Modeling of Climate

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Abstract

- This project deals with climate models and their construction. In particular, it talks about the construction of climate models using differential equations. The project begins with an explanation of what a climate model is, and what must be considered when making one. Following that, some climate models have been discussed. The EBM and an added adjustment (The Latitude dependent model) have been talked about, with a description of how they are made. Following that, the project details the use of the Lotka-Volterra prey-predator model to describe the interplay of temperature and relative humidity in a particular region. Two more models have been described very briefly, ended by the components of a climate model and the ways in which it is validated.

Contents

Introduction

Every time we look at the weather report in the news it always makes mention of the forecast. Climate models are the main reason the forecasts can exist in the first place. While most commercially used climate models are highly complex, there are also low-level models that can be constructed using ordinary differential equations. This project aims to shed light on the ways in which ODEs can be used to create and implement these models, while also giving an overview of what makes a climate model as a whole. Some high-level climate models have also been discussed very briefly .

PREREQUISITES

Here we have provided a short description of some important tools, specifically, the ode45 MATLAB function

The Runge – Kutta (RK) Method of solving Differential Equations

- This is a prerequisite to the discussion of climate models discussed, as at least one of them has made use of the ODE45 MATLAB function. Hence, the order 4 RK method has been described here.
- The RK method is applicable for ODEs of the type y' = f(x, y), where $y(x_0) = y_0$ (say). Hence, this method is useful for IVPs.
- The implementation is as follows
- $K_1 = h*f(x_n , y_n)$
- $K_2 = h*f(x_n+(h/2)), y_n + (K_1/2))$
- $K_3 = h*f(x_n+(h/2)), y_n + (K_2/2))$
- $K_4 = h*f(x_n+(h/2)), y_n + K_3)$
- $K = (1/6)*(K_1+2K_2+2K_3+K_4)$
- $y_{n+1} = y_n + K$

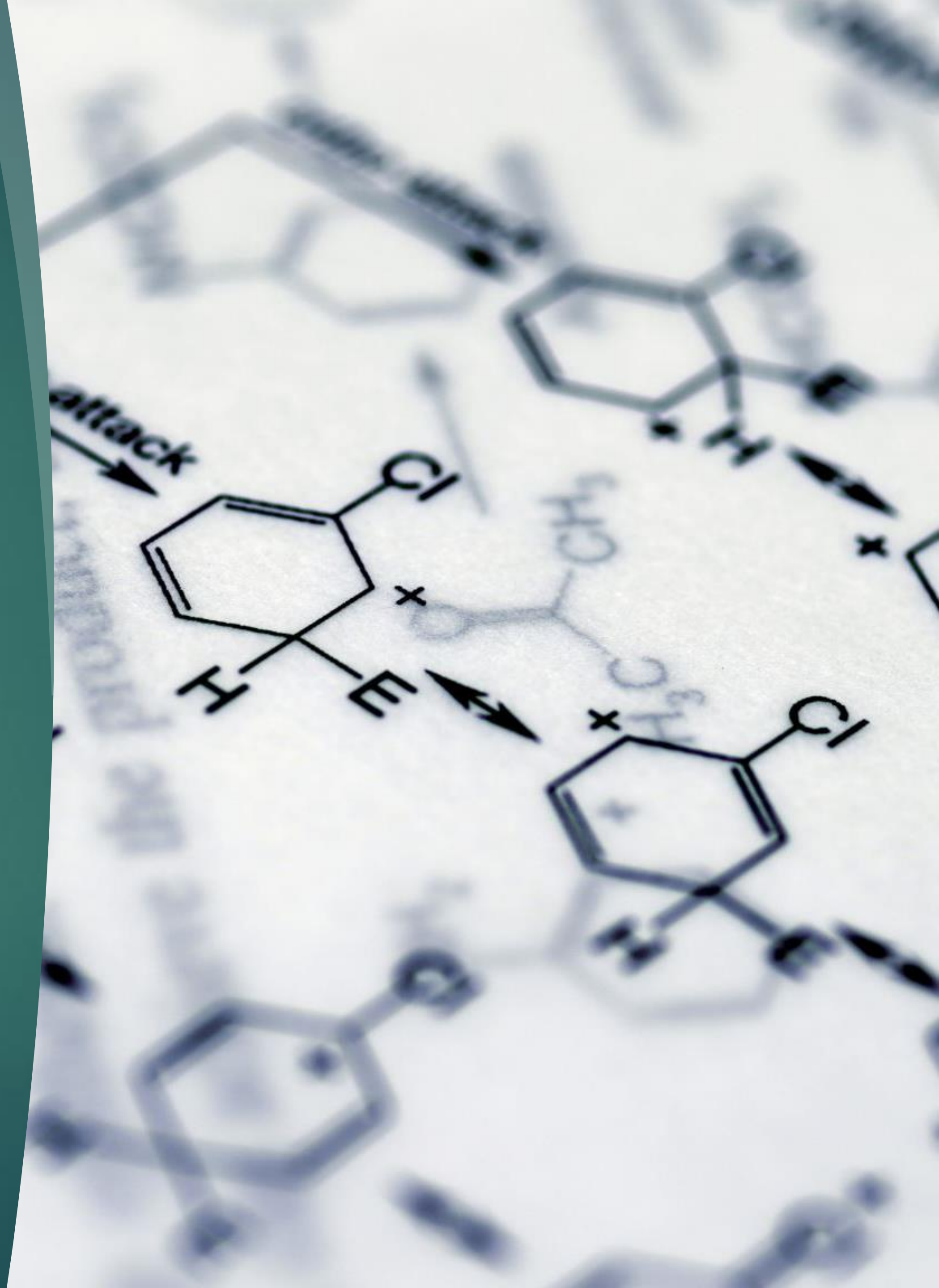
The order 4 RK method (Continued)

- The main task in this algorithm is to find k. Initially, we set n to 0 and use the initial values after setting a fixed step size h.
- After calculating K, using the formula described previously, we calculate the next values of (x,y) by incrementing them by h and k respectively.
- Then n is incremented and the process continues.
- This in turn gives a numerical approximation of the solution of the differential equation in question
- This is useful if the ODE in question does not have a direct solution

WHAT IS A CLIMATE MODEL?

