

UNIT - III

Continuous probability distribution :-

Normal distribution :-

Normal distribution was discovered by Karl Friedrich Gauss so that it is also called Gaussian distribution.

The Normal distribution is limiting case of binomial distribution under the conditions.

i) No. of trials n is very large. i.e., $n \rightarrow \infty$

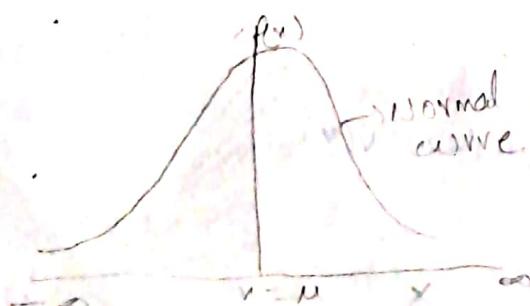
ii) Neither p nor q is very small.

A continuous R.V 'x' is said to have normal distribution if its density function is given by

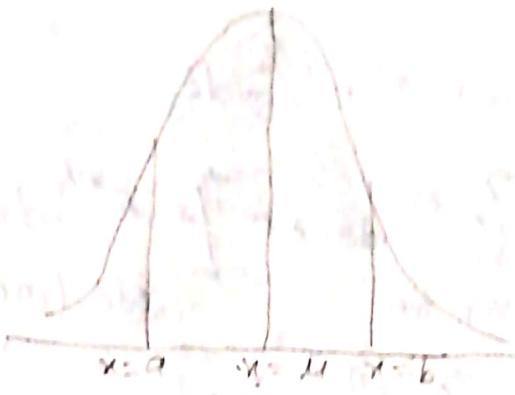
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where μ, σ are mean & standard deviations of x .

The graph of the R.V 'x' and density function $f(x)$ is called Normal curve. The total area bounded by Normal curve and x axis is equal to 1.



$$\int_{-\infty}^{\infty} f(x) dx = 1 = 100\%$$



$$P(a \leq x \leq b) = \int_a^b f(x) dx = \text{Area under the curve below } (x=a, x=b)$$

Normal Distribution case of B:D :-

Formulae:-

$$1) \int_{-\infty}^{\infty} f(x) dx = \begin{cases} 2 \int_0^{\infty} f(x) dx & , f(x) \text{ even} \\ 0 & , f(x) \text{ odd} \end{cases}$$

$$2) \text{ If } \int_a^b f(x) dx = 0, \text{ then } a=b \quad (f(x) \neq 0)$$

$$3) \int_0^{\infty} e^{-x^2/2} dx = \sqrt{\pi}/2$$

$$4) \int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}.$$

$$5) i) \Gamma_n = \int e^{-x} x^{n-1} dx.$$

ii) $\Gamma_n = (n-1) \Gamma(n-1)$ if n is the function.

$$iii) \Gamma_{1/2} = \sqrt{\pi} \quad \Gamma 1 = 1.$$

Mean of Normal distribution:-

$$\mathbb{E}(x) = \int_{-\infty}^{\infty} x \cdot f(x) dx.$$

$$\mathbb{E}(x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} x \cdot e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2} dx.$$

$$\text{let } \frac{x-\mu}{\sigma} = z \Rightarrow [x = \mu + \sigma z]$$

$$\frac{dx}{\sigma} = dz \quad \therefore dz = \sigma dz$$

$$\begin{aligned} E(x) &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z) e^{-z^2/2} \sigma dz \\ &= \frac{1}{\sqrt{2\pi}} \left\{ \mu \int_{-\infty}^{\infty} e^{-z^2/2} dz + \sigma \int_{-\infty}^{\infty} z e^{-z^2/2} dz \right\}, \end{aligned}$$

even func odd func.

$$E(x) = \frac{1}{\sqrt{2\pi}} \left\{ \mu \int_0^{\infty} e^{-z^2/2} dz + 0 \right\},$$

$$E(x) = \frac{\mu}{\sqrt{2\pi}} \cdot \sqrt{\frac{\pi}{2}}$$

$$= \frac{\mu\sqrt{\pi}}{\sqrt{2}\cdot\sqrt{2\pi}}$$

$$\boxed{\therefore E(x) = \mu}$$

Variance of $N(\mu, \sigma^2)$:

$$V(x) \leq E[(x-\mu)^2] = E(x^2) - \mu^2$$

$$E[(x-\mu)^2] = \int (x-\mu)^2 f(x) dx,$$

$$V(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2 e^{-1/2} \left(\frac{x-\mu}{\sigma}\right)^2 dx,$$

$$\text{let } z = \frac{x-\mu}{\sigma} \Rightarrow x-\mu = \sigma z$$

$$dz = \frac{dx}{\sigma}$$

$$dx = \sigma dz.$$

$$V(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z)^2 \cdot e^{-z^2/2} \sigma dz$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 \cdot e^{-z^2/2} dz$$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^{\infty} z^2 \cdot e^{-z^2/2} dz.$$

$$\text{let } z^2/2 = t$$

$$z^2 = 2t.$$

$$Z = \sqrt{2} \sqrt{E}$$

$$dZ = \sqrt{2} - \frac{1}{\sqrt{2} \sqrt{E}} dt$$

$$dZ = \frac{1}{\sqrt{2} \sqrt{E}} dt$$

$$V(x) = \frac{8\sigma^2}{\sqrt{2\pi}} \int_0^\infty e^{-t} dt \cdot \frac{1}{\sqrt{2\sqrt{E}}} dt$$

$$= \frac{4\sigma^2}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{1/2} dt$$

$$= \frac{8\sigma^2}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{3/2} dt$$

$$\Rightarrow \frac{8\sigma^2}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{3/2} dt$$

$$\Rightarrow \frac{8\sigma^2}{\sqrt{\pi}} \Gamma(5/2)$$

$$\Rightarrow \frac{8\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{2} \Gamma(3/2)$$

$$\Rightarrow \frac{8\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{2} \sqrt{\pi}$$

$$\therefore V(x) = \sigma^2$$

Mode of N.D :-

Mode is a value of 'x' for which $f(x)$ is maximum.

We get mode by $f'(x)=0, f''(x)<0$ at mode.

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-1/2} \left(\frac{x-\mu}{\sigma}\right)^2$$

d.w.r.t to x'

$$f'(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-1/2} \left(\frac{x-\mu}{\sigma}\right)^2 \cdot \frac{d}{dx} \left(-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2\right)$$

$$f'(x) = -\frac{1}{2} [f(x)] \cdot 2 \left(\frac{x-\mu}{\sigma}\right) \frac{1}{\sigma}$$

$$f'(x) = -f(x) \left(\frac{x-\mu}{\sigma}\right) \rightarrow 0$$

$$\therefore f''(x) = 0$$

$$f(x) \left(\frac{x-\mu}{\sigma^2} \right) = 0$$

$$\frac{x-\mu}{\sigma^2} = 0$$

$$\therefore x = \mu$$

∴ want to 'x'

$$f''(x) = - \left[f'(x) \left(\frac{x-\mu}{\sigma^2} \right) + f(x) \frac{1}{\sigma^2} \right]$$

at $x = \mu$,

$$f''(\mu) = - \left[0 + f(\mu) \frac{1}{\sigma^2} \right]$$

$$f''(\mu) = - \frac{1}{\sigma \sqrt{2\pi}} \cdot \frac{1}{\sigma^2} < 0$$

∴ Mode = μ .

Median of N.D :-

Let 'm' be median of N.D' $x \in [-\infty, \infty]$.

$$\int_{-\infty}^m f(x) dx = \int_m^{\infty} f(x) dx = \frac{1}{2}$$

$$\int_{-\infty}^m f(x) dx = \frac{1}{2}$$

Suppose, $-\infty < \mu < m$,

$$\int_{-\infty}^{\mu} f(x) dx + \int_{-\mu}^m f(x) dx = \frac{1}{2} \rightarrow ①$$

(consider,

$$\int_{-\infty}^{\mu} f(x) dx = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\mu} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2} dx$$

$$\text{let } \frac{x-\mu}{\sigma} = z \quad \text{if } x = -\infty \Rightarrow z = -\infty$$

$$\frac{dx}{\sigma} = dz \quad \text{if } x = \mu \Rightarrow z = 0$$

$$dx = \sigma dz$$

$$\Rightarrow \int_{-\infty}^{\mu} f(x) dx = \frac{1}{\sigma\sqrt{2\pi}} \cdot \int_{-\infty}^0 e^{-z^2/2} dz$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^0 e^{-z^2/2} dz$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty e^{-z^2/2} dz$$

$$\Rightarrow \frac{1}{\sigma\sqrt{2\pi}} \sqrt{\frac{\pi}{2}} \Rightarrow \gamma_2.$$

From ①,

$$\frac{1}{2} + \int_u^m f(x) dx = \gamma_2.$$

$$\int_u^m f(x) dx = \gamma_2 - \gamma_1 = 0$$

then $m = u$.

$$\therefore \text{median} = u.$$

Mean deviation :-

$$M.D = \int_{-\infty}^{\infty} |x - u| f(x) dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x - u) e^{-z^2/2} \frac{(x-u)^2}{\sigma^2} dz$$

$$\text{let } \frac{x-u}{\sigma} = z \Rightarrow x-u = \sigma z$$

$$\frac{dx}{\sigma} = dz \Rightarrow dx = \sigma dz$$

$$M.D = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} |\sigma z| e^{-z^2/2} (\sigma dz)$$

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |z| e^{-z^2/2} dz.$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \int_0^{\infty} |z| e^{-z^2/2} dz.$$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_0^\infty e^{-z^2/2} dz$$

$$\text{let } z^2/2 = t \Rightarrow z^2 = 2t$$

$$z = \sqrt{2t}$$

$$dz = \sqrt{2} \cdot \frac{1}{2\sqrt{t}} dt$$

$$dz = \frac{1}{\sqrt{2}\sqrt{t}} dt$$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_0^\infty \sqrt{2} \cdot \sqrt{t} \cdot e^{-t} \cdot \frac{1}{\sqrt{2}\sqrt{t}} dt.$$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{1/2} dt. \quad \Gamma(1/2) = \sqrt{\pi}$$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{1/2} dt. \quad \Gamma(1) = 1$$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \cdot \Gamma(1)$$

