

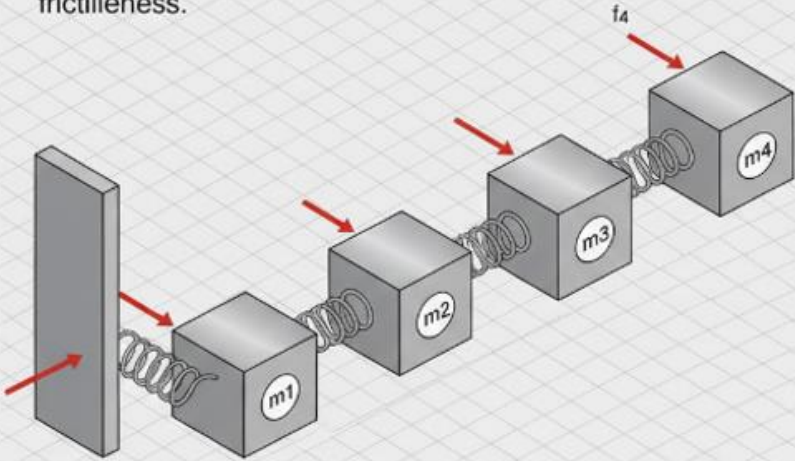
Mathematics Lab TEST

Name: Srushti .Joshi
 Roll No:1913
 SRN:01fe24bar014
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Question

Static Equilibrium Question

Refer the diagram below showing a 4-mass, system. All surfaces are frictionless.



- Given the following values:
- Spring Stiffnesses: $k_1 = 20 \text{ Nm}$, $k_2 = 20 \text{ Nm}$, $k_3 = 30 \text{ Nm}$,
- External Forces: $f_1 = 15 \text{ N}$ $f_2 = 0 \text{ N}$ $f_4 = 20 \text{ N}$

- Construct the 4x4 Stiffness Matrix K for this system.
- Calculate the displacement of each mass from its equilibrium position.

Take $k_4 = 30$, $f_3 = 0$

Solution

i) Identify the parameters and mathematical concept
 parameter = x_1, x_2, x_3, x_4
 force = f_1, f_2, f_3, f_4

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ii) Solve analytically

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SRUSHTI. G. JOSHI.
01F24BAP014.

Q)

Construct 4×4 stiffness matrix.

$$F_1 = k_1 x_1 + k_2 (x_1 + x_2)$$

$$F_2 = k_2 (x_2 - x_1) + k_3 (x_2 - x_3)$$

$$F_3 = k_3 (x_3 - x_2) + k_4 (x_3 - x_4)$$

$$F_4 = k_4 (x_4 - x_3)$$

Matrix

$$\begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$\begin{bmatrix} 40 & -20 & 0 & 0 \\ -20 & 50 & -30 & 0 \\ 0 & -30 & 60 & -30 \\ 0 & 0 & -30 & 30 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 0 \\ 20 \end{bmatrix}$$

$x_1 = 1.75 \text{ m}$
 $x_2 = 2.75 \text{ m}$
 $x_3 = 3.47 \text{ m}$
 $x_4 = 4.08 \text{ m}$

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CODE:

```
import numpy as np
k1 = 20
k2 = 20
k3 = 30
k4 = 30
K = np.array([
    [k1+k2, -k2, 0, 0],
    [-k2, k2+k3, -k3, 0],
    [0, -k3, k3+k4, -k4],
    [0, 0, -k4, k4]
], dtype=float)
F = np.array([15, 0, 0, 20], dtype=float)
x = np.linalg.solve(K, F)
print("Global stiffness matrix [K]:")
print(K)
print("\n Force vectors {F}:")
print(F)
print("\n Displacement {x} (m):")
print(x)
```

iii) GeoGebra Screenshot / Program Execution

```
) import numpy as np

# Spring stiffnesses (N/m)
k1 = 20
k2 = 20
k3 = 30
k4 = 30

# Global stiffness matrix (4x4)
K = np.array([
    [k1 + k2,   -k2,       0,       0],
    [-k2,       k2 + k3,   -k3,      0],
    [0,         -k3,       k3 + k4,  -k4],
    [0,         0,        -k4,      k4]
], dtype=float)

# External force vector (N)
F = np.array([15, 0, 0, 20], dtype=float)

# Solve for displacements
x = np.linalg.solve(K, F)

# Print results
print("Global Stiffness Matrix [K]:")
print(K)

print("\nForce Vector {F}:")
print(F)

print("\nDisplacements {x} (m):")
print(x)
```

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iv) Results and analysis from the graph

... Global Stiffness Matrix [K]:

```
[ [ 40. -20.  0.  0.]  
  [-20.  50. -30.  0.]  
  [  0. -30.  60. -30.]  
  [  0.   0. -30.  30.]]
```

Force Vector {F}:

```
[15.  0.  0. 20.]
```

Displacements {x} (m):

```
[1.75      2.75      3.41666667 4.08333333]
```