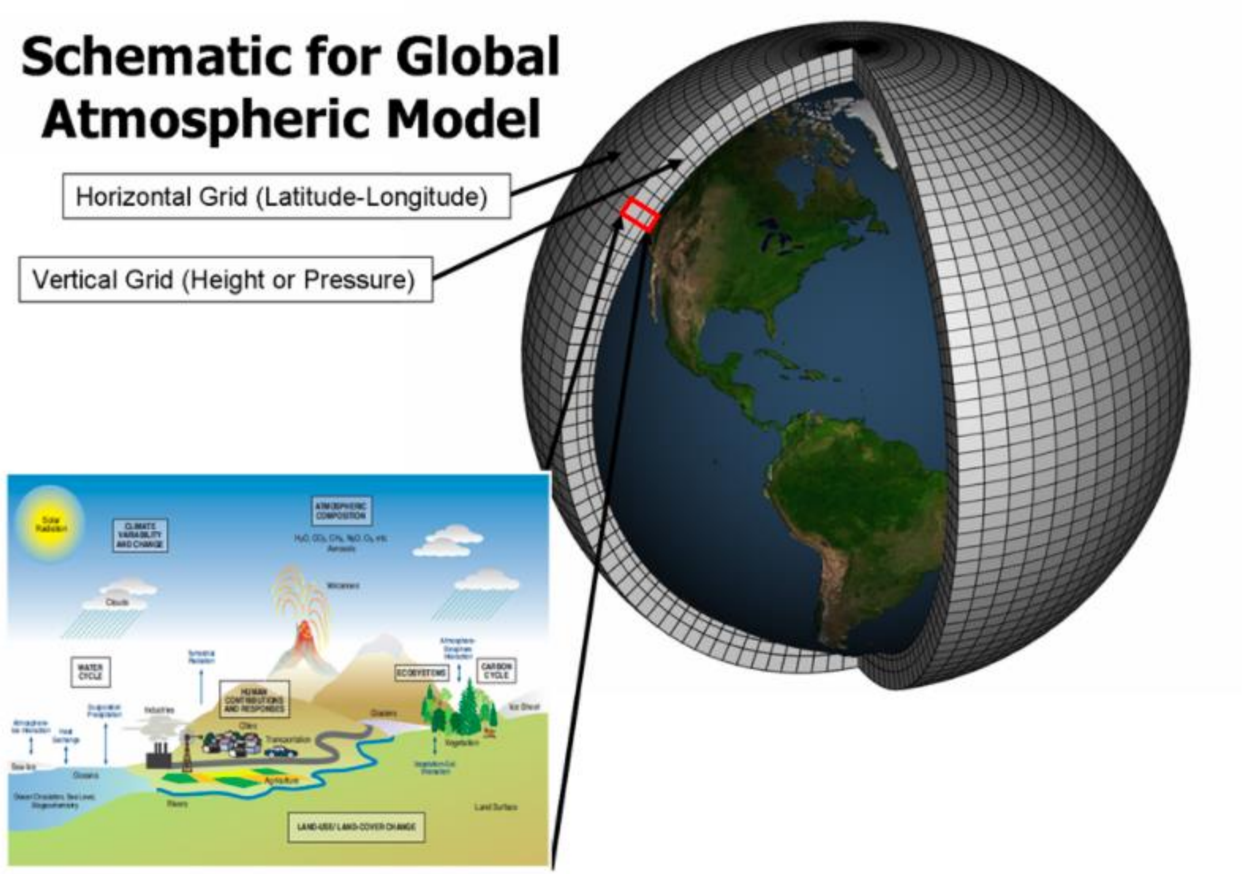
What is a climate model?

- **A climate model is a mathematical depiction of the climate system based on physical, biological, and chemical concepts in general.**
- These laws' equations are so complicated that they must be solved numerically.
- As a result, climate models produce a discrete space and time solution, which means that the findings obtained are averages over regions whose size is determined by model resolution and for certain times.



- **The models themselves come in different forms** – from those that just cover one particular region of the world or part of the climate system, to those that simulate the atmosphere, oceans, ice and land for the whole planet.
- The output from these models drives forward climate science, helping scientists understand how human activity is affecting the Earth's climate. **These advances have underpinned climate policy decisions on national and international scales for the past five decades.**
- Furthermore, many processes are still not well-known enough for detailed behaviour to be included in models.
- **Many climate models have been built to do climate forecasts, that is, to simulate and analyse climatic changes as a result of greenhouse gas and aerosol emissions.**

- Because of the complexity of the climate system and limitation of computing power, a model cannot possibly calculate all of these processes for every cubic meter of the climate system.
- **Instead, a climate model divides up the Earth into a series of boxes or "grid cells".** A global model can have dozens of layers across the height and depth of the atmosphere and oceans.
- The image shows a 3D representation of what this looks like.
- **The model then calculates the state of the climate system in each cell – factoring in temperature, air pressure, humidity and wind speed.**



Hierarchy of models

- You can think of a climate model hierarchy as **a ladder connecting our understanding of basic physical principles to the Earth system in all its complexity.**
- All model types, when used correctly, can provide useful information on the behaviour of the climate system.
- **There is no one-size-fits-all model that can be used for everything. This is why there are so many different climate models, constituting what is known as the spectrum or hierarchy of models.**
- One type of model may be chosen depending on the purpose or question. The optimum type of model to utilize is determined by the goal or inquiry. Combining the results of multiple types of models, on the other hand, is frequently the greatest method to acquire a detailed knowledge of the dominating processes in action.