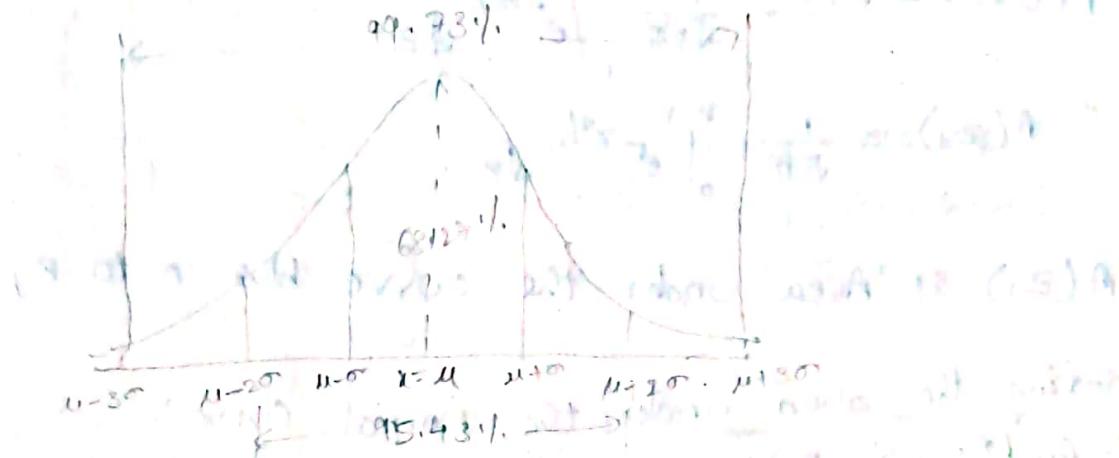
$$\Rightarrow \frac{\sqrt{2}}{\sqrt{\pi}} \sigma \Rightarrow \frac{4}{5} \sigma.$$

Chief characteristics of N.D:-

- i) The graph of Normal distribution $y=f(x)$ in XY plane is called normal curve.
 - ii) The curve is bell shaped and symmetrical about mean line $x=\mu$. and the tails on the right and left sides of mean extends to infinity.
- 3) i) The area under the normal curve represents total population.
- ii) Mean, median & mode of distribution are coincide at $x=\mu$.
i.e.; mean = median = mode.

4) X-axis is asymptote to the curve. And normal curve is unimodal.

5)



Area b/w $\mu - \sigma$ & $\mu + \sigma$ is 68.27%.

i.e., $P(\mu - \sigma < x < \mu + \sigma) = 68.27\%$.

$P(\mu - 2\sigma < x < \mu + 2\sigma) = 95.43\%$.

$P(\mu - 3\sigma < x < \mu + 3\sigma) = 99.73\%$.

6) Normal Variable-

The variable $Z = \frac{x-\mu}{\sigma}$ is called normal variable.

Standard Normal Variable-

Normal variable Z is called standard normal variable if $\mu=0$, $\sigma=1$.

Area under the Normal Curve-

If the r.v 'x' lies b/w μ & x_1 , then probability of 'x' lies b/w μ & x_1 is

$$P(x_0 < x < x_1) = \int_{\mu}^{x_1} f(x) dx \\ = \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu}^{x_1} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

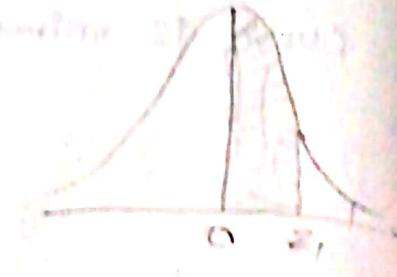
$$\text{let } Z = \frac{x-\mu}{\sigma} \quad | \quad \text{if } x=\mu \Rightarrow Z=0$$

$$dx = \frac{dZ}{\sigma} \quad | \quad \text{if } x=x_1 \Rightarrow Z_1 = \frac{x_1-\mu}{\sigma}$$

$$P(0 < Z < z_1) = \frac{1}{\sqrt{2\pi}} \int_0^{z_1} e^{-z^2/2} dz$$

$$P(0 < Z < z_1) = \frac{1}{\sqrt{2\pi}} \int_0^{z_1} e^{-z^2/2} dz$$

$$A(z_1) = \frac{1}{\sqrt{2\pi}} \int_0^{z_1} e^{-z^2/2} dz$$

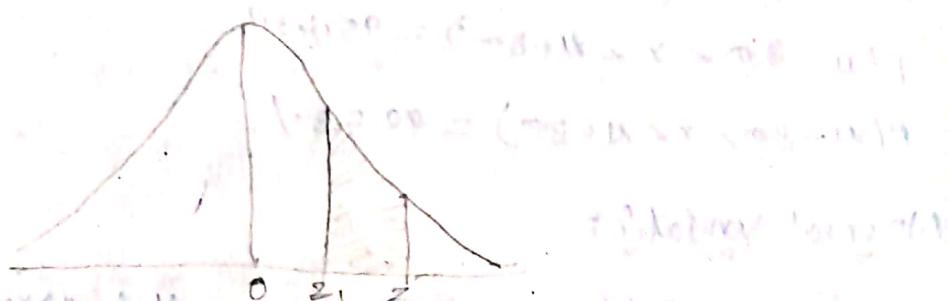


$A(z_1)$ is Area under the curve b/w 0 to z_1 ,

Finding the area under the Normal Curve :-

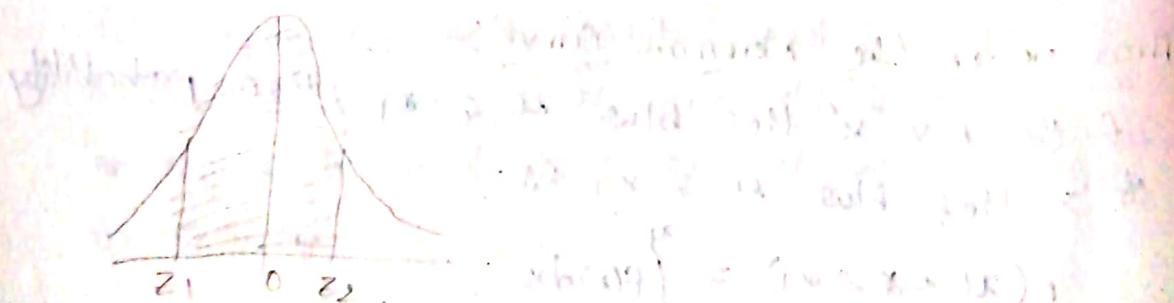
1) for $P(z_1 < Z < z_2)$:
Case - 1 :- If $z_1 > 0, z_2 > 0$,

$$\text{for } P(z_1 < Z < z_2) \rightarrow P(z_1 < Z < 0) + P(0 < Z < z_2)$$



$$\begin{aligned} P(z_1 < Z < z_2) &= P(0 < Z < z_2) - P(0 < Z < z_1) \\ &= A(z_2) - A(z_1) \end{aligned}$$

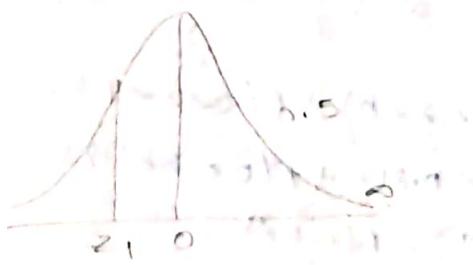
Case - 2 :- If $z_1 < 0, z_2 > 0$



$$\begin{aligned} P(z_1 < Z < z_2) &= P(z_1 < Z < 0) + P(0 < Z < z_2) \\ &= P(0 < Z < z_1) + P(0 < Z < z_2) \\ &= A(z_1) + A(z_2) \end{aligned}$$

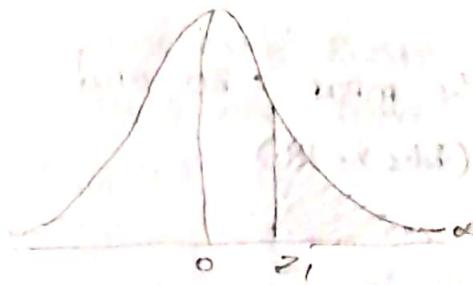
2) for $P(z > z_1)$:

Case-I: if $z_1 < 0$.



$$\begin{aligned}P(z > z_1) &= 0.5 + P(z_1 < z < 0) \\&= 0.5 + P(0 < z < z_1) \\&= 0.5 + A(z_1).\end{aligned}$$

Case-II: if $z_1 > 0$.



$$\begin{aligned}P(z > z_1) &= 0.5 - P(0 < z < z_1) \\&= 0.5 - A(z_1).\end{aligned}$$

1) Normally distributed variable with mean $\mu = 1$, standard deviation $\sigma = 3$. Find (i) $P(3.43 \leq x \leq 6.19)$

(ii) $P(-1.43 \leq x \leq 6.19)$.

i) $P(3.43 \leq x \leq 6.19)$

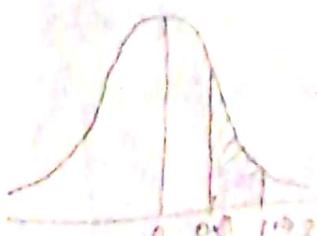
$$z = \frac{x-\mu}{\sigma}$$

$$\text{if } x = 3.43 \Rightarrow z = \frac{3.43-1}{3} = 0.81$$

$$\text{If } x = 6.19 \Rightarrow z = \frac{6.19-1}{3} = \frac{5.19}{3} = 1.73.$$

$$\begin{aligned}P(0.81 \leq z \leq 1.73) &= P(0 < z < 1.73) - P(0 < z < 0.81) \\&= A(1.73) - A(0.81)\end{aligned}$$

$$\begin{aligned}&= 0.4582 - 0.2910 \\&= 0.1672\end{aligned}$$



$$\text{ii) } P(-1.43 \leq z \leq 6.19).$$

$$\text{If } x = -1.43 \Rightarrow z = \frac{-1.43 - 1}{3} \Rightarrow \frac{-2.43}{3} \Rightarrow -0.81.$$

$$\text{If } x = 6.19 \Rightarrow z = \frac{6.19 - 1}{3} \Rightarrow \frac{5.19}{3} \Rightarrow 1.73.$$

$$\begin{aligned} P(-0.81 \leq z \leq 1.73) &= P(-0.81 \leq z < 0) + P(0 \leq z \leq 1.73) \\ &= P(0 \leq z \leq 0.81) + P(0 \leq z \leq 1.73) \\ &= A(0.81) + A(1.73) \\ &= 0.2910 + 0.4582 \\ &= 0.7492 \end{aligned}$$

