Project Report

SC1.440.Dynamical Processes in Complex Networks

Sudden transitions in coupled opinion and epidemic dynamics with vaccination in a network

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0.1 Objectives

1. **Investigate Opinion Dynamics:**
    - Utilize continuous opinion dynamics to model public attitudes towards vaccination.
    - Analyze how opinion dynamics influence vaccination uptake and disease spread.
2. **Model Epidemic Dynamics:**
    - Implement the Susceptible-Infected-Susceptible (SIS) model to simulate disease spread.
    - Explore the impact of vaccination campaigns on epidemic dynamics using differential equations.
3. **Explore Transition Points:**
    - Identify critical transition points in disease control strategies influenced by public opinion and vaccine effectiveness.
    - Employ Monte Carlo simulations to study sudden transitions in coupled opinion and epidemic dynamics.

0.2 Introduction

Statistical physics explores macroscopic phenomena emerging from microscopic interactions, relevant across diverse fields including social, epidemic, and vaccination dynamics. Physicists study theoretical and practical aspects of collective phenomena, including scenarios arising from coupled vaccination and opinion dynamics. Public opinion significantly influences vaccination campaign outcomes, as observed in instances like the 2010 H1N1 vaccine rollout in France.

Despite highly effective vaccines, vaccine-preventable diseases persist due to opinion dynamics. While studies have addressed the impact of vaccination behavior on disease spread, there's a lack of research on vaccination opinions as continuous variables. To bridge this gap, our study investigates emergent scenarios of an epidemic model with vaccination coupled with social dynamics using continuous opinions. Continuous opinions are suitable for modeling vaccination dynamics as they capture opinion strengthening, gradual changes, and a spectrum of opinions in the population.

0.3 Theory

We will be using SISV model and Erdos Reyni network in this. So we will be discussing about them in brief.

0.3.1 Susceptible-Infectious-Susceptible (SIS) model

The Susceptible-Infectious-Susceptible (SIS) model is a classic epidemiological model used to study the spread of infectious diseases within a population where individuals can both recover from infection and become susceptible again. In this model, individuals move between two states: susceptible (S) and infectious (I).

The dynamics of the SIS model can be described using a system of ordinary differential equations (ODEs). Let's denote:

- $S(t)$ as the number of susceptible individuals at time t
- $I(t)$ as the number of infectious individuals at time t
- N is the population size

The basic SIS model can be described by the following set of differential equations:

$$\frac{dS}{dt} = \lambda - \beta\frac{SI}{N} + \gamma I$$
$$\frac{dI}{dt} = \beta\frac{SI}{N} - \gamma I$$

where

- λ - The constant rate of recruitment of susceptible individuals into the population
- β - transmission rate: the rate at which susceptible individuals become infected upon contact with infectious individuals
- γ - recovery rate: the rate at which infectious individuals recover and become susceptible again

0.3.2 Susceptible-Infectious-Vaccinated (SISV) model

To incorporate vaccine dynamics into the SIS model, we can introduce a vaccination rate (μ) and a parameter representing the effectiveness of the vaccine ($0 \leq \epsilon \leq 1$). The vaccine dynamics involve moving individuals from the susceptible state to a vaccinated state. We'll denote $V(t)$ as the number of vaccinated individuals at time t.

The modified set of differential equations for the SIS model with vaccination becomes:

$$\begin{aligned}
\frac{dS}{dt} &= \lambda - \beta\frac{SI}{N} - \mu S \\
\frac{dI}{dt} &= \beta\frac{SI}{N} - \gamma I \\
\frac{dV}{dt} &= \mu S - \epsilon\beta\frac{SV}{N}
\end{aligned}$$

Parameters:

1. **Transmission Rate (β)**: This rate represents the probability of transmission of the disease from an infectious individual to a susceptible individual per unit time.

2. **Recovery Rate (γ)**: This rate represents the rate at which infectious individuals recover and return to the susceptible state per unit time.

3. **Vaccination Rate (μ)**: This rate represents the rate at which susceptible individuals are vaccinated per unit time.

