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Optimal drive of electric vehicles using an inversion-based trajectory generation approach

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Abstract: This paper considers an energy management problem for an electric vehicle and explains how to solve it with an inversion-based approach. Modeling assumptions are discussed and yield the formulation of a simple, yet representative, optimal control problem to solve. This tutorial problem can serve as a benchmark for future works and other optimization techniques as it comprises the effects of gravity, aerodynamics and the equations of the electric drive. The purpose of this paper is to expose, step by step, how to compute the optimal trajectories using a presented inversion-based approach, in which the state variables, the control, and the adjoint variables are analytically parameterized by a single unknown which satisfies a high order two-point boundary value problem. Numerical results are provided.

1. INTRODUCTION

Over the last decades, to maximize the energy efficiency of automotive vehicles, engine control technologies have been addressing with great efforts the important problems of transient control of energy production while limiting pollutant emissions (see Jankovic et al. [1998], Van Nieuwstadt et al. [2000], Eriksson et al. [2002], Guzzella and Sciarretta [2007], Shaver [2009], Chiang et al. [2007], Chauvin [2006], Hillion [2009], Lepreux [2009], Leroy [2010], Moulin [2010] among others). An also important problem is the one of energy management by means of high-level vehicle driving optimization. This is true for internal combustion engines, hybrid and electric vehicles as well (see e.g. Brahma et al. [2000], Gong et al. [2008], Lin et al. [2003]). In a typical such problem, the path to follow is known in advance, as the beginning and endpoint reaching times are, while the question is to find the most efficient way to travel along this path in terms of energy savings. The goal is thus to optimize the acceleration of the vehicle with respect to its energy consumption while meeting drivability requirements.

This problem and related ones (see e.g. Guzzella and Sciarretta [2007], Sciarretta and Guzzella [2007] and references therein) have been given more and more importance as enabling embedded sensors have become available. In particular, radar ranging devices, GPS, inertial measurement units (IMU), navigation and cartographic systems with altimetry, are now considered in numerous applications (see the rendez-vous problem studied in Sciarretta and Guzzella [2005]).

In this paper, we study such a problem in which a car powered by a DC-type motor is considered. This tutorial problem can be addressed with various optimization techniques. We hope it can serve as a benchmark for future research. We give several details and modeling assumptions. Then, we propose a method to solve the proposed optimal control problem (OCP). It relies on the inversion based methodology to address such problems (see e.g. Murray et al. [2003]). The steps bringing a high order two-point boundary value problem are detailed. In a sequence, the states variables, the control, and the adjoint variables are analytically parameterized by a single unknown. Numerical results obtained with a freely available software package are given. They illustrate the merits of the proposed method, which, thanks to its relatively low computational burden (the Intel Core 2 Duo CPU usage being largely below 1 sec.), could be included in future feedback strategies, e.g. through a receding horizon approach (as in e.g. Borhan et al. [2009]).

The paper is organized as follows. In Section 2, we expose the OCP and present the model under consideration. In Section 3, we expose the solution method, by underlining the invertibility of the primal and dual systems invoked in the calculus of variations of the OCP. Finally, in Section 4, we give some conclusions and sketch future directions of research.

2. SYSTEM MODELING AND OPTIMAL CONTROL PROBLEM

The vehicle under consideration is powered by a DC-type motor. However, permanent-magnet synchronous motors can be modeled by similar equations, under certain assumptions (Guzzella and Sciarretta [2007]). We now briefly expose modeling steps yielding a two-states, single-input dynamical system, and propose motivations for an OCP to be solved.

A sketch of the vehicle with its power-train including a battery, a DC/DC converter, a motor, and a transmission is presented in Fig. 1.

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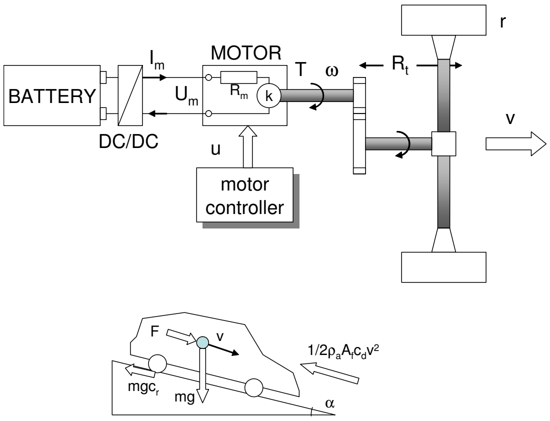

Fig. 1. Schematics of the full electric powertrain considered.

