized.

In single-agent settings MMD's performance is competitive with PPO in Atari and MuJoCo environments. And, in the multi-agent setting the performance of tabular MMD is on par with CFR, but worse than CFR+.

3.2.4 Equivalence of MMD and MDPO

- MDPO with an added entropy term is equivalent to MMD with negative entropy mirror map and uniform magnet.

CHAPTER 4

MODIFIED UPDATES FOR MIRROR-DESCENT BASED METHODS

We propose applying a few modifications to the methods discussed in the previous section and investigate the effect of these modifications in terms of their performance and convergence behaviors in two-player zero-sum games. We first introduce the proposed modifications to be applied on the methods discussed in the previous section, and then provide a description of how these modifications fit into those algorithms.

4.1 Neural Replicator Dynamics (NeuRD)

Neural Replicator Dynamics (NeuRD) [2] is a model-free sample-based algorithm that is an application of Replicator Dynamics to Policy Gradients. Replicator Dynamics is an idea from Evolutionary game theory (EGT) that defines operators to update the dynamics of a population in order to maximize some pay-off defined by a fitness function.

The single-population replicator dynamics is defined by the following system of differntial equations:

$$\dot{\pi}(a) = \pi(a)[u(a, \pi) - \bar{u}(\pi)], \forall a \in \mathcal{A} \tag{4.1}$$

Hennes et.al, [2] show equivalence between Softmax Policy gradients 2.1.1.2 and continuous-time Replicator Dynamics [2, THEOREM 1, on p5].

NeuRD can be implemented as a single line change to the SPG algorithm. This can be seen as applying a fix to the update of SPG to make it more responsive to changes in a non-stationary environment. Since the typical neural network representation of policies use softmax projection on the logits, the idea of fixing the gradient updates can be applied more generally to algorithms beyond SPG. The NeuRD loss has been adapted into other algorithms to improve performance or induce convergence in competitive and cooperative settings. Chhablani et.al, [3] showed improved performance in identical-interest games by applying the NeuRD fix to COMA [4]. Perolat et.al, [5] used a NeuRD based loss function along with adaptive regularization [6] to induce last-iterate convergence in Stratego.

4.1.1 MMD-N

4.1.2 MDPO-N

4.2 Extragradient updates

The Extragradient method was first introduced by G.M.Korpelevich [7] as a modification of gradient descent methods in solving saddle point problems. Extragradient is a classical method for solving smooth and strongly convex-concave bilinear saddle point problems with a linear rate of convergence. Extragradient and Optimistic Gradient Descent Ascent methods have been shown to be approximations of proximal-point method for solving saddle point methods [8].

CHAPTER 5

EXPERIMENTS

We now evaluate our proposed methods empirically in various settings. Through the experiments, we aim to answer the following questions:

- How does the addition of the NeuRD-fix, Extragradient updates, Optimism affect the convergence rate of these algorithms in solving for QREs, and Nash equilibrium?
- What is the last-iterate vs average-iterate convergence behavior of these algorithms in the presence of these modifications?
- Do these performance improvements scale well with the size of the game?

5.1 Experimental Domains

In answering the above questions, we consider both tabular and function approximation settings. For the tabular experiments we evaluate the performance of these algorithms for convergence to QRE and Nash equilibrium. methods using the following environments - Perturbed RPS.

For the function approximation setting, we evaluate the algorithms on Kuhn Poker, Abrupt Dark Hex, and Phantom Tic-tac-toe. Kuhn Poker is a smaller extensive form game that allows for more introspection and accurate measurement of performance. Whereas, Abrupt Dark Hex, and Phantom TTT are large games that test the stability of these modifications in settings with a large state-space.

5.2 Evaluation Methods

Typically, to measure the convergence rate of an algorithm we can use either a a measure of distance from an equilibrium if the equilibrium point is known or, a measure of exploitability as an indication of the policies reaching the equilibrium.

For tabular settings since the equilibrium solutions are known, we use both exact exploitability and KL-divergence to the equilibrium to evaluate the algorithms. For PerturbedRPS, we use the QRE solutions computed through Gambit and, we also derive the unique Nash Equilibrium (please refer to **??** in the appendix for the derivation) for PerturbedRPS.

-Define exact exploitability

For the function approximation settings we use the above metrics for the smaller games Kuhn Poker, and 2x2 Dark Hex which also have equilibrium solutions computed through Gambit. For the larger games, since exact exploitability computation is not possible and we do not know the equilibrium solution, we use an approximate measure of exploitability by using a DQN to approximate the best response computation similar to [1].