4. **Effectiveness of the Vaccine (ϵ)**: This parameter represents the effectiveness of the vaccine in reducing the probability of infection among vaccinated individuals.

5. **Recruitment Rate (λ)**: This rate represents the rate at which new susceptible individuals enter the population per unit time.

0.3.3 Erdos Reyni Network

Erdős-Rényi graphs are a class of random graphs named after Hungarian mathematicians Paul Erdős and Alfréd Rényi. These graphs are characterized by two parameters: the number of vertices, denoted by n, and the probability of edges between any pair of vertices, denoted by p. There are two common models within the Erdős-Rényi framework:

- $G(n, p)$ Model: In this model, a graph is generated by connecting each pair of vertices with probability p, independently of the other pairs.
- $G(n, M)$ Model: In this model, a graph is generated by randomly selecting M edges from the set of all possible edges in the complete graph with n vertices.

The behavior of Erdős-Rényi graphs undergoes a phase transition as the parameter p varies. When p is small, the graph is typically disconnected into small components. As p increases, a giant connected component emerges, eventually leading to a connected graph. This transition is known as the Erdős-Rényi phase transition.

1. Probability of an Edge in $G(n, p)$ Model:

$$P(E_{ij}) = p$$

2. Expected Number of Edges in $G(n, p)$ Model:

$$E(M) = \binom{n}{2} p$$

3. Probability of a Connected Graph in $G(n, p)$ Model:

$$P(\text{connected}) = \begin{cases} 0 & \text{if } p \leq \frac{\ln n}{n} \\ 1 & \text{if } p \geq \frac{\ln n}{n} \end{cases}$$

4. Critical Probability for Connectivity:

$$p_c = \frac{\ln n}{n}$$

0.4 Model

In the model, we consider a **fully-connected population** with N individuals. Each person/agent i in this society carries an opinion o_i, which is a real number such that $o_i \in [-1, 1]$.

1. Positive values indicate a favorable position towards the vaccination campaign.
2. Negative values indicate an unfavorable position towards the vaccination campaign.

3. Opinions tending to $+1$ and -1 indicate extremist individuals.

4. Opinions near 0 represent neutral or undecided agents.

The agents are classified as follows:

1. Opinion states:
   (a) Pro-vaccine ($o_i > 0$)
   (b) Anti-vaccine ($o_i < 0$)
2. Epidemic states:
   (a) Susceptible (S)
   (b) Infected (I)
   (c) Vaccinated (V)

0.4.1 Model Dynamics and Transitions

We define the initial density of positive opinions as D, a model parameter. The density of negative opinions at the beginning is $1-D$. Each Susceptible agent i takes the vaccine with probability γ_i, which varies among individuals (heterogeneous). This parameter represents individuals' engagement in the vaccination campaign, measuring their tendency to get vaccinated. If an individual does not take the vaccine, they can become infected with probability λ upon contact with an Infected individual, following the standard SIS model. An Infected individual becomes Susceptible again with probability α upon recovery. Vaccinated agents are not permanently immune; a vaccinated agent becomes Susceptible again with a rate ϕ, representing the resusceptibility probability.

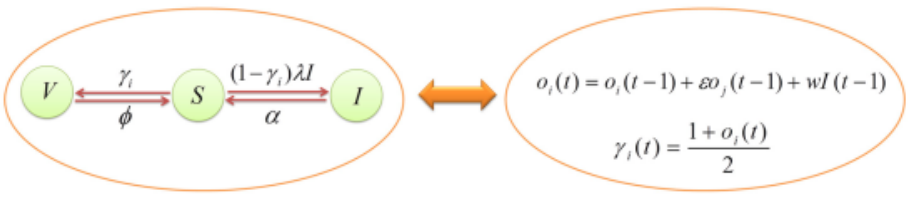

Figure 1: Coupled vaccination and continuous opinion dynamics schematics

Individuals can transition among epidemic compartments as follows:

- **Susceptible to Vaccinated ($S \rightarrow V$):** A Susceptible individual becomes Vaccinated with probability γ_i.
- **Susceptible to Infected ($S \rightarrow I$):** A Susceptible individual becomes Infected with probability $(1-\gamma_i)\lambda$ upon contact with an Infected agent.
- **Infected to Susceptible ($I \rightarrow S$):** An Infected individual recovers and becomes Susceptible again with probability α.
- **Vaccinated to Susceptible ($V \rightarrow S$):** A Vaccinated individual becomes Susceptible again with probability ϕ.