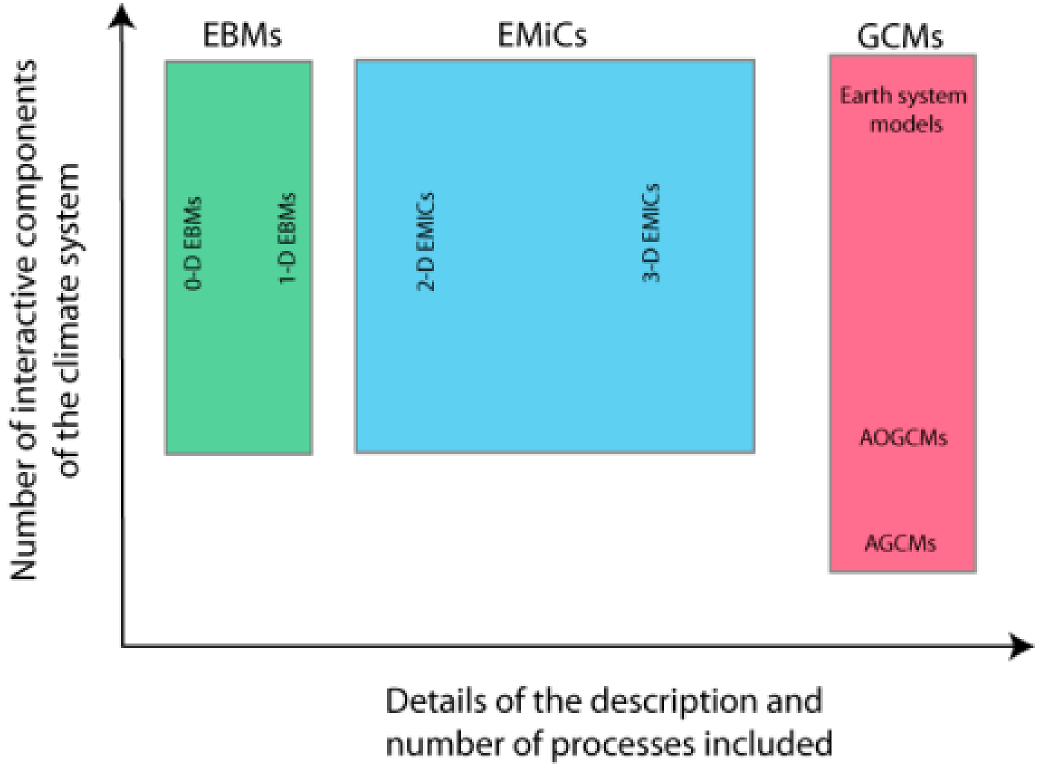
Types of Climate Models

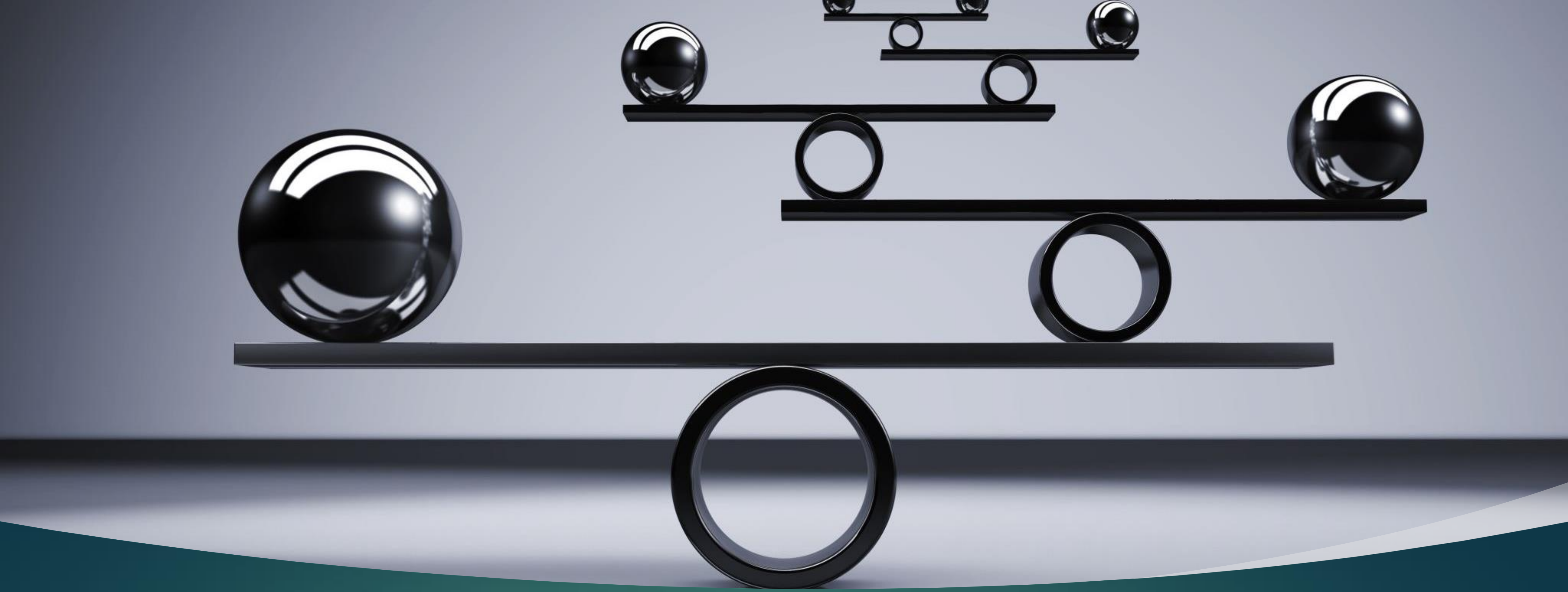
Modelers must first identify which variables or processes will be considered and which will be treated as constants. This gives a method of categorizing the models based on the interactively represented components.

