

$$b) P(A \cap B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B) \cdot P(A|B)}{P(B)}$$

$$= \frac{4/3}{1/3} = \frac{4}{1} = \frac{2}{1}$$

Q.5)

$$P\left(\bigcup_{i=1}^n A_i\right) \geq \sum_{i=1}^n P(A_i) - (n-1)$$

Mathematical induction method is used to prove this
for any event, $E \in S, 0 \leq P(E) \leq 1$

Consider the two events $(n=2)$. Then
 $P(A_1 \cup A_2) \leq 1 \quad \because A_1 \cup A_2$ is an event

By using addition theorem

$$P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq 1$$

$$P(A_1) + P(A_2) \leq 1 + P(A_1 \cap A_2)$$

we can write the above equation

$$P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1$$

The statement is true for $n=2$

let us assume that statement is true for $n=k$.

i.e. $P\left(\bigcap_{i=1}^k A_i\right) \geq \sum_{i=1}^k P(A_i) - (k-1)$

then

$$P\left(\bigcap_{i=1}^{k+1} A_i\right) = P\left(\bigcap_{i=1}^k (A_i \cap A_{k+1})\right) \geq$$

Q. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Sol:- Let A and B be any two events. If $A \cup B$ is the union of these mutually exclusive events.

$$A \cup B = (A \cap B') \cup (A' \cap B) \cup (A \cap B) \quad \dots (1)$$

By axiom 2:

$$P(A \cup B) = P(A \cap B') + P(A \cap B) + P(A' \cap B) \quad \dots (2)$$

Now,

$$A = (A \cap B^c) \cup (A \cap B)$$

$$B = (A^c \cap B) \cup (A \cap B)$$

Therefore

$$P(A) = P(A \cap B^c) + P(A \cap B) \quad \dots (3)$$

$$\text{and } P(B) = P(A^c \cap B) + P(A \cap B) \quad \dots (4)$$

By equation 3 and 4, we get

$$P(A) + P(B) = P(A \cap B^c) + P(A^c \cap B) + 2P(A \cap B) \quad \dots (5)$$

Compare eq (2) and (5),

$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$

$$\text{So, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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Cond

Probability
Theory

Q. Explain Bayes theorem.

- Bayes theorem is a method to revise the probability of an event given additional information.
Bayes theorem calculates a conditional probability called
- a posterior or revised probability

- Bayes theorem is a result in probability theory that relates conditional probabilities. If A and B denote two events, $P(B|A)$ denotes the conditional probability of A occurring, given that B occurs. The two conditional probabilities $P(A|B)$ and $P(B|A)$ are in general different.

- Bayes theorem gives a relation between $P(A|B)$ and $P(B|A)$. An important application of Bayes theorem is that it gives a rule how to update or refine the strength of evidence-based belief in light of new evidence or knowledge.

- A prior probability is an initial probability value originally envisaged before any additional information is obtained.

- A posterior probability is a probability value that has been refined by using additional information that is later obtained.

eg:

conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{--- ①}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \quad \text{--- ②}$$

Jan 0 & 0

$$P(\alpha \mid \beta) \cdot P(\beta) = P(\beta \mid \alpha) \cdot P(\alpha)$$

$$\left[P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)} \right] \\ (\alpha \text{ } 43) \text{ } \& \text{ } 99) \text{ } \& \text{ } 95$$

Ans) Two boxes A, and B, containing 100 and 200 light bulbs respectively.

Q. What is Random Variable ? What is it Explain discrete random variable

Ans:-

The distribution function $F(x)$ or the density $f(x)$ completely characterizes the behavior of a random variable X . The concept of a random variable will enable us to replace the original probability space with one in which we set of numbers.