0.4.2 Opinion Dynamics Equations

While the use of discrete opinions (yes or no) is common, employing continuous opinions is more appropriate for several reasons:

1. Continuous opinions allow for the reinforcement of viewpoints through interactions with peers.
2. Opinions are dynamic and subject to change over time, making continuous representation more realistic.
3. Real-world opinions are often multifaceted and nuanced, necessitating the incorporation of extreme and moderate views.

We assume, based on kinetic models of collective opinion formation, that the **opinion dynamics are governed by the equation**:

$$o_i(t) = o_i(t-1) + \varepsilon o_j(t-1) + wI(t-1) \tag{1}$$

which considers that the opinion of each agent i at an instant t depends on:

1. Their previous opinion.
2. A peer pressure exerted by a randomly selected agent j, modulated by a stochastic variable ε uniformly distributed in the interval [0, 1], that introduces heterogeneity in the pairwise interactions.
3. The proportion of Infected agents $I(t-1)$ modulated by an individual's risk perception parameter w.

We assume that **the vaccination probability γ_i of an agent i is proportional to his opinion about vaccination** o_i, as follows:

$$\gamma_i(t) = \frac{1 + o_i(t)}{2} \tag{2}$$

The above equation ensures that $0 \leq \gamma_i \leq 1$ since $-1 \leq o_i \leq 1$ and also introduces a high level of heterogeneity in the personal vaccination hesitancy.

0.5 Methodology

Monte Carlo simulations are conducted within the framework of an agent-based system, as individuals (agents) are the focal point of social theory . We consider **populations with $N = 10^4$ agents**. Time is measured in Monte Carlo steps (mcs), where each step involves visiting each of the N agents. To maintain simplicity and without sacrificing generality, we fix the **recovery probability** $\alpha = 0.1$ in all simulations. Random initial conditions are used.

In order to control the initial conditions, we introduce the parameter D. **D represents the initial fraction of pro-vaccine agents**, where $o_i(t = 0) > 0$. Equation (1) indicates that cases with $D > 0.5$ lead to a consensus where $o_i = 1$ for all i since the overall opinion is positive, ω is positive, and $I(t - 1)$ is positive. This implies that every agent will certainly vaccinate, effectively halting the epidemic spread. Consequently, we are particularly interested in scenarios where the initial majority opposes vaccination, i.e., $D < 0.5$.

0.6 Results

0.6.1 Baseline Simulations

We depict the densities of Infected (I) and Vaccinated (V) individuals, along with the average opinion:

$$m = \frac{\sum_{i=1}^{N} o_i}{N}$$

With $D = 0.2$, the initial majority holds a negative view on vaccination.

