Example 14·2. *A machinist is making engine parts with axle diameters of 0·700 inch. A random sample of 10 parts shows a mean diameter of 0·742 inch with a standard deviation of 0·040 inch. Compute the statistic you would use to test whether the work is meeting the specifications. Also state how you would proceed further.*

Solution. Here we are given :

$$\mu = 0{\cdot}700 \text{ inches}, \bar{x} = 0{\cdot}742 \text{ inches}, s = 0{\cdot}040 \text{ inches and } n = 10$$

Null Hypothesis, $H_0 : \mu = 0{\cdot}700$, *i.e.*, *the product is conforming to specifications.*

Alternative Hypothesis, $H_1 : \mu \neq 0{\cdot}700$

Test Statistic. Under H_0, the test statistic is :

$$t = \frac{\bar{x} - \mu}{\sqrt{S^2/n}} = \frac{\bar{x} - \mu}{\sqrt{s^2/(n-1)}} \sim t_{(n-1)}$$

Now

$$t = \frac{\sqrt{9}(0{\cdot}742 - 0{\cdot}700)}{0{\cdot}040} = 3{\cdot}15$$

How to proceed further. Here the test statistic 't' follows Student's t-distribution with $10 - 1 = 9$ d.f. We will now compare this calculated value with the tabulated value of t for 9 d.f. and at certain level of significance, say 5%. Let this tabulated value be denoted by t_0.

(*i*) If calculated 't' *viz.*, $3{\cdot}15 > t_0$, we say that the value of t is significant. This implies that $\bar{x}$ differs significantly from μ and H_0 is rejected at this level of significance and we conclude that the product is not meeting the specifications.

(*ii*) If calculated $t < t_0$, we say that the value of t is not significant, *i.e.*, there is no significant difference between $\bar{x}$ and μ. In other words, the deviation $(\bar{x} - \mu)$ is just due to fluctuations of sampling and null hypothesis H_0 may be retained at 5% level of significance, *i.e.*, we may take the product conforming to specifications.

$= 1 - 0 \cdot 2231 \times 3 \cdot 625 = 0 \cdot 19126$

Example 7·25. *A manufacturer of cotter pins knows that 5% of his product is defective. If he sells cotter pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality?* **[Kanpur Univ. B.Sc. 1993]**

Solution. We are given $n = 100$.

Let $p =$ Probability of a defective pin $= 5\% = 0 \cdot 05$

$\therefore \quad \lambda =$ Mean number of defective pins in a box of 100

$= np = 100 \times 0 \cdot 05 = 5$

Since 'p' is small, we may use Poisson distribution.

Probability of x defective pins in a box of 100 is

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x\,!} = \frac{e^{-5} 5^x}{x\,!}\,;\; x = 0, 1, 2, \ldots$$

Probability that a box will fail to meet the guaranteed quality is

$$P(X > 10) = 1 - P(X \leq 10) = 1 - \sum_{x=0}^{10} \frac{e^{-5} 5^x}{x\,!} = 1 - e^{-5} \sum_{x=0}^{10} \frac{5^x}{x\,!}$$

Example 7·26. *Six coins are tossed 6,400 times. Using the Poisson dis-*

Example 12·23. *The means of two single large samples of 1000 and 2000 members are 67·5 inches and 68·0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2·5 inches ? (Test at 5% level of significance).*

Solution. We are given :

$$n_1 = 1000,\ n_2 = 2000;\ \bar{x}_1 = 67{\cdot}5 \text{ inches},\ \bar{x}_2 = 68{\cdot}0 \text{ inches}.$$

Null hypothesis, $H_0 : \mu_1 = \mu_2$ and $\sigma = 2{\cdot}5$ inches, *i.e.*, the samples have been drawn from the same population of standard deviation 2·5 inches.

Alternative Hypothesis, $H_1 : \mu_1 \neq \mu_2$ (Two tailed.)