- whenever you run and experiment , flip a coin , roll a die , pick a card , you assign a number to represent the value to the outcome that you get . This assign is called a random variable

- A random variable is a variable X that assigns a real number $[x]$, for each and every outcome of a random experiment . If S is the sample space containing all the 'n' outcomes $[c_1, c_2, c_3, \dots, c_n]$ of random experiment and X is a random variable

defined as a function $X(\omega)$ on Ω , then for every outcome ω_i (where $i = 1, 2, 3, \dots, n$) that is in Ω the random variable $X(\omega_i)$ will assign a real value x_i .

1.20 Light

→ Discrete Random Variable:-

The random variable is called a discrete random variable if it is defined over a sample space having a finite or a countable infinite number of sample points. In this case, random variable takes on discrete values and it is possible to enumerate all the values it may assume.

- A discrete random variable can only have a specific (or finite) number of numerical values.

We can have infinite discrete random variable if we think about things that we know have an uncounted number. Think about the number of stars in the universe. We know that there are not a specific number that we have a very big count so this is an example of an infinite discrete random variable.

Another example would be with individuals with short names. If you were to insult me, I talk at the start of year, you could only insult the amount you would tell at the end of year.

Q. Explain difference between discrete and continuous random variable

A.

① It uses countable set

C

It uses set of interval

B

② F is set of all subset of n

F is made from combination of n with set operation

③ For a set A $\in F$

$$P(A) = \sum_{\omega \in A} p(\omega)$$

$$P(A) = \int_A f(x) dx$$

④ distribution function (cdf):

$$F_X(x) = \sum_{\omega \leq x} p(\omega)$$

④ distribution function (cdf):