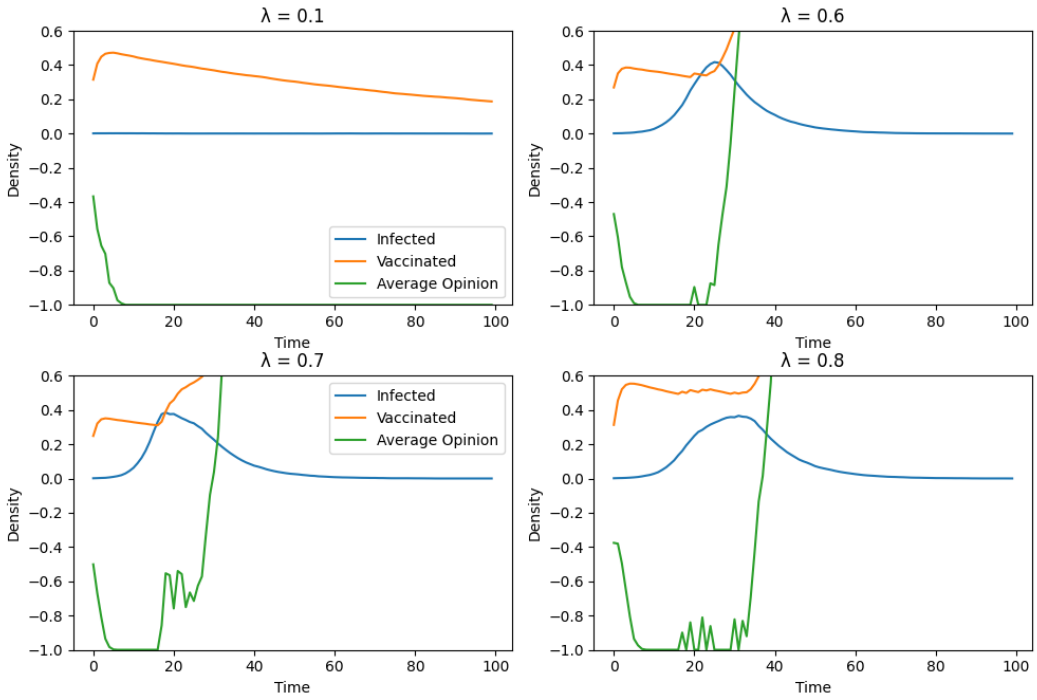

Figure 2: Tuning and plotting different temporal evolution for $I(t)$, $V(t)$ and $m(t)$

Tuning λ to generate diverse temporal evolutions for $I(t)$, $V(t)$, and $m(t)$

Parameters used are: $D = 0.20$, $w = 0.90$, $\alpha = 0.1$, $\phi = 0.01$, and $N = 10^4$.

- In the plot, we observe that in the absence of an external field, social pressure drives the system towards a consensus ($o_i = -1$ for all i). This occurs when the infection probability ($\lambda = 0.1$) results in the disease-free phase I (DFI), causing the vaccine coverage to diminish after an initial increase.
- For moderately transmissible diseases ($\lambda = 0.6$), an initial outbreak leads to a permanent endemic phase where $o_i = -1$ for all i, yet vaccine coverage persists due to sustained risk perception.
- Temporal evolutions with $\lambda = 0.7$ exhibit bistable solutions, indicating two distinct stable steady states. The observed plot for $m(t)$ resembles

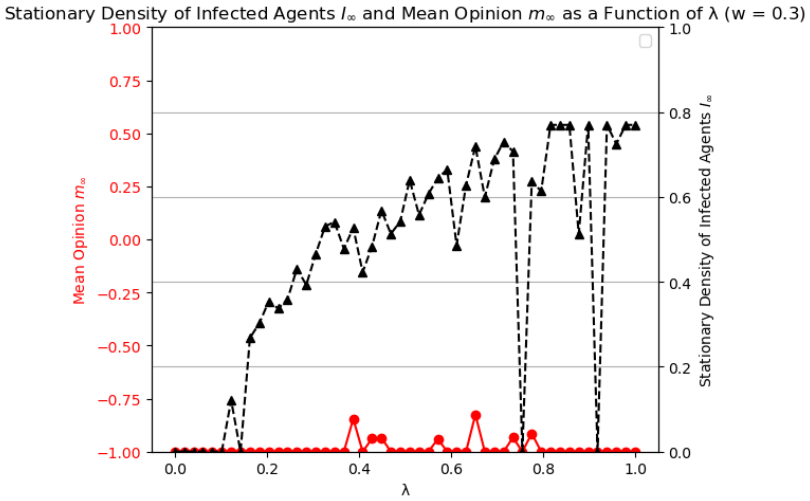

Figure 3: w = 0.3

the average of the lambda values $\lambda = 0.6$ and $\lambda = 0.8$.

- We observe a spread similar to that of a highly contagious disease for $\lambda = 0.8$, prompting a large-scale outbreak. This overwhelms initial negative opinions and causes a surge in vaccine coverage, halting epidemic contagion (DFII).

