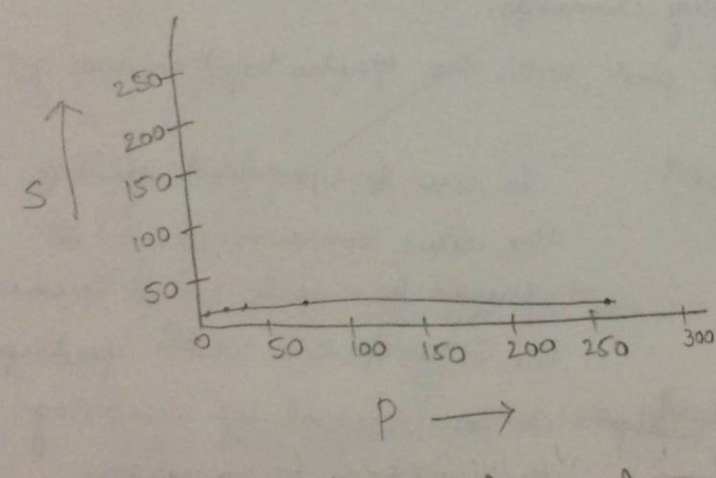
Homework-6

Problem 5.5:- when considering the example 5.1, the standard speedup for adding 'n' numbers using 'n' processing elements is

$$S = \frac{\Theta(n)}{T_p} \quad \text{Since } T_p = \Theta(\log n) \quad \therefore S = \Theta\left(\frac{n}{\log n}\right)$$

Speedup S is defined as the ratio of serial runtime of the best sequential algorithm for solving a problem to the time taken by the parallel algorithm to solve the same problem on 'p' processing elements. The p processing elements used by the parallel algorithm are assumed to be identical to the one used by the sequential algorithm.

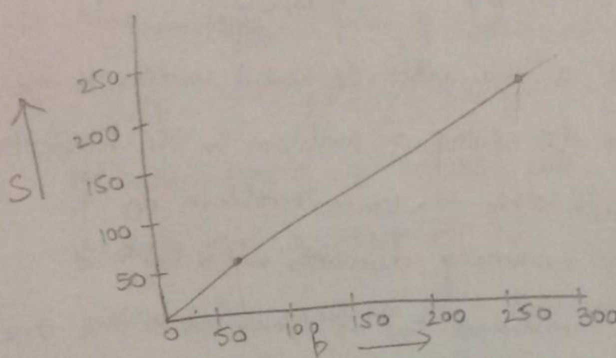
Below is the graph obtained for standard speedup:



The Value of Scaled speedup remains constant though the number of processors is increased. Though there are many processors but while doing summation in figure 5.2 in text book, for summing 16 elements there are a total of 4 communica steps to be considered, the speedup doesnot depend on the num of processors, but its only dependent on number of processing elements

Given scaled speedup $= \dfrac{pW}{T_p(pW, p)}$ W : base problem size for a single processing element

when plotted graph for p from 1 to 256, below is the speedup curve obtained: where $T_p = (n/p - 1) + 11 \log p$
$n = 256$



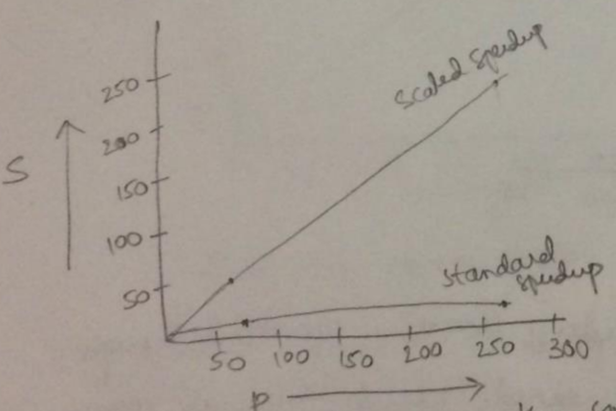
$$\text{Speedup} \propto \frac{p}{(n/p - 1) + 11 \log p}$$

$$\propto \frac{p^2}{(n-p) + 11 p \log p}$$

The above value is always less than p, so we cannot get a linear graph.

In this case, scaled speedup is the speedup that can be calculated when the problem size is increased with the number of processing elements.

For comparision, lets plot both the graphs together, we get



In case of standard speedup, the value remains constant whereas in case of scaled speedup, the value of the overall speedup can be increased by increasing the number of processors.

For standard speedup, the speedup does not depend on the number of processors and its value remains constant though the number of processors is increased whereas the value of scaled speedup can be increased by increasing the number of processors.