$$F_X(x) = \int_{-\infty}^x f(x) dx$$

Q. 60) 66) 67) 68)

~~~~~

By equations (3) and (4), we get

$$P(A) + P(B) = P(A \cap B^c) + P(A^c \cap B) + 2P(A \cap B)$$

Compare the equation (2) and (5),  
P(A) + P(B) = P(A \cup B) + P(A \cap B)

So,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\begin{aligned} P(A \cap B^c) &= \frac{9}{15} \\ P(A \cap B) &= P(A) - \frac{9}{15} \\ &= \frac{1}{5} \end{aligned}$$

on first dice exceeds that

(d)

**Q.22** For any three arbitrary events A, B, C such that :

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$$

Ans. : IITNU : Nov-99, 14

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$$\begin{aligned} P(A \cup B \cup C) &= P[(A \cup B) \cup C] \\ &= P(A \cup B) + P(C) - P[(A \cup B) \cap C] \\ &= P(A) + P(B) - P(A \cap B) + P(C) \\ &\quad - P[(A \cap C) \cup (B \cap C)] \\ &= P(A) + P(B) + P(C) - P(A \cap B) \\ &\quad - P(A \cap C) - P(B \cap C) - P[(A \cap C) \cap (B \cap C)] \\ &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\ &\quad - P(B \cap C) + P(A \cap B \cap C) \end{aligned}$$

heads in successive tosses. Is this a

(n)

TTHHH, TTHH,

THH, THHHH,

staining as many

s.

$P(B) - P(A \cap B)$

b. To write  $A \cup B$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\begin{aligned} &= \frac{1}{5} + \frac{2}{3} - \frac{1}{15} \\ &= \frac{3+2 \times 5}{15} - \frac{1}{15} \\ &= \frac{13-1}{15} = \frac{12}{15} \end{aligned}$$

$$\begin{aligned} &P(A^c \cap B) \\ &\dots (1) \end{aligned}$$

$$P(A^c \cap B)$$

$$P(A^c \cap B)$$

\* A posterior probability is a probability value that has been revised by using additional information that is later obtained.

\* Suppose that  $B_1, B_2, B_3 \dots B_n$  partition the outcomes of an experiment and that A is another event. For any number, k, with  $1 \leq k \leq n$ , we have the formula :

$$P(B_k/A) = \frac{P(A/B_k) \cdot P(B_k)}{\sum_{i=1}^n P(A/B_i) \cdot P(B_i)}$$

- Q.43** Two boxes  $B_1$  and  $B_2$  contain 100 and 200 light bulbs respectively. The first box ( $B_1$ ) has 15 defective bulbs and the second  $B_2$ . Suppose a box is selected at random and one bulb is picked out.  
 a) What is the probability that it is defective?  
 b) Suppose we test the bulb and it is found to be defective. What is the probability that it came from box 1?

Ans. :

- a) Probability that it is defective : Box  $B_1$  has 85 good and 15 defective bulbs. Similarly box  $B_2$  has 195 good and 5 defective bulbs.  
 Let D = "Defective bulb is picked out".

Then,

$$P(D/B_1) = \frac{15}{100} = 0.15, \quad P(D/B_2) = \frac{5}{200} = 0.025.$$

Since a box is selected at random, they are equally likely.

$$P(B_1) = P(B_2) = \frac{1}{2}$$

Thus  $B_1$  and  $B_2$  form a partition and using above equation, we obtain

$$\begin{aligned} P(D) &= P(D/B_1) P(B_1) + P(D/B_2) P(B_2) \\ &= 0.15 \times \frac{1}{2} + 0.025 \times \frac{1}{2} = 0.0875 \end{aligned}$$

Thus, there is about 9% probability that a bulb picked at random is defective.

- b) Probability that it came from box 1 :

$$P(B_1/D) = \frac{P(D/B_1) P(B_1)}{P(D)} = \frac{0.15 \times 1/2}{0.0875} = 0.8571$$

**Q.44** A mechanical factory production line is manufacturing bolts using three machines, A, B and C. The total output, machine A is responsible for 25%, machine B for 35% and machine C for the rest. The machines that 5% of the output from machine A is defective, 4% from machine B and random 2% from machine C. A bolt is chosen at random what from the production line and found to be woman defective. What is the probability that it came Ans. : from

- i. machine A ii. machine B iii. machine C ?

Ans. : Let

$$D = \{\text{bolt is defective}\},$$

$$A = \{\text{bolt is from machine A}\},$$

$$B = \{\text{bolt is from machine B}\},$$