Q) If x is normal variate if the mean = 30 and

Std. deviation 5 then find i) $P(26 \leq x \leq 40)$

$$\text{i) } P(x \geq 45) \quad \text{iii) } P(x \leq 45)$$

$$\text{i) Mean } (\mu) = 30$$

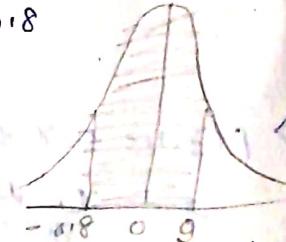
$$\text{s.d. } (\sigma) = 5.$$

$$\text{i) } P(26 \leq x \leq 40)$$

$$\text{If } x = 26 \Rightarrow z = \frac{26 - 30}{5} \Rightarrow \frac{-4}{5} \Rightarrow -0.8$$

$$\text{If } x = 40 \Rightarrow z = \frac{40 - 30}{5} \Rightarrow \frac{10}{5} \Rightarrow 2.$$

$$P(26 \leq x \leq 40) \Rightarrow P(-0.8 \leq z \leq 2)$$



$$\Rightarrow P(-0.8 \leq z \leq 0) + P(0 \leq z \leq 2)$$

$$\Rightarrow P(0 \leq z \leq 0.8) + P(0 \leq z \leq 2)$$

$$\Rightarrow A(0.8) + A(2)$$

$$\Rightarrow 0.2881 + 0.4772$$

$$\Rightarrow 0.7653$$

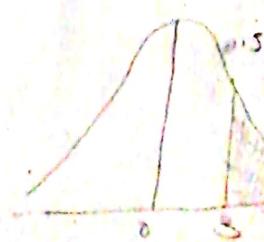
$$\text{ii) } P(x \geq 45)$$

$$\text{If } x = 45 \Rightarrow z = \frac{45 - 30}{5} \Rightarrow \frac{15}{5} \Rightarrow 3.$$

$$P(x \geq 45) = P(z \geq 3)$$

$$\Rightarrow 0.5 - P(0 \leq z \leq 3) \Rightarrow 0.5 - A(3)$$

$$\Rightarrow 0.5 - 0.4987 \Rightarrow 0.0013$$



$$\text{iii) } P(x \leq 45)$$

$$\text{if } x=45 \Rightarrow z = \frac{45-30}{5} \Rightarrow 3.$$

$$P(X \leq 45) \Rightarrow 0.5 + P(0 \leq Z \leq 3)$$

$$2) \quad 0.15 + A(3)$$

$$^2) \quad 0.15 + 0.4987.$$

$$\Rightarrow 0.9987.$$



3) In a sample of 1000 cases, mean is 14.5 and std deviation is 3.5. Assume the distribution to be normal. Find how many students scored below 18.

(i) How many score above 10.

iii) How many score below 18.

$$\text{Mean}(\mu) = 14$$

$$S.D(\sigma) = 8.5$$

Let x = score of student.

$$i) P(18 \leq x < 15).$$

$$z = \frac{x - \mu}{\sigma}$$

$$\text{If } x=12 \Rightarrow z = \frac{12-14}{2-5} \Rightarrow -\frac{2}{3} \Leftrightarrow -0.666\ldots$$

$$\text{If } x = 15 \Rightarrow z = \frac{45-14}{9.5} \Rightarrow \frac{1}{2.5} \Rightarrow \frac{10}{25} \Rightarrow 0.4$$

$$\begin{aligned}
 P(-0.8 < Z < 0.4) &= P(-0.8 < Z < 0) + P(0 < Z < 0.4) \\
 &= A(-0.8) + A(0.4) \\
 &= 0.2881 + 0.1554 \\
 &= 0.4435
 \end{aligned}$$

No. of students to score below 12 & 15 PS

$$\Rightarrow 0.4435 \times 1000$$

→ 443-5

\Rightarrow 444 students.

$$\text{ii)} P(X > 18)$$

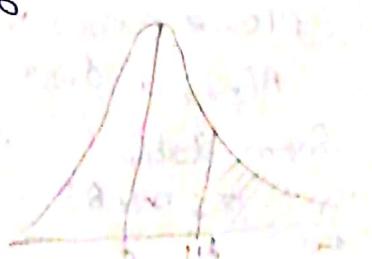
$$\text{if } x=18 \Rightarrow z = \frac{18-14}{8.5} = \frac{4}{8.5} \approx 1.6$$

$$P(z > 1.6) = 0.5 - P(0 < z < 1.6)$$

$$= 0.5 - A(4.6)$$

$$= 6.5 = 0.4452$$

→ 0.0548



$$\text{No. of students score above } 18 \Rightarrow 0.0548 \times 1000 \\ \Rightarrow 54.8 \\ \Rightarrow 55.$$

iii) $P(X < 18)$.

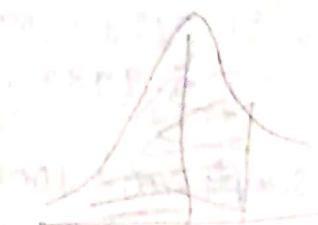
$$\text{If } X=18 \Rightarrow z=1.6.$$

$$P(z < 1.6) = 0.5 + P(0 < z < 1.6)$$

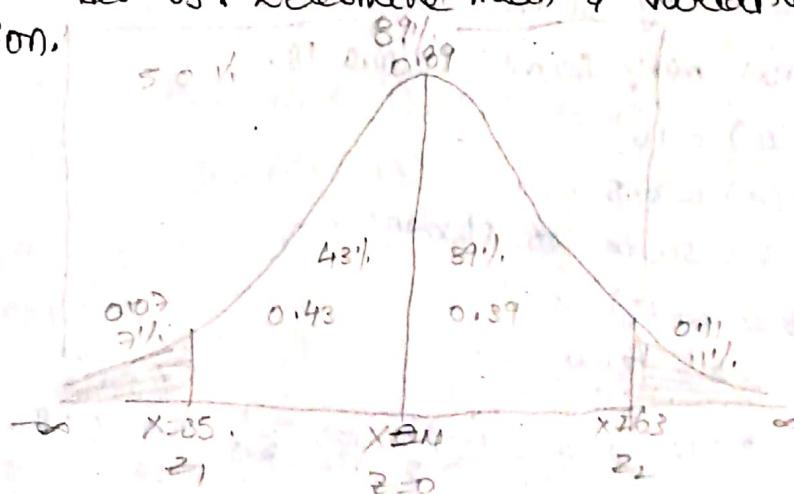
$$= 0.5 + A(1.6)$$

$$\Rightarrow 0.5 + 0.4452$$

$$\Rightarrow 0.9452.$$



4) In a Normal dist. 71% of items under 35 and 89% of items under 63. Determine mean & variance of distribution.



$$P(X \leq 35) = 71\% = 0.71$$

$$P(X \leq 63) = 89\% = 0.89$$

$$P(X \geq 63) = 1 - P(X \leq 63)$$

$$= 1 - 0.89 = 0.11$$

$$Z = \frac{x-\mu}{\sigma}$$

$$\text{if } x=35 \Rightarrow \frac{35-\mu}{\sigma} \Rightarrow -z_1 \text{ (say)}$$

$$\text{if } x=63 \Rightarrow \frac{63-\mu}{\sigma} \Rightarrow z_2 \text{ (say)} \quad \left. \begin{array}{l} \\ \end{array} \right\} \rightarrow ①$$

From diagram,

$$P(0 < z < z_2) = 0.89$$

$$A(z_2) = 0.89$$

From table

$$z_2 = 1.83$$

$$P(-z_1 < z < 0) = 0.43.$$

$$A(-z_1) = 0.43.$$

$$A(z_1) = 0.43.$$

From table

$$z_1 = 1.48.$$

From ①.

$$\frac{35}{\sigma} - \frac{\mu}{\sigma} = -1.48,$$

$$-\frac{63}{\sigma} - \frac{\mu}{\sigma} = 1.23$$

$$\underline{\underline{-\frac{28}{\sigma} = -2.71.}}$$

$$\boxed{\therefore \sigma = 10.33.}$$

From ①

$$\frac{35-\mu}{\sigma} = -1.48.$$

$$\frac{35-\mu}{10.33} = -1.48$$

$$35-\mu = -1.48 \times 10.33.$$

$$\boxed{\mu = 50.28.}$$

5)

In Normal dist of stems under 45 & 81% of stems over 64. Find Mean & Variance of distribution.

$$P(x < 45) = 0.31$$

$$P(x > 64) = 81\% = 0.08$$

$$Z = \frac{x-\mu}{\sigma}$$

$$\text{If } x = 45 \Rightarrow \frac{45-\mu}{\sigma} = -2, (\text{say}).$$

$$\text{If } x = 64 \Rightarrow \frac{64-\mu}{\sigma} = Z_2, (\text{say})$$

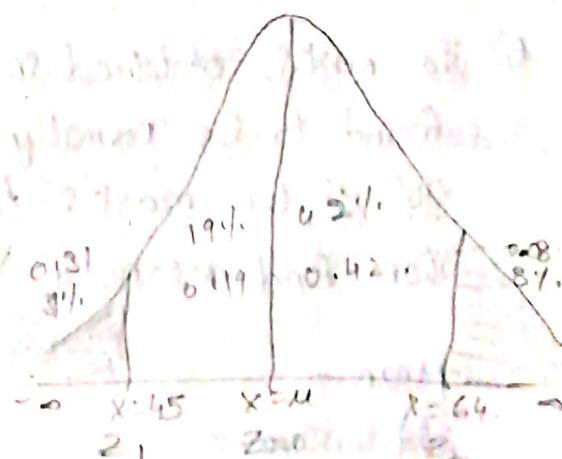
From diagram

$$P(0 < Z < Z_2) = 0.42$$

$$A(Z_2) = 0.42$$

From table

$$\boxed{Z_2 = 1.41.}$$



$$P(-z_1 < z < 0) = 0.19$$

$$A(-z_1) = 0.19$$

$$A(z_1) = 0.19$$

From table

$$z_1 = 0.5.$$

From ①.

$$\frac{45 - \mu}{\sigma} = -0.50$$

$$\begin{array}{r} \frac{64}{\sigma} - \frac{\mu}{\sigma} = 1.41 \\ + \\ \hline \frac{-19}{\sigma} = -1.91 \end{array}$$

$$\sigma = \frac{-19}{-1.91}$$

$$\boxed{\sigma = 9.94.}$$

From ②

$$\frac{45 - \mu}{\sigma} = -0.5,$$

$$\frac{45 - \mu}{9.94} = -0.5.$$

$$45 - \mu = -0.5 \times 9.94.$$

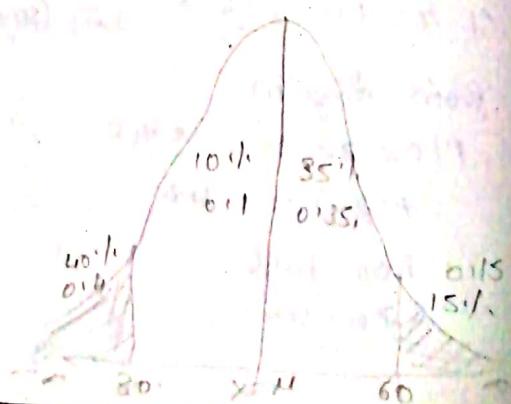
$$\mu = 45 + 0.5 \times 9.94.$$

$$\boxed{\mu = 55.4.}$$

- 6) The marks obtained in Statistics in a certain examination found to be normally distributed. If 15% of student got ≥ 60 marks, 40% of students got < 30 marks. Then find Mean & Variance of distribution.

