Lecture Notes 10/02/2020

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1 The Exponential Function

Definition:

$$f(x) = a^x$$

Where $x > 1$ and $a \neq 0$
Its domain is $(-\infty, \infty)$ and its range is $(0, \infty)$

Proposition: $f(x) = a^x$
When $a > 1$

$$\lim_{x \to -\infty} a^x = 0$$

$$\lim_{x \to \infty} a^x = \infty$$

When $0 < a < 1$

$$\lim_{x \to -\infty} a^x = \infty$$

$$\lim_{x \to \infty} a^x = 0$$

1.1 Evalutating Limits of Exponential Functions

Evaluate $\lim_{x\to\infty} \frac{2}{5^x}$

Take the two out

$$2\frac{1}{5^x}$$

Direct substitution

$$2(\frac{1}{5^\infty})$$

Simplify

$$2(0) = 0$$

Evaluate $\lim_{x\to\infty} \frac{2-5x}{2+3(5)^x}$

Divide by the highest degree of x

$$\frac{5^x(\frac{2}{5^x}-1)}{5^x(\frac{2}{5^x}+3)}$$

Simplify and direct substitution

$$\frac{1(\frac{2}{5^\infty}-1)}{1(\frac{2}{5^\infty}+3)}=\frac{(0-1)}{(0+3)}=-\frac{1}{3}$$

Evaluate $\lim_{x\to-\infty} e^{7x}sin(x)$
Break into two functions

$$h(x)=e^{7x}, g(x)=sin(x)$$

Use the Squeeze Theorem

$$-1\leq sin(x)\leq 1$$

$$-e^{7x}\leq e^{7x}sin(x)\leq e^{7x}$$

Note: $h(x)=e^{7x}$ can bee seen as the amplitude of $g(x)=sin(x)$
Now take the limits

$$\lim_{x\to-\infty} -e^{7x}=0$$

$$\lim_{x\to\infty} e^{7x}=0$$

We get

$$\lim_{x\to-\infty} e^{7x}sin(x)=0$$

1.1.1 Cool identities

$$a^{x+y}=a^x+a^y$$

$$a^{x-y}=\frac{a^x}{a^y}$$

$$(a^x)^y=a^{xy}$$

$$(ab)^x=a^xb^x$$