$$C = \{\text{bolt is from machine C}\}.$$

Given data :  $P(A) = 0.25, P(B) = 0.35, P(C) = 0.4$ .

$$P(D|A) = 0.05, \quad P(D|B) = 0.04, \quad P(D|C) = 0.02.$$

From the Bayes' Theorem :

$$\begin{aligned} P(A/D) &= \frac{P(D/A) \times P(A)}{P(D/A) \times P(A) + P(D/B) \times P(B) + P(D/C) \times P(C)} \\ &= \frac{0.05 \times 0.25}{0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.4} \\ &= \frac{0.0125}{0.0125 + 0.014 + 0.008} \end{aligned}$$

$$P(A/D) = 0.3621$$

Similarly :

$$\begin{aligned} P(B/D) &= \frac{P(D/B) \times P(B)}{P(D/A) \times P(A) + P(D/B) \times P(B) + P(D/C) \times P(C)} \\ &= \frac{0.04 \times 0.35}{0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.4} \\ &= \frac{0.014}{0.0125 + 0.014 + 0.008} = \frac{0.014}{0.0345} \end{aligned}$$

$$P(B/D) = 0.4057$$

$$\begin{aligned} P(C/D) &= \frac{P(D/C) \times P(C)}{P(D/A) \times P(A) + P(D/B) \times P(B) + P(D/C) \times P(C)} \\ &= \frac{0.02 \times 0.4}{0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.4} \\ &= \frac{0.008}{0.0125 + 0.014 + 0.008} = \frac{0.008}{0.0345} \end{aligned}$$

$$P(C/D) = 0.2318$$

cal factory production line is using three machines, A, B and C for 35% and machine C for 5%. The total output from machine A is responsible for 35% and machine C for 5% of the output from machine B and machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from machine B? iii. machine C?

(it is defective),

is from machine A),

is from machine B),

is from machine C).

$P(B) = 0.35, P(C) = 0.4.$

$P(D|B) = 0.04, P(D|C) = 0.02.$

Item :

$$P(D/A) \times P(A)$$

$$+ P(D/B) \times P(B) + P(D/C) \times P(C)$$

$$0.05 \times 0.25$$

$$0.04 \times 0.35 + 0.02 \times 0.4$$

$$+ 0.008$$

$$= 0.008$$

$$P(D/B) \times P(B)$$

$$P(D/B) \times P(B) + P(D/C) \times P(C)$$

$$0.35$$

$$0.35 + 0.02 \times 0.4$$

$$\frac{0.014}{0.08} = \frac{0.014}{0.0345}$$

$$P(D/C) \times P(C)$$

$$P(D/B) \times P(B) + P(D/C) \times P(C)$$

$$0.4$$

$$0.35 + 0.02 \times 0.4$$

$$\frac{0.008}{0.08} = \frac{0.008}{0.0345}$$

**Q.45** At a certain university, 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favour of women. If a student is selected at random from among all those over six feet tall, what is the probability that the student is a woman?

Ans. : Let us assume following :

$$M = \{\text{Student is Male}\},$$

$$F = \{\text{Student is Female}\},$$

$$T = \{\text{Student is over 6 feet tall}\}.$$

Given data :

$$P(M) = 2/5,$$

$$P(F) = 3/5,$$

$$P(T|M) = 4/100$$

$$P(T|F) = 1/100.$$

We require to find  $P(F|T)$  ?

Using Bayes' Theorem we have :

$$P(F|T) = \frac{P(T|F) P(F)}{P(T|F) P(F) + P(T|M) P(M)}$$

$$= \frac{\frac{1}{100} \times \frac{3}{5}}{\frac{1}{100} \times \frac{3}{5} + \frac{4}{100} \times \frac{2}{5}} = \frac{\frac{3}{500}}{\frac{3}{500} + \frac{8}{500}}$$

$$P(F|T) = \frac{3}{11}$$

**Q.46** A pair of dice is rolled. If the sum of 9 has appeared, find the probability that one of the dice shows 3.

Ans. : Let A = The event that the sum is 9

B = The event the one of dice shows 3.

Exhaustive cases =  $6^2 = 36$ .

Favorable cases of the event A = (3, 6), (6, 3), (4, 5), (5, 4).

So  $P(A) = 4/36$

$$P(A) = \frac{1}{9}$$

Favorable case for the event  $A \cap B = (3, 6), (6, 3)$

$$\text{Hence } P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

$$\text{But } P(A \cap B) = P(A) \times P(B/A)$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B/A) = \frac{1/18}{1/9} = \frac{1}{18} \times \frac{9}{1}$$

$$P(B/A) = 1/2$$

**Q.47** If  $A_1$  and  $A_2$  are quality like exclusive and exhaustive events and  $P(B/A_2) = 0.3$ , find  $P(A_1/B)$ .

Ans. : Since  $A_1$  and  $A_2$  are equally likely,  $P(A_1) = P(A_2)$ . Further, they are mutually exclusive and exhaustive.

$$P(A_1) + P(A_2) = 1$$

$$P(A_1) = P(A_2) = 0.5$$

