UNIT–IV

Estimation & Tests of Hypotheses.

Short Answer questions

1) Define Null Hypothesis and Alternative Hypothesis

Sol:- Null Hypothesis :- A definite statement about the population parameter for applying the test of significance is called null hypothesis, which is usually a hypothesis of no difference and is denoted by H_0

Alternative Hypothesis :- Any hypothesis which is complementary to the null Hypothesis is called an alternative hypothesis and is denoted by H_1

2) Define Type I error and Type II error

Type I error :- if we reject the null hypothesis when it is true, and it is also known as producer list

$$P[\text{reject } H_0 \text{ when it is true}] = \alpha$$

Type II error :- if we accept the null hypothesis when it is wrong, and it is also known as consumer list.

$$P[\text{accept } H_0 \text{ when it is wrong}] = \beta$$

(3) if a Sample number is '500' and The S.D is '15' find maximum error with 95% confidence.

Sol:-

Maximum error $E = Z_{\alpha/2}\, \sigma/\sqrt{n}$

$Z_{\alpha/2} = 1.96$ at 95%.

$\sigma = 15$

$n = 500$

$E = 1.96 \dfrac{15}{\sqrt{500}}$

$E = 1.31$

(4) The mean & S.D of population are 11795 & 14504 respectively if n=50 & find 95% Confidence Interval for The mean

Sol:-

$$\text{Interval} = \left(\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

$\bar{x} = 11795$ $\quad \sigma = 14,504$

$$= \left(11795 - (1.96)\frac{14054}{\sqrt{50}},\ 11795 + (1.96)\frac{14054}{\sqrt{50}}\right)$$

$$= (7899.42,\ 15690.57)$$

(5) if we can assert with 95% that The maximum error is 0.05 and p=0.2 find Sample Size

Sol:-

Max error $E = Z_{\alpha/2}\sqrt{\dfrac{PQ}{n}}$

$P = 0.2$ $\quad Q = 1-P = 1-0.2 = 0.8$

$$0.05 = 1.96\sqrt{\frac{(0.2)(0.8)}{n}}$$

$$n = \frac{(0.2)(0.8) \times (1.96)^2}{(0.05)^2} = 246$$

(6) A Random Sample of size '100' has a S.D of '5'. What can u say about the max error with 95% Confidence

Sol: Given That $n=100$ $\sigma=5$

$$Z_{\alpha/2} = 1.96 \quad (95\% \text{ Confidence})$$

Max error $E = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$= 1.96 \frac{5}{\sqrt{100}} = 0.98$$

(1) (a) Construct 95% Confidence Interval for True proportion of Computer literates if ~~47~~ out of 150 persons from rural areas are Computer literates

proportion $p = \frac{x_1}{n_1} = \frac{47}{150} = 0.313$ $\quad Q = 1-P$

$$= 1-0.313$$

$$= 0.687$$

Confidence Interval

$$\left(p - Z_{\alpha/2}\sqrt{\frac{PQ}{n}} \; , \; p + Z_{\alpha/2}\sqrt{\frac{PQ}{n}}\right)$$

$$\left(0.313 - (1.96)\sqrt{\frac{(0.313)(0.687)}{150}} \; , \; 0.313 + 1.96\sqrt{\frac{(0.313)(0.687)}{150}}\right)$$

$$(0.313 - 0.0742 \; , \; 0.313 + 0.0742)$$

$$(0.2388 \; , \; 0.3872)$$

b) A Sample of size 9 was taken from a population gave $s^2 = 10.9$, $\bar{x} = 15.8$ obtain 99% Confidence Interval for μ

$$\left(\bar{x} \pm Z_{\alpha/2} \, s/\sqrt{n}\right)$$

$$\left(15.8 - 2.58\left(3.3/\sqrt{9}\right), \; 15.8 + 2.58\left(3.3/\sqrt{9}\right)\right)$$

$$(12.96, \; 18.6)$$

Critical Value (significant values):- The value of test statistics separates a rejection region and the acceptance region is called the Critical value.

Critical value of z	level of Significance (α)		
	1%	5%	10%
Two tailed	2.58	1.96	1.64
Right tailed	2.33	1.64	1.28
Left tailed	-2.33	-1.64.	-1.28

long Answer questions

2) The mean height of 90 Students in a Class is 180cm test 90% level whether the Sample has been drawn from a population mean is 170 Cms and standard deviation '35'.

Sol:- Given data. Sample mean $\bar{x} = 180$

Sample Size $n = 90$

population mean $\mu = 170$

population S.D $\sigma = 35$

Null Hypothesis: H_0:- The Sample has been drawn from a population $(\bar{x} = \mu)$

Alternative Hypothesis H_1:- The Sample has not drawn from a population $(\bar{x} \neq \mu)$ (Two Tailed)

Test Statistic:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{180 - 170}{35/\sqrt{90}} = \frac{10 \times 9.48}{35} = 2.71$$

$Z_{cal} =$

level of significance:- $\alpha = 10\%$

$Z_{tab} = 1.64$

Conclusion:- $|Z_{cal}| > Z_{tab}$ $(|2.71| > 1.64)$

So Alternative Hypothesis accepted

The sample is not drawn from same population

(3) The means of two large samples of sizes 1000 & 2000 member are 67.5 Inches & 68.0 Inches Can the sample be regarded as drawn from the same population of S.D 2.5 Inches.