Test Statistic. Under H_0, the test statistic is (since samples are large)

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0, 1)$$

Now $$Z = \frac{67{\cdot}5 - 68{\cdot}0}{2{\cdot}5 \times \sqrt{\frac{1}{1000} + \frac{1}{2000}}} = \frac{-0{\cdot}5}{2{\cdot}5 \times 0{\cdot}0387} = -5{\cdot}1$$

Conclusion. Since $|Z| > 3$, the value is highly significant and we reject the null hypothesis and conclude that samples are certainly not from the same population with standard deviation 2·5.

Example 12·1. *A dice is thrown 9,000 times and a throw of 3 or 4 is observed 3,240 times. Show that the dice cannot be regarded as an unbiased one and find the limts between which the probability of a throw of 3 or 4 lies.*

Solution. If the coming of 3 or 4 is called a success, then in usual notations we are given

$$n = 9{,}000;\ X = \text{Number of successes} = 3{,}240$$

Under the null hypothesis (H_0) *that the dice is an unbiased one,* we get

P = Probability of success = Probability of getting a 3 or 4 $= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

Alternative hypothesis, $H_1 : p \neq \frac{1}{3}$, (*i.e.*, dice is biased).

We have $Z = \dfrac{X - nP}{\sqrt{nQP}} \sim N(0, 1)$, since n is large.

$$\text{Now} \qquad Z = \frac{3240 - 9000 \times 1/3}{\sqrt{9000 \times (1/3) \times (2/3)}} = \frac{240}{\sqrt{2000}} = \frac{240}{44{\cdot}73} = 5{\cdot}36$$

Since $|Z| > 3$, H_0 is rejected and we conclude that the dice is almost certainly biased.

Since dice is not unbiased, $P \neq \frac{1}{3}$. The probable limits for 'P' are given by :

$$\hat{P} \pm 3\sqrt{\hat{P}\hat{Q}/n} = p \pm 3\sqrt{pq/n},$$

where $\hat{P} = p = \dfrac{3240}{9000} = 0{\cdot}36$ and $\hat{Q} = q = 1 - p = 0{\cdot}64$.

Hence the probable limits for the population proportion of successes may be taken as

$$\hat{P} \pm 3\sqrt{\hat{P}\hat{Q}/n} = 0{\cdot}36 \pm 3 \cdot \sqrt{\frac{0{\cdot}36 \times 0{\cdot}64}{9000}} = 0{\cdot}36 \pm 3 \times \frac{0{\cdot}6 \times 0{\cdot}8}{30\sqrt{10}}$$

$$= 0{\cdot}360 \pm 0{\cdot}015 = 0{\cdot}345 \text{ and } 0{\cdot}375.$$

Hence the probability of getting 3 or 4 almost certainly lies between 0·345 and 0·375.

12·7·3. Procedure for Testing of Hypothesis. We now summarise below the various steps in testing of a statistical hypothesis in a systematic manner.

1. *Null Hypothesis*. Set up the Null Hypothesis H_0 (see § 12·5, page 12·6).

2. *Alternative Hypothesis*. Set up the Alternative Hypothesis H_1. Ths will enable us to decide whether we have to use a single-tailed (right or left) test or two-tailed test.

3. *Level of Significance*. Choose the appropriate level of significance (α) depending on the reliability of the estimates and permissible risk. This is to be decided before sample is drawn, *i.e.*, α is fixed in advance.

4. *Test Statistic (or Test Criterion)*. Compute the test statistic

$$Z = \frac{t - E(t)}{S.E.(t)}$$

under the null hypothesis.

5. *Conclusion*. We compare z the computed value of Z in step 4 with the significant value (tabulated value) z_α, at the given level of significance, 'α'.

If $|Z| < z_\alpha$, *i.e.*, if the calculated value of Z (in modulus value) is less than z_α we say it is not significant. By this we mean that the difference $t - E(t)$ is just due to fluctuations of sampling and the sample data do not provide us sufficient evidence against the null hypothesis which may therefore, be accepted.

If $|Z| > z_\alpha$, *i.e.*, if the computed value of test statistic is greater than the critical or significant value, then we say that it is significant and the null hypothesis is rejected at level of significance α *i.e.*, with confidence coefficient $(1 - \alpha)$.