

Mathematics Lab Assessment No. 1

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Question

Problem on Springs #1

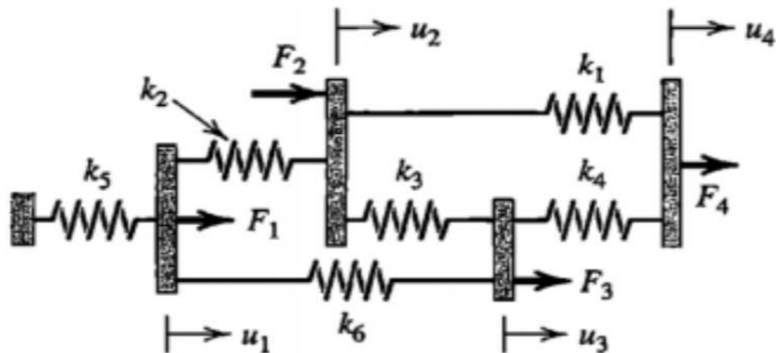


Fig 1: Springs and blocks arrangement

Springs and blocks arrangement is shown in Fig 1. The displacement of springs allowed in horizontal direction only and the blocks are considered as rigid and connected linear springs with stiffness. **Write the system of equilibrium of equations for the applied forces $F_1=2F_2=20$ kN, $F_3=F_4=30$ kN. Evaluate the displacement fields for the applied loads**

$$\begin{aligned} k_1 &= 100 \text{ N/m} & k_2 &= 200 \text{ N/m} & k_3 &= 300 \text{ N/m} \\ k_4 &= 500 \text{ N/m} & k_5 &= 400 \text{ N/m} & k_6 &= 150 \text{ N/m} \end{aligned}$$

Solution

i) Identify the parameters and mathematical concept

k1 to k6	Stiffness of individual springs
k	Global Stiffness Matrix (System Model)
b	External Forces applied to nodes
x	Nodal Displacements (The Result)

CONCEPT : SPRING MASS

ii) Solve analytically

1)

Free-body diagram showing four masses m_1, m_2, m_3, m_4 connected by springs $k_1, k_2, k_3, k_4, k_5, k_6$. Forces F_1, F_2, F_3, F_4 are applied to the masses. Displacements u_1, u_2, u_3, u_4 are indicated.

Equations of motion:

$$F_1 = (k_2 + k_5 + k_6)u_1 - u_2 k_2 - k_6 u_3$$

$$F_2 = -k_2 u_1 + (k_1 + k_2 + k_3)u_2 - k_3 u_3 - k_1 u_4$$

$$F_3 = -k_6 u_1 - k_3 u_2 + (k_3 + k_4 + k_5)u_3 - k_4 u_4$$

$$F_4 = -u_2 k_1 - k_4 u_3 + (k_1 + k_4)u_4$$

Matrix equation $Kx = F$:

$$K = \begin{bmatrix} (k_2 + k_5 + k_6) & -k_2 & 0 & 0 \\ -k_2 & (k_1 + k_2 + k_3) & -k_3 & -k_1 \\ 0 & -k_3 & (k_3 + k_4 + k_5) & -k_4 \\ 0 & -k_1 & -k_4 & (k_1 + k_4) \end{bmatrix}$$

Row-1:

$$k_{11} = 200 + 150 + 400 = 750$$

$$k_{12} = -200$$

$$k_{13} = -150$$

$$k_{14} = 0$$

Row-2:

$$k_{21} = -200$$

$$k_{22} = 100 + 200 + 300 = 600$$

$$k_{23} = -300$$

$$k_{24} = -100$$

Row-3:

$$k_{31} = -150$$

$$k_{32} = -300$$

$$k_{33} = 500 + 150 + 300 = 950$$

$$k_{34} = -500$$

Row-4:

$$k_{41} = 0$$

$$k_{42} = -100$$

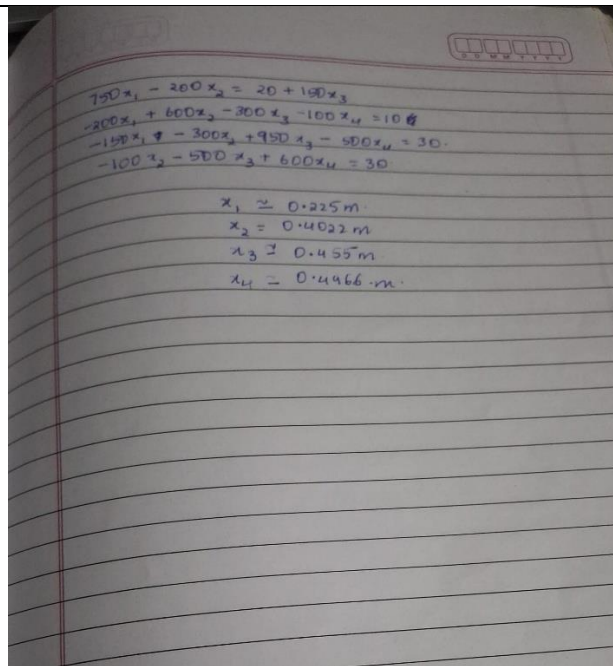
$$k_{43} = -500$$

$$k_{44} = 100 + 500 = 600$$

System of equations:

$$\begin{bmatrix} 750 & -200 & -150 & 0 \\ -200 & 600 & -300 & -100 \\ 150 & -300 & 950 & -500 \\ 0 & -100 & -500 & 600 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ 30 \\ 30 \end{bmatrix}$$

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$$\begin{aligned}
 750x_1 - 200x_2 &= 20 + 100x_3 \\
 -200x_1 + 600x_2 - 300x_3 - 100x_4 &= 10 \\
 -150x_1 + 300x_2 + 950x_3 - 500x_4 &= 30 \\
 -100x_2 - 500x_3 + 600x_4 &= 30
 \end{aligned}$$

$$\begin{aligned}
 x_1 &\approx 0.225 \text{ m} \\
 x_2 &= 0.4022 \text{ m} \\
 x_3 &\approx 0.455 \text{ m} \\
 x_4 &= 0.4466 \text{ m}
 \end{aligned}$$

iii) GeoGebra Screenshot / Program Execution

```

import numpy as np

# Spring stiffness values
k1 = 100
k2 = 200
k3 = 300
k4=500
k5=400
k6=150

# Construct the global stiffness matrix
k = np.array([
    [(k2+k6+k5), -k2, -k6,0],
    [-k2, (k1+k2+k3), -k3,-k1],
    [-k6, -k3, (k4+k6+k3), -k4],
    [0, -k1, -k4, (k1+k4)]
], dtype=float)

# External force vector
b = np.array([20,10,30,30], dtype=float)

# Solve for displacements x1, x2, x3
x = np.linalg.solve(k, b)

# Print results
print("Global stiffness matrix [K]:")
print(k)

print("\nExternal force vector {b}:")
print(b)

print("\nDisplacements {x} (m):")
print(x)

```

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	<p>iv) Results and analysis from the graph analytical solutions are often impractical.</p> <hr/> <p>Global stiffness matrix [K]:</p> $\begin{bmatrix} 750. & -200. & -150. & 0. \\ -200. & 600. & -300. & -100. \\ -150. & -300. & 950. & -500. \\ 0. & -100. & -500. & 600. \end{bmatrix}$ <p>External force vector {b}:</p> $[20. \ 10. \ 30. \ 30.]$ <p>Displacements {x} (m):</p> $[0.225 \quad 0.40215736 \ 0.45545685 \ 0.4965736 \]$