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ABSTRACT

During retail stage of food supply chain (FSC), food waste and stock-outs occur mainly due to inaccurate sales forecasting which leads to inappropriate ordering of products. The daily demand for a fresh food product is affected by external factors, such as seasonality, price reductions and holidays. In order to overcome this complexity and inaccuracy, the sales forecasting should try to consider all the possible demand influencing factors. The objective of this study is to develop a Seasonal Autoregressive Integrated Moving Average with external variables (SARIMAX) model which tries to account all the effects due to the demand influencing factors, to forecast the daily sales of perishable foods in a retail store. With respect to performance measures, it is found that the proposed SARIMAX model improves the traditional Seasonal Autoregressive Integrated Moving Average (SARIMA) model.

Keywords: SARIMAX; Seasonal Autoregressive Integrated Moving Average with External Variables; Sales Forecasting; Food Retail Industry; Perishable Foods; Time Series Sales; Influencing Factors; Food Waste

INTRODUCTION

Discount retail stores have been a noticeable feature of German retail market since the 1980s. In particular, the growth in number of discount retail stores have significantly increased after reunification of Germany. Recently, there is a growing trend of increasing varieties of fruits and vegetables with year-around availability across all the German discount retail outlets rather than just in their traditional growing season. In order to attract customers and remain competitive in the market, the fruits and vegetables are exported from foreign countries and stocked for longer periods. Particularly, increase in number of retail stores, availability of varieties of fruits and vegetables (in stock) with short shelf-lives, frequent price variations, and different storage conditions increase the complexity and results in huge amount of food waste. In Germany, the retail sector produces the food waste of around 0.5 million tons per year (Kranert et al., 2012). Although the retail sector contributes only 5% of the total food waste in food supply chain, mostly they are avoidable food waste (wasting food which is fit for consumption). The quantity of food waste that occurs in the home (61%) is partially due to the management decisions in the retail sector (e.g. frequent promotions) that stimulate the consumer’s eagerness to purchase, and distract them to equate their demand with the purchase (Arunraj et al., 2014; Gooch et al., 2010). Hence, the proper decision making in the retail sector can help the suppliers and consumers to avoid the food waste. The role of sales forecasting in reducing the food waste in retail stores is a significant topic of discussion in the recent food waste related studies (Mena et al., 2011; Mena et al., 2014). According to Mena et al. (2011) and Stenmarck et al. (2011), the improvement of forecast accuracy is one of the essential remedial measures to reduce the food waste in the retail sector of food supply chain.

Presently, the food retail companies use from simple informal methods to complex scientific approaches to forecast the sales. The forecastability of demand highly depends on the volatility of demand (Gilliland & Sglavo, 2010). If the demand is stable and smooth, it can be forecasted precisely even with simple statistical methods. At the same time, if the demand is irregular and random, it is irrational to anticipate greater accuracy. The forecasting process can be improved not only by making it more accurate, but also by increasing its efficiency by using fewer resources (Gilliland & Sglavo, 2010; Gilliland, 2011). Therefore, there is a need for trade-off between forecast accuracy and complexity of the system in terms of data processing and modeling for forecast (Ali et al., 2009). While selecting a forecasting model, the major factors to be taken into account are time horizon to forecast, technical resources and cost affordability, availability and requirement of data, variability and consistency of data, accuracy, timing, and form (Herbig et al., 1993). To make a forecast of demand in the retail industry, the mentionable advantages of time series forecasting models are easy implementation and better interpretation with reasonable accuracy, when compared to other competitive approaches (Liu et al., 2001; Cools et al., 2009; Lee & Hamzah, 2010; Shukla & Jharkharia, 2013). The time series forecasting models use the past movements of variables in order to predict their future values. When the time series exhibits a seasonal variation, the SARIMA model is usually applied. It has the capability to incorporate both seasonal and non-seasonal factors in a multiplicative model. However, the SARIMA model is not sufficient to forecast the time series which are influenced by the external factors. In order to improve the forecast accuracy, the external demand influencing factors have to be incorporated in the forecasting model. In the past studies, some of the literature identified this issue in the time series forecasting models (Aburto & Weber, 2007; Bratina & Faganel, 2008; Cools et al., 2009; Lee & Hamzah, 2010; Ramanathan & Muyldermans, 2010; Chikobvu & Sigauke, 2012;Chadsuthi et al., 2012; Peter & Silvia, 2012; Kongcharoen & Kruangpradit, 2013; Nasiru et al., 2013; Trancart et al., 2013; Hamjah, 2014). In this study, a SARIMA model with the external factors (SARIMAX) is proposed to overcome the disadvantage of the traditional SARIMA model, in forecasting the daily sales of fresh foods in a retail store. To investigate the applicability of this model, the daily sales data of a perishable food (e.g. banana) from a discount retail store in Lower Bavaria, Germany are used in this study. From economical perspective, the SARIMAX model can help the retail managers to forecast the sales with better accuracy, so that the stock-outs and the food waste can be reduced.

The structure of this study is as follows: Section 2 presents a recent literature review on SARIMAX models and the factors affecting the demand and forecast accuracy. Section 3 presents development of the SARIMAX model. Section 4 presents the data collection and analysis. Section 5 describes implementation of the SARIMAX model in a discount retail store. Section 6 presents results and discussion. Finally, the paper is concluded in section 7.

LITERATURE REVIEW

In earlier studies, the traditional and hybrid SARIMA models were frequently applied to forecast the demand in food retail industry. Aburto & Weber, (2003, 2007) presented an additive hybrid SARIMA and neural network to forecast demand in a Chilean supermarket. In their forecasting model, the errors of SARIMA process was modeled by neural network, where payment, intermediate payment, before holidays, holidays, festivals, school vacation, climate, price variables were considered as input neurons. They compared their proposed model with naïve, seasonal naïve, unconditional average, SARIMAX, and several neural network models. Bratina & Faganel (2008) developed an ARMAX model (Auto Regressive Moving Average with external variables) to forecast daily demand for beer on Slovenian market. The sales of beer usually depends on the weather, so the temperature was used in their model to capture the sales during summer (yearly seasonality). The other external variables such as New Year and price promotions were also regressed as dummy variables. Lee & Hamzah (2010) developed an ARIMAX model to forecast monthly sales of Muslim boys' clothes in Indonesia. This model integrates ARIMA model with calendar variation effect during Islamic Eid holidays using linear regression. The results of their study showed that the proposed ARIMAX model had better forecast at out-sample data when compared to decomposition method, SARIMA, and neural network. Cornelsen & Normand (2012) used an ARIMAX model, not to forecast, but to estimate the impact of smoking ban on bars in Ireland. In their model, the demand in bars is explained by ARIMA model with relative prices in bars, prices of alcohol sold in off-licences and the aggregate retail sales (to represent general economic activity and incomes) as external variables. Shukla & Jharkharia (2013) applied ARIMA model to forecast the demand for vegetable (onion and potato) on daily basis in an Indian wholesale vegetable market. Da Veiga et al., (2014) applied ARIMA and Holt-Winters (HW) models to forecast a time series of a group of perishable dairy products. In their study, the HW model performed better than the ARIMA model based on the performance metrics such as MAPE and Theil's U statistics.

