Lecture Notes 09/30/2020

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1 Differentials

Definition:

$$dy = f'(x)dx$$

Remark: If you divide both sides by dx, you get the Leibniz notation for derivative; $\frac{dy}{dx}$

For example: Find dy when $f(x) = \sqrt{x^2 + 1}$

$$f'(x) = \frac{x}{\sqrt{x^2 + 1}}$$
$$dy = f'(x)dx \rightarrow dy = \frac{x}{\sqrt{x^2 + 1}}dx$$

1.1 Product Rule for Differentials

y is a composite function; $y = f(x)g(x)$, so using the product rule gets us:

$$dy = g(x)df + f(x)dg$$

There's a proof for this in the lecture notes

1.2 Quotient Rule for Differentials

Like the product rule, y is a composite function; $\frac{f(x)}{g(x)}$

$$dy = \frac{g(x)df - f(x)dg}{(g(x))^2}$$

There's also an epic proof for this in the lecture notes.

Now we get an example problem: Find the differential, dw, of $w = x^{15}cos(2x)$

The derivative of $f(x)$, df, is:

$$df = 15x^{14}$$

Likewise, the derivative of $g(x)$, dg, is:

$$dg = -2sin(2x)$$

Now, just plug df and dg into the product rule equation.
Remark: You're probably gonna have to memorize the product and quotient rule equations.

$$dw = g(x)df + f(x)dg \rightarrow dw = 15x^{14}cos(2x)dx - 2x^{15}sin(2x)dx$$

Note: Remember to put in dx

2 Error

Definiton: Your relative error as a percentage is:

$$\frac{\Delta Q}{Q}$$

Where: Q is the quantity measured and ΔQ is the actual error

Currently, I'm not sure exactly what all this is. Maybe error for linearization? Anyways, let's do an example:
Talking about the volume of a sphere with a radius of .7 inches, the max error is .01 inches.

First, the derivative of the formula for volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

$$V' = 4\pi r^2$$

Now, the values from the formulas.

$$r = .7, dr = .01$$

Now we plug everything in (into the differential formula?).
Note: dV is the same thing as ΔV?

$$dV = V'(r)dr \rightarrow dv = 4\pi(.7)^2 * .01 = .0616in^3$$

Now for the percentage error. The formula for percentage error of the volume, V, is:

$$\frac{\Delta V}{V} \rightarrow \frac{4\pi r^3 dr}{\frac{4}{3}\pi r^2}$$

Simplify to get:

$$\frac{3dr}{r}$$

Plug in your values.

$$\frac{3(.01)}{(.7)} \rightarrow .0429(4.29\%)$$