The physical behavior of the atmosphere, ocean, and sea ice must be reflected in the bulk of climate studies. Furthermore, the terrestrial and marine carbon cycles, dynamic vegetation, and ice sheet components are increasingly being added, resulting in Earth-system models.

The intricacy of the processes that are included is a second means of distinguishing various models.

- At one end of the spectrum, General Circulation Models (GCMs) attempt to account for all of the system's important features at the highest resolution possible.
- The acronym GCM was coined since one of these models' primary goals is to realistically replicate the three-dimensional structure of winds and currents. Atmospheric General Circulation Models (AGCMs) and Ocean General Circulation Models (OGCMs) are the two types (OGCMs). The acronyms AOGCM (Atmosphere Ocean General Circulation Model) and the larger CGCM (Coupled General Circulation Model) are commonly used in climate research that include interaction atmospheric and oceanic components.





ENERGY BALANCE MODEL

1. Energy balance models

- EBMs are among the most basic climate models. Budyko and Sellers introduced them virtually simultaneously.
- We can make **a basic model of the temperature of the Earth by assuming that it receives incoming insolation (solar energy) but radiates some of it back into space.**
- **These models have only one dependant variable: the Earth's near-surface air temperature T, which eliminates the climate's dependency on the wind field, ocean currents, and the Earth's rotation.**
- Later the EBMs were equipped by the hydrological cycle to study the feedbacks in the atmosphere–ocean–sea ice system.

- Variables included in the modelling are - **T**, the temperature of the Earth's surface averaged over the entire globe.
- Physical parameters such as **R**, the radius of the Earth, **S**, the energy flux density (also referred to as the energy flux)—the amount of **energy** (W) flowing through a flat surface of area 1 m2 .
- One of the most useful examples of a simple but powerful model is the one- or zero-dimensional energy balance model. As a starting point, a zero-dimensional model of the radiative equilibrium of the Earth is introduced

$$(1-a)S\pi R2 = 4\pi R2\epsilon\sigma T^4$$

- The left-hand side represents the incoming energy from the Sun (size of the disk is equal to the shadow area πR2), while the right-hand side represents the outgoing energy from the Earth.
- T is calculated from the Stefan–Boltzmann law assuming a constant radiative temperature; S is the solar constant (the incoming solar radiation per unit area), about 1367 Wm−2; and a is the Earth's average planetary albedo, measured to be 0.3. R is Earth's radius = 6.371×106 m, σ is the Stefan–Boltzmann constant $=5.67\times10^{-8}K^{-4}m^{-2}s^{-2}$ and ϵ is the effective emissivity of Earth (about 0.612). The geometrical constant πR2 can be factored out, giving:

$$(1-a)S = 4\pi\sigma T^4$$

Solving this, we get the temperature.

$$T = \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}.$$

- Temperatures based on the local energy balance without a heat capacity would vary between:

$$T_{\max} = \sqrt[4]{\frac{(1-\alpha)S}{\epsilon\sigma}} = \sqrt{2} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = \sqrt{2} \cdot 288\,\mathrm{K} = 407\,\mathrm{K}.$$

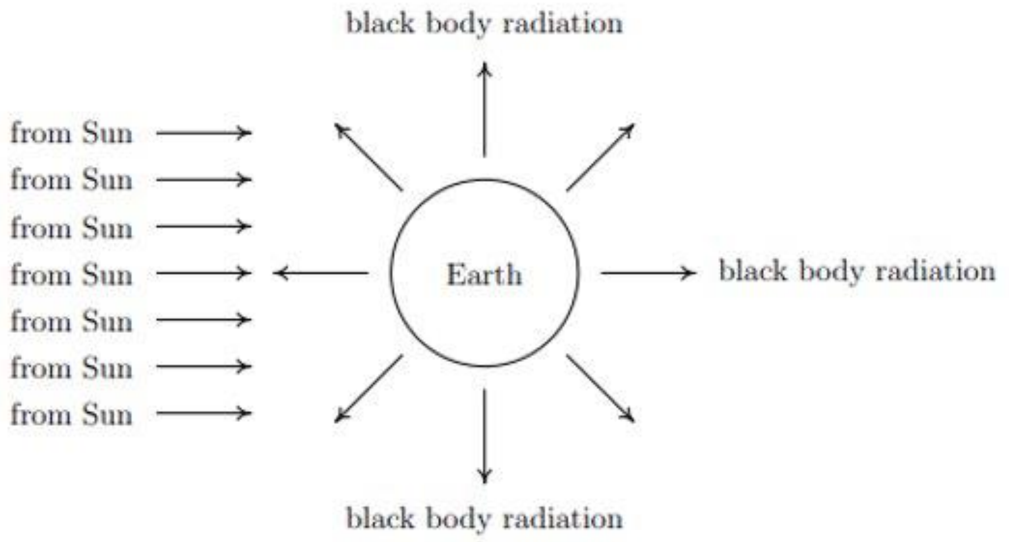
- Using the Stefan – Boltzmann Law, and factoring in the albedo (reflectivity of the Earth), as well as the effect of greenhouse gases, namely the greenhouse constant denoted by epsilon, we have the following equation to use as our model

Changes in heat storage = absorbed solar radiation - emitted terrestrial radiation

$$C\frac{dT}{dt} = (1-\alpha)Q - \varepsilon\sigma T^4$$

- The first term on the right is **incoming heat absorbed by the Earth and its atmosphere system.**
- The second term is **heat radiating out as if the Earth were a blackbody** with all of the outgoing longwave radiation (OLR) escaping to space.

$$C\frac{dT}{dt} = (1-\alpha)Q - \varepsilon\sigma T^4$$



- In order to take the geographical distribution of temperature at the Earth’s surface into account, zero-dimensional EBMs can be extended to include one (generally the latitude) or two horizontal dimensions. An additional term Δtransp is then included, representing the net effect of heat input and output associated with horizontal transport:

$$C_E \frac{\partial T_{s,i}}{\partial t} = \left((1-\alpha_p)\frac{S_0}{4} - A\uparrow \right) + \Delta transp$$