Apart from forecasting the market demand, the SARIMAX models were also used as forecasting tools in diverse fields of application. Cools et al. (2009) developed ARIMAX and SARIMAX models to forecast the daily traffic counts. The seasonality in the daily traffic data and the effects of holiday at different site locations were analyzed in their study. The inclusion of weekly seasonality and holiday effects at different site locations revealed that both ARIMAX and SARIMAX models are better frameworks. Adanacioglu & Yercan (2012) analyzed the seasonal price variation of tomato and developed a SARIMA model to forecast the monthly tomato prices at wholesale level in Antalya, Turkey. Chikobvu & Sigauke (2012) predicted daily peak electricity demand in South Africa using SARIMA and regression-SARIMA. They considered weekdays and holidays as external variables in regression-SARIMA model. Their results suggest that the regression-SARIMA model can be used to identify the importance of predictor variables. They also notified the regression-SARIMA model can be improved further by including weather parameters into the model. Peter & Silvia (2012) employed ARIMA and

ARIMAX models to forecast quarterly gross domestic product (GDP) per capita. They used unemployment rate as external input variable. Based on the forecast error results, they concluded that ARIMAX model predicts better than ARIMA model. Trancart et al. (2013) forecasted the migration of silver eel using SARIMAX model from two fishing sites in Brittany (north-western France). The purpose of their forecasting model is to reduce mortality of migratory fish and to optimize the shutdowns of turbines. The external factors such as rainfall, atmospheric pressure, and water temperature were used as covariates in their proposed SARIMAX model. Deutsche bundesbank (2013) applied regression-ARIMA (ARIMAX) model to forecast the monthly volume of coins in circulation that were issued in Germany. The dummy variables for months and holidays were considered as external variables. Kongcharoen & Kruangpradit (2013) used ARIMAX model to forecast the monthly exports from Thailand to its trading partners. They used composite leading indicator as external variable to improve the performance of ARIMA. Nasiru et al. (2013) applied ARIMAX model to predict monthly currency in circulation in Ghana, where the months were considered as external dummy variables. Hamjah (2014) used ARIMA model along with climatic variables as regressors to measure the climatic effects on different types of pulse crops production in Bangladesh. They considered sun shine, minimum and maximum temperature, rainfall, relative humidity and cloud coverage as climatic variables in their forecasting model. The above mentioned literature review is summarized in Table 1.

Factors Affecting Demand and Forecast Accuracy

Generally, there are two characteristics of forecasting (i) expected demand and (ii) degree of accuracy (Stevenson, 2007). In food retail industry, the expected demand can be a function of different kinds of structural variations like trend or seasonality. The regular demand patterns of customers may be affected by price variations and weather. The demand peaks may occur due to promotions and holidays. Besides, the demand cannot be measured directly in food retail stores, because the customers are not placing orders, they just buy what are on the shelves. Hence, the actual sales are assumed to be true demand (in this case of assumption, when the stock-outs occur, the true demand will be underestimated).

The important factors which affects the expected demand can be classified and explained as follows (Vorst et al., 1998):

i. Trend and seasonal demand patterns
ii. Price variations and reductions: Changes in market may lead to price variations which in turn affect the customer demand patterns. The price reductions may be planned or unplanned, encourage the customers to purchase more and make the demand more volatile. If the price reductions are not planned well, they lead to unnecessary warehousing and food waste (Armstrong, 2001).
iii. Holidays: Changes in demand due to holidays and festivals depend on location, religion, demography, and cultural habits of customers. During the festival season, the retail stores located in or near touristic places and country borders may have higher demand variability due to tourist visits and cross-border shopping respectively.
iv. Weather: Extreme weather such as rainfall, snowfall, very hot and cold temperatures disturbs the customer purchase behavior. It keeps the customers in home or forces them to visit their nearby retail stores. If the weather variables are used in the sales forecasting, the quality of weather forecast also influences the forecast accuracy.
v. Substitution and cannibalization: Generally, the effects of substitution and cannibalization are inversely proportional to the forecast accuracy. For example, when a similar type of product is stock-out or has promotions, it may affect the sales of the considered product. Apart from this, the introduction of new products and private labelled products with lower price also varies the customer demand patterns.

In addition to these factors, if the new distribution channels evolve and/or the existing distribution channels expand in FSC, there may be a change in demand (EyeOn, 2013). From the above mentioned factors, the price reduction of a product and/or other related products and changes in weather patterns may produce short-term shocks in time series (variation in seasonality) with or without leading effect. On the other hand, the changes in distribution channels and introduction of new products may lead to long-term variation (shift in existing products demand).

The forecast accuracy is eventually limited by the nature of the time series being forecast. In other words, the forecast accuracy depends on

i. Data availability: The availability of complete and longer historical data is important to identify and understand the external factors which affects the sales.
ii. Data quality: The forecast model highly depends on quality of input data (training and testing data). However, the quality of estimation-period data also plays a crucial role to forecast better (e.g. quality of weather forecast data during prediction).

iii. Forecast horizon: Long forecast horizon may increase uncertainty leading to high inaccuracy in forecast.

Generally, the sales forecast inaccuracies result in two types of problems (Agnew & Thornes, 1995): understocking and overstocking. The understocking leads to stock-outs as well as lower customer confidence and deterioration of market image which are difficult to quantify. The overstocking leads to insufficient shelf space, shrinkage, and food waste, especially in perishable foods. The factors like short shelf-life and bad product quality usually intensifies the amount of food waste. In food retail stores, there is always a tension between availability and wastage of perishable foods, due to their short shelf life. The factors influencing demand and forecasting accuracy can be summed up together to visualize the reasons for the food waste in retail industry as shown in Figure 1 (EyeOn, 2013). As shown in this figure, when there is a simultaneous increase in pressure on all the outside “factor ballons”, the central ballon has to accept all the air, i.e. when there is a change in trend and seasonality, longer forecast horizon, frequent price variations and reductions, extreme weather, short shelf-life of product, poor product quality, change in distribution channels or introduction of new products, and limited data availability, the food waste happens in huge amount.