Stationary density of Infected agents I_∞ averaged only over surviving runs and mean opinion m_∞ as a function of λ

Parameters used are: $D = 0.1$, $\phi = 0.01$, $\alpha = 0.1$, and $N = 10^4$.

- The plot reveals a new first-order phase transition in I_∞ with an increment in risk perception ($w = 0.3$).
- Despite this, the usual continuous active-absorbing phase transition persists, driven solely by epidemic factors.
- For $w = 0.7$, a second discontinuous active-absorbing phase transition emerges.

To understand the discontinuity in I_∞, we revisit the equation:

$$o_i(t) = o_i(t-1) + \varepsilon o_j(t-1) + wI(t-1)$$

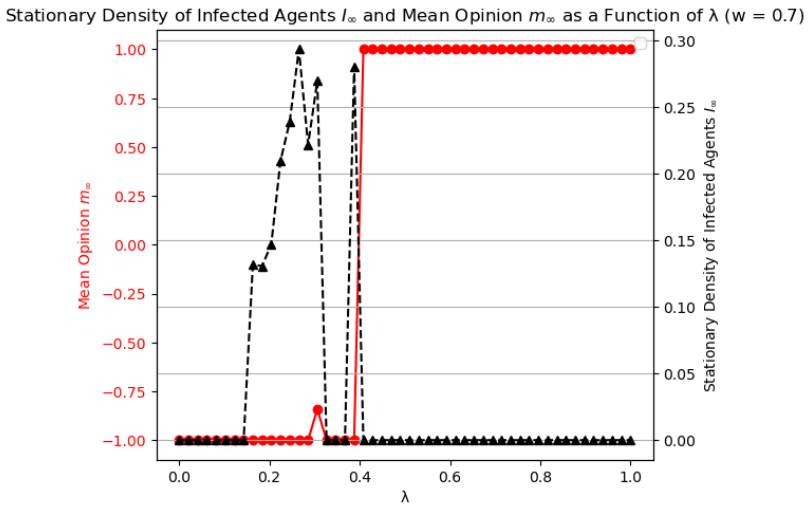

Figure 4: w = 0.7

In the absence of the external field (wI), this equation drives opinions to one of the extremes ($o_i = \pm 1$) due to the bounded nature of opinions. This bound gives rise to weighted interactions. For instance, if $o_i = -0.9$ for all i and $wI = 0.3$, any agent's opinion in the next time step is $o_i' = -0.9 - 0.9 + 0.3$. As $\varepsilon \in [0, 1]$, the maximum possible decrement in the opinion is 0.1 for $\varepsilon \geq \frac{4}{9}$, while the maximum increment is 0.3 when $\varepsilon = 0$. Although increments are less probable, they contribute noticeably.

A strong external field (wI) amplifies these increments, leading agents' opinions slightly higher than 0. This increment triggers a shift towards overall positive opinions ($o_i = 1$ for all i), resulting in $\gamma_i = 1$ and instant vaccination initiation.

0.6.2 Erdos-Renyl Network Simulation

The opinion dynamics equation $o_i(t+1) = o_i(t) + \varepsilon o_j(t) + wI(t)$ assumes a fully-connected population. To incorporate network connections, we modify the equation to incorporate dependencies on the fraction of infected neighbors, $I_{\text{neighbor}}(t)$, thus providing local information. The revised equation takes the form:

$$o_i(t+1) = o_i(t) + \varepsilon o_j(t) + wI_{\text{neighbor}}(t)$$

.

This involves considering an ER network as our population and verifying the opinion dynamics results.

Mathematical Model for Opinion Dynamics

The mathematical model explores how public opinion can influence the course of an epidemic. The equation tracks how an individual's opinion can be influenced by:

- Their own current opinion ($O_i(t)$)
- The influence of their neighbors ($\sum_{j=1} A_{ij}O_j(t)$) weighted by the adjacency matrix (A_{ij})
- A general influence term ($\sum_{j=1} wK_jA_{ij}(t)$) modulated by the risk perception parameter (w)

Overall, the model suggests that an individual's opinion about a certain topic can be impacted by their own perspective, how those around them view it, and their perceived risk associated with the topic.