$$\text{Mean} =$$

$$\text{Std. Deviation} =$$



- 7) The marks obtained in mathematics by 3000 students is normally distributed with mean 78%. and standard deviation 11%. determine (i) how many students got marks above 90%. (ii) what was the highest mark obtained by lowest 10% of students given what limits did middle 60% of students lie.
- 8) Given that the mean height of students in a college is 155 cm and S.D 15. what is the probability that mean height of 36 students is less than 157 cm.
- 9) If X is normally distributed with mean μ & variance S.D $\sigma = 0.1$, then find $P(X \geq 0.01)$

7) Let X = Marks percentage of students.

$$\text{Mean}(\mu) = 78\% = 0.78.$$

$$\text{S.D}(\sigma) = 11\% = 0.11.$$

$$i) P(X > 90\%)$$

$$\text{if } X = 90\% = 0.9 \Rightarrow Z = \frac{X-\mu}{\sigma} = \frac{0.9-0.78}{0.11} = 1.09.$$

$$P(X > 0.9) = P(Z > 1.09)$$

$$\Rightarrow 0.5 - P(0 < Z < 1.09)$$

$$\Rightarrow 0.5 - A(1.09)$$

$$\Rightarrow 0.5 - 0.3621$$

$$\Rightarrow 0.1379.$$

$$\text{No. of students got marks above } 90\% = 0.1379 \times 1000 \\ \Rightarrow 138 \text{ students.}$$

$$ii) P(-2 < Z < 0) = 0.4,$$

$$A(-2) = 0.4$$

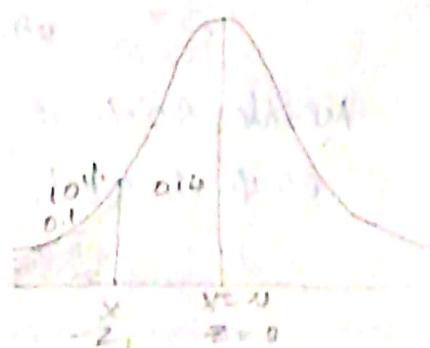
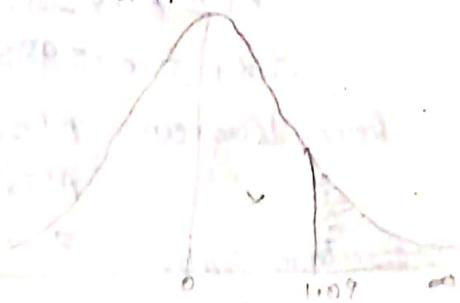
from tables,

$$-Z_1 = 1.29$$

$$\boxed{Z_1 = -1.29},$$

$$\frac{X_1 - \mu}{\sigma} = -1.29,$$

$$\frac{X_1 - 0.78}{0.11} = -1.29,$$



$$x_1 - 0 \cdot 78 = -1 \cdot 29 \times 0.11$$

$$x_1 = 0.6381 \Rightarrow 63.81\% \cong 64\%$$

Highest marks obtained by lower 10% of student
is 64%.

ii)

Middle 90% means leaving 5%
area on both sides of normal
curve.

From diagram:

$$P(-2 < Z < 0) = 0.45$$

$$A(-z_1) = 0.45$$

$$-z_1 = 1.65$$

$$z_1 = -1.65 \quad (\text{from Tables})$$

$$\frac{x_1 - \mu}{\sigma} = -1.65$$

$$\frac{x_1 - 0.78}{0.11} = -1.65$$

$$x_1 - 0.78 = -1.65 \times 0.11$$

$$x_1 = 0.78 - (-1.65 \times 0.11)$$

$$\therefore x_1 = 0.5985 \Rightarrow 59.85\% \Rightarrow 60\%$$

From diagram, $P(0 < Z < z_2) = 0.45$

$$A(z_2) = 0.45$$

From Tables $z_2 = 1.65$

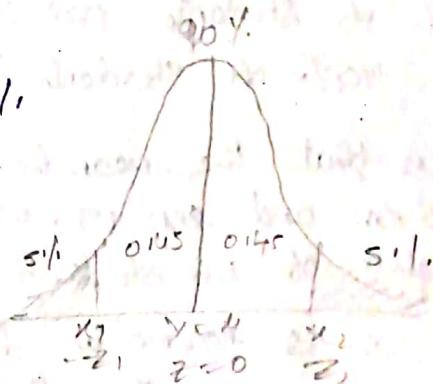
$$\frac{x_2 - \mu}{\sigma} = 1.65$$

$$\frac{x_2 - 0.78}{0.11} = 1.65$$

$$\therefore x_2 = 0.78 + (1.65 \times 0.11)$$

$$\therefore x_2 = 0.9615 \Rightarrow 96.15\% \Rightarrow 96\%$$

Middle 90% of students got marks b/w
60% & 90%.



8) Mean(μ) = 155 cm

$s\sqrt{\sigma} = 15$ cm,

No. of students (n) = 36.

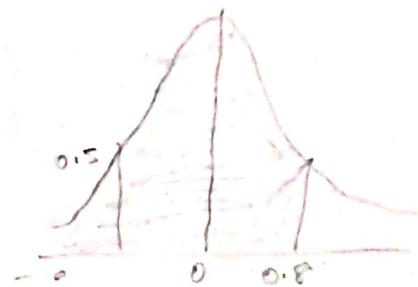
Sample mean (\bar{x}) = 157

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

To find $P(\bar{x} < 157)$

If $\bar{x} = 157 \Rightarrow z = \frac{157 - 155}{15/\sqrt{36}}$

$$\Rightarrow \frac{2 \times 6}{15} = \frac{12}{15} = 0.8.$$



$$P(z < 0.8) = 0.5 + P(0 < z < 0.8)$$

$$\Rightarrow 0.5 + A(0-0.8)$$

$$\Rightarrow 0.5 + 0.2881$$

$$\Rightarrow 0.7881$$

9)

$\mu = 2$

$\sigma = 0.1$

To find $P(|x-2| \geq 0.01)$

$$|x-2| \leq a \Leftrightarrow -a \leq x-2 \leq a$$

$$|x-2| > a \Leftrightarrow x \in (-\infty, -a] \cup [a, \infty)$$

* $P(|x-2| \geq 0.01) = 1 - P(|x-2| < 0.01) \rightarrow \text{Q1.}$

$$|x-2| < 0.01$$

$$-0.01 < (x-2) < 0.01$$

$$2 - 0.01 < x < 2 + 0.01$$

$$1.99 < x < 2.01$$

If $x = 1.99 \Rightarrow z = \frac{1.99 - 2}{0.1} \Rightarrow \frac{-0.01}{0.1} = -0.1$

If $x = 2.01 \Rightarrow z = \frac{2.01 - 2}{0.1} \Rightarrow \frac{0.01}{0.1} = 0.1$

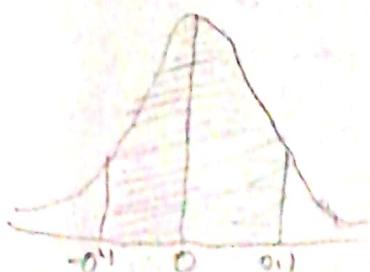
$$P(|x-2| < 0.01) = P(1.99 < x < 2.01)$$

$$= P(-0.1 < z < 0.1)$$

$$= P(-0.1 < z < 0) + P(0 < z < 0.1)$$

$$= A(-0.1) + A(0.1)$$

$$= 2A(0.1) \Rightarrow 2A(0.0398) \Rightarrow 0.0796.$$



Normal approximation to Binomial distribution:-

Let X be a Random variable of Binomial distribution.
 $\& n = \text{no. of trials}$, $p = \text{prob. of success}$ and
 $q = \text{prob. of failure}$, $\mu = np$, $\sigma^2 = npq$
 $\sigma = \sqrt{npq}$

To find prob. of x lies b/w x_1 & x_2

$$\text{i.e., } P(x_1 \leq x \leq x_2)$$

$$\text{if } x=x_1 \Rightarrow z_1 = \frac{(x_1 - \mu) - \mu}{\sigma}$$

$$\text{if } x=x_2 \Rightarrow z_2 = \frac{(x_2 + \mu) - \mu}{\sigma}$$

$$P(x_1 \leq x \leq x_2) = P(z_1 \leq z \leq z_2).$$

Find the probability that out of 100 patients b/w 84 & 95 inclusive will survive from heart operation given that the chances of survival is 0.9.

$$P = 0.9 \quad n = 100$$

$$q = 1 - p \Rightarrow 1 - 0.9 = 0.1$$

Let $X = \text{no. of survivors}$.

$$\mu = np = (100)(0.9) = 90$$

$$\sigma = \sqrt{npq} \Rightarrow \sqrt{90(0.1)} \Rightarrow \sqrt{9} \Rightarrow 3.$$

To find $P(84 \leq x \leq 95)$

$$\text{if } x_1 = 84 \Rightarrow z_1 = \frac{(x_1 - \mu) - \mu}{\sigma} \Rightarrow \frac{(84 - 90) - 90}{3} = -2.16.$$

$$\text{if } x_1 = 95 \Rightarrow z_2 = \frac{(x_2 + \mu) - \mu}{\sigma} \Rightarrow \frac{(95 + 90) - 90}{3} = 1.83.$$

$$P(84 \leq x \leq 95) = P(-2.16 \leq z \leq 1.83)$$

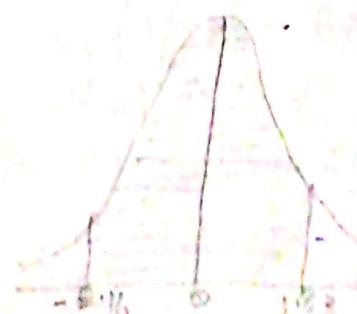
$$\Rightarrow P(-2.16 \leq z \leq 0) + P(0 \leq z \leq 1.83).$$

$$\Rightarrow A(-2.16) + A(1.83)$$

$$\Rightarrow A(2.16) + A(1.83)$$

$$\approx 0.4846 + 0.4664$$

$$\approx 0.952.$$



Q) 8 coins are tossed together. Find the probability that getting 1 to 4 heads in a single toss.

$$P = \frac{1}{2}, n = 8$$

$$q = 1 - p \Rightarrow 1 - \frac{1}{2} = \frac{1}{2}$$

Let x = no. of heads.

$$\mu = np = 8 \left(\frac{1}{2}\right) = 4$$

$$\sigma = \sqrt{npq} \Rightarrow \sqrt{8 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)} = \sqrt{2}$$

To find $P(1 < x < 4)$.