DEVELOPMENT OF SARIMAX MODEL

A non-seasonal ARIMA (p,d,q) model represents a time series with *p* autoregressive terms, *q* moving average terms and *d* non-seasonal differences. It can be expressed as (Cools et al., 2009; Aburto & Weber, 2007)

$$\phi_p(B)(1-B)^d Z_t = c + \theta_q(B)\varepsilon_t \qquad (1)$$

where, B - Delay or lag operator, time series observation lag k period is symbolized $B^k X_t = X_{t-k}$
$\phi_p(B)$ - Autoregressive operator of p-order
$(1-\phi_1(B)-\phi_2(B^2)-\cdots-\phi_p(B^p))$
$\theta_q(B)$ - Moving average operator of q-order
$(1-\theta_1(B)-\theta_2(B^2)-\cdots-\theta_q(B^q))$
$(1-B)^d$- Differencing operator of order d to remove non-seasonal stationarity
Z_t - Sales of a product at time t
ε_t - Residual error in SARIMA model
c - Constant
The SARIMA model can be represented as (Cools et al., 2009; Aburto & Weber, 2007)

$$\phi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D Z_t = \theta_q(B)\Theta_Q(B^S)\varepsilon_t \qquad (2)$$

where, $\Phi_p(B)$ - Seasonal autoregressive operator with p-order
$\Theta_q(B)$ - Seasonal moving average operator with q-order
$(1-B)^D$ - Seasonal differencing operator of order D
$(1-B)^d$ - Differencing operator of order d
S - Seasonal length (e.g. in quarterly data s=4 and in monthly data s=12)

The distinct advantage of SARIMA approach is its capability to handle stationary and non-stationary time series with seasonal elements. The generation of time series forecasts using SARIMA is better if no outlying data occur. Based on the behavior of the time series, outliers could have potential impact on the estimates on the model parameters. The outlying data in a time series may often point out significant events or exceptions and provide useful information to the management. Therefore, it is important to consider external variables, which deliver meaningful answers to the outlying data. As an alternative of modeling a time series Y_t with only a combination of past values, Y_t can be explained by both SARIMA and external variables (regressors). In this study, the SARIMAX model is used to forecast the daily time series using Box-Jenkins SARIMA approach and multiple linear regression (MLR).

The SARIMAX model is a SARIMA model with external variables, called SARIMAX (p,d,q) $(P,D,Q)_S$ (X), where X is the vector of external variables. The external variables can be modeled by multi linear regression equation is expressed as

$$Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \cdots + \beta_k X_{k,t} + \omega_t \qquad (3)$$

where, $X_{1,t}, X_{2,t}, \ldots . X_{k,t}$ are observations of *k* number of external variables corresponding to the dependent variable Y_t; $\beta_0, \beta_1, \ldots . \beta_k$ are regression coefficients of external variables; ω_t is a stochastic residual, i.e. the residual series that is independent of input series. The residual series ω_t can be represented in the form of ARIMA model as follows (Peter & Silvia, 2012)

$$\omega_t = \frac{\theta_q(B)\Theta_Q(B^S)}{\phi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D}\varepsilon_t \qquad (4)$$

The general SARIMAX model equation can be obtained by substituting Equation 4 in Equation 3 (Cools et al., 2009; Aburto & Weber, 2007)

$$Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \cdots + \beta_k X_{k,t} + \left(\frac{\theta_q(B)\Theta_Q(B^S)}{\phi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D} \varepsilon_t \right) \quad (5)$$

In this case of model, the regression coefficient can be interpreted as in usual and easier way (Hyndman, 2010).

The general SARIMAX model consists of five iterative steps (Box et al., 2008; Nasiru et al., 2013; Shukla & Jharkharia, 2013):

(1) *Model identification:* This step involves selection of the order of differencing (d), order of seasonal differencing (D), seasonal length (S), non-seasonal autoregressive order (p), seasonal autoregressive order (P), non-seasonal moving average order (q), and seasonal moving average order (Q). Autocorrelations function (ACF) and partial autocorrelations function (PACF) are used to identify the model.

(2) *Parameter estimation:* The parameters of the identified model from step 1 are estimated.

(3) *Diagnosis the fitness of model:* The model is diagnosed using Ljung-Box Q statistic to check the adequacy. If the residuals are not normally distributed, go to step 4. Otherwise, proceed to step 5.

(4) *Inclusion of external variables:* The relevant external variables are included into the SARIMA model using linear regression. To diagnose the model, go to step 4.

(5) *Forecasting and validation:* The diagnosed model is validated using out-sample. The validated model is used for forecasting the future values.

Performance Measures

Gilliland & Sglavo (2010) suggested the use of a simple benchmark model (random walk, seasonal random walk, or moving average model) to mark the acceptable level of forecasting performance. If the proposed model does not perform up to the acceptable level, then it can be substituted by the benchmark model itself. Gilliland (2011) recommended a forecast value added (FVA) analysis to evaluate the performance of forecasting process. In their FVA analysis, the performance measures of forecasting models are compared to determine that the addition of factors is having effect on the forecasting performance or not. The FVA analysis insists the trade-off between the amounts of resources utilized on forecasting and the amounts of paybacks from them.

For the purpose of evaluation, the out-sample data is used to provide a fair cross validation. In this study, mean absolute percentage error (MAPE) and root mean squared error (RMSE) are used as the performance measures of forecast accuracy. The MAPE is expressed as

$$\frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t - \widehat{Y}_t}{Y_t}\right| \quad (6)$$

and the RMSE is expressed as (Liu et al., 2001)

$$\sqrt{\frac{\sum_{t=1}^{n}(Y_t - \hat{Y}_t)^2}{n}} \quad (7)$$

where, $\widehat{Y}_t$ is the forecast and n is the number of observations.

In this study, Theil's U statistic is usually used to gauge the ability of a forecasting model to be more accurate than a naïve model which is considered as a point of reference (Da Veiga et al., 2014). The Theil's U statistic is presented in two specifications (U_1, U_2) and are defined as

$$U_1 = \frac{\sqrt{\frac{\sum_{t=1}^{n}\left(Y_t - \widehat{Y}_t\right)^2}{n}}}{\sqrt{\frac{\sum_{t=1}^{n}(Y_t)^2}{n}} + \sqrt{\frac{\sum_{t=1}^{n}\left(\widehat{Y}_t\right)^2}{n}}} \quad (8)$$

$$U_2 = \frac{\sqrt{\frac{\sum_{t=1}^{n-1}\left(\frac{Y_{t+1} - \widehat{Y_{t+1}}}{Y_t}\right)^2}{n}}}{\sqrt{\frac{\sum_{t=1}^{n-1}\left(\frac{Y_{t+1} - Y_t}{Y_t}\right)^2}{n}}} \quad (9)$$

where, Y_{t+1}, $\widehat{Y_{t+1}}$ are actual and forecast values for the time period $(t+1)$. The U_1 statistic lies between 0 and 1. When it is closer to zero, indicates that the forecast accuracy is good. The U_2 statistic will take a value of 1 under naïve forecasting method. The value of U_2 less than one indicates greater the forecast accuracy.

DATA COLLECTION AND ANALYSIS

This section presents the data collection and analysis. In this study, the daily sales data of banana (in kilograms) is collected from a typical discount retail store in the region of Lower Bavaria, Germany (from December 2009 to August 2014) to investigate and evaluate the forecasting model. Banana is selected for this purpose of investigation, because of its short perishable time (shelf life of 2 – 3 days) and its availability in market throughout the year. In

Germany, banana is the most popular imported fruit and the second highly consumed fruit, next to apple.