$$= \frac{0.5 \times 0.2}{0.5 \times 0.2 + 0.5 \times 0.3}$$

$$= \frac{0.1}{0.1 + 0.15} = \frac{0.1}{0.25}$$

$$P(A_1/B) = 0.4$$

**Q.48** 10% of the bulbs produced are of red colour and 2% are red and defective. If a bulb is picked up at random, find the probability of its being defective if it is red.

Ans. :

Let A and B be the events that the bulb is red and defective, respectively.

$$P(A) = 10/100 = 1/10$$

$$P(A \cap B) = 2/100 = 1/50$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/50}{1/10}$$

Thus the probability of the picked bulb being defective, if it is red, is 1/5.

## 1.6 : Random Variables and Distributions : Concept of Variable

**Q.49** Define discrete sample space.

Ans. : If a sample space contains a finite number of possibilities or an unending sequence of elements as there are whole numbers, it is called discrete sample space.

$$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16} + \frac{4}{16}$$

$$= \frac{1+4+6+4}{16} = \frac{15}{16}$$

$$F(4) = f(0) + f(1) + f(2) + f(3) + f(4)$$

$$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16}$$

$$= \frac{1+4+6+4+1}{16} = \frac{16}{16} = 1$$

**Q.55** A shipment of 8 similar microcomputers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives.

Ans. : Let  $X$  be a random variable whose values  $x$  are the possible numbers of defective computers purchased by the school. Then  $x$  can be any of the numbers 0, 1, and 2. Now

Number of ways of choosing any 2 :

$${}^8C_2 = \frac{8!}{((8-2)!2!)} = 28$$

(This will be the denominator)

$$P[0] = {}^3C_0 \times {}^5C_2 / 28$$

$$= (3! / ((3-0)!0!) \times (5! / ((5-2)!2!))) / 28$$

$$= 5/14$$

$$P[1] = {}^3C_1 \times {}^5C_1 / 28 = 15/28$$

$$P[2] = {}^3C_2 \times {}^5C_0 / 28 = 3/28$$

**Q.56** If a random variable  $X$  takes the values 1, 2, 3 and 4 such that  $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$ , derive the probability distribution function of  $X$ .

[JNTU : March-17, Marks 5]

Assume  $P(X = 3) = \alpha$ . By the given equation

$$1) \quad P(X = 1) = \frac{\alpha}{2} \quad P(X = 2) = \frac{\alpha}{3} \quad P(X = 4) = \frac{\alpha}{5}$$

a probability distribution (and mass function)

$$\sum P(x) = 1$$

$$P(1) + P(2) + P(3) + P(4) = 1$$

$$\frac{\alpha}{2} + \frac{\alpha}{3} + \alpha + \frac{\alpha}{5} = 1$$

$$1 \Rightarrow \frac{61}{30}\alpha = 1 \Rightarrow \alpha = \frac{30}{61}$$

$$P(X = 1) = \frac{15}{61}; \quad P(X = 2) = \frac{10}{61}; \quad P(X = 3) = \frac{1}{61}$$

$$P(X = 4) = \frac{6}{61}$$

- The probability distribution given is

|      |       |       |       |      |
|------|-------|-------|-------|------|
| X    | 1     | 2     | 3     | 4    |
| P(X) | 15/61 | 10/61 | 30/61 | 6/61 |

### 1.7 : Continuous Probability Statistical Independence

**Q.57** What is Continuous probability?

Ans. : Continuous probability distribution deals with continuous data or random variables. The continuous variables deal with different kinds of

**Q.58** What is a discrete probability distribution? What two conditions determine a discrete probability distribution?

Ans. : • Discrete distribution is a distribution where sample space is a random variable in such a case can take values. For example: number of people in a family, number of coins etc.

• Discrete Probability distribution values a random variable can take corresponding probabilities of the values.

• The two conditions are

- Sum of all probabilities should be 1.
- Each probability must be greater than or equal to zero.

**Q.59** Explain difference between continuous random variable.

Ans. :

| Sr. No. | Discrete              | Continuous                    |
|---------|-----------------------|-------------------------------|
| 1.      | It uses countable set | It uses uncountable set on R. |