MATLAB Codes for EBM

```
span = [0 5];

C = 2.912; % Heat capacity in W/yr

alpha = 0.3; % Albedo of Earth

epsilon = 0.8; % Assumed value of Greenhouse constant

boltzconst = (5.67)*(10^(-8)); % Boltzmann Constant

Q = 341.9;

T0 = 286.9; % 20th century average global temperature
in Kelvin, as per NOAA USA

Tprime = @(t,T) (Q*(1-alpha)) - (boltzconst*(T^4));

[t,T] = ode45(Tprime, span, T0);

figure;

plot(t,T);grid on;xlabel('Time');ylabel('Global
Temperature');
```

LATITUDE – DEPENDENT MODEL

A Latitude Dependent Model

- The model is a time-dependent Energy Balance Model (EBM) in which the surface temperature depends on latitude and time and allows for the evaluation of non-uniform climate engineering strategies.
- The resulting partial differential equation is solved using a Green's function approach.
- This model offers an efficient analytical approach to design strategies that counteract climate change on a latitudinal basis to overcome regional disparities in cooling.

• Glaciers

- Glaciers are incorporated into this model via an adjustment to the albedo function. The lower the albedo, the more radiation from the Sun that gets absorbed by the planet, and temperatures will rise.
- We assume that ice exists only above the ice line: η
- Here, as ice is more reflective than water or land, the albedo will be larger for latitudes above the ice line.

Albedo:

$$\alpha_\eta(y) = \alpha(y, \eta)$$

Albedo function:

$$\alpha(y,\eta) = \begin{cases} \alpha_1, & y < \eta; \\ \alpha_2, & y > \eta; \\ \frac{1}{2}(\alpha_1 + \alpha_2), & y = \eta, \end{cases}$$

• Distribution of Insolation

This insolation is the primary driver of the climate system. Here we will examine the geometric factors that determine insolation.

The tropics receive more energy from the sun on an annual basis than do the polar regions. This difference is considered by modeling the energy absorbed by the surface via term:

$$Qs(y)\big(1-\alpha(y,\eta)\big)$$

where s(y) is the distribution of insolation over latitude, normalized so that

$$\int_0^1 s(y)\,dy = 1.$$

• Meridional Heat Transport

- When we consider the meridional structure, the earth receives more energy than it emits in tropical latitudes, while the opposite is the case in polar latitudes.
- To attain a local balance, there must be transport of energy from the tropics to the polar latitudes, either by the atmosphere or by the ocean.
- Based on Budyko's model, the meridional transport term is: $C(T - \overline{T})$ where C is constant and T is:

$$\overline{T} = \overline{T}(t) = \int_0^1 T(t, y)\, dy.$$

- Budyko's model is then given by: $$R\frac{\partial T(t,y)}{\partial t} = Qs(y)\big(1 - \alpha(y,\eta)\big) - (A + BT) - C\left(T - \overline{T}\right).$$

LOTKA – VOLTERRA CLIMATE MODEL

Using the Lotka Volterra Models to look at climate

We can begin by looking at the Lotka - Volterra Models of population growth. One may think that this has nothing to do with climate change, but this model can be extended to be used for this as well. Let us begin by looking at the model itself for population, and then see how it can be extended.

Here, we can consider the prey-predator model, described on the next page.

The Lotka-Volterra Prey-Predator Model

- Consider the prey-predator problem. Let us say that there are two variables x and y, each representing the population of rabbits and foxes in an enclosed area. We make a couple of assumptions at this point:
- 1. There is plenty of food supply for the prey in the environment
- 2. The environment does not Favour predator or prey.

- So, we can safely say that the population of the prey, that is, x, has dependencies on the following:
- 1. Its own growth rate
- 2. The growth rate of the predators
- 3. The current prey population

- Thus, we get a system of ODEs

$$\frac{dx}{dt} = x(\alpha_1 - \beta_1 y)$$

$$\frac{dy}{dt} = y(\alpha_2 - \beta_2 x)$$

- At the same time, the consideration of the second factor must also be looked into, because of the fact that the predators will kill and eat the prey, and if there are more predators present in the habitat, then the prey will diminish accordingly. Thus, we end up with another system of differential equations.

$$\frac{dx}{dt} = x(\epsilon_1 - \sigma_1 x - \alpha_1 y)$$

$$\frac{dy}{dt} = y(\epsilon_2 - \sigma_2 y - \alpha_2 x)$$

Configuring the Lotka-Volterra model for climate

- Just as we did for population, we can rework our assumptions for climate
- 1. Environment does not favor temperature or humidity
- 2. There is no deficiency of water in the environment

- The dependencies will be set accordingly

- So what happens here is that the increase in temperature results in more water evaporating from the surface, which in turn increases humidity. Humidity leads to precipitation, which decreases the air temperature. Thus humidity and air temperature regulate each other and have boundary values, aside from their individual growth rates. Hence, the usage of the Lotka-Volterra model for climate is apt.

Configuring the Lotka-Volterra model for climate (Continued)

- We can come to how we can use this for describing climate. We can use the Lotka Volterra model to describe the interplay of humidity and air temperature. This model can in turn help us understand the relation between these two climate-determining factors.