2.1 DC-type motor

The equivalent electric circuit of a DC motor armature reads

$$I_m R_m = U_m - \kappa\omega \tag{1}$$

where I_m is the armature current, R_m is the armature resistance, U_m is the armature voltage, κ is the speed constant, and ω is the rotational speed of the motor. Losses other than ohmic are neglected in this model.

The motor torque production is described by

$$\mathcal{T} = \kappa I_m \tag{2}$$

The electric power consumption P_m is calculated by expressing I_m, U_m as a function of $\mathcal{T}, \omega$,

$$P_m = I_m U_m = \omega\mathcal{T} + \frac{R_m}{\kappa^2}\mathcal{T}^2 \tag{3}$$

Define a control variable u as a percent torque demand,

$$\mathcal{T} = u\tau \tag{4}$$

where τ is the motor maximum torque. This parameter changes with motor speed, particularly if flux-weakening strategies are applied. For simplicity, this dependency is neglected here.

2.2 Transmission and vehicle model

A single-gear transmission without losses is considered here, including the final gear. Rigid wheels without slip are further assumed, from whence,

$$F = \frac{u\tau R_t}{r}, \quad \omega = \frac{vR_t}{r} \tag{5}$$

where F is the traction force at the wheels, v is the vehicle longitudinal speed, R_t is the transmission ratio, and r is the wheel radius.

Newton's third law is applied to the longitudinal direction of the vehicle body, under the action of rolling resistance, aerodynamic, and gravity forces (see Fig. 1), leading to

$$m\dot{v} = \frac{u\tau R_t}{r} - \frac{1}{2}\rho_a A_f c_d v^2 - mgc_r - mg\sin(\alpha(x)), \quad \dot{x} = v \tag{6}$$

where m is the vehicle mass, ρ_a is the external air density, A_f is the vehicle frontal area, c_d is the aerodynamic drag coefficient, c_r is the rolling resistance coefficient, α is the road slope as a function of the position x along the road, and g is the acceleration of gravity.

Equations (6) can be rewritten as

$$\dot{v} = h_1 u - h_2 v^2 - h_0 - \gamma(x), \quad \dot{x} = v \tag{7}$$

where

$$h_0 \triangleq gc_r, \quad h_1 \triangleq \frac{\tau R_t}{mr}, \quad h_2 \triangleq \frac{c_d A_f \rho_a}{2m} \tag{8}$$

and

$$\gamma(x) \triangleq g\sin(\alpha(x)) \tag{9}$$

The power consumption (3) can be rewritten as

$$P_m = b_1 uv + b_2 u^2 \tag{10}$$

where

$$b_1 \triangleq \frac{\tau R_t}{r}, \quad b_2 \triangleq \frac{R_m}{\kappa^2}\tau^2 \tag{11}$$

2.3 Optimal control problem

Considering the preceding equations, we desire to find a control strategy that minimizes the power consumption under the constraints that the vehicle must reach a destination point at a distance D in a given time T, with a zero velocity, starting from a given point, at rest. This yields the formulation of the following optimal control problem (OCP) where an integral cost and end-points state constraints come into play

$$\begin{cases} \min \int_0^T \left(b_1 u(t)v(t) + b_2 u^2(t)\right) dt \\ \dot{v} = h_1 u - h_2 v^2 - h_0 - \gamma(x) \\ \dot{x} = v \\ x(0) = 0, \quad x(T) = D \\ v(0) = v(T) = 0 \end{cases} \tag{12}$$

Note that, with respect to the optimal control of the propulsion in an ICE-base vehicle (Guzzella and Sciarretta [2007], Sciarretta and Guzzella [2005]), the term $b_2 u^2$ further appears in the power consumption to be minimized, while the state equations keep the same shape. As a further remark, in the case the electric motor would be controlled by means of its voltage, a slightly different formulation would result, implying the same shape of the cost function (with u now representing the voltage ratio with respect to the battery voltage and with a negative b_1) and an additional term in the speed state equation, which is linear in v. As it is easy to see, the methods illustrated below keep their validity to these cases as well.