Problem 5.6:- In this case the problem size is scaled up according to isoefficiency function on $\Theta(p\log p)$ where p is number of processing elements. $T_p = (n/p - 1) + 11\log p$

we also have Isoefficient Scaled speedup $= \dfrac{pW\log p}{T_p(pW\log p, p)}$

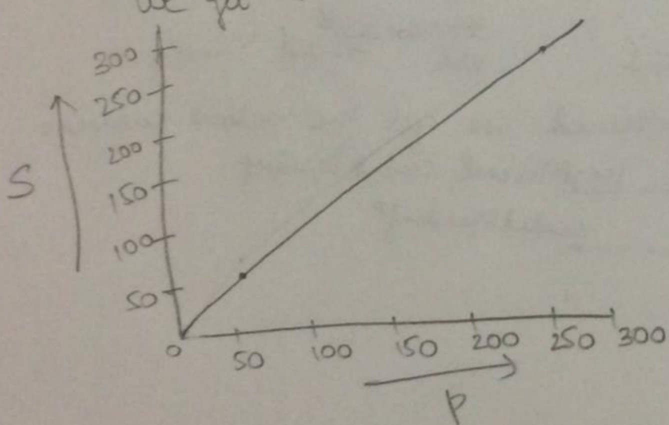
problem size $= p\log p$ when substituted in T_p gives us.

$$T_p = \left(\frac{p\log p}{p} - 1\right) + 11\log p = 12\log p - 1 \quad (\text{neglecting } -1)$$

on substituting in Isoefficient scaled speedup we can reduce the expression to $\dfrac{pW\cancel{\log p}}{12\cancel{\log p}}$

$\therefore$ Scaled speedup is proportional to the number of processors (p)

we get the below graph:



p	S
1	1
4	4
16	16
64	64
256	256

From the above graph, we can understand that the speedup increases linearly with the number of processors.

Using Isoefficient function, we can make the scaled speedup to increase linearly as the number of processors increases.

Problem 5.7:-

Efficiency, by definition is the ratio of speedup(s) to the number of processors(p) i.e., $E = \frac{S}{P}$

efficiency curve for standard speedup is $E = \frac{\theta\left(\frac{n}{\log n}\right)}{n}$

if the efficiency is calculated for 'n' elements with 'n' processors then we get $E = \theta\left(\frac{1}{\log n}\right)$

p	E
1	1
4	0.7213
16	0.2305
64	0.0926
256	0.0182

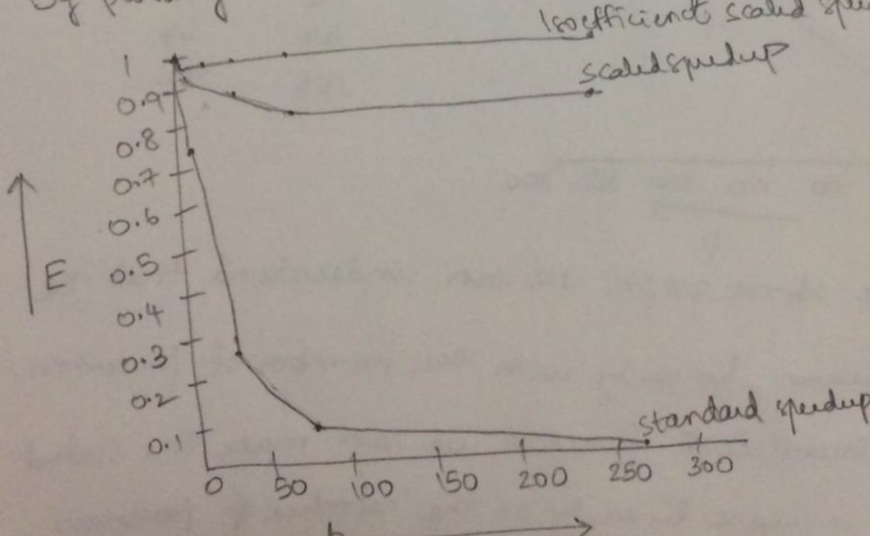
efficiency for scaled speedup is $E = \frac{p}{p\left[\left(\frac{n}{p}-1\right)+11\log p\right]}$

$$E = \frac{1}{n/p - 1 + 11\log p} = \frac{p}{n - p + 11\,p\log p}$$

p	E
1	1
4	0.93
16	0.86
64	0.8
256	0.76

efficiency for isoefficient scaled speedup is $E = \frac{p}{p} \simeq 1$

By plotting the values obtained we get, the below curves:



plotting efficiency curves corresponding to the speedup curves

Problem 5.8:- Though the number of processing elements are increased, if the total workload is not increased then the speedup does not increase linearly with the number of processing elements and when the efficiency is calculated which is the ratio of speedup to the number of processors. The efficiency decreases gradually. From the problems 5.5 and 5.7, it can be observed that the scaled speedup does not increase linearly but it has values less than the linear graph. So, scaled speedup does not solve the problem of decreasing efficiency though the number of processors are increased. The scaled speedup curve is linear only if the isoefficiency function of a parallel system is linear.