Based on the second system of equations in the prey-predator model slide, we can generate the following model for relative humidity(H_R) and air temperature(T_{air}).

$$\frac{dH_R}{dt} = H_R(\epsilon_1 - \sigma_1 H_R - \alpha_1 T_{air}) \; ; \; H_R(0) = H_0, \; H_0 > 0$$

$$\frac{dT_{air}}{dt} = T_{air}(\epsilon_2 - \sigma_2 T_{air} - \alpha_2 H_R) \; ; \; T_{air}(0) = T_0, \; T_0 > 0$$

MATLAB Codes

We can define the system of ODEs in a separate file, titled system_ex.m

- The values of the coefficients have been derived in the source paper using statistical methods and are beyond the scope of this project

```
function yprime = system_ex (t , y ) % Defining the ODES
yprime = zeros (2 ,1) ;
yprime (1) = 0.04725*y(1) - 0.00063084*y(1)*y(1) - 0.00024*y(1)*y(2);
yprime (2) = 0.04041*y(2) - 0.00053737*y(2)*y(2) - 0.00018*y(1)*y(2);
```

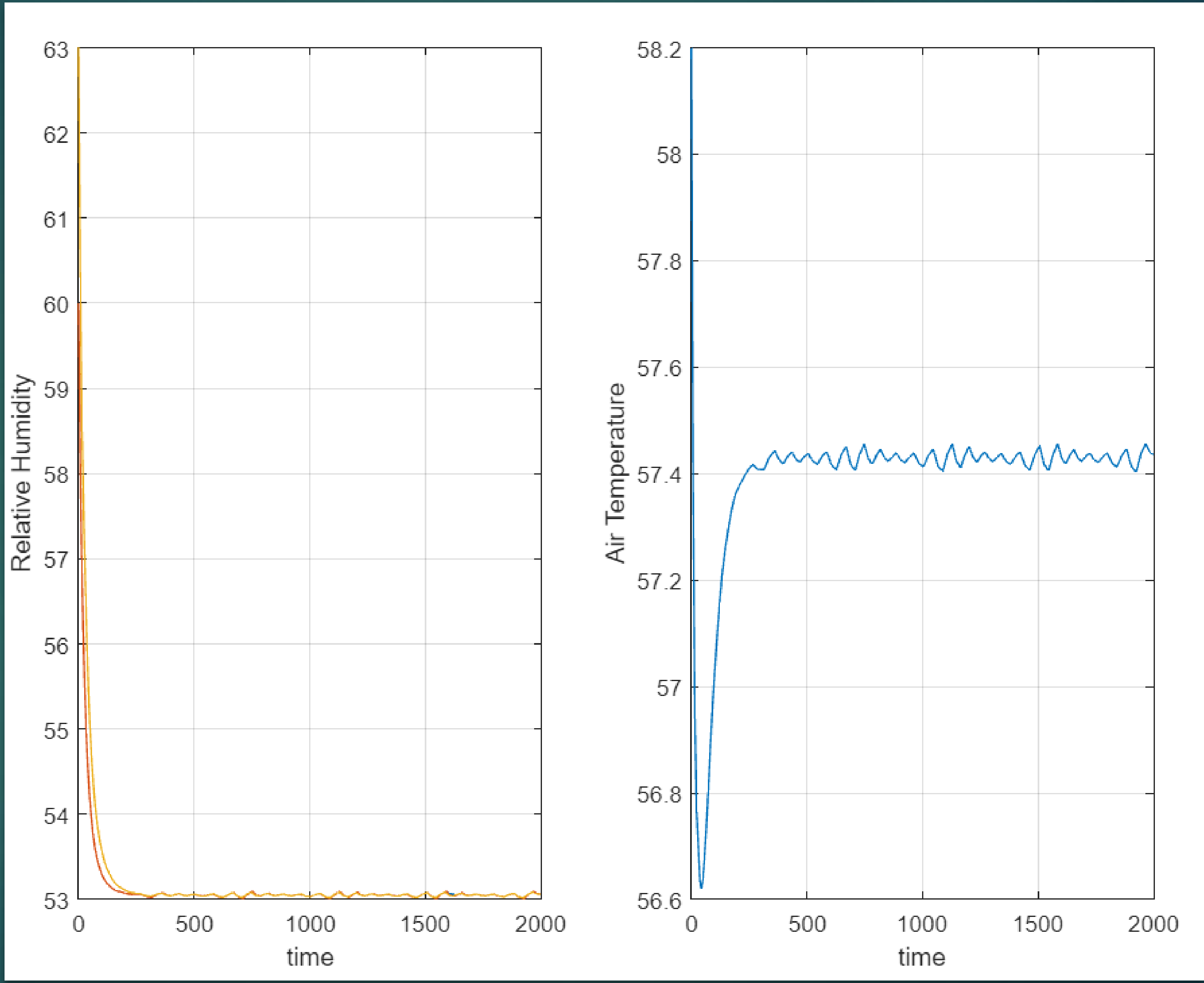
With this system now defined in a separate .m file, we can apply the ODE45 method to numerically solve this system of differential equations. Upon previous testing, we found that there was no direct solution to this system of differential equations, hence the numerical method was used. This is done in a separate code in the next slide.

MATLAB CODES (Part 2)

```
[t,y] = ode45(@system_ex,[1,1626],[63,58.2]);
figure(1);
plot(t,y(:,1));xlabel("time");ylabel("Relative Humidity");
figure(2);
plot(t,y(:,2));xlabel("time");ylabel("Air Temperature");
```

This second code is used to just solve the previously mentioned system of ODEs and plot the solution. We can adjust the initial conditions and the time span for the plot. Changing the initial conditions can lead to a different set of plots altogether.

Plots



63
62
61
60
59
58
57
56
55
54
53
Relative Humidity
0
500
1000
1500
2000
time
58.2
58
57.8
57.6
57.4
57.2
57
56.8
56.6
Air Temperature
0
500
1000
1500
2000
time

Observations

By looking at the parameters and the system of ODEs itself, we can see that it has been assumed that only humidity and air temperature have a dynamic effect on each other, while the initial conditions are static.

It would be fair to assume that these static initial conditions are affected by geographical parameters, such as altitude, that do not change regularly. The coefficients are also constants, which may be subject to change as well.

Intermediate complexity models

- **EMICs, like EBMs, offer some simplifications, but they always include a depiction of the Earth's geography, i.e. they provide more than global averages or huge boxes.** Second, they have a far **larger number of degrees of freedom than EBMs**. As a result, unlike some simpler models, the parameters of EMICs cannot be simply modified to reproduce the observable characteristics of the climate system.