3) Find the prob. of getting even number on the face in 3 to 5 times in throwing 10 dice together.

$$P = \frac{3}{6} \Rightarrow \frac{1}{2}, n = 10$$

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

Let x = no. of times getting even number

$$\mu = np =$$

$$\sigma = \sqrt{npq}$$

$$P(3 < x < 5).$$

Exponential Distribution:-

If x is a continuous R.V probability density function.

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0, \lambda > 0. \\ 0, & \text{elsewhere.} \end{cases}$$

1) Mean :-

$$\mathbb{E}(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$\mathbb{E}(x) = 0 + \int_0^{\infty} x \cdot \lambda \cdot e^{-\lambda x} dx$$

$$\mathbb{E}(x) = \lambda \left\{ x \left(\frac{e^{-\lambda x}}{-\lambda} \right) - 1 \cdot \frac{e^{-\lambda x}}{\lambda^2} \right\} \Big|_0^{\infty}$$

$$\mathbb{E}(x) = \lambda \{ [0 - 0] - [0 - 1/\lambda^2] \},$$

$$\mathbb{E}(x) = \text{mean} = 1/\lambda$$

2) Variance :-

$$\mathbb{E}(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx.$$

$$\mathbb{E}(x^2) = 0 + \lambda \int_0^{\infty} x^2 \cdot \lambda \cdot e^{-\lambda x} dx$$

$$\Rightarrow \lambda \left\{ x^2 \left(\frac{e^{-\lambda x}}{-\lambda} \right) - (2x) \left(\frac{e^{-\lambda x}}{\lambda^2} \right) + 2 \left(\frac{e^{-\lambda x}}{\lambda^3} \right) \right\} \Big|_0^{\infty}$$

$$\Rightarrow \lambda \{ 0 + 0 + 0 \} - [0 + 0 - 2/\lambda^2]$$

$$\mathbb{E}(x^2) = \lambda \cdot 2/\lambda^2 \Rightarrow 2/\lambda^2$$

$$V(x) = \mathbb{E}(x^2) - \mu^2 \Rightarrow 2/\lambda^2 - 1/\lambda^2$$

$$\therefore V(x) = 1/\lambda^2$$

- 1) The length of time a person speaks over phone follows exponential distribution with mean 6. What is the probability that person will talk for (i) More than 8 mins
(ii) Between 4 & 8 mins.

Mean = $\lambda = 1/\lambda$

$$\lambda = 1/6.$$

$$f(x) = \begin{cases} 1/6 e^{-x/6}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Let x = length of time.

$$\text{i)} P(x \geq 8) = \int_8^\infty f(x) dx \Rightarrow \frac{1}{6} \int_8^\infty e^{-x/6} dx$$
$$\Rightarrow \frac{1}{6} \left[\frac{e^{-x/6}}{-1/6} \right]_8^\infty \Rightarrow -\left[e^{-\infty} - e^{-8/6} \right] \Rightarrow e^{-4/3}.$$

$$\text{ii)} P(4 < x < 8) = \int_4^8 f(x) dx$$
$$\Rightarrow \frac{1}{6} \int_4^8 e^{-x/6} dx \Rightarrow \frac{1}{6} \left[\frac{e^{-x/6}}{-1/6} \right]_4^8$$
$$\Rightarrow e^{-4/6} - e^{-8/6}$$
$$\Rightarrow e^{-2/3} - e^{-4/3}.$$

2)

The mileage which a car owners get with a certain kind of radial tire is R.V having exponential dist. with mean 40000 km - find the prob that one of these tyres will last (i) atleast 20000 km.
(ii) almost 30,000 km

Mean = 40000 = $1/\lambda$

$$\lambda = \frac{1}{40000}$$

$$\therefore f(x) = \begin{cases} \frac{1}{40000} e^{-x/40000}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Let x = length of time.

$$\text{i)} P(x \geq 20000)$$

3) If $X \leq e^\lambda$ with $P(X \leq 1) = P(X > 1)$ find $\text{Var}(X)$.

Exponential distribution.

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \lambda > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Given,

$$P(X \leq 1) = P(X > 1)$$

$$P(X \leq 1) = 1 - P(X \leq 1)$$

$$\therefore P(X \leq 1) = 1/2.$$

$$\int_{-\infty}^1 f(x) dx = 1/2$$

$$\lambda \int_0^1 e^{-\lambda x} dx = 1/2$$

$$\lambda \left[\frac{e^{-\lambda x}}{-\lambda} \right]_0^1 = 1/2$$

$$\Rightarrow -[e^{-\lambda} - 1] = 1/2$$

$$-e^{-\lambda} + 1 = 1/2$$

$$1 - 1/2 = e^{-\lambda}$$

$$e^{-\lambda} = 1/2$$

Apply 'log'

$$\log e^{-\lambda} = \log(1/2)$$

$$\therefore \lambda \log e = \log \frac{1}{2}$$

$$-\lambda = -\log 2$$

$$\lambda = \log 2$$

$$\text{Var}(x) = \frac{1}{\lambda^2} \Rightarrow \frac{1}{(\log 2)^2}$$

4) If X is exp-distributed with par. λ , find the val of k , there exists $\frac{P(X > k)}{P(X \leq k)} = a$

Given

$$P(X > k) = a \cdot P(X \leq k).$$

$$P(X > k) = a [1 - P(X \leq k)]$$

$$P(x > k) = a - a P(x > k)$$

$$(1+a) P(x > k) = a.$$

$$P(x > k) = \frac{a}{1+a}$$

$$\int_k^\infty f(x) dx = \frac{a}{1+a}$$

$$\lambda \int_k^\infty e^{-\lambda x} dx = \frac{a}{1+a}$$

$$\lambda \left[\frac{e^{-\lambda x}}{-\lambda} \right]_k^\infty = \frac{a}{1+a}$$

$$-\left[e^{-\infty} - e^{-\lambda k} \right] = \frac{a}{1+a}$$

$$e^{-\lambda k} = \frac{a}{1+a}$$

To get apply on B.S.

$$\log e^{-\lambda k} = \log \left[\frac{a}{1+a} \right]$$

$$-\lambda k \log e = \log \left(\frac{a}{1+a} \right)$$

$$k = \frac{-1}{\lambda} \log \left(\frac{a}{1+a} \right)$$

$$\therefore k = \frac{1}{\lambda} \log \left(\frac{1+a}{a} \right)$$

Continuous Uniform Distribution:-

If X is a continuous R.V if it follows prob. density function,

$$f(x) = \begin{cases} k, & \text{if } a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases}$$

w.k.t

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_a^b k dx = 1$$

$$k[b-a] = 1$$

$$k[b-a] = 1$$

$$k = 1/(b-a)$$

∴ Density function :-

$$\boxed{f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases}}$$

Mean:-

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx.$$

$$= \int_a^b x \cdot \frac{1}{b-a} dx \Rightarrow \frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b \Rightarrow \frac{1}{2(b-a)} (b^2 - a^2)$$

$$E(x) = \frac{(b+a)(b-a)}{2}$$

$$E(x) = \text{mean}(x) = \frac{b+a}{2}$$

Variance:-

$$V(x) = E(x^2) - \mu^2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx.$$

$$E(x^2) = \frac{1}{b-a} \cdot \int_a^b x^2 dx.$$

$$= \frac{1}{b-a} \left[\frac{x^3}{3} \right]_a^b$$

$$\begin{aligned}
 E(x^2) &= \frac{1}{3(b-a)} (b^3 - a^3) \\
 &= \frac{(b-a)(b^2 + ab + a^2)}{3(b-a)} \\
 &= \frac{b^2 + ab + a^2}{3}.
 \end{aligned}$$

$$\begin{aligned}
 V(x) &= \frac{b^2 + ab + a^2}{3} - \left(\frac{b+a}{2}\right)^2 \\
 &= \frac{1}{12} [4b^2 + 4ab + 4a^2 - 3b^2 - 6ab - 3a^2] \\
 &\Rightarrow \frac{1}{12} [b^2 - 2ab + a^2] \Rightarrow \frac{(b-a)^2}{12}.
 \end{aligned}$$

$$V(x) = \frac{(b-a)^2}{12}.$$

Density function:-

Distribution function:-

$$F(x) = P(x \leq x) = \int_{-\infty}^x f(x) dx. \text{ if } a \leq x \leq b.$$

$$\begin{aligned}
 F(x) &= \int_a^x f(x) dx \\
 &= \int_a^x \frac{1}{b-a} dx \Rightarrow \frac{1}{b-a} [x]_a^x \\
 &\Rightarrow \frac{x-a}{b-a}, \quad a \leq x \leq b.
 \end{aligned}$$

$$F(x) = \begin{cases} 0 & , -\infty < x < a \\ \frac{x-a}{b-a} & , a \leq x \leq b \\ 1 & , b < x < \infty \end{cases}$$

$$F(-\infty) = 0$$

$$F(\infty) = 1.$$

Note :-

if $x \in (-a, a)$

$$f(x) = \begin{cases} \frac{1}{2a} & , -a < x < a \\ 0 & , \text{ otherwise} \end{cases}$$

- 1) A electric trains on a certain lines run every half an hour b/w midnight & 6:00 am in the morning. what is the prob. that a man entering the station at random time during this period will have to ~~wait~~ wait atleast 20 mins.

Let X = waiting time of a man.

$$f(x) = \begin{cases} \frac{1}{30-0}, & \text{if } 0 < x < 30 \\ 0, & \text{otherwise.} \end{cases}$$

Prob of waiting time atleast 20 min.

$$\begin{aligned} P(X \geq 20) &= \int_{20}^{30} f(x) dx = \int_{20}^{30} \frac{1}{30} dx = \frac{1}{30} [x]_{20}^{30} \\ &= \frac{10}{30} = \frac{1}{3} \end{aligned}$$

- 2) Buses arrived to a specified bus stop at 15 mins intervals, starting at 7.am, i.e., 7 am, 7:15 am, etc.. If a passenger arrives at same busstop at a random time which is uniformly distributed b/w 7:am and 7:30 am. Find the prob. that he waits (a) less than 5 mins (b) atleast 12 mins for a bus.