3. SOLUTION METHOD

The preceding OCP (12) can be solved by numerous methods. Among these are Dynamic programming (see e.g Bertsekas [2001], Bryson [1999], Sundström and Guzzella [2009]), direct methods (e.g. collocation as exposed in Hargraves and Paris [1987]), and indirect methods (a.k.a. adjoint methods). We now briefly outline the main characteristics of these methods.

Dynamic programming is a very efficient method when applied to systems of low dimensions, such as the one considered here. Its main advantage is that is provides a feedback solution, because it generates a field of extremals covering every possible initial condition, at once. On the other hand, its computational burden is relatively high, and it can reveal troublesome to generalize to higher dimensions systems. Recently, it has been applied on several problems related to the one considered in this article (see e.g. Guzzella and Sciarretta [2007]).

In the field of trajectory generation problems, leaving out the previously discussed approach of dynamic programming, two families of numerical techniques are commonly used (see von Stryk and Bulirsch [1992a]). The direct methods imply a discretization of the optimal control problem, yielding a nonlinear program (NLP). On the other hand, indirect methods are based on the solution of necessary conditions for optimality, as derived by the calculus of variations. While direct methods have been the workhorse of control engineers (as advocated in Hargraves and Paris [1987], Betts [2001, 1998]), indirect methods are usually reported to produce higher accuracy solutions, although being relatively instable. Both approaches can be cascaded to take advantages of these properties (see Bulirsch et al. [1993], von Stryk and Bulirsch [1992b], Shen and Tsiotras [1999]).

In the following, we explain how to setup such an indirect approach using commonly and freely available software. Interestingly, this method takes advantage of the geometric structure of the system differential equations. As a result the computational burden is alleviated.

3.1 Geometric structure of the system equations

Geometric tools of nonlinear control theory (see Isidori [1989], Nijmeijer and van der Schaft [1990] for an overview of this field) have long been used for feedback linearization of control-affine systems. The induced changes of variables readily solve the inverse problems of computing inputs corresponding to a prescribed behavior of outputs. When trajectory optimization is desired, further techniques are needed.

As it has been noted in Milam [2003], Petit et al. [2001b], it is advantageous to use the geometric structure of the dynamics to solve an OCP. In particular, the *invertibility* or partial invertibility (i.e. the system may have a zero dynamics) in the input-ouput sense of geometric control of nonlinear systems (see again Isidori [1989], Nijmeijer and van der Schaft [1990]), is a key property in this context. The reason for this is as follows. In general collocation methods, coefficients are used to approximate with basis functions both states and inputs (see Hargraves and Paris [1987]). While it was known since Seywald [1994] that it is numerically efficient to eliminate the control, it was emphasized in Petit et al. [2001b], Milam [2003] that it is possible to reduce the problem further. In details, the numerical impact of the relative degree (as defined in Isidori [1989]) of the output chosen to cast the optimal control problem into a NLP was emphasized.

Choosing outputs with maximum relative degrees is the key to efficient variable elimination that lowers the number of required coefficients (see for example Petit et al. [2001a], Ross et al. [2002], Fahroo and Ross [2002], El-Kady [2003], Neckel et al. [2003], Carson III et al. [2006]). In differential equations, in constraints, and in cost functions, unnecessary variables are substituted with successive derivatives of the chosen outputs. When combined to a NLP solver (such as NPSOL by Gill et al. [1998], e. g.), this can induce drastic speed-ups in numerical solving (see Nieuwstadt [1996], Steinbach [1997], Agrawal and Faiz [1998], Milam et al. [2000], Ross and Fahroo [2002], Oldenburg and Marquardt [2002], Murray et al. [2003], Bhattacharya [2006]).

Interestingly, this (at least partial) invertibility plays a similar role in indirect methods. In Chaplais and Petit [2003], the case of single-input single-output (SISO) systems with a n-dimensional state was addressed. It was shown that r the relative degree of the primal system also plays a role in the adjoint (dual) dynamics. It appears that the two-point boundary value problem (TPBVP) implied by the stationarity conditions of the calculus of variations can be rewritten by eliminating many variables. Only $n-r$ variables are required. In the case of full feedback linearisability, the primal and adjoint dynamics take the form of a $2n$-degree differential equation in a single variable: the linearizing output. The adjoint variables are computed and eliminated. In Chaplais and Petit [2008], the general case of multi-inputs multi-outputs (MIMO) systems was addressed. Noting n the dimension of the state, m the number of inputs, and r the total relative degree. It was shown how to determine a $2n$ dimensional necessary state-space form equation for the primal and adjoint dynamics using a reduced number of variables $(m+2(n-r))$. Adjoint states corresponding to the linearizable part of the dynamics can be explicitly computed and eliminated from stationarity conditions. This is the reason for the substantial reduction of the computational burden associated to the numerical solving of the TPBVP.