5.3 Results

5.3.1 Tabular Experiments

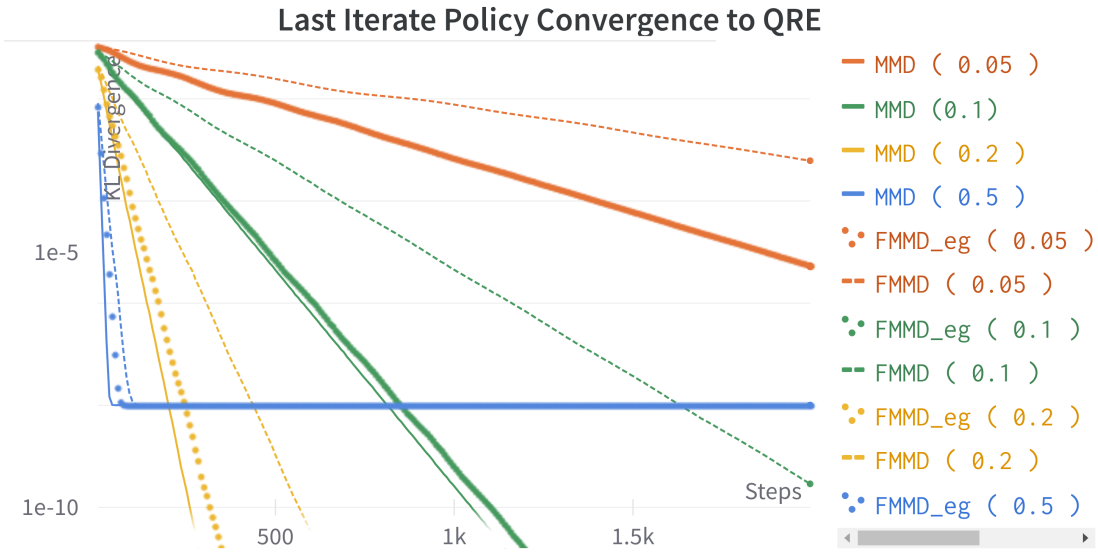
Below, we give an overview of Last, and Average iterate convergences of 3 different algorithms - MMD, MDPO, SPG, and their variants obtained by applying combinations of the modified updates. We give the convergence results for the more interesting behaviors observed, a more exhaustive list of all combinations and their convergence behaviors can be found in the appendix.

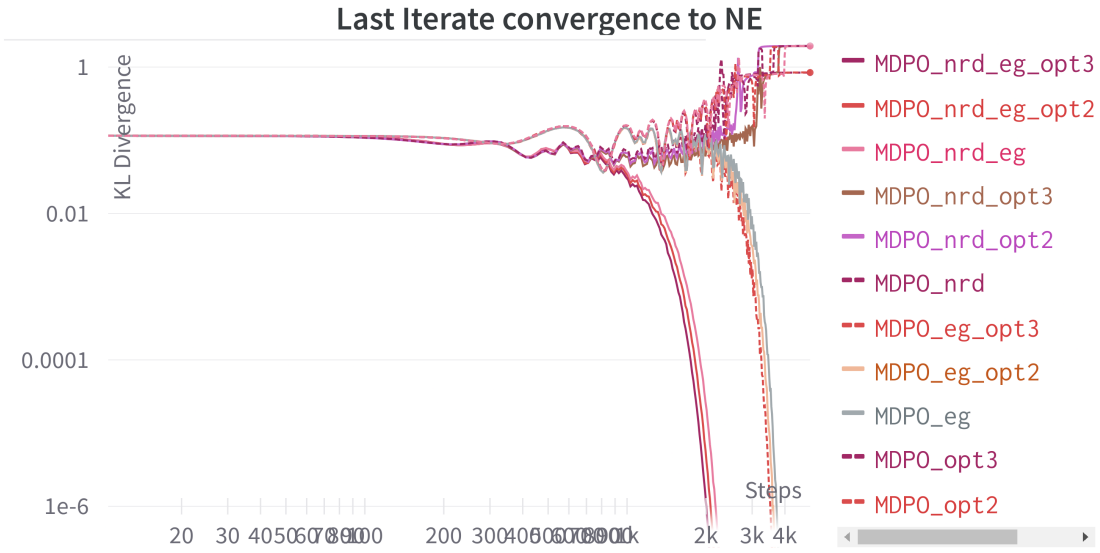
Algorithm	Nash		QRE (α=0.5)	
	Avg	Last	Avg	Last
MMD (cf)	y	y	y	y
FMMD (bf)	y	y	y	y
FMMD-N	y	x	x	x
FMMD-EG	y	y	y	y
FMMD-N-EG	y	y	x	x
FMMD-N-OPT	y	x	x	x
MDPO	x	x	-	-
MDPO-N	x	x	-	-
MDPO-EG	y	y	-	-
MDPO-N-EG	y	y	-	-
MDPO-EG-OPT	y	y	-	-
MDPO-N-EG-OPT	y	y	-	-