We employ a discrete opinion dynamics model represented by the equation:

$$O_i(t+1) = O_i(t) + \frac{\epsilon}{K_i}\sum_{j=1}^{N} A_{ij}O_j(t) + \frac{w}{K_i}\sum_{j=1}^{N} A_{ij}I_j(t)$$

where:

- $O_i(t)$ represents the opinion of the i-th individual at time t.
- w denotes the risk perception parameter.
- ϵ is a modulated parameter.
- K_i indicates the degree of the i-th node.
- A represents the adjacency matrix.

Tuning λ to generate diverse temporal evolutions for $I(t)$, $V(t)$, and $m(t)$

Parameters used are: $D = 0.20$, $w = 0.90$, $\alpha = 0.1$, $\phi = 0.01$, and $N = 10^4$.

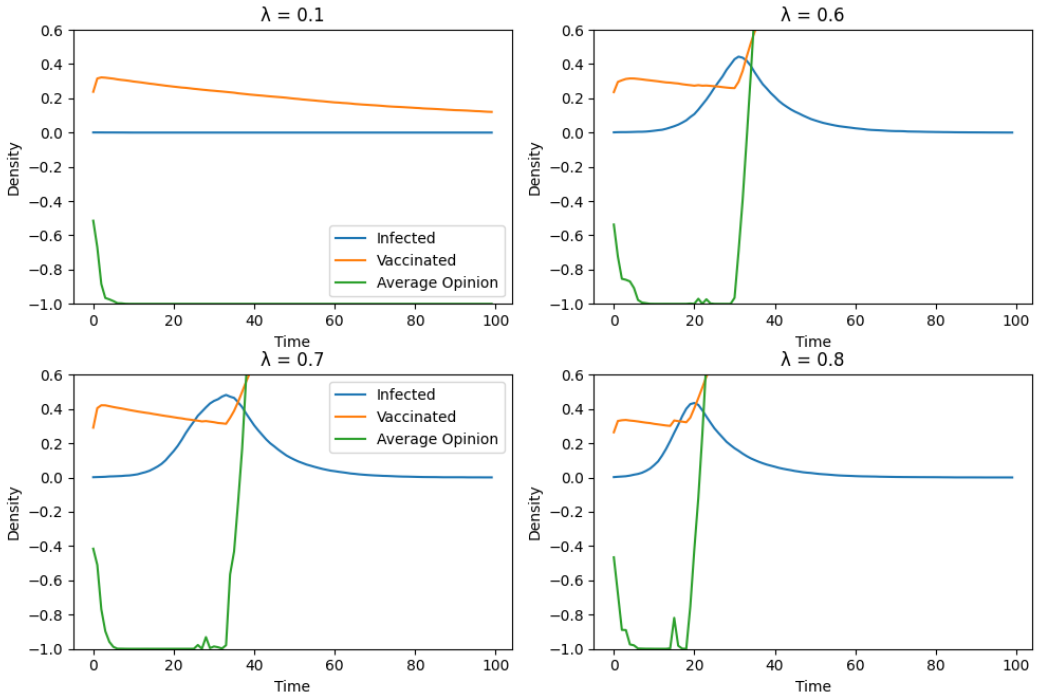

Figure 5: Tuning λ and and plotting different temporal evolution for I(t) , V (t) and m(t)

