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DEPARTMENT OF MATHEMATICS

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### Mathematics Lab Assessment No. 1

Name: Srushti G Joshi  
Division: A  
SRN: 01fe24bar014

Question

### Problem on Springs #1

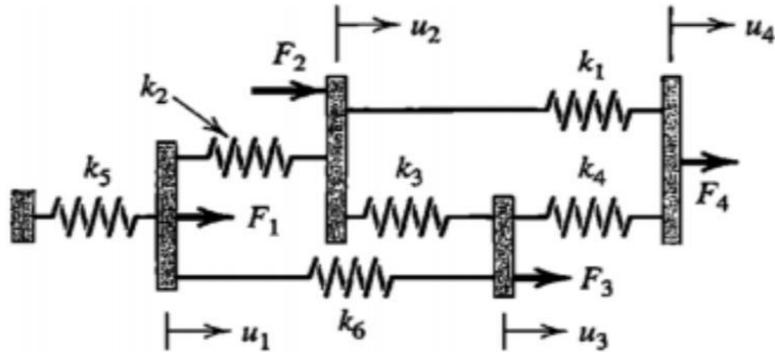


Fig 1: Springs and blocks arrangement

Springs and blocks arrangement is shown in Fig 1. The displacement of springs allowed in horizontal direction only and the blocks are considered as rigid and connected linear springs with stiffness. **Write the system of equilibrium of equations for the applied forces  $F_1=2F_2=20 \text{ kN}$ ,  $F_3=F_4=30 \text{ kN}$ . Evaluate the displacement fields for the applied loads**

$$\begin{array}{lll} k_1 = 100 \text{ N/m} & k_2 = 200 \text{ N/m} & k_3 = 300 \text{ N/m} \\ k_4 = 500 \text{ N/m} & k_5 = 400 \text{ N/m} & k_6 = 150 \text{ N/m} \end{array}$$

Solution

i) Identify the parameters and mathematical concept

k1 to k6	Stiffness of individual springs
k	Global Stiffness Matrix (System Model)
b	External Forces applied to nodes
x	Nodal Displacements (The Result)

CONCEPT : SPRING MASS

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ii) Solve analytically

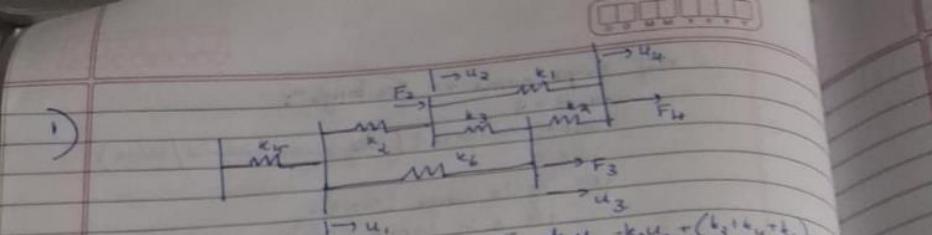


Diagram showing a 4-mass spring system. Masses  $m_1, m_2, m_3, m_4$  are arranged in a horizontal line. Springs with stiffnesses  $k_1, k_2, k_3, k_4$  connect adjacent masses. External forces  $F_1, F_2, F_3, F_4$  are applied to each mass respectively.

Equations of motion:

$$F_1 = (k_2 + k_3 + k_4)u_1 - u_2k_2 - k_6u_3 \quad F_3 = k_6u_1 - k_3u_2 + (k_3 + k_4 + k_5)u_3 - u_4k_4$$

$$F_2 = -k_2u_1 + (k_1 + k_2 + k_3)u_2 - k_3u_3 - k_1u_4 \quad F_4 = -k_4u_3 + (k_4 + k_5 + k_6)u_4$$

$$K = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

$$\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

$$F_4 = -u_2k_1 - k_4u_3 + (k_1 + k_4)u_4 \quad \text{Row 2}$$

Row 1:

$$K_{11} = 200 + 150 + 400 = 750 \quad K_{21} = -200$$

$$K_{12} = -200 \quad K_{22} = 100 + 200 + 300 = 600$$

$$K_{13} = -150 \quad K_{23} = -300$$

$$K_{14} = 0 \quad K_{24} = -100$$

Row 3:

$$K_{31} = -150 \quad K_{41} = 0$$

$$K_{32} = -300 \quad K_{42} = -100$$

$$K_{33} = 500 + 150 + 300 = 950 \quad K_{43} = -500$$

$$K_{34} = -500 \quad K_{44} = 100 + 500 = 600$$

Row 4:

$$\begin{bmatrix} 750 & -200 & -150 & 0 \\ -200 & 600 & -300 & -100 \\ 150 & -300 & 950 & -500 \\ 0 & -100 & -500 & 600 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ 30 \\ 80 \end{bmatrix}$$

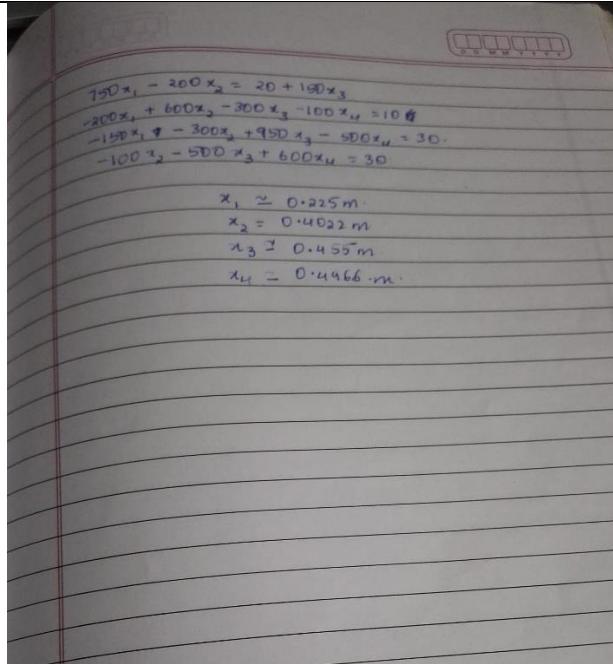
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Handwritten notes on a lined notebook page:

$$100x_1 - 200x_2 = 20 + 100x_3$$

$$-200x_1 + 600x_2 - 300x_3 - 100x_4 = 10 \quad | \quad \text{Multiplying by } -1$$

$$-100x_1 + 300x_2 + 950x_3 - 500x_4 = 30$$

$$x_1 \approx 0.225 \text{ m}$$

$$x_2 = 0.4022 \text{ m}$$

$$x_3 \approx 0.455 \text{ m}$$

$$x_4 \approx 0.4966 \text{ m}$$

**iii) GeoGebra Screenshot / Program Execution**

```

import numpy as np

# Spring stiffness values
k1 = 100
k2 = 200
k3 = 300
k4=500
k5=400
k6=150

# Construct the global stiffness matrix
k = np.array([
    [(k2+k6+k5), -k2, -k6, 0],
    [-k2, (k1+k2+k3), -k3, -k1],
    [-k6, -k3, (k4+k6+k3), -k4],
    [0, -k1, -k4, (k1+k4)]], dtype=float)

# External force vector
b = np.array([20, 10, 30, 30], dtype=float)

# Solve for displacements x1, x2, x3
x = np.linalg.solve(k, b)

# Print results
print("Global stiffness matrix [K]:")
print(k)

print("\nExternal force vector {b}:")
print(b)

print("\nDisplacements {x} (m):")
print(x)
```

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iv) Results and analysis from the graph  
analytical solutions are often impractical.

Global stiffness matrix [K]:

$$\begin{bmatrix} 750. & -200. & -150. & 0. \\ -200. & 600. & -300. & -100. \\ -150. & -300. & 950. & -500. \\ 0. & -100. & -500. & 600. \end{bmatrix}$$

External force vector {b}:

$$[20. \ 10. \ 30. \ 30.]$$

Displacements {x} (m):

$$[0.225 \quad 0.40215736 \quad 0.45545685 \quad 0.4965736]$$