Seasonality

In retail industry, a thorough understanding of the seasonal sales patterns can be very useful to increase the accuracy of sales forecasting. The retail stores are usually closed on Sundays in Germany, so the daily sales data from Monday to Saturday are used to represent the sales pattern in a week, i.e. the seasonal length is taken as 6. To observe the sales pattern in week, the median sales from Monday to Saturday are plotted in Figure 2. From this figure, it is evident that the first and second highest sales day for banana are Friday and Saturday. The median sales of banana during regular months are presented in Figure 3. With reference to the literature review, there are many influencing factors which affect the forecasting and ordering system. This study takes into account some of the controllable factors such as price reduction and holiday effects, which are explained in following subsections. Due to lack of data, cannibalization and weather effects are not taken into consideration.

Holiday Effects

The holiday effects can be divided into four different categories such as regular holidays, Christmas, Easter and school vacations. The regular holidays are nine official holidays in Lower Bavaria, Germany (except Easter and Christmas holidays). The stores are closed during regular holidays, Easter and Christmas. The missing sales data during holidays and festivals are filled with the estimated values from trend using linear interpolation, in order to maintain the alignment of periodicity. To evaluate the holiday effect, it is classified into before holiday, after holiday, before Christmas, after Christmas, before Good Friday, after Good Friday, after Easter Monday and school vacations.

Price Reduction Effects

Apart from holiday effects, price reduction is an important factor which influences the sales. In this study, two types of price reductions are assumed to be offered in retail stores: (i) discount price reduction and (ii) promotional price reduction. The discount price reduction can be defined as the price reduction that is done by the store manager on banana after its two or three days of shelf life. The weighted average discounted sales in a day can be determined by dividing the sum of the products of percentage of discounts and discounted sales with the sum of percentage discounts. The promotional price reduction can be defined as the price reduction during promotional offers or advertisements. This can be expressed by the percentage of price reduction in a day during promotional period.

APPLICATION OF SARIMAX MODEL

In this section, the SARIMAX model is applied to a German case study to forecast the daily sales of banana. For the purpose of cross-validation, the time series dataset are classified into training data and testing data. The training and testing data contain 80% and 20% percent of the dataset respectively. The training data of banana sales are investigated to identify the auto-regressive, moving average and differencing orders of the SARIMAX model. The autocorrelation and partial autocorrelation plots for banana sales are shown in Figure 4 and 5 respectively. To determine the requirement of differencing, the stationarity of time series can be checked using Augmented Dickey-Fuller test (ADF test) (Hyndman & Athana-sopou-los, 2013). From the ADF test results, it is found that the time series of sales of banana is stationary, i.e. the p-value is less than 0.05. After investigating ACF and PACF plots, SARIMA (0,0,3) (1,0,1) (6) is selected as the best forecasting model that fit the training dataset well with minimum values of performance measures through trial and error. Based on Equation 2, the selected SARIMA model can be written as

$$(1 - 0.998B^6)\ Z_t = 61.05 + (1 + 0.361B + 0.340B^2 + 0.171B^3 + 0.128B^4)\ (1 - 0.962B^6) + \varepsilon_t \quad (10)$$

Using the method of maximum likelihood, the parameters of SARIMA model are estimated. The p-values of the parameters of the model shows that the seasonal and non-seasonal AR and MA values significant at 5% level. In Table 2, the performance measures of the model for in-sample and out-sample are presented. The adjusted coefficient of determination (adjusted R^2) for this model is determined as 0.386. The model is diagnosed using the Ljung-Box Q statistic which indicates that the residuals are not white noise, as the p-value of test statistic does not exceed the 5% level of significance. This is due to the effect of external variables such as holidays and price reduction on the sales.

The forecasting performance of existing SARIMA model can be enhanced by adding the external variables. Therefore, the predicted sales using SARIMA model is regressed against the response variable along with the considered external variables. The daily sales of banana is fitted using SARIMA and linear regression model, where the errors of SARIMA model are explained by the independent variables such as holidays, price reductions, and months. The classified effects of holiday such as before holiday effect, after holiday effect, before Christmas, after Christmas, before Good Friday, after Good Friday, after Easter Monday and school vacations are represented as eight dummy variables (0 or 1) in this model, i.e. if there is an effect, then 1, otherwise 0. The price reduction effect are represented by the discounted sales and the percentage of price reduction due to promotion. The months are represented as eleven dummy variables (0 or 1). January is taken as reference month. In this study, the models are built using SPSS 22. In Table 3, the coefficients of the SARIMAX model factors are estimated by ordinary least square method and the regression results are presented. Using the parameters from Table 3, the SARIMAX can be represented as

$$(1 - 0.998B^6)\,(Y_t - 1.712 * \text{Promotional price reduction} + 31.117 * \text{after Christmas} - 24.538 * \text{before Good Friday} - 33.418 * \text{after Good Friday} + 11.740 * \text{March} - 11.773 * \text{June}) = 61.05 + (1 + 0.170B + 0.107B^2 + 0.122B^3)\,(1 - 0.981B^6) + \varepsilon_t \quad (11)$$

In Equation 11, the intercept can be interpreted as a forecast value of the daily sales of banana in January, neither before holidays nor after holidays, with mean percentage of promotion and discounted sales. From Table 3, it is obvious that first order seasonal autoregressive and moving average; and first and third order moving average have statistically significant effects on the daily sales of banana at 5% level of significance except second order moving average which is statistically significant at 10% level of significance. *Promotion* and *before holiday* variables are highly significant variables at 1% level. For example, *promotional price reduction* has positive coefficient of 1.712, suggesting a 1% reduction of price due to promotion results in 1.712 Kg increase in banana sales when other variables remains constant. *Before holiday* has positive coefficient of 20.56, i.e. the sales of banana increases by 20.56 Kg on the days before regular holidays when other variables remains constant. *After Good Friday* variable has a positive and significant coefficient of 33.4 at 5% level of significance, i.e. the sales of banana increases by 33.4 Kg on the day after Good Friday when other variables remains constant. *After Christmas*, *before Good Friday*, *March*, *June* variables have significant effect at 7% level of significance. *After Christmas* and *March* variables have negative effects on the daily sales of banana.

The variables such as *discounted sales*, *before Christmas*, *after Easter Monday*, *after holiday*, *February*, *April*, *May*, *July*, *August*, *September*, *October*, *November* and *December* are insignificant at 10% level of significance. To ensure the adequacy, the SARIMAX is diagnosed using Ljung-Box Q statistic. The test statistic shows that there is no significant difference from white noise, as the p-value exceeds the 5% significance level. The ACF and PACF plots of residuals shown in Figure 6 reveals that the residuals are uncorrelated. The fitted SARIMAX model is applied to forecast the out-sample test dataset and the results are discussed in next section. After including the external variables, the performance of the SARIMAX model increases gradually.

