Linear Algebra

1.

(a)
$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 5 \\ 1 & -1 & 0 & -1 \\ -1 & 1 & 1 & 5 \end{array}\right]$$

$R_2 \to -R_1 + R_2$
$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 5 \\ 0 & 1 & -2 & -6 \\ -1 & 1 & 1 & 5 \end{array}\right]$$

$R_3 \to R_1 + R_3$
$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 5 \\ 0 & +1 & -2 & -6 \\ 0 & -1 & 3 & 10 \end{array}\right]$$

$R_3 \to R_2 + R_3$
$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 5 \\ 0 & 1 & -2 & -6 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_2 \to 2R_3 + R_2$
$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \to -2R_3 + R_1$
$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \to 2R_2 + R_1$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right].$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

(b)
$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ -1 & 1 & 1 & -1 \\ 2 & 3 & 8 & 4 \end{array}\right]$$

$R_2 \to R_1 + R_2$
$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 2 & 4 & 2 \\ 2 & 3 & 8 & 4 \end{array}\right]$$

$R_3 \to -2R_1 + R_3$
$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 2 & 4 & 2 \\ 0 & 1 & +2 & -2 \end{array}\right]$$

$R_2 \to \frac{R_2}{2}$
$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & -2 \end{array}\right]$$

$R_3 \to -R_2 + R_3$
$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

$R_1 \to -R_2 + R_1$
$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

$x_1 + x_3 = 2$

$x_2 + 2x_3 = 1$

$\boxed{x_3 = -3} \Rightarrow x_2 + 2(-3) = 1$

$x_1 - 3 = 2$

$\boxed{x_1 = 5}$

$x_2 - 6 = 1$

$\boxed{x_2 = 7}$

2. $\begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & -2 & 3 & 5 \\ -1 & -2 & -1 & -7 \end{bmatrix}$