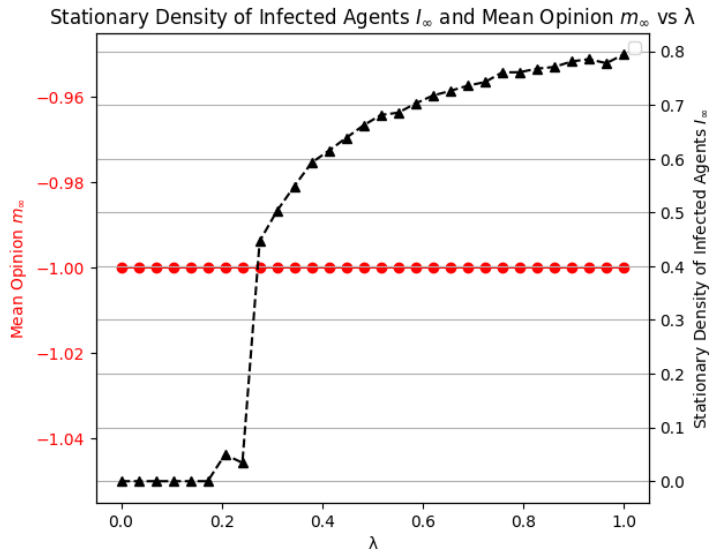

Figure 6: Density plot as function of λ

Stationary density of Infected agents I_∞ averaged only over surviving runs and mean opinion m_∞ as a function of λ

Parameters used in the model are as follows: $\epsilon = 1$, $w = 0.8$, $\alpha = 0.1$, and $\phi = 0.01$.

We observe that the results obtained are slightly similar to that obtained in a fully connected graph.

- Community Structure: ER networks typically lack strong community structures, potentially leading to more uniform opinion dynamics across the network.
- Epidemic Spread: Disease or opinion spread might be more uniform due to the random nature of connections in ER networks.
- Information Transmission: Information may propagate more slowly in ER networks, affecting the speed of convergence to consensus opinions or epidemic spread rates.

0.7 Novelty

We have identified a few areas for potential improvement and future research directions:

0.7.1 Heterogeneous Conviction Levels

Currently, all agents in our model possess the same degree of conviction. However, it is evident from existing literature that individuals on the refusal side (anti-vaccine) tend to be more strongly convicted in their opinions compared to pro-vaccine agents. To address this, we can modify the opinion dynamics equation to incorporate heterogeneous conviction levels among agents. This could involve introducing a parameter representing the conviction level of each agent, which influences their opinion update rule. For example, a more strongly convicted agent may have a larger impact on their peers' opinions:

$$o_i(t+1) = o_i(t) + \varepsilon o_j(t) + wI(t) \cdot \text{Conviction}_i$$

0.7.2 Dynamic Parameter Adjustment

Exploring the effects of dynamically adjusting model parameters based on real-time data or external factors could enhance the model's predictive capabilities. For instance, we can introduce adaptive vaccination strategies that respond to changing epidemic conditions or public opinion dynamics. This could involve modifying the vaccination probability $\gamma_i(t)$ to be a function of time, incorporating feedback mechanisms based on current infection rates or perceived risk levels:

$$\gamma_i(t) = f(\text{epidemic conditions}, \text{public opinion}, t)$$

where f represents a function that dynamically adjusts $\gamma_i(t)$ based on the evolving epidemic conditions and public sentiment.

0.7.3 Behavioral Dynamics Modeling

Integrating more nuanced behavioral dynamics into the model, such as social influence mechanisms, decision-making processes, and information dissemination channels, could yield a more comprehensive understanding of vaccination behavior and epidemic spread. This could involve extending the model to incorporate additional factors influencing opinion formation and vaccination decisions, such as social networks, media influence, and cognitive biases.

We can extend the existing equations governing opinion dynamics and vaccination behavior to account for these factors. For example, we could

introduce a term in the opinion dynamics equation that represents the influence of media exposure or social network interactions on individuals' opinions:

$$o_i(t) = o_i(t-1) + \varepsilon o_j(t-1) + wI(t-1) + \text{Media}(t)$$

where Media(t) represents the influence of media exposure at time t.

0.8 Conclusion

In conclusion, our study investigated the interplay between disease risk perception, vaccination attitudes, and disease spread within an SIS epidemic model coupled with vaccination and opinion dynamics. Through continuous opinions and Monte Carlo simulations, we found the emergence of a first-order phase transition alongside the active-absorbing phase transition in the SIS model, indicating critical thresholds in epidemic outcomes. We also observed an initial increase in pro-vaccine individuals can reduce epidemic outbreaks, but sustained long-term infection chains may occur due to dynamics of vaccine effectiveness and coverage.

0.9 Acknowledgment

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0.10 References

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