In the case of our OCP (12), the 2-states with 1 input system is (fully) invertible. It is in fact already under a cascade form, x being the *linearizing output* (see Isidori [1989], Nijmeijer and van der Schaft [1990]). We take advantage of this property and determine an equivalent form of the TPBVP (17). It takes the form of a fourth order differential equation in a *single* variable to be solved along with appropriate boundary conditions.

We now explain how to proceed.

3.2 Stationarity conditions from calculus of variations

Consider the OCP (12) and introduce the Hamiltonian

$$H = b_1 uv + b_2 u^2 + \lambda_1 (h_1 u - h_2 v^2 - h_0 - \gamma(x)) + \lambda_2 v \tag{13}$$

where λ_1 and λ_2 are two adjoint variables.

Following e.g. Bryson [1999], the calculus of variations, in the absence of state or input constraints yields the following stationarity conditions, $\forall t \in [0, T]$,

$$\frac{\partial H}{\partial u} = 0 \tag{14}$$

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial v} \tag{15}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial x} \tag{16}$$

These take the form

$$
\begin{aligned}
u &= -\frac{\lambda_1 h_1 + b_1 v}{2b_2} \\
\dot{\lambda}_1 &= -b_1 u + 2\lambda_1 h_2 v - \lambda_2 \\
\dot{\lambda}_2 &= \lambda_1 \frac{d\gamma}{dx}
\end{aligned}
$$

Together with the primal equations found in (12), they constitute the following TPBVP

$$
\left.
\begin{aligned}
&\dot{v} = -\frac{\lambda_1 h_1 + c_1 v}{2b_2} h_1 - h_2 v^2 - h_0 - \gamma(x) \\
&\dot{x} = v \\
&\dot{\lambda}_1 = b_1 \frac{\lambda_1 h_1 + c_1 v}{2b_2} + 2\lambda_1 h_2 v - \lambda_2 \\
&\dot{\lambda}_2 = \lambda_1 \gamma'(x) \\
&x(0) = 0, \quad x(T) = D \\
&v(0) = v(T) = 0
\end{aligned}
\right\} \tag{17}
$$

This set of 4 first-order differential equations in the 4 unknowns $(x, v, \lambda_1, \lambda_2)$ with 4 endpoint conditions, must be solved to eventually determine the optimal control

$$
u = -\frac{\lambda_1 h_1 + c_1 v}{2b_2}
$$

3.3 Determination of a high order form of the TPBVP

As will now appear, the TPBVP (17) can be rewritten in a much more compact form.

All the variables, including the primal variables x, v, u and the dual variables λ_1, λ_2 can be rewritten, under the assumption that the stationarity conditions hold, using x and its derivatives (see Equations (18),(19),(20),(21) below). Then, we will determine the fourth-order differential equation satisfied by x (see Equation (22)), arising from

$$
\frac{d^2}{dt^2}\left(\frac{\partial H}{\partial u}\right) = 0
$$

In view of the computer implementation, it will be rewritten under the factorized form (22).

Explicit derivation of the primal and dual variables along extremals From the beginning, it is known that the system dynamics, which are already under a cascade form, are invertible using x as linearizing output. In other words, x, v, and u can be written using x and its derivatives. From this, it is a general result, see Chaplais and Petit [2008], that all the dual variables can also be determined from x and its derivatives.