|    |                                                                                                        |
|----|--------------------------------------------------------------------------------------------------------|
| 2. | $F$ is set of all subset of $\Omega$ . $F$ is made from sub-intervals of $\Omega$ with set operations. |
| 3. | For a set $A \in F$ ,<br>$P(A) = \sum_{\omega \in A} p(\omega)$                                        |
| 4. | Distribution function (Cdf):<br>$F_X(x) = \sum_{\omega \leq x} p(\omega)$                              |

For a set  $A \in F$ ,

$$P(A) = \int_A f_X(x) dx$$

Distribution function (Cdf):  
 $F_X(x) = \int_{-\infty}^x f_X(t) dt$

### Q.60 Explain statistical Independence

Ans.: Two variates A and B are statistically independent if the probability  $P(A|B)$  of A given B satisfies  $P(A|B) = P(A)$

in which case the probability of A and B is just

$$P(AB) = P(A \cap B) = P(A) P(B)$$

• Statistical independence means one event conveys no information about the other; statistical dependence means there is some information.

• Statistically independent is not the same as mutually exclusive: if A and B are mutually exclusive, then they can't be independent, unless one of them is probability 0 to start with :

$$P(r)(A \cap B) = 0 = Pr((A) \Pr((B))$$

### Q.61 Three cards are drawn in succession from a deck without replacement. Find the probability distribution for the number of spades.

Ans.:  $X$  = number of spades in the three draws.

Let S and N stand for a spade and not a spade, respectively.

$$P(X=0) = P(NNN)$$

$$= (39/52) (38/51) (37/50)$$

$$= 703/1700$$

$$P(X=1) = P(SNN) + P(NSN) + P(NNS)$$

$$= 3(13/52)(39/51)(38/50)$$

$$= 741/1700$$

$$P(X=3) = P(SSS)$$

$$= (13/52)(12/51)(11/50)$$

- Q.62 From a box containing 4 black green balls, 3 balls are drawn in such ball being replaced in the box draw is made. Find the probability d the number of green balls.
- Ans.: • Let  $X$  denotes the number of the three draws.  
 • Let G and B stand for the colors black, respectively.

| Sample Event | X |
|--------------|---|
| BBB          | 0 |
| GBB          | 1 |
| BGB          | 1 |
| BBG          | 1 |
| BGG          | 2 |
| GBG          | 2 |
| GGB          | 2 |
| GGG          | 3 |

- Q.63 A traffic engineer is interested in the number of vehicles reaching a particular crossroads per minute during periods of relatively low traffic flow. finds that the number of vehicles X probability distribution:

| x        | 0    | 1    | 2    | 3 |
|----------|------|------|------|---|
| $P(X=x)$ | 0.37 | 0.39 | 0.19 |   |

Calculate the expected value, the standard deviation of the random variable  $X$ .

Ans.: The expected value, the variance

| x | $x^2$ | $P(X=x)$ |
|---|-------|----------|
| 0 | 0     | 0.37     |
| 1 | 1     | 0.39     |
| 2 | 4     | 0.04     |
| 3 | 9     | 0.09     |
| 4 | 16    | 0.01     |

Let  $X$  be a random variable with the following probability distribution

|            |       |       |       |
|------------|-------|-------|-------|
| $X$        | -3    | 6     | 9     |
| $P(X = x)$ | $1/6$ | $1/2$ | $1/3$ |

then evaluate  $E(2X + 1)^2$

Ans. : We know that :

$$E(2X + 1)^2 = E(4X^2 + 4X + 1) = E4(X^2) + E4(X) + 1$$

$$E(X) = (-3)\frac{1}{6} + (6)\frac{1}{2} + (9)\frac{1}{3} = \frac{11}{12}$$

$$E(X^2) = (-3)^2 \frac{1}{6} + (6^2) \frac{1}{2} + (9^2) \frac{1}{3} = \frac{93}{2}$$

$$E(2X + 1)^2 = (4)(93/2) + (4)(11/2) + 1 = 209$$

Q.66 A class contains of 6 girls and 10 boys. If a committee of 3 is chosen at random from the class, find the probability that (i) 3 boys are selected, (ii) exactly 2 girls are selected.

Ans. : Total number of student ( $n$ ) = 16

$$\begin{aligned} \text{Then } n(s) &= \text{number of ways of choosing 3 from 16} \\ &= 16C_3 \end{aligned}$$

