MAT 256 EXAM 1 NOTES

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1 Domain of a function

1.1 Rational Functions

Basically, the denominator can never equal zero.

For example:
$\frac{1}{x^2+5x+6}$ has a domain of $x \neq -2$ and $x \neq -3$ because the bottom polynomial is equal to 0 when x is either -2 or -3. Solve the polynomial yourself and see.

1.2 Radical Functions

For these, never let what's inside the square root equal anything less than zero, because math gets weird when you have a negative in the square root.

For example:
$\sqrt{x-3}$ has a domain of $[3, \infty)$ because anything smaller than 3 makes the square root negative.

1.3 Combined

Take both principles and apply them to the function:
$\frac{\sqrt{15-3x}}{x+2}$

The domain is $(-\infty, -2) \cup (-2, 5]$ because *EXACTLY* -2 makes the denominator 0 and anything bigger than 5 makes the square root negative.

2 Odd and Even Functions

Just plug in $-x$ for x in your function. If:

$f(-x) = f(x)$ (your original function), then the function is even.

If you end up with $f(-x) = -f(x)$, then your function is odd.

Otherwise, it's neither.

3 Composite Functions

FILL IN LATER

4 Limits

4.1 Numerically

$\lim_{x\to 3} \frac{1}{3-x}$
Just plug in numbers that are slightly below 3 (2.9999) and numbers that are above 3 (3.00001). You can put it in your calculator and look at the table. *Easy peasy*

4.2 Graphically

Just look at it lmao. $\lim_{x\to 0^-}$ is the limit from the left and $\lim_{x\to 0^+}$ is the limit from the right.

4.3 Limits Involving Infinity

If it's a rational function, divide the denominator by the highest degree of x, something something square roots, then simplify.
It's the wizard thing that blew your mind.

5 Squeeze Theorem

Definition: $\lim_{x\to a} g(x) = L$ if $f(x) <= g(x) <= h(x)$ and both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} h(x) = L$
You need two helper functions ($f(x)$ and $h(x)$).

How to do it:

6 Continuity

There are three conditions a function must meet in order to be continuous at a point, p:

- it must be defined at p
- the limit as x approaches p must exist
- the limit must equal the value of the function, $f(x)$, at the point ($f(p)$)

7 Derivatives

7.1 Definition of a Derivative

$f'(x) = \frac{f(x+h)-f(x)}{h}$
PLUG IN $(h+x)$ **ANYWHERE YOU SEE AN** x **IN THE ORIGINAL FUNCTION**

7.2 Average Rate of Change

Basically the slope of whatever your function is, i guess. $\frac{y_2-y_1}{x_2-x_1}$

7.3 Tangent Lines

To find the tangent line at a given point:

1. get the derivative of the original function
2. if you're given a point, or at least an x value, plug it into the derivative (**THIS GETS YOU THE SLOPE,** (m))
3. if you're not given a y value, plug the x value into the original equation
4. use point slope formula $(y - y_1 = m(x - x_1))$ to get the equation of the tangent line
5. simplify

7.4 Important Derivative Formulas

Product rule:
$\frac{d}{dx} f(x) * g(x) = f'(x)g(x) + f(x)g'(x)$

Quotient rule:
$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x)f'(x)-f(x)g'(x)}{g(x)^2}$

Trig stuff:
$\frac{d}{dx} sin(x) = cos(x)$
$\frac{d}{dx} cos(x) = -sin(x)$
$\frac{d}{dx} - sin(x) = -cos(x)$
$\frac{d}{dx} - cos(x) = sin(x)$
It's like a loop