From the system dynamics, one has

$$
v = \dot{x} \tag{18}
$$

$$
\begin{aligned}
u &= \frac{1}{h_1}\left(\dot{v} + h_2 v^2 + h_0 + \gamma(x)\right) \\
&= \frac{1}{h_1}\left(\ddot{x} + h_2.(\dot{x})^2 + h_0 + \gamma(x)\right)
\end{aligned} \tag{19}
$$

Then, from the stationarity conditions, starting with $\frac{\partial H}{\partial u} = 0$, one obtains

$$
\begin{aligned}
\lambda_1 &= -\frac{1}{h_1}\left(b_1 v + 2b_2 u\right) \\
&= -\frac{1}{h_1}\left(b_1 \dot{x} + 2\frac{b_2}{h_1}\left(\ddot{x} + h_2.\left(\dot{x}\right)^2 + h_0 + \gamma(x)\right)\right)
\end{aligned} \tag{20}
$$

Then, $\frac{d}{dt}\left(\frac{\partial H}{\partial u}\right) = 0$ yields

$$
\begin{aligned}
\lambda_2 &= -b_1 u + 2\lambda_1 h_2 \dot{x} + 2\frac{b_2}{h_1^2}\left(x^{(3)} + 2h_2 \dot{x}\ddot{x} + \dot{x}\gamma'(x)\right) + \frac{b_1}{h_1}\ddot{x} \\
&= \frac{-b_1}{h_1}\left(h_2.\left(\dot{x}\right)^2 + h_0 + \gamma(x)\right) \\
&\quad + 2\lambda_1 h_2 v + \frac{2b_2}{h_1^2}\left(x^{(3)} + 2h_2\dot{x}\ddot{x} + \gamma'(x)\dot{x}\right)
\end{aligned} \tag{21}
$$

Finally, from the stationarity equation (recall that $\frac{d}{dt}\left(\frac{\partial H}{\partial u}\right) = 0$ holds for all $t \in [0, T]$)

$$
\frac{d^2}{dt^2}\left(\frac{\partial H}{\partial u}\right) = 0
$$

we obtain the high (fourth) order differential equation to be satisfied by x. It is rewritten under the factorized form (22)

$$
\begin{aligned}
Ax^{(4)} &+ B(\dot{x})x^{(3)} + C(x, \dot{x}, \ddot{x})x^{(2)} \\
&+ D(x, \dot{x}, \ddot{x}, x^{(3)})\dot{x} - h_1 \lambda_1(x, \dot{x}, \ddot{x})\gamma'(x) = 0
\end{aligned} \tag{22}
$$

with (after some easy computations)

$$
\begin{aligned}
A &= 2\frac{b_2}{h_1} \\
B &= 4\frac{b_2 h_2}{h_1}\dot{x} \\
C &= 2\frac{b_2}{h_1}\gamma'(x) + 4\frac{h_2 b_2}{h_1}\ddot{x} - 2h_2 b_1 \dot{x} + 2h_1 h_2 \lambda_1 \\
D &= 2\gamma''(x)\frac{b_2}{h_1}\dot{x} - b_1\gamma'(x) + 2h_1 h_2\left(-b_1 u + 2\lambda_1 h_2 \dot{x} - \lambda_2\right)
\end{aligned}
$$

The endpoints conditions associated to the single variable fourth-order differential equations are

$$
x(0) = 0 = \dot{x}(0) = \dot{x}(T) = 0, \quad x(T) = D \tag{23}
$$

A simple particular case For sake of checking the analytical computations performed above and their forthcoming numerical implementation, it is instructive to consider the particular case where the aerodynamic friction and the slope of the road are null, i.e. $h_2 = 0$, $\gamma(x) = 0$. Then, the fourth order differential equation (22) is simply

$$
x^{(4)} = 0 \tag{24}
$$

In other words, $[0, T] \ni t \mapsto x(t)$ is the interpolating polynomial of third order, whose coefficients are readily determined from the boundary conditions (23).

The adjoint variables are

$$
\lambda_1 = -\frac{1}{h_1}\left(b_1\dot{x} + 2\frac{b_2}{h_1}(\ddot{x} + h_0)\right) \tag{25}
$$

$$
\lambda_2 = -\frac{b_1}{h_1}h_0 + 2\frac{b_2}{h_1^2}x^{(3)} \tag{26}
$$

In details, the variables histories are

$$
\begin{aligned}
x(t) =& 3\frac{D}{T^2}t^2 - 2\frac{D}{T^3}t^3 \\
v(t) =& 6\frac{D}{T^2}t - 6\frac{D}{T^3}t^2 \\
u(t) =& \frac{1}{h_1}\left(6\frac{D}{T^2} - 12\frac{D}{T^3}t + h_0\right) \\
\lambda_1(t) =& -\frac{1}{h_1}\big(b_1(6\frac{D}{T^2}t - 6\frac{D}{T^3}t^2) \\
& + 2\frac{b_2}{h_1}((6\frac{D}{T^2} - 12\frac{D}{T^3}t) + h_0)\big) \\
\lambda_2(t) =& \left(-h_0\frac{b_1}{h_1} - 24\frac{b_2 D}{h_1^2 T^3}\right)
\end{aligned}
$$