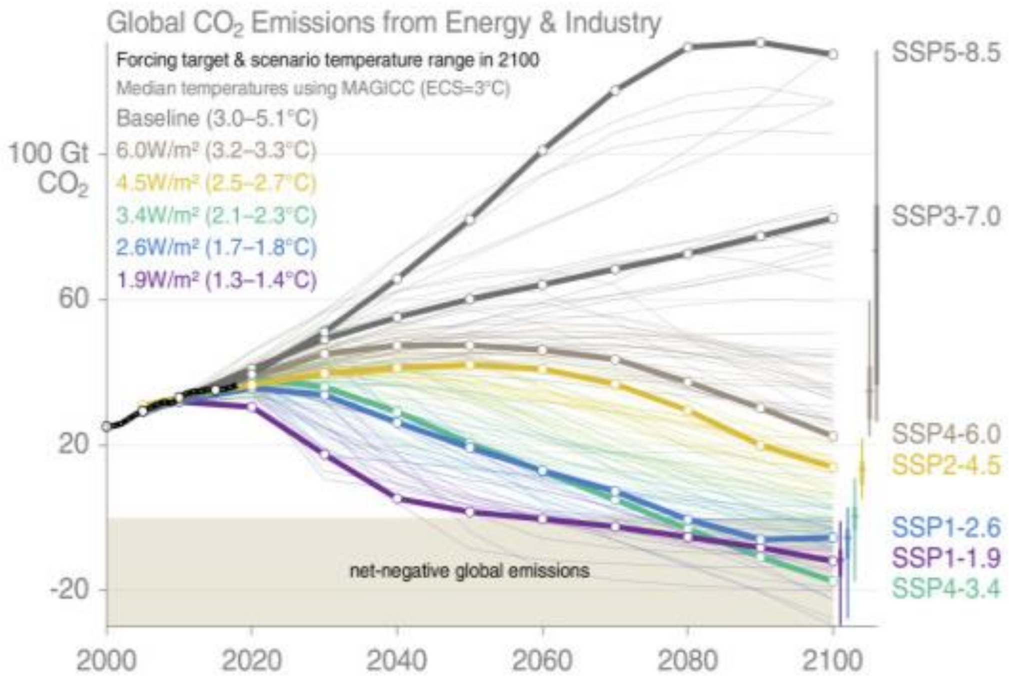
- Some EMICs, on the other hand, incorporate components that are quite similar to those designed for GCMs, but with a coarser numerical grid so that computations may be completed quickly enough to run a large number of reasonably long simulations.

- Other components are often simplified, **such as the atmosphere, because it is the component of linked climate models that relies the most on computer time.**
- Future directions for EMICs are anticipated to include assessing uncertainties and serving as a forerunner for incorporating new earth systems. They also lend themselves to the formation of ensembles to constrain parameters and assess earth systems due to their speed. In the realm of climate stabilisation research, EMICs have recently taken the lead.
- McGuffie and Henderson-Sellers argued in 2001 that EMICs would be "as essential" as GCMs in the climate modelling field in the future; while this is no longer the case, their importance has not reduced. Finally, since climate science has come under greater scrutiny, models' ability to not only project but also to explain has become increasingly crucial.

General circulation models

- A GCM's equations produce "weather" in the form of **daily rainfall, temperature, and wind values**. Most of these outputs, however, are not preserved during a GCM run. Saving such enormous volumes of data is simply not feasible.
- Instead, **summary statistics such as average monthly rainfall and average temperature are saved.** This allows you to save a profile of your run without taking up a lot of storage space with unnecessary data. **The output of a GCM is frequently reduced further to provide a trace of global mean temperature rise.**
- GCM runs, on the other hand, may not always save the data that is most important to biological investigation.

- Extreme events, like as drought or strong storms, may cause organisms to respond in ways that aren't recorded in average monthly statistics. Biological investigations may rely on commonly archived information or collaborate with climatologists to save or extract more biologically relevant outputs from GCM runs.



- GCM simulations take a lot of computer time because of the enormous number of processes they contain and their very high resolution.
- On the fastest computers, for example, a one-hundred-year experiment takes several weeks to complete. Longer simulations with higher resolution become more cheap as computing power develops, delivering more regional details than prior models.

Components of a Climate Model

We've covered three of the main components:

1. **atmosphere**
2. **ocean**
3. **ice sheets**

Atmosphere:

We can form 7 equations with 7 variables: 3 components of velocity (u, v, w), temperature (T), pressure (P), specific humidity (q) and density (ρ).

- 1. Using Newton's second law

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\vec{\nabla}p - \vec{g} + \vec{F}_{fric} - 2\vec{\Omega}\times\vec{v}$$

- 2. Using Conservation of Mass

$$\frac{\partial\rho}{\partial t} = -\vec{\nabla}.(\rho\vec{v})$$

- 3. Conservation of the mass of water vapor
- (E and C are evaporation and condensation rates resp.)

$$\frac{\partial\rho q}{\partial t} = -\vec{\nabla}.(\rho\vec{v}q) + \rho(E - C)$$

- 4. First Law of Thermodynamics

$$Q = C_p\frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt}$$

(Cp is specific heat)

- 5. The equation of State

$$p = \rho R_g T$$

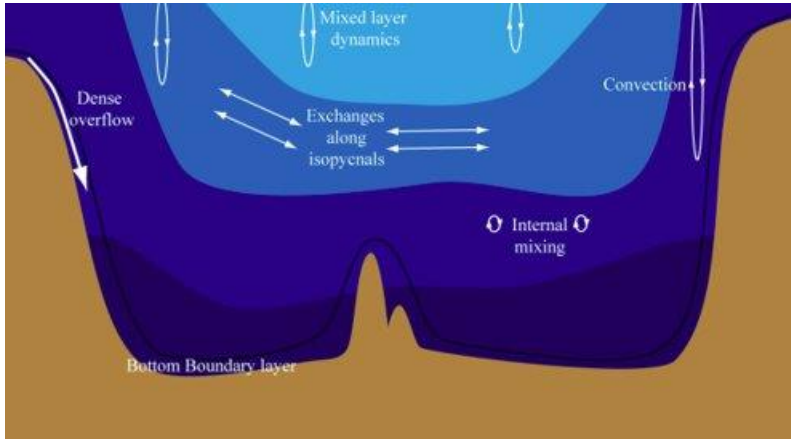
Ocean:

Now, instead of specific humidity an equation of salinity is required.