Sol:- Given data

first sample mean $\bar{x}_1 = 67.5$

second sample mean $\bar{x}_2 = 68$

pop S.D $= \sigma_1 = \sigma_2 = \sigma = 2.5$

$n_1 = 1000 \quad n_2 = 2000$

Null Hypothesis H_0: Two samples drawn from the same population i.e ($\mu_1 = \mu_2$)

Alternative Hypothesis H_1 Two samples not drawn from the same population i.e ($\mu_1 \neq \mu_2$)

Test statistic
$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}} = \frac{67.5 - 68}{\sqrt{\frac{(2.5)^2}{1000} + \frac{(2.5)^2}{2000}}}$$
$$= -5.16$$

level of significance $\alpha = 5\% \quad Z_\alpha = 1.96$

Conclusion: $|Z|_{cal} > Z_{tab}$ $(|-5.16| > 1.96)$

Alternative hypothesis accepted

The samples are drawn from the same population

4) A manufacturer Claimed that at least 98%
of The Steel pipes which he supplied to a
factory Conformed to specification. ~~test this Claim~~
~~at a significance level of 0.05~~. An examination

~~population proportion $P = \frac{98}{100} = 0.98$~~

of 500 pieces of pipes revealed That '30' were
detective. Test this Claim at a Significance
level of 0.05

Sol:-

population proportion $P = \frac{98}{100} = 0.98$

500 pipes '30' are defective So non defective
pipes $= 500 - 30 = 470$

Sample proportion of non defective pipes $p = \frac{x_1}{n_1} = \frac{470}{500} = 0.94$

$n_1 = 500$

Null Hypothesis H_0:- There is no diff b/w
Sample proportion with population proportion
i.e $(p = P)$

Alternative Hypothesis H_1 There is diff b/w
Sample proportion with population proportion
i.e $(p \neq P)$

Test Statistic:-

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$P = 0.98$ $\quad Q = 1 - P$

$= 1 - 0.98$

$= 0.02$

$$z_{cal} = \frac{0.94 - 0.98}{\sqrt{\frac{(0.98)(0.02)}{500}}} = -2.02$$

level of significance: $\alpha = 0.05$

$= 5\%$

$z_{tab} = 1.96$

Conclusion:- $|z_{cal}| > z_{tab}$ $\quad (|-2.02| > 1.96)$

Alternative Hypothesis accepted

There is diff b/w Sample proportion with pop proportion.

(5) In a Certain City 125 men in a Sample of 500 were found to be Smokers. In another City The number of Smokers was 375 in a random Sample of 1000. Does this indicate that there is a greater population smokers in the second City then the first City?

Sol:-

Smokers proportion In first City $p_1 = \frac{x_1}{n_1} = \frac{125}{500} = 0.25$

Smokers proportion In second City $P_2 = \frac{x_2}{n_2} = \frac{375}{1000} = 0.375$

Null Hypothesis H_0: There is no diff b/w Two City Smokers proportion $(p_1 = p_2)$

Alternative Hypothesis H_1:- There is a greater population Smokers in the second City then the first city $(p_2 > p_1)$ (Right tailed)

Test Statistic

$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$p = \frac{x_1 + x_2}{n_1 + n_2} = \frac{125 + 375}{500 + 1000} = \frac{500}{1500} = 0.\dot{3}3$$

$$q = 1 - p$$
$$= 1 - 0.33$$
$$= 0.67$$

$$z = \frac{(0.25) - (0.375)}{\sqrt{(0.33)(0.67)\left(\frac{1}{500} + \frac{1}{1000}\right)}} = 1.506$$

<u>level of Significance</u>: $\alpha = 5\%$ (Right-tailed)

$$z_{tab} = 1.64$$

<u>Conclusion</u>:- $|z_{cal}| < z_{tab}$ $(|1.506| < 1.64)$

Null Hypothesis accepted

There is a greater population Smokers in the Second City then The first City

(6) The owner of a machine shop must decide which of two snack vending machines to install in this shop. if each is tested '250' times the first machine fails to work 13 times and second machine fails to work '7' times. test at 0.05 level of significance whether the difference b/w the corresponding sample proportions is significant

Sol:-

first sample size $n_1 = 250$

Second sample size $n_2 = 250$

first sample proportion to fails machine $p_1 = \frac{x_1}{n_1} = \frac{13}{250} = 0.052$

second sample proportion $p_2 = \frac{x_2}{n_2} = \frac{7}{250} = 0.028$

$$p_1 = 0.052 \quad p_2 = 0.028$$

Null Hypothesis H_0:- $p_1 = p_2$

Alternative Hypothesis: H_1:- $p_1 \neq p_2$

Test Statistic

$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$p = \frac{x_1 + x_2}{n_1 + n_2} = \frac{13 + 7}{250 + 250} = \frac{20}{500} = 0.04$$

$$q = 1 - 0.04 = 0.96$$

$$z_0 = \frac{0.052 - 0.028}{\sqrt{(0.04)(0.96)\left(\frac{1}{250}+\frac{1}{250}\right)}} = 1.38$$

$z_{cal} = 1.38$

<u>level of Significance:</u>- $\alpha = 5\%$

$z_{tab} = 1.96$

<u>Conclusion:</u> $|z_{cal}| < z_{tab}$ $(|1.38| < 1.96)$

Null Hypothesis accepted

There is no diff b/w Sample proportions