3.4 Numerical implementation

The high order TPBVP (22)-(23) can be solved using the freely distributed code COLNEW by Ascher et al. [1995] which is implemented in the standard `bvode` routine in the Scilab software package. Interestingly, new software packages are now available (Kitzhofer et al. [2009]) in the Matlab environment. Before presenting some obtained results, we would like to mention some significant impact on convergence that dealing with such a high order representation of the system of differential equations has (see also Chaplais and Petit [2008] for more discussions).

Performance of solvers on high order representations of TPBVP In [Ascher et al., 1995, section 5.6] numerical schemes for solving boundary value problems for high order differential equations are studied. A collocation scheme is proposed along with various implementations. A first convergence result for linear boundary value problems is proven [1]. Note p the regularity of the coefficients of the linear differential system, and m its order. Approximate solutions are sought after among piecewise polynomials of degree $k+m$. There are k collocation points, and h corresponds to the mesh size. Under an orthogonality condition on the collocation points, the following error estimates are derived in [Ascher et al., 1995, theorem 5.140]. At the mesh points x_i

$$|u^{(j)}(x_i) - u_\pi^{(j)}(x_i)| = O(h^p), \quad O \leq j \leq m-1 \tag{27}$$

where u is the exact solution of the TPBVP problem and u_π is the approximate solution obtained through the collocation scheme for the high order system. Outside the mesh points, one has

$$|u^{(j)}(x) - u_\pi^{(j)}(x)| = O(h^{k+m-j}) + O(h^p), \quad O \leq j \leq m-1 \tag{28}$$

Interestingly, if one chooses to use the proposed collocation method on an equivalent state-space form, (27) remains unchanged, but (28) is replaced by

$$|y(x) - y_\pi(x)| = O(h^{k+1}) + O(h^p) \tag{29}$$

where y (resp. y_π) is the exact (resp. approximate collocation) solution of the equivalent state-space form TPBVP (y is the concatenation of the derivatives of u from order 0 to $m-1$ see [Ascher et al., 1995, pages 220-222]). In terms of convergence, the upper bound of (28) is better than (29). If p is large enough, it clearly appears that *the collocation method for the high order system is more accurate* than the collocation method for the state-space form at points outside the mesh.

Some numerical results The differential equation (22) with boundary conditions (23) in the single unknown x is implemented in Scilab.

The obtained results are reproduced in Figure 2. The following value for the parameters were considered

$$b_1 = 10^3; b_2 = 10^3; h_0 = 0.1; h_1 = 1; h_2 = 10^{-3};$$

with

$$D = 10; T = 1$$

and

$$\gamma(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3; \tag{30}$$

with

$$p_0 = 3; p_1 = 0.4; p_2 = -1; p_3 = 0.1;$$

The computational burden is very low [2]. On a standard laptop equipped with an Intel Core 2 Duo 2.80 MHz with 4 GB of RAM, the whole computation, starting from 0 has initial guess, took less than 200 ms. The routine `bvode` was called with the following parameters: $ncomp = 1$ number of differential equations, $m = 4$, $aleft = 0$, $aright = T$, 7 collocation points per subinterval, 1 subinterval in the initial mesh, a single tolerance of 10^{-7} on the differential equation. The factors A, B, C and D appearing in (22) were analytically differentiated with respect to their arguments as the routine `bvode` makes frequent call to their derivatives.

As a check, the computed value of the Hamiltonian (13), which is constant along the extremals in theory, ranges from -3630425.5 to -3630425.2. This stresses the accuracy of the employed method. The cost value is, after a trapezoidal integration, 1228586.7

The obtained solution presents the expected features the optimal should have: a steady increase of speed followed by a decrease to reach the zero velocity at the endpoint. The velocity history is not symmetrical, due to the asymmetry of the slope profile over time.