i) 3 boys are selected : We can write  ${}^{10}C_3$  here.

$$n(E) = {}^{10}C_3 \text{ Therefore}$$

$$P(E) = \frac{{}^{10}C_3}{16C_3} = \frac{10 \times 9 \times 8}{16 \times 15 \times 14} = \frac{3}{14}$$

ii) exactly 2 girls are selected

$$\text{Then } n(E) = {}^6C_2 \times {}^{10}C_1$$

$$P(E) = \frac{{}^6C_2 \times {}^{10}C_1}{16C_3} = \frac{15}{56}$$

Q.67 Three groups of students contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys; one student is selected at random from each group. Find the probability of selecting 1 girl and 2 boys.

Ans. : If G denotes selection of girls and B denotes selection of a boy then the various cases of selection can be GBB respectively from the three groups, or BBG respectively from the three groups.

$S_0, P(GBB) = P(G \text{ from group I}) \times P(B \text{ from group II})$   
 $\times P(B \text{ from group III})$

$$P(GBB) = \frac{3}{4} C_1 \times \frac{2}{4} C_1 \times \frac{3}{4} C_1 = \frac{3}{2} \times \frac{2}{4} \times \frac{3}{4} = \frac{9}{32}$$

Similarly,

$$P(BGB) = \frac{1}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{3}{32}$$

$$P(BBG) = \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{1}{32}$$

Required probability =  $P(GBB) + P(BGB) + P(BBG)$

$$= \frac{9}{32} + \frac{3}{32} + \frac{1}{32} = \frac{13}{32}$$

**Q.68** A batch of 100 manufactured components is checked by an inspector who examines 10 components selected at random. If none of the 10 component's is defective, the inspector accepts the whole batch. Otherwise, the batch is subjected to further inspection. What is the probability that a batch containing 10 defective components will be accepted?

Ans. : Let the N denote the number of ways of indiscriminately selecting 10 components from a batch of 100 components. Then N is given by

$$N = C(100, 10)$$

$$= \frac{100!}{(100-10)! \times 10!} = \frac{100!}{90! \times 10!}$$

- Let E denote the event "the batch containing 10 defective components is accepted by the inspector".
- The number of ways that E can occur is the number of ways of selecting 10 components from the 90 non-defective components and no components from the 10 defective component's.
- This number, N(E) is given by

$$\begin{aligned} N(E) &= C(90, 10) \times C(10, 0) = C(90, 10) \\ &= \frac{90!}{(90-10)! \times 10!} = \frac{90!}{80! \times 10!} \end{aligned}$$

The probability of event E is given by :

$$\begin{aligned} P(E) &= \frac{N(E)}{N} = \frac{90!}{80! \times 90!} \times \frac{90! \times 10!}{100!} = \frac{90! \times 90!}{100! \times 80!} \\ P(E) &= 0.3305 \end{aligned}$$

**Q.69** Two coins are tossed. Let A denote the event "at most one head on the two tosses" and B denote the event "one head and one tail". Are A and B independent events? Ans. : The sample space of the experiment is HT, TH, TT.

Now events are defined as follows :

$$A = \{HT, TH, TT\}$$

$$\text{and } A \cap B = \{HT, TH\}$$

Thus :

$$P(A) = \frac{3}{4}$$

$$P(A \cap B) = \frac{2}{4} = \frac{1}{2}$$

$$P(A) P(B) = \frac{8}{3}$$

Since  $P(A \cap B) = P(A) P(B)$ , we conclude that A and B are not independent.

**Q.70** A die is thrown 3 times. If considered as success find the atleast 2 success.

Ans. :

$$\begin{aligned} P &= \frac{1}{6}, q = \frac{5}{6}, n = 3 \\ P(\text{at least 2 success}) &= P(X \geq 2) = P \\ &= 3C_2 \left(\frac{1}{6}\right)^2 \frac{5}{6} + 3C_3 \left(\frac{1}{6}\right)^3 \end{aligned}$$

### Fill in the Blanks for Mid

- Q.1 A \_\_\_\_\_ variable takes which is determined by the random experiment.
- Q.2 A pictorial representation manipulations with events using \_\_\_\_\_ diagrams.
- Q.3 Statistical data, generated in be very useful for studying the distribution if presented tabular and graphic display plot