- 3) A passenger arrives at a local railway platform at 10 am knowing that the local train will arrive at some time uniformly distributed b/w 10 am and 10:30 am. What is the prob. that (i) he will have to wait longer than 10 mins (ii) if at 10:15 am the train has not yet arrived, what is the prob. that he will have to wait atleast 10 additional mins?

- 4) If x is uniformly distributed over $(0, 10)$ find the prob. that (i) $x < 2$ (ii) $x > 8$ (iii) $3 < x < 9$.

- 5) If x is uniformly distributed with mean 2 & variance $\frac{4}{3}$ then find $P(X < 0)$

6) If x is uniformly distributed over $(-a, a)$ where $a > 0$, find ' a' . Such that $P(x \geq 1) = \frac{1}{3}$.

7) A R.V 'x' has uniform distribution $(-3, 3)$ and $|x+2| \leq 2$

- $P(x < 2)$.
- $P(|x| \leq 2)$
- $P(|x-2| \leq 2)$ add.
- Find 'k'. such that $P(x > k) = \frac{1}{3}$ $P(0 < x < 4) = \int_0^4 f(x) dx$
 $\Rightarrow \int_0^3 f(x) dx$.

Q) Let x = passenger arrives to bus stop at sometime to get bus.

$$f(x) = \begin{cases} \frac{1}{30-0}, & 0 \leq x \leq 30 \\ 0, & \text{otherwise} \end{cases}$$

i) prob. that he waits < 5 mins = prob. that he arrives bus stop at b/w $7:10 - 7:15$ & $7:25 - 7:30$,

$$\Rightarrow P(10 < x < 15) + P(25 < x < 30),$$

$$\Rightarrow \int_{10}^{15} f(x) dx + \int_{25}^{30} f(x) dx \Rightarrow \int_{10}^{15} \frac{1}{30} dx + \int_{25}^{30} \frac{1}{30} dx$$

$$\Rightarrow \frac{1}{30} [x]_{10}^{15} + \frac{1}{30} [x]_{25}^{30} \Rightarrow \frac{1}{30} [15-10] + \frac{1}{30} [30-25]$$

$$\Rightarrow \frac{5}{30} + \frac{5}{30} \Rightarrow \frac{10}{30} \Rightarrow \frac{1}{3}$$

ii) prob. that he waits ≥ 12 mins = prob. that he arrives bus stop b/w $7:00 - 7:08$ & $7:15 - 9:18$.

$$\Rightarrow P(0 < x < 8) + P(15 < x < 18),$$

$$\Rightarrow \int_0^8 f(x) dx + \int_{15}^{18} f(x) dx \Rightarrow \int_0^8 \frac{1}{30} dx + \int_{15}^{18} \frac{1}{30} dx$$

$$\Rightarrow \frac{1}{30} [x]_0^8 + \frac{1}{30} [x]_{15}^{18} \Rightarrow \frac{8}{30} + \frac{3}{30} \Rightarrow \frac{11}{30} = \frac{1}{3}$$

3) Let x = passenger arrival time to railway station at some time.

$$f(x) = \begin{cases} \frac{1}{30-0}, & 0 \leq x \leq 30 \\ 0, & \text{otherwise.} \end{cases}$$

i) prob. that he waits ≥ 10 mins = prob. that he arrives railway station b/w $10:10 - 10:30$

$$\Rightarrow P(10 < X < 30)$$

$$\Rightarrow \int_{10}^{30} f(x) dx \Rightarrow \int_{10}^{30} \frac{1}{30} dx = \frac{1}{30} [30 - 10] \Rightarrow 2/3.$$

1) To find prob. that the passenger has to wait atleast 10 additional mins given that the train has not arrived at 10:15 am.

$P(\text{he has to wait atleast 10 additional mins}) \text{ given that he has already waited 15 mins} = P(X > (15+10) / X > 15).$

$$\Rightarrow P(X > 25 / X > 15) = \frac{P(X > 25 \cap X > 15)}{P(X > 15)} \Rightarrow \frac{P(X > 25)}{P(X > 15)}$$

$$P(X > 25) = \frac{\int_{25}^{30} f(x) dx}{\int_{15}^{30} f(x) dx} \Rightarrow \frac{\frac{1}{30} [x]_{25}^{30}}{\frac{1}{30} [x]_{15}^{30}} = \frac{30 - 25}{30 - 15} \Rightarrow \frac{5}{15} = 1/3.$$

4) Given $(0, 10) = (a, b)$.

$$f(x) = \begin{cases} \frac{1}{10}, & 0 < x < 10 \\ 0, & \text{otherwise} \end{cases}$$

$$\text{i)} P(X < 2) = \int_0^2 f(x) dx \Rightarrow \frac{1}{10} \int_0^2 1 dx \Rightarrow \frac{1}{10} [x]_0^2 \Rightarrow 2/10 = 1/5.$$

$$\text{ii)} P(X > 8) = \int_8^{10} f(x) dx \Rightarrow \frac{1}{10} \int_8^{10} 1 dx \Rightarrow \frac{1}{10} [10 - 8] = 2/10 = 1/5.$$

$$\text{iii)} P(3 < X < 9) \Rightarrow \int_3^9 f(x) dx \Rightarrow \frac{1}{10} [9 - 3] \Rightarrow 6/10 = 3/5.$$

5) Given

$$\text{Mean} = 1$$

$$\frac{a+b}{2} = 1$$

$$a+b=2$$

$$\sqrt{var} = 4/3$$

$$\frac{(b-a)^2}{12} = 4/3$$

$$(b-a)^2 = 16$$

$$b-a = \pm 4$$

$$b-a = 4$$

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow b+a=2$$

$$b-a=4$$

$$2b=6$$

$$b=3$$

$$a=2-3$$

$$a=-1$$

$$(-1, 3)$$

$$\Rightarrow \begin{array}{l} b+a=2 \\ b-a=4 \end{array}$$

$$\hline 2b=6$$

$$b=3$$

$$a=2-3$$

$$a=-1$$

$$a=3$$

$$\therefore \text{Interval} \Rightarrow (-1, 3)$$

$$f(x) = \begin{cases} \frac{1}{3+1}, & -1 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

$$P(X < 0) = \int_{-1}^0 f(x) dx \Rightarrow \int_{-1}^0 \frac{1}{4} dx \Rightarrow \frac{1}{4}[x]_{-1}^0 \Rightarrow \frac{1}{4}(0 - (-1)) = \frac{1}{4}$$

6) Interval $\Rightarrow (-\alpha, \alpha)$; $\alpha > 0$

$$f(x) = \begin{cases} \frac{1}{\alpha - (-\alpha)}, & -\alpha < x < \alpha \\ 0, & \text{otherwise} \end{cases} \Rightarrow \begin{cases} \frac{1}{2\alpha}, & -\alpha < x < \alpha \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Given } P(X \geq 1) = \frac{1}{3}$$

$$\int_{-\infty}^{\infty} f(x) dx = \frac{1}{3}$$

$$\int_{-\infty}^{\infty} f(x) dx = \frac{1}{3} \Rightarrow \int_{-\infty}^{\infty} \frac{1}{2\alpha} dx = \frac{1}{3}$$

$$\frac{1}{2\alpha} [x]_{-1}^0 \Rightarrow \frac{1}{3}$$

$$\frac{\alpha - 1}{2\alpha} = \frac{1}{3}$$

$$3\alpha - 3 = 2\alpha$$

$$\therefore \alpha = 3$$

Gamma Distribution :-

The continuous r.v 'X' is said to follow gamma distribution if the density function,

$$f(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1}, & \alpha, \lambda > 0, 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

* $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$

$$\Gamma_n = (n-1)\Gamma(n-1) \cdot \text{if } n \text{ is +ve function.}$$

$$\Gamma_n = (n-1)! \text{ if } n \text{ is +ve integral.}$$

$$\Gamma_1 = 1$$

$$\Gamma_2 = \sqrt{\pi}$$

Mean :-

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

$$\Rightarrow \int_0^{\infty} x \frac{\lambda^x}{\Gamma(\alpha)} e^{-\lambda x} \cdot x^{\alpha-1} dx$$

$$\Rightarrow \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^{\infty} e^{-\lambda x} \cdot x^{\alpha+1} dx$$

Let $\lambda x = t$
 $x = t/\lambda \Rightarrow dx = \frac{dt}{\lambda}$

$$E(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^{\infty} e^{-t} \cdot t^{\alpha} \cdot \frac{dt}{\lambda}$$

$$E(x) = \frac{1}{\lambda \Gamma(\alpha)} \int_0^{\infty} e^{-t} \cdot t^{\alpha} dt$$

$$= \frac{1}{\lambda \Gamma(\alpha)} \int_0^{\infty} e^{-t} \cdot t^{(\alpha+1)-1} dt$$

$$= \frac{1}{\lambda \Gamma(\alpha+1)} \Rightarrow \frac{1}{\lambda \Gamma(\alpha+1)} \alpha \cdot \Gamma(\alpha)$$

$\therefore \text{Mean} = \frac{\alpha}{\lambda}$

If $\beta/\lambda = \beta$, $\therefore \text{Mean} = \alpha\beta$

Variance :-

$$V(x) = E(x^2) - \mu^2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$E(x^2) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^{\infty} x^2 e^{-\lambda x} \cdot x^{\alpha-1} dx$$

$$E(x^2) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^{\infty} e^{-\lambda x} \cdot x^{\alpha+1} dx$$

Let $\lambda x = t$
 $x = t/\lambda \Rightarrow dx = \frac{dt}{\lambda}$

$$E(x^2) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^{\infty} e^{-t} \cdot t^{\alpha+1} \cdot \frac{dt}{\lambda}$$

$$E(x^2) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \frac{1}{\lambda^{\alpha+1}} \int_0^{\infty} e^{-t} \cdot t^{\alpha+1} dt$$

$$E(x^2) = \frac{1}{\lambda^2 \Gamma(\alpha+1)} \int_0^{\infty} e^{-t} \cdot t^{(\alpha+2)-1} dt$$

$$\Rightarrow \frac{1}{\lambda^{\alpha}} \Gamma(\alpha)$$

$$\Rightarrow \frac{1}{\lambda^{\alpha}} (\alpha+1) \Gamma(\alpha+1)$$

$$\Rightarrow \frac{1}{\lambda^{\alpha}} (\alpha+1) \alpha \Gamma(\alpha)$$

$$E(X^2) = \frac{\alpha^2 + \alpha}{\lambda^2}$$

$$V(X) = E(X^2) - \mu^2 \\ = \frac{\alpha^2 + \alpha}{\lambda^2} - \frac{\alpha^2}{\lambda}$$

$$\boxed{V(X) = \frac{\alpha}{\lambda^2}}$$

$$\text{if } \lambda = \beta \text{ then } \boxed{V(X) = \alpha \beta^2}$$

- 1) In a certain city, the daily consumption of electric power in millions of kWh can be treated as a R.V. having Gamma distribution with parameters $\lambda = 1/2, \alpha = 3$. If the power plant of this city daily capacity of 12 mwh, what is the prob. that the power supply will be inadequate on any given day?