4. CONCLUSION

In this paper, an optimal control problem for an electric vehicle has been considered and treated by an inversion-based trajectory generation approach. The dynamics and the cost functional under consideration have been formulated in a relatively simple and compact form, for sake of proposing a benchmark problem for future works. Yet, numerous efforts have been made to make this formulation representative of true vehicle dynamics. The inversion-based method has proven effective and accurate. Interestingly, the case of state constraints, which is of importance for applications and usually difficult to address both from a theoretical and a practical viewpoint, can be treated using saturation functions in the presented dynamics (the reader can refer to Graichen et al. [2010], Graichen and Petit [2009] for details) which are directly incorporated in

[1] These approaches are then extended to the nonlinear case using quasi-linearization and a Newton method to solve the nonlinear problem. Roundoff errors depend on which functions basis is used for collocation. This is beyond the scope of this remark; interested readers can refer to [Ascher et al., 1995, section 5.6.4].

[2] The interested reader can contact the authors to have access to the implementation code.

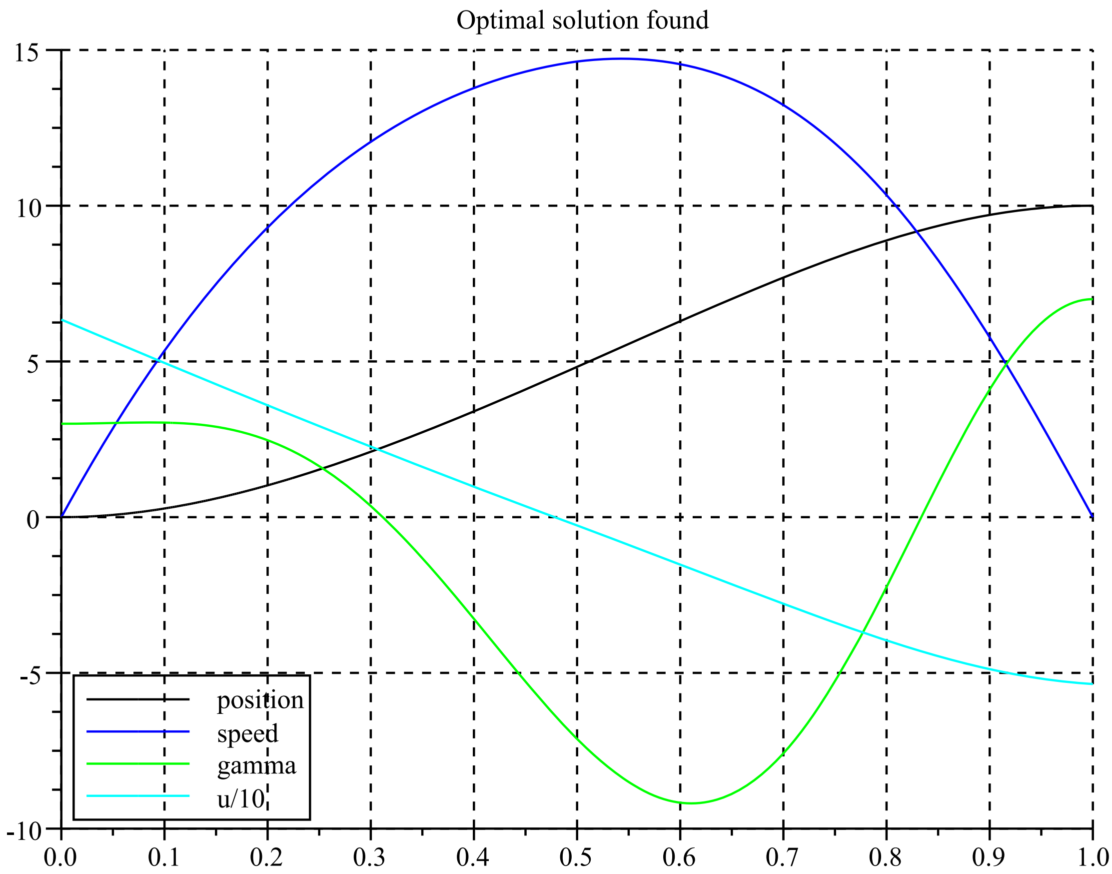

Fig. 2. Numerical results

the calculus of variations. Further works will be focused on considering more general cases (most likely constrained as previously discussed), over longer time horizons, to account for complex trajectories involving various type of roads, and numerous stops and starts. Addressing uncertainties is also a topic of importance, and could be done using several approaches, including observers, and receding horizon techniques, to name a few.

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