RESULTS AND DISCUSSION

The increase in adjusted R^2 value of the forecasting model usually implies the improvement in fitness. As shown in Table 4, the proposed SARIMAX model improves the adjusted R^2 from 0.386 to 0.613 i.e. the SARIMAX model with price reduction, holiday effects, and month effects can be able to explain nearly 61% of variation. Conversely, the SARIMA model explains only 38% of variation. By including the promotional price reduction to the SARIMA model, the adjusted R^2 increases from 0.386 to 0.533. It increases further to 0.573 by incorporating the holiday effects. Finally, it increases to 0.613, when the month effects are included. As shown in Table 4, the MAPE and RMSE of the SARIMA and SARIMAX models are estimated using the aforementioned Equations 6 and 7 respectively. In this table, the decrease in MAPE and RMSE unambiguously proves that the incorporation of the extra external variables improves the forecast accuracy of the SARIMAX model.

As mentioned earlier, a benchmark model is developed to act as a baseline model against the proposed model. In the retail store which is under study, the store manager uses a naïve model to forecast demand. The naïve model is a seasonal random walk model, forecasts the demand using the sales data of the same day-of-the week from previous week. Whenever there is a promotion and/or holiday in the previous week, the additional sales due to promotion and holiday are deducted from the corresponding sales. At the same time, when there is a promotion and/or holiday in the forecasting week, the additional sales due to promotion and/or holiday are added to the corresponding forecasts. For the purpose of comparison, this naïve model is used as a benchmark model in this study. The approximate additional sales during promotional price reductions and different holidays are estimated using linear regression and expert opinion from store managers. In this case, the linear regression follows two steps: (i) smoothing and detrending the sales data and (ii) estimating the coefficients of promotion and holidays. In order to compare the SARIMAX model with the benchmark model, Theil's statistics are estimated using Equations 8 and 9. Theil's U_1 statistic values for the SARIMAX and the benchmark forecasting model are 0.003 (close to zero), indicates that the models are good. Theil's U_2 statistic value for the SARIMAX and the benchmark forecasting model are 0.60 and 1.004 respectively. This specifies that the SARIMAX model has greater accuracy than the benchmark forecasting model.

In this study, the FVA analysis begins with the assumption that the benchmark model is not having any effect on the forecasting performance, i.e. the baseline model. Table 5 shows the FVA analysis, where the forecast accuracy is calculated using MAPE. Although the benchmark model considers some external factors (promotions, holidays, and months), its performance is comparatively poorer than the SARIMA model. The SARIMA is adding value by improving the forecast accuracy 5.3 percentage points. The SARIMA with promotional price reduction is adding value by improving the forecast accuracy 3.3 percentage points. The SARIMA with promotional price reduction and holiday effects is adding value by improving the forecast accuracy 0.8 percentage points. The SARIMA with promotional price reduction, holiday, and month effects improves the forecast accuracy by 1.7 percentage points. Among all the factors, promotion and discount contributes higher percentage points of improvement.

Prediction Interval

With an assumption that the forecast errors are uncorrelated and normally distributed, the prediction interval at 95% for the next observation in a time series is

$$\widehat{Y}_t = 1.96\hat{\sigma} \tag{12}$$

where, $\hat{\sigma}$ is an estimate of the standard error of the prediction. In Figure 7, the graphical plot of the prediction intervals of out-sample forecasts using the SARIMAX model is given. To assess the quality of the prediction, the real sales data are plotted (stars) with lower and upper 95% prediction intervals (black lines).

CONCLUSION

The important assumption of time series forecasting is the current demand is a function of the past sales. However, the past sales pattern was influenced by many external factors such as level, trend, seasonality, price variations, promotions, holidays, and festivals. The traditional SARIMA forecasting model considers only level, trend and seasonality. In this study, a SARIMA model regressed with the external variables such as price reductions, holiday effects, and month effects is proposed to overcome the disadvantage of the traditional SARIMA model. The distinct advantage of the SARIMAX model lies in its explanation of the outlying data, which is not done by the SARIMA model. For the purpose of evaluation, an appropriate SARIMA model with external factors was developed and applied to predict the daily sales of banana in a German discount retail store. The comparison of adjusted R^2, MAPE and RMSE results of the models shows that the performance of the SARIMAX model is better than the SARIMA model for out-sample data. The results of FVA analysis shows that the SARIMA model improves the forecast accuracy by ~5 percentage points when compared to the benchmark model (store manager's naïve forecasting model). The inclusion of promotional price reduction, holiday effects, and month effects to the SARIMA model further improves the forecast accuracy by ~6 percentage points. The $U_1$and U_2 values of Theil's U statistic also suggest that the proposed SARIMAX model is highly accurate and is better than the benchmark and SARIMA model.

Both understocking and overstocking due to the forecast inaccuracy have a negative impact not only on customer service and profit, but also on environment as a food waste. In order to improve the forecast accuracy, the external demand influencing factors have to be considered in the forecasting model. What emerges from the proposed forecasting model will provide right information for the management to make proper ordering decisions, which in turn improves the sales performance and reduces the food waste. From a technical perspective, the automation and implementation of the SARIMAX model can reduce the efforts of the retail managers while making the ordering decisions for varieties of products. In addition to this, the dynamic nature of SARIMAX model for daily forecasting is an important feature, which can help to understand the undefined change in future demand. As a future scope of research, the consideration of additional demand influencing factors may improve the forecasting accuracy further.

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Appendix

Table 1 Review of recent ARIMA, SARIMA, ARIMAX, and SARIMAX applications

Source	Model	External variables	Product	Frequency
Aburto & Weber, (2003, 2007)	Hybrid SARIMA and Neural Networks	payment, intermediate payment, before holidays, holidays, festivals, school vacation, climate, and price	Vegetable oil demand	Daily
Bratina & Faganel (2008)	ARMAX	Temperature, New Year, and price promotion	Beer demand	Daily
Cools et al. (2009)	ARIMAX and SARIMAX	Holidays	Traffic	Daily
Lee & Hamzah (2010)	ARIMAX	Eid holidays	Muslim boys' clothes demand	Monthly
Adanacioglu & Yercan (2012)	SARIMA	-	Tomato price	Monthly
Chikobvu & Sigauke (2012)	SARIMAX	Weekdays and holidays	Electricity demand	Daily
Cornelsen & Normand (2012)	ARIMAX	Relative prices in bars, prices of alcohol sold in off-licences, and the aggregate retail sales	Total demand in a bar	Monthly
Peter & Silvia (2012)	ARIMAX	Unemployment rate	GDP per capita	Quarterly
Trancart et al. (2013)	SARIMAX	Rainfall, atmospheric pressure, and water temperature	Migration of silver eel	Yearly
Deutsche Bundesbank (2013)	ARIMAX	Months and holidays	Coin circulation volume in Germany	Monthly
Kongcharoen & Kruangpradit (2013)	ARIMAX	Composite leading indicator	Exports from Thailand	Monthly
Nasiru et al. (2013)	ARIMAX	Months	Currency circulation volume in Ghana	Monthly
Shukla & Jharkharia (2013)	ARIMA	-	Onion and potato demand	Daily
Hamjah (2014)	ARIMAX	Sun shine, minimum and maximum temperature, rainfall, relative humidity, and cloud coverage	Pulse crops production	Monthly
Da Veiga et al., (2014)	ARIMA and HW	-	Dairy products demand	Monthly