Given $\lambda = 1/2, \alpha = 3$

X = power consumption in a city.

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1}$$

$$f(x) = \left(\frac{1}{2}\right)^3 \cdot \frac{1}{\Gamma(3)} e^{-x/2} \cdot x^2, 0 < x < \infty$$

$$= \frac{1}{2^3} \cdot \frac{1}{2!} x^2 e^{-x/2}, 0 < x < \infty$$

$$f(x) \Rightarrow \frac{x^2 e^{-x/2}}{16}, 0 < x < \infty$$

$$P(X > 12) = \int_{12}^{\infty} f(x) dx \Rightarrow \int_{12}^{\infty} \frac{x^2 e^{-x/2}}{16} dx$$

$$\Rightarrow \frac{1}{16} \left[x^2 \left(\frac{e^{-x/2}}{-1/2} \right) - 2x \left(\frac{e^{-x/2}}{-1/4} \right) + 2 \left(\frac{e^{-x/2}}{-1/8} \right) \right]_{12}^{\infty}$$

$$\Rightarrow \frac{1}{16} \left\{ (0+0+0) - e^{-6} (-8(12)^2 - 4(24) + (-16)) \right\}$$

$$\Rightarrow \frac{e^{-6}}{16} [288 + 96 + 16] \Rightarrow \frac{e^{-6}}{16} [400] \Rightarrow 25e^{-6}.$$

Q) The daily consumption of milk in a city, in excess of 20,000 litres is approximately distributed as a gamma variate with parameters $\alpha = 2$, $\lambda = \frac{1}{10,000}$. The city has daily stock of 30,000 litres. What is the prob. that the stock is insufficient on particular day?

$$\lambda = \frac{1}{10,000}, \alpha = 2.$$

x = excess of NMC consumption. / y = daily consumption.

$$y = x + 20,000 \text{ then } \Rightarrow x = y - 20,000.$$

$$f(x) = \frac{1}{(10,000)^2} \frac{1}{2} e^{-x/10,000} \cdot x$$

$$f(x) = \frac{x}{(10,000)^2} e^{-x/10,000}, 0 < x < \infty.$$

$$P(Y > 30,000) = P(x + 20,000 > 30,000)$$

$$= P(x > 10,000)$$

$$= \int_{10,000}^{\infty} f(x) dx \Rightarrow \int_{10,000}^{\infty} \frac{x}{(10,000)^2} e^{-x/10,000} dx$$

$$\Rightarrow \frac{1}{(10,000)^2} \int_{10,000}^{\infty} x \cdot e^{-x/10,000} dx \Rightarrow \frac{1}{(10,000)^2} \left[x \left(\frac{e^{-x/10,000}}{-1/10,000} \right) - \left(\frac{e^{-x/10,000}}{(-1/10,000)^2} \right) \right]_{10,000}^{\infty}$$

$$\Rightarrow \frac{(10,000)^2}{(10,000)^2} \left[10,000 \cdot e^{-x/10,000} \Big|_{10,000}^{\infty} - e^{-x/10,000} \Big|_{10,000}^{\infty} \right]$$

$$\Rightarrow (10,000) - \left((-10,000)^2 e^1 - e^1 \right)$$

$$\Rightarrow \frac{1}{e} (10,000^2 + 1) \Rightarrow$$

Sampling Distributions:-

Population: The aggregate or totality of statistical data corresponding to observations whether it is either finite or infinite is called population.

Ex:- i) population of heights of Indians.

ii) No. of students in a village those are classified according to blood group.

Size of population:- The no. of observations in the population is known as size of population. It is denoted by ' N '.

Sample :- A small portion of population which gives population characteristics is known as sample (or) The subset of population is known as sample.

Size of Sample:- The no. of observations in sample is known as size of sample - It is denoted by ' n '.

Large Sample:- If the size of sample $n \geq 30$, then sample is called Large sample.

Small Sample:- If the size of sample $n < 30$, then the sample is called Small sample.

Sampling:- The process of selection of sample from the population is known as Sampling.

* There are 3 types of sampling distributions

i) Probability Sampling Distributions:-

i) Random Sampling distribution

ii) Stratified " "

iii) Systematic " "

a) Non-probability sampling distributions

- i) purposive sampling distribution
- ii) sequential

Note:-

i) No. of samples with replacement from population
 $= N^n$

ii) No. of samples without replacement from population $= {}^N C_n$

iii) Sampling with replacement considered as infinite population.

iv) Sample without replacement finite population.

Symbols

Population

i) population mean = μ

$$\mu = \frac{1}{N} \sum x_i$$

ii) pop. variance

$$\sigma^2 = \frac{1}{N} \sum (x_i - \mu)^2$$

iii) N = size of population

Sample

i) Sample mean = \bar{x}

$$\bar{x} = \frac{1}{n} \sum x_i$$

ii) Sample variance

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

iii) n = size of sample

Parameters

The statistical measurements of all observations of populations is known as parameters.

Ex:- μ, σ^2 .

Statistics

The statistical measurements of all units selected in a sample is known as statistics.

Ex:- \bar{x}, s^2 .

Sampling distributions of mean :-

Let $x_1, x_2, x_3, \dots, x_n$ be n random samples with size n drawn from population of size N with mean μ & standard deviation σ . Let \bar{x} be mean of sample (without replacement).
 i) Infinite population :- (Sampling with replacement):-

i) Mean of Pop = $\mu = \frac{1}{n} \sum x_i$

ii) Variance of Pop = $\sigma^2 = \frac{1}{n} \sum (x_i - \mu)^2$

iii) Mean of Sampling distribution of means = $\mu_{\bar{x}}$

$$\boxed{\mu_{\bar{x}} = \mu}$$

iv) Variance of Sampling distribution of means = $\sigma_{\bar{x}}^2$

$$\sigma_{\bar{x}}^2 = \frac{1}{\text{total samples}} [\sum (\bar{x}_i - \mu_{\bar{x}})^2] \quad (\text{or})$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

v) Finite population :- (sampling without Replacement).

i) Mean of Pop. = μ

ii) Variance of pop = σ^2

iii) Mean of sampling dist. of means = $\mu_{\bar{x}}$

$$\mu_{\bar{x}} = \mu$$

iv) Variance of Sampling distr. of means = $\sigma_{\bar{x}}^2$

$$\sigma_{\bar{x}}^2 = \frac{1}{N-n} (\bar{x}_i - \mu_{\bar{x}})^2 \quad (\text{or}) \quad \sigma_{\bar{x}}^2 = \left(\frac{N-n}{N-1} \right) \frac{\sigma^2}{n}$$

Finite population correction factor :-

$$\frac{N-n}{N-1}$$

Central limit theorem :-

If \bar{x} is central limit Sample mean of with size n drawn from a population having mean μ & Standard deviation σ , then the Sampling distribution of mean follows normal distribution by

$$\boxed{Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$

Standard error of sample mean :-

$$S.E.(\bar{x}) = \frac{\sigma}{\sqrt{n}}$$

Note:-

- i) for infinite population, the sampling distribution of sum's or mean's has the mean $\mu_{\bar{x}_1 + \bar{x}_2}$ and S.D $\sigma_{\bar{x}_1 + \bar{x}_2}$ is defined by

$$\mu_{\bar{x}_1 + \bar{x}_2} = \mu_{\bar{x}_1} + \mu_{\bar{x}_2}$$

$$\sigma_{\bar{x}_1 + \bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- ii) Sampling distribution of differences of means has the mean $\mu_{\bar{x}_1 - \bar{x}_2}$ & S.D $\sigma_{\bar{x}_1 - \bar{x}_2}$ are defined by

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_{\bar{x}_1} - \mu_{\bar{x}_2}$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- i) A population consists of 5 numbers 2, 3, 6, 8 & 11. Consider all possible samples of size 2 which can be drawn from this population. Then find (i) mean of population

- (ii) Standard deviation of population. (iii) Mean of sampling distribution of means (iv) S.D of sampling distribution of means.

Given population is 2, 3, 6, 8 & 11

Population size (N) = 5

Sample size (n) = 2

- i) Mean of population (μ):-

$$\mu = \frac{2+3+6+8+11}{5} \Rightarrow \frac{30}{5} = 6.$$

- ii) Variance of population (σ^2):-

$$\sigma^2 = \frac{1}{N} \sum (x_i - \mu)^2$$

$$= \frac{1}{5} [(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2]$$

$$\Rightarrow \frac{1}{5} [16+9+0+4+25]$$

$$\Rightarrow \frac{1}{5} (54)$$

$$\Rightarrow 10.8$$

$$\sigma = \sqrt{10.8} \Rightarrow 3.$$

iii) No. of samples with replacement $t = N^2 = 5^2 = 25$

Sample S = $\{(2, 2), (2, 3), (2, 6), (2, 8), (2, 11), (3, 2), (3, 3), (3, 6), (3, 8), (3, 11), (6, 2), (6, 3), (6, 6), (6, 8), (6, 11), (8, 2), (8, 3), (8, 6), (8, 8), (8, 11), (11, 2), (11, 3), (11, 6), (11, 8), (11, 11)\}$

All samples are $\{2, 2.5, 4, 5, 6.5, 2.5, 3, 4.5, 5.5, 7, 4, 4.5, 6, 7, 8.5, 5.5, 5, 7, 8, 9.5, 6.5, 7, 8.5, 9.5, 11\}$

Mean of sampling distributions of means

$$\mu_{\bar{x}} = \frac{2 + 2.5 + 4 + 5 + \dots + 9.5 + 11}{25}$$

$$\mu_{\bar{x}} = 6.$$

iv) S.D of sampling distribution.

$$\begin{aligned} \sigma_{\bar{x}}^2 &= \frac{1}{\text{total}} \sum (\bar{x}_i - \mu_{\bar{x}})^2 \\ &= \frac{1}{25} [(2-6)^2 + (2.5-6)^2 + \dots + (11-6)^2] \\ &\quad \vdots 5, 4 \\ \sigma_{\bar{x}} &= \sqrt{5.4} \Rightarrow 2.8 \text{ a.} \end{aligned}$$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{10.8}{\sqrt{2}} \Rightarrow 5.4. \quad (\omega, k-1)$$

v) Solve above problem for without replacement.

