

DEPARTMENT OF MATHEMATICS

Mathematics Lab TEST

Name: Srushti .Joshi

Roll No:1913

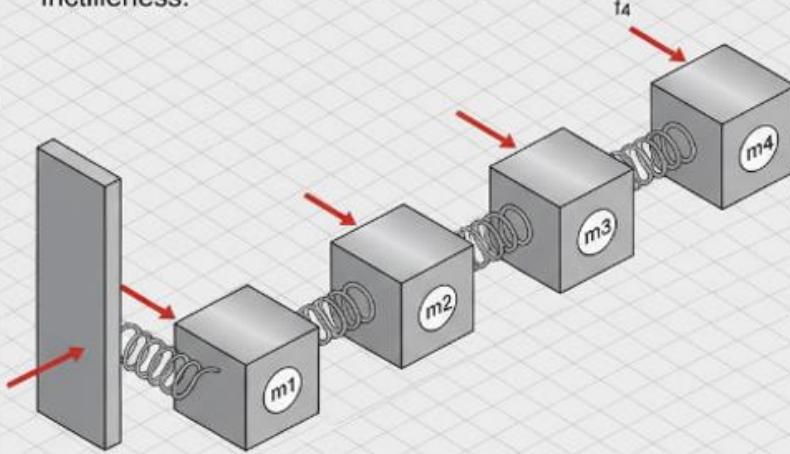
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Question

Static Equilibrium Question

Refer the diagram below showing a 4-mass, system. All surfaces are frictionless.



- Given the following values:
- Spring Stiffnesses: $k_1 = 20 \text{ Nm}$, $k_2 = 20 \text{ Nm}$, $k_3 = 30 \text{ Nm}$,
- External Forces: $f_1 = 15 \text{ N}$ $f_2 = 0 \text{ N}$ $f_4 = 20 \text{ N}$

1. Construct the 4×4 Stiffness Matrix K for this system.

2. Calculate the displacement of each mass from its equilibrium position.

Take $k_4 = 30$, $f_3 = 0$

Solution i) Identify the parameters and mathematical concept
 parameter = x_1, x_2, x_3, x_4
 force = f_1, f_2, f_3, f_4

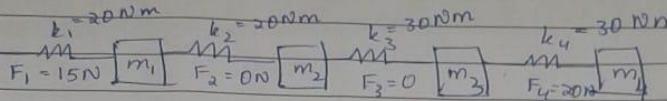
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ii) Solve analytically

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SRUSHTI. G. JOSHI.
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Q) 

Construct 4×4 stiffness matrix.

$F_{4F} = k_4(x_4)$ $F_1 = k_1x_1 + k_2(x_1 + x_2)$
 $k_4x_1 = F_{4F}$ $F_2 = k_2(x_2 - x_1) + k_3(x_2 - x_3)$
 $F_3 = k_3(x_3 - x_2) + k_4(x_3 - x_4)$
 $F_4 = k_4(x_4 - x_3)$

Matrices

$$\begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$\begin{bmatrix} 40 & -20 & 0 & 0 \\ -20 & 50 & -30 & 0 \\ 0 & -30 & 60 & -30 \\ 0 & 0 & -30 & 30 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 0 \\ 20 \end{bmatrix}$$

$x_1 = 1.75 \text{ m}$
 $x_2 = 2.75 \text{ m}$
 $x_3 = 3.47 \text{ m}$
 $x_4 = 4.08 \text{ m}$

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```

CODE:
import numpy as np
k1 = 20
k2 = 20
k3 = 30
k4 = 30
K = np.array([
    [k1+k2, -k2, 0, 0],
    [-k2, k2+k3, -k3, 0],
    [0, -k3, k3+k4, -k4],
    [0, 0, -k4, k4]
], dtype=float)
F = np.array([15, 0, 0, 20], dtype=float)
x = np.linalg.solve(K, F)
print("Global stiffness matrix [K]:")
print(K)
print("\n Force vector [F]:")
print(F)
print("\n Displacement {x} (m):")
print(x)

```

iii) GeoGebra Screenshot / Program Execution

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```
import numpy as np

# Spring stiffnesses (N/m)
k1 = 20
k2 = 20
k3 = 30
k4 = 30

# Global stiffness matrix (4x4)
K = np.array([
    [k1 + k2, -k2, 0, 0],
    [-k2, k2 + k3, -k3, 0],
    [0, -k3, k3 + k4, -k4],
    [0, 0, -k4, k4]
], dtype=float)

# External force vector (N)
F = np.array([15, 0, 0, 20], dtype=float)

# Solve for displacements
x = np.linalg.solve(K, F)

# Print results
print("Global Stiffness Matrix [K]:")
print(K)

print("\nForce Vector {F}:")
print(F)

print("\nDisplacements {x} (m):")
print(x)
```

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iv) Results and analysis from the graph

... Global Stiffness Matrix [K]:

```
[[ 40. -20.  0.  0.]
 [-20.  50. -30.  0.]
 [ 0. -30.  60. -30.]
 [ 0.  0. -30.  30.]]
```

Force Vector {F}:

```
[15.  0.  0.  20.]
```

Displacements {x} (m):

```
[1.75      2.75      3.41666667 4.08333333]
```