Table 2 Performance measures of the model for in-sample and out-sample

Forecasting Method	In-sample		Out-sample	
	MAPE	RMSE	MAPE	RMSE
SARIMA	28.32	24.01	28.93	20.60

Table 3 Summary statistics of SARIMAX model

	Estimate	**SE**	**t-value**	**Significance**
Constant	49.402	9.929	4.976	0.000
Independent variables				
MA1	-0.170	0.063	-2.713	0.007
MA2	-0.107	0.063	-1.695	0.091
MA3	-0.122	0.063	-1.940	0.053
AR, Seasonal	0.997	0.000	2260.439	0.000
MA, Seasonal	0.981	0.024	41.121	0.000
Promotional price reduction	1.712	0.163	10.521	0.000
After Christmas	-31.117	16.608	-1.874	0.062
Before Good Friday	24.538	13.009	1.886	0.060
After Good Friday	33.418	16.924	1.975	0.049
Before holiday	20.562	5.807	3.541	0.000
March	-11.740	6.181	-1.899	0.059
June	11.773	6.325	1.861	0.064

Table 4 Forecasting performance measures of SARIMA and SARIMAX models

Forecasting Methods	**Adjusted R^2**	**MAPE**	**RMSE**
SARIMA	0.386	28.928	20.606
SARIMA + Promotional price reduction	0.533	25.571	18.027
SARIMA + Promotional price reduction + Holidays	0.573	24.748	17.455
SARIMA + Promotional price reduction + Holidays + Months	0.613	23.059	16.973

Table 5 FVA analysis

Forecasting Methods	**Forecast Accuracy (%)**	**FVA (% points)**
Benchmark model (Seasonal random walk model + Promotional price reduction+ Holidays)	65.757	-
SARIMA	71.072	5.315
SARIMA + Promotional price reduction	74.429	3.357
SARIMA + Promotional price reduction + Holidays	75.252	0.823
SARIMA + Promotional price reduction + Holidays + Months	76.941	1.689

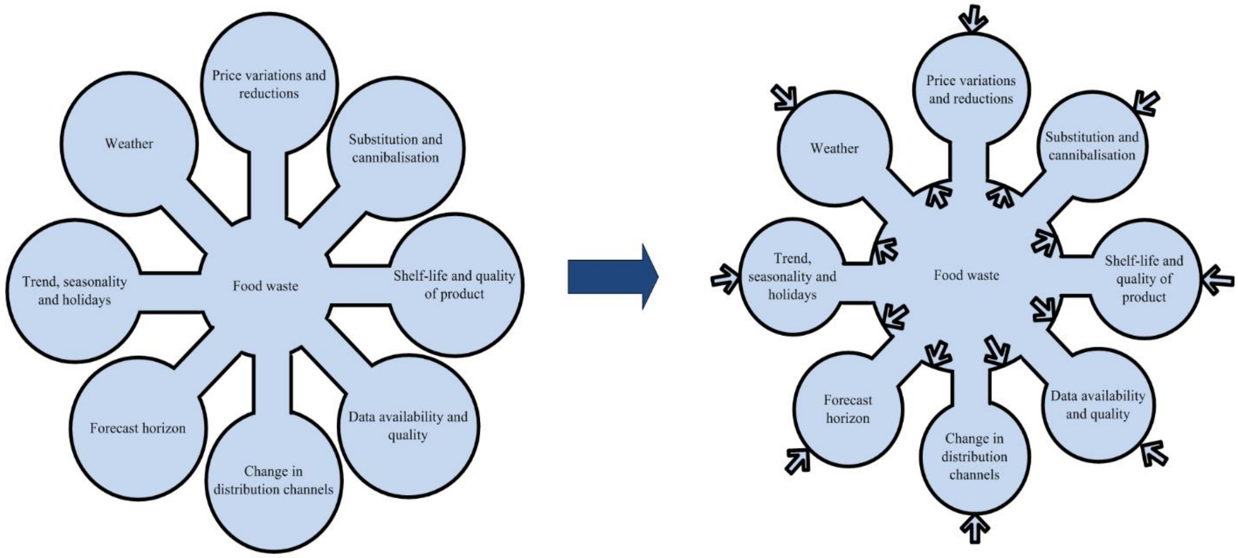

Figure 1 Factors influencing food waste (EyeOn, 2013)