Given population is 2, 3, 6, 8 & 11.

i) (i) questions are same as above

ii) No. of samples without replacement $t = N C_n = 5 C_2 = 10$

Samples = $\{(2, 3), (2, 6), (2, 8), (2, 11), (3, 6), (3, 8), (3, 11), (6, 8), (6, 11), (8, 11)\}$

All samples means = { 8.5, 4, 5, 6.5,
 4.5, 5.5, 7,
 7, 8.5, 9.5 }.

Mean of Sampling distribution of means =

$$\mu_{\bar{x}} = \frac{8.5 + 4 + 5 + \dots + 9.5 + 7}{10} \Rightarrow 6.$$

iv) $\sigma_x^2 = \frac{1}{\text{total}} \sum (x_i - \mu_{\bar{x}})^2$

$$\sigma_x^2 = \frac{1}{10} [(8.5 - 6)^2 + (4 - 6)^2 + \dots + (9.5 - 6)^2].$$

$$\sigma_x^2 = 4.05$$

$$\sigma_x = \sqrt{4.05} \Rightarrow 2.01.$$

(or) $\sigma_x^2 = \left(\frac{N-n}{N-1} \right) \sigma^2 \Rightarrow \left(\frac{5-2}{5-1} \right) \frac{10.8}{2} = \left(\frac{3}{4} \right) \left(\frac{10.8}{2} \right) = 4.05$

- Q) A population is 5, 10, 14, 18, 13, 24. Consider all possible samples of size '2' which can be drawn without replacement from this population. Find (i) Mean of population
 (ii) S.D of population (iii) Mean of sampling dist. of means
 (iv) S.D of sampling dist. of means.

- Q) If the population is 3, 6, 9, 15 & 27. (i) Find all possible samples of size '3' that can be taken without replacement from the finite population. (ii) Mean of sampling dist. of means (iii) S.D of all means

Given population is 3, 6, 9, 15, 27

Population size (N) = 5

Sample size (n) = 3

Mean of population (μ):

$$\mu = \frac{3+6+9+15+27}{5} = \frac{60}{5} = 12$$

Variance of population.

$$\sigma^2 = \frac{1}{N} \sum (x_i - \mu)^2$$

$$= \frac{1}{5} [(3-12)^2 + (6-12)^2 + (15-12)^2 + (27-12)^2]$$

$$\Rightarrow \frac{1}{5} [81+36+9+9+225] \Rightarrow \frac{1}{5} (360) = 72$$

$$\sigma^2 = \frac{72}{2} = 36$$
$$\sigma = \sqrt{36} = 6$$

Population standard deviation = 6
Population standard error = $\frac{6}{\sqrt{100}} = 0.6$

Mean = 100.0
Standard deviation = 6
Population standard error = 0.6
Margin of error = 1.96 * 0.6 = 1.176
Upper limit = 100 + 1.176 = 101.176
Lower limit = 100 - 1.176 = 98.824

Margin of error can be calculated by multiplying standard deviation with Z value. As we know that Z value depends upon the confidence level. If confidence level is 95% then Z value is 1.96 and if confidence level is 99% then Z value is 2.58. Margin of error is also called as standard error.

Population standard error is used to calculate the margin of error. Margin of error is used to calculate the confidence interval. Confidence interval is used to calculate the confidence level.

8) A Random Sample of size 64 is taken from a normal population with mean $\mu = 51.4$ and $\sigma = 6.8$. What is the probability that the mean of sample will
 i) exceed 52.9 ii) fall b/w 50.5 & 52.3
 iii) less than 50.6.

9) The mean height of students is 165cm

9) A Random sample of size 100 is taken from infinite population having mean $\mu = 76$ $\sigma^2 = 256$. what is the probability that \bar{x} lies b/w 75 & 78.

10) The mean breaking strength of copper wire is 575 lbs with S.D 8.3 lbs. How large a sample must be used in order that there will be one chance in 100 that mean \bar{x} strength of sample is less than 572 lbs

11) 8 masses are measured as 62.34, 20.48, 35.97 kg
 $S.D = 0.54, 0.21, 0.46$ kgs, find mean and S.D of sum of masses.

8) Mean = 51.4, Sample size (n) = 64
 $S.D(\sigma) = 6.8$

i) To find $P(\bar{x} > 52.9)$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \Rightarrow$$

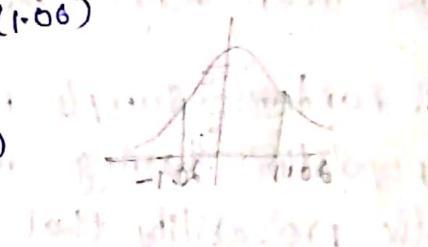
$$\text{if } \bar{x} > 52.9 \Rightarrow z = \frac{52.9 - 51.4}{6.8/\sqrt{64}} \Rightarrow 1.87$$

$$\begin{aligned} P(\bar{x} > 52.9) &= P(z > 1.87) \\ &= 0.5 - P(0 < z < 1.87) \\ &= 0.5 - A(1.87) \\ &= 0.5 - 0.4693 \Rightarrow 0.0307 \end{aligned}$$

$$\text{ii)} P(50.5 < \bar{x} < 52.3)$$

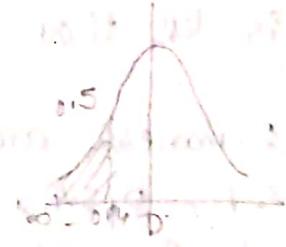
$$\text{If } \bar{x} = 50.5 \Rightarrow z = \frac{50.5 - 51.4}{6.8/\sqrt{64}} = \frac{-0.9}{-0.125} \Rightarrow \approx -1.13$$

$$\text{If } \bar{x} = 52.3 \Rightarrow z = \frac{52.3 - 51.4}{6.8/\sqrt{64}} = \frac{1.06}{0.125} \approx 1.13.$$

$$\begin{aligned} P(-1.13 < z < 1.13) &= P(-1.06 < z < 1.06) + P(0 < z < 1.06) \\ &\Rightarrow P(0 < z < 0.94) + A(1.06) \\ &\Rightarrow A(1.06) + A(1.06) \\ &\Rightarrow 2A(1.06) \\ &\Rightarrow 2(0.3554) \\ &\Rightarrow 0.7108. \end{aligned}$$


$$\text{iii)} P(\bar{x} < 50.6)$$

$$\text{If } \bar{x} = 50.6 \Rightarrow z = \frac{50.6 - 51.4}{6.8/\sqrt{64}} = -0.94$$

$$\begin{aligned} P(\bar{x} < 50.6) &= P(z < -0.94) \\ &\Rightarrow 0.5 - P(-0.94 < z < 0) \\ &\Rightarrow 0.5 - P(0 < z < 0.94) \\ &\Rightarrow 0.5 - A(0.94) \\ &\Rightarrow 0.5 - 0.3264 \\ &\Rightarrow 0.1736. \end{aligned}$$


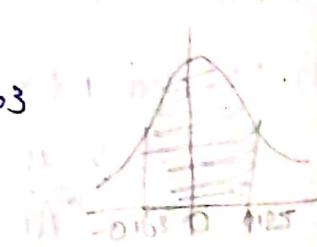
$$9) \text{ Mean} = \mu = 76.$$

$$\text{S.D} (\sigma) = \sqrt{256} = 16. \quad \text{sample size} (n) = 100.$$

$$\text{i)} P(75 < \bar{x} < 78)$$

$$\text{If } \bar{x} = 75 \Rightarrow z = \frac{75 - 76}{16/\sqrt{100}} = -0.63$$

$$\text{If } \bar{x} = 78 \Rightarrow z = \frac{78 - 76}{16/\sqrt{100}} = 1.25.$$

$$\begin{aligned} P(-0.63 < z < 1.25) &= P(-0.63 < z < 0) + P(0 < z < 1.25) \\ &= A(-0.63) + A(1.25) \\ &= 0.2857 + 0.3944 \\ &\Rightarrow \end{aligned}$$


$$10) \text{ Mean}(\mu) = 575 \text{ lbs}$$

$$\text{S.D}(\sigma) = 8.3 \text{ lbs}$$

$$\text{If } \bar{x} = 572 \text{ lbs} \Rightarrow z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \Rightarrow \frac{572-575}{8.3/\sqrt{n}} \Rightarrow \frac{-3\sqrt{n}}{8.3}$$

$$P(\bar{x} < 572) = \frac{1}{100} = 1\% \Rightarrow 0.01$$

from diagram.

$$P(-z_1 < z < 0) = 0.49$$

$$A(-z_1) = 0.49$$

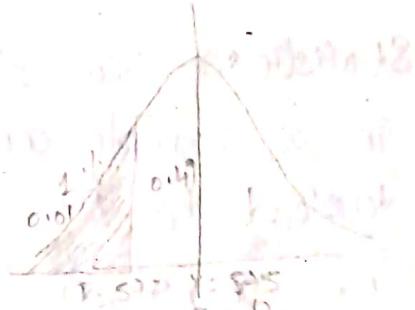
$$z_1 = -2.33$$

$$\frac{-3\sqrt{n}}{8.3} = -2.33$$

$$\sqrt{n} = \frac{2.33 \times 8.3}{3}$$

$$n = \left(\frac{2.33 \times 8.3}{3} \right)^2$$

$$n = 48$$



11) Assume masses,

$$\mu_{\bar{x}_1} = 62.34$$

$$\mu_{\bar{x}_2} = 20.48$$

$$\mu_{\bar{x}_3} = 35.97$$

$$\begin{aligned} \text{Mean of sum of masses} &\Rightarrow \mu_{\bar{x}_1} + \mu_{\bar{x}_2} + \mu_{\bar{x}_3} = \mu_{\bar{x}_1 + \bar{x}_2 + \bar{x}_3} \\ &\Rightarrow 62.34 + 20.48 + 35.97 \\ &\therefore \end{aligned}$$

$$\text{S.D of sum of masses } \sigma_{\bar{x}_1 + \bar{x}_2 + \bar{x}_3} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2 + \sigma_{\bar{x}_3}^2}$$

$$= \sqrt{(62.34)^2 + (20.48)^2 + (35.97)^2}$$