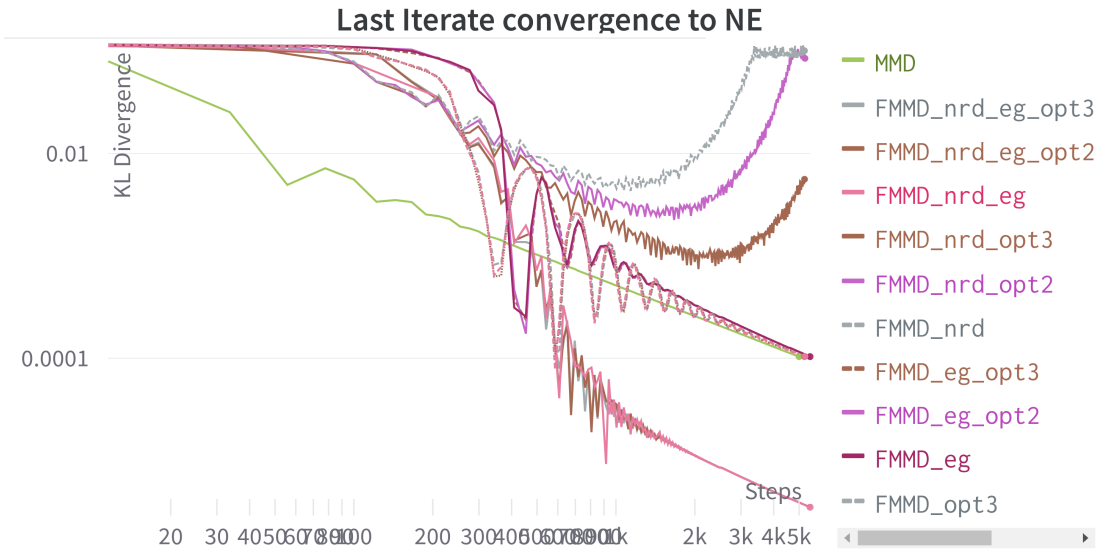
TABLE I

Convergence in Perturbed RPS

Apart from the final convergence results, we also make a few observations regarding the speed of convergence for these variants. From **??**, we note the following




Last Iterate convergence to NE
KL Divergence
1
0.01
0.0001
1e-6
Steps
20 30 40 50 60 70 80 90 100 200 300 400 500 600 700 800 900 1k 2k 3k 4k
MDPO_nrd_eg_opt3
MDPO_nrd_eg_opt2
MDPO_nrd_eg
MDPO_nrd_opt3
MDPO_nrd_opt2
MDPO_nrd
MDPO_eg_opt3
MDPO_eg_opt2
MDPO_eg
MDPO_opt3
MDPO_opt2


Last Iterate convergence to NE
KL Divergence
0.01
0.0001
Steps
20 30 40 50 60 70 80 90 100 200 300 400 500 600 700 800 900 1k 2k 3k 4k 5k
MMD
FMMD_nrd_eg_opt3
FMMD_nrd_eg_opt2
FMMD_nrd_eg
FMMD_nrd_opt3
FMMD_nrd_opt2
FMMD_nrd
FMMD_eg_opt3
FMMD_eg_opt2
FMMD_eg
FMMD_opt3

- MDPO with extragradient updates have superlinear last iterate convergence to Nash equilibrium.
- MMD with NeuRD fix and extragradient

5.3.2 Neural Experiments

Based on the observations from the Tabular NFG experiments, we evaluate the most promising combinations of these algorithms in the function approximation setting. As discussed in [1], behavioral form MMD can be implemented as a reinforcement learning algorithm with minor changes to PPO due to its similarity. We implemented MMD, in a similar way as described in [1] by modifying the PPO implementation in RLLib [9]. We also use RLLib's OpenSpiel adapter with some modifications to use information states as inputs as opposed to observations. We train these reinforcement learning agents in self-play for the environments mentioned above.

- Implementation details (RLLib, PPO modifications, GAE) - Neural network architecture, hyperparameters

APPENDICES

Appendix A

SOME ANCILLARY STUFF

Ancillary material should be put in appendices.

Appendix B

SOME MORE ANCILLARY STUFF

[?]

CITED LITERATURE

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