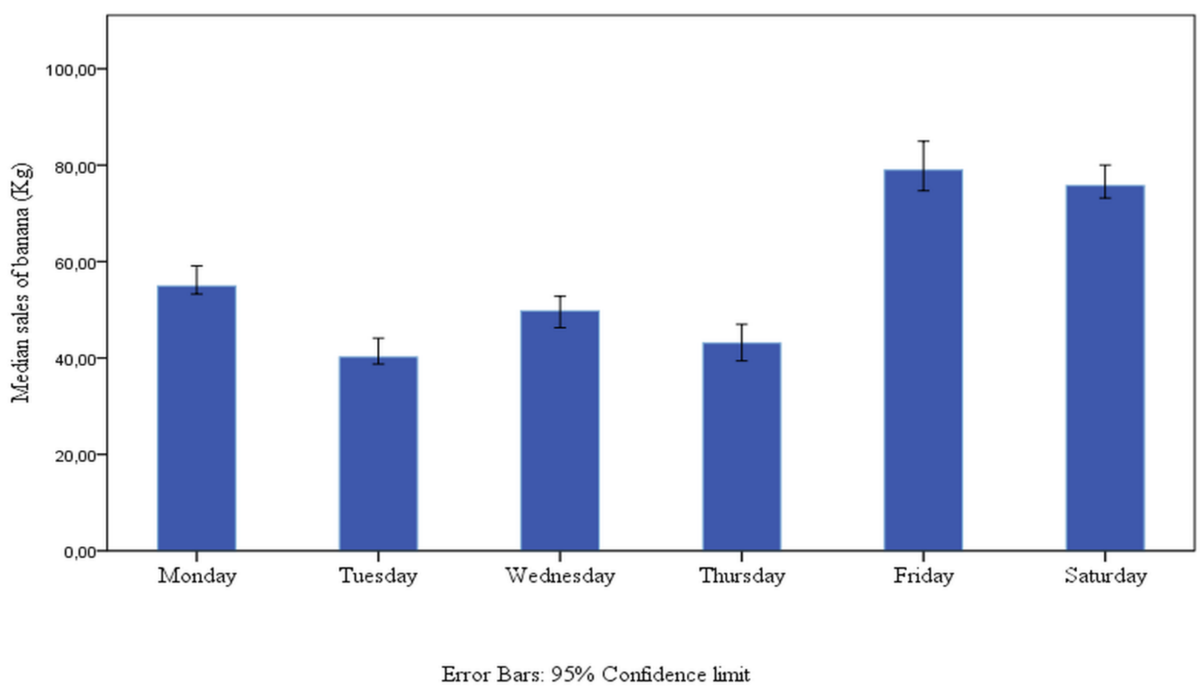

Figure 2 Median sales of banana from Monday to Saturday

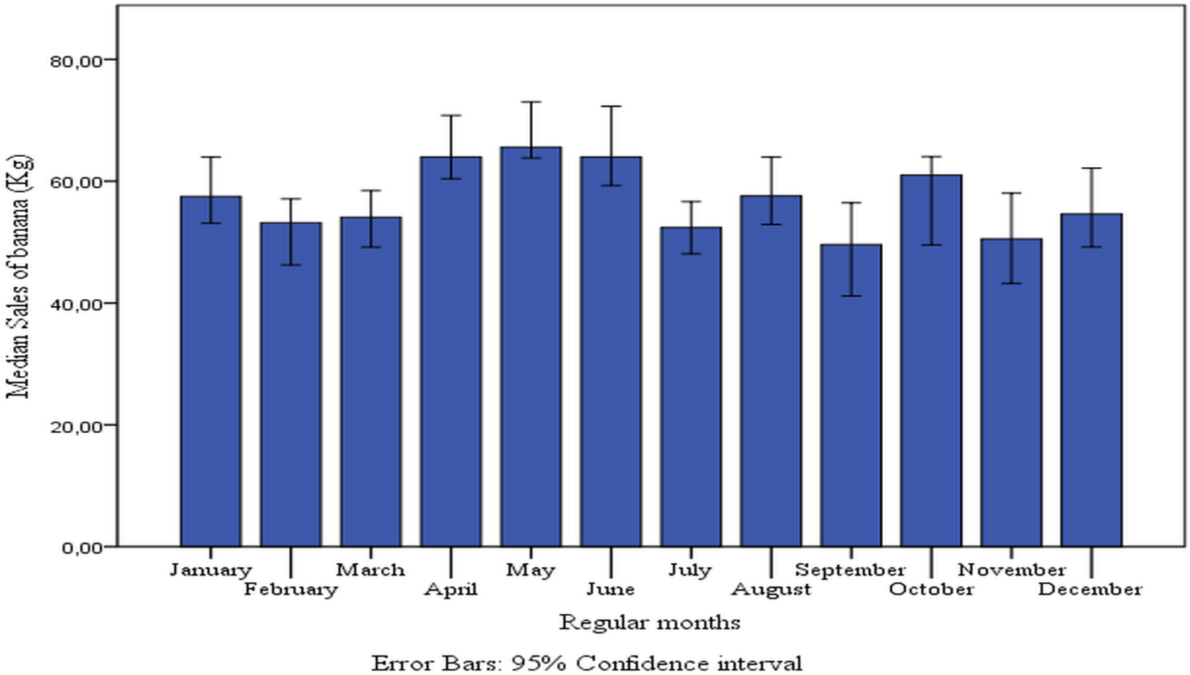

Figure 3 Median sales of banana during regular months

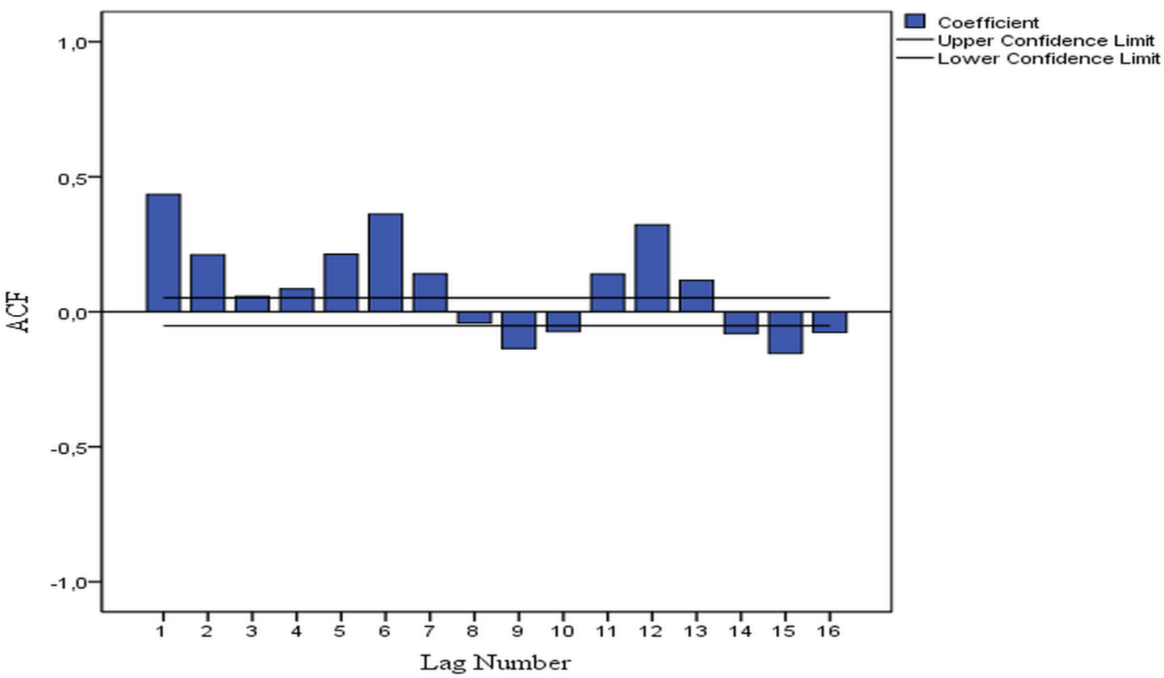

Figure 4 Autocorrelation plots for sales of banana