The only significant heat source in the ocean is the absorption of solar radiation. Thus, there is an exponential decay of the solar irradiance in model. $\frac{dT}{dt} = F_{sol} + F_{diff}$ F_{sol} refers to the absorbed solar radiation in ocean

There is no source or sink of salinity inside the ocean, thus $\frac{dS}{dt} = F_{diff}$

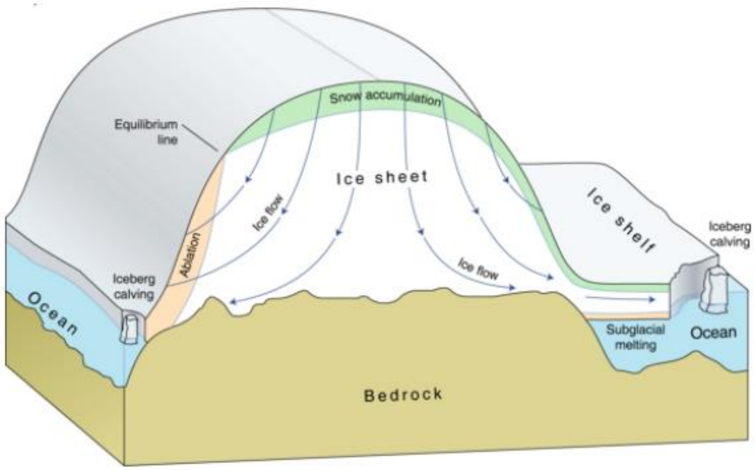
Schematic representation of some small-scale processes that have to be parameterized in ocean models:



Ice Sheets:

- ice-sheet models can be decomposed into two major components: a dynamic core that computes the flow of the ice and a thermodynamic part that estimates the changes in ice temperatures, snow accumulation, melting, etc. The ice velocity can be computed using the complete three-dimensional equation.

$$\frac{\partial H}{\partial t} = -\vec{\nabla}.(\vec{v}_m H) + M_b$$



- Vm is the depth-averaged horizontal velocity field and Mb is the mass balance accounting for snow accumulation as well as basal and surface melting
- The melting at the ice base is deduced from the balance between the heat conduction in the ice and in the ground, considering the geothermal heat flux.
- Conditions at the ice base, have a large impact on the ice velocity as they reduce the stresses greatly, compared to the situation where the ice is well below the freezing point.

Testing the Validity of Models

- Once a climate model is set up, it can be tested via a process known as “hind-casting.”
- This process runs the model from the present time backwards into the past. The model results are then compared with observed climate and weather conditions to see how well they match.
- This testing allows scientists to check the accuracy of the models and, if needed, revise its equations.

The verification and validation of weather and climate models consider many criteria, such as:

- The correctness of a set of equations to represent phenomena
- The accuracy of the representation of those equations with discrete mathematics suitable for digital computers
- The correctness of the implementation on the computers
- The construction, by coupling, of comprehensive models from component models in which functions and physical processes have been represented in a split or granular fashion
- The ability of component and coupled models to represent observations with correct physical, chemical, and biological processes
- The ability of the coupled model to represent the conservation of energy, mass, and momentum

Use of climate models:

Climate models are important tools for improving our understanding and predictability of climate behavior on seasonal, annual, decadal, and centennial time scales.

Models investigate the degree to which observed climate changes may be due to natural variability, human activity, or a combination of both.

Their results and projections provide essential information to better inform decisions of national, regional, and local importance, such as water resource management, agriculture, transportation, and urban planning.

- **Climate modelling can also be used for diagnosis and prognosis.**
- Detection and attribution are two examples of diagnostic uses. Detection and attribution include proving that a detected change is statistically significant before assigning it to non-natural causes, such as the role of anthropogenic forcing in twentieth-century climate change.
- Prognostic climate modelling uses current or historical data (ocean structure, radiative forcing, etc.) to forecast future climate, such as global warming trends. Seasonal/interannual variability, decadal prediction, and 21st century scenarios are all timescales for projection.

- All model types, when used correctly, can provide useful information on the behavior of the climate system.
- **There is no one-size-fits-all model that can be used for everything. This is why there are so many different climate models, constituting what is known as the spectrum or hierarchy of models.**
- One type of model may be chosen depending on the purpose or question. The optimum type of model to utilize is determined by the goal or inquiry. Combining the results of multiple types of models, on the other hand, is frequently the greatest method to acquire a detailed knowledge of the dominating processes in action.

Significance of differential equations in climate modelling

- Modeling is using **preset differential mathematical equations to represent the forces that control a situation.** By changing variables in the equation one can one can envision a future outcome.
- Mathematical models of differential equations can be used to solve problems and generate models.
- The typical dynamic variable is time, and if it is the only dynamic variable, the analysis will be based on an ordinary differential equation (ODE) model. When, in addition to time, geometrical considerations are also important, partial differential equation (PDE) models are used.
- PDE models, **together with their boundary and initial conditions**, arguably constitute the most sophisticated and challenging models in today's science.

Conclusion

- We have thus looked at the implementation of climate models using differential equations. We began by looking at the need and the various uses of climate models
- We started by examining the use and the requirements that go into the formulation of a climate model.
- We have thus gone through multiple climate models and the way they can be constructed using differential equations.
- One major limitation that can be seen in most of the models discussed here is that they fail to account for changes in geographical features and/or effects of human activity over the years

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