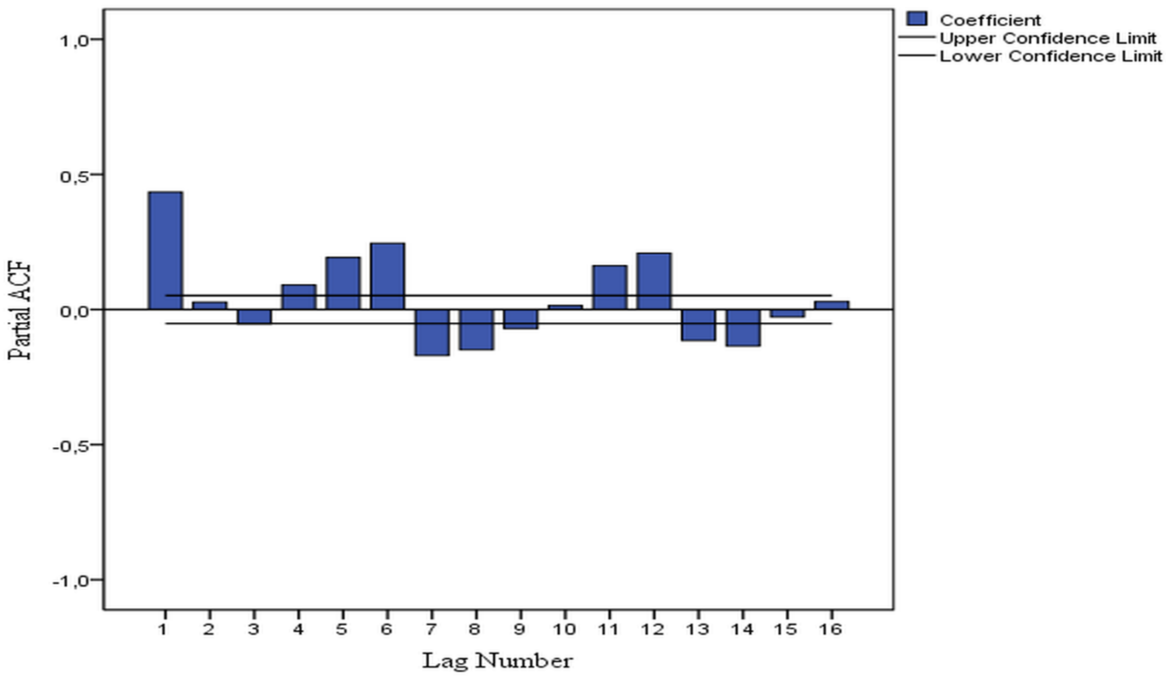

Figure 5 Partial autocorrelation plots for sales of banana

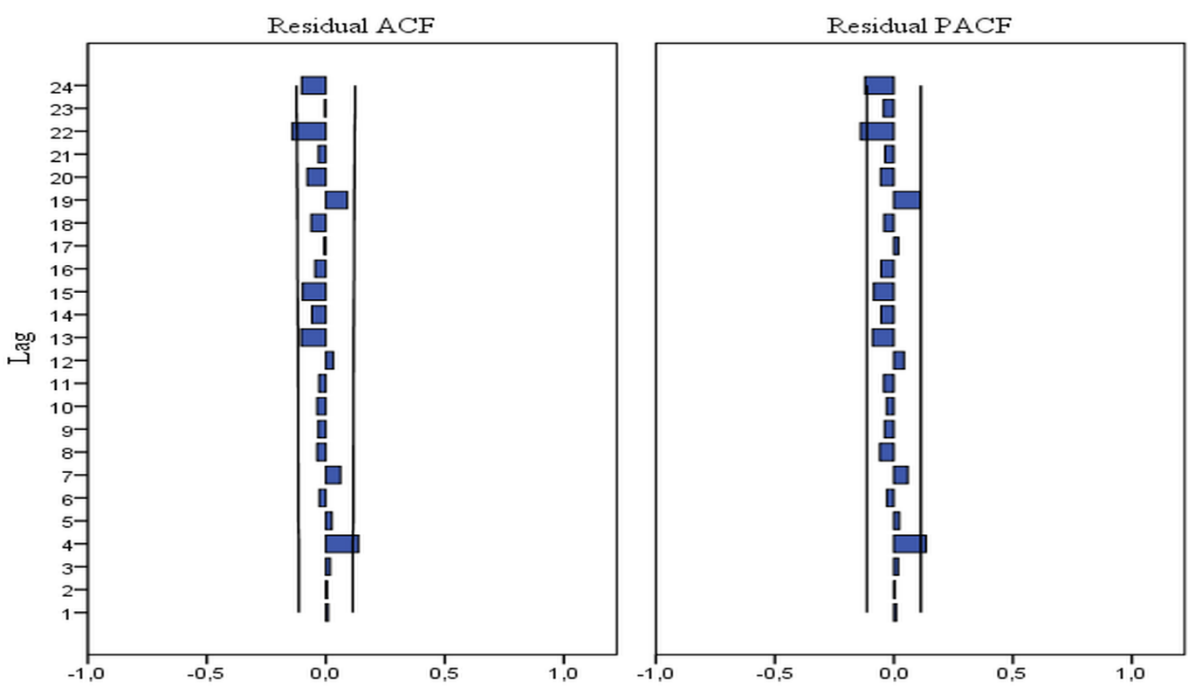

Figure 6 ACF and PACF plots of residuals

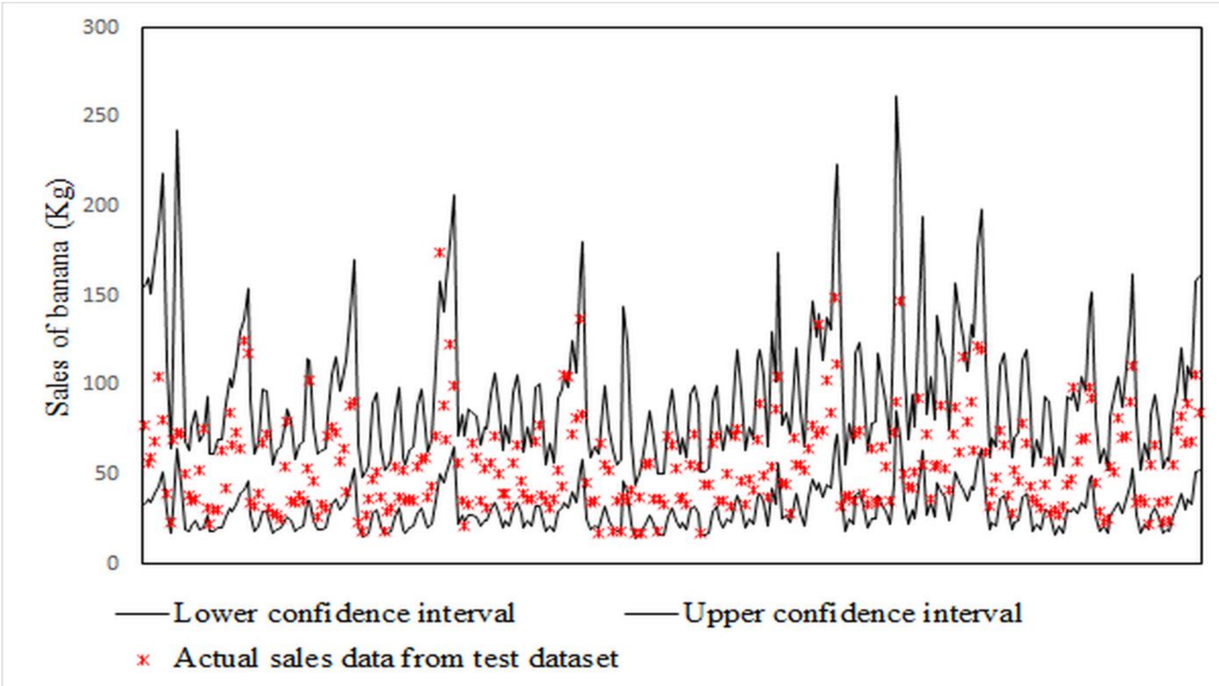

Figure 7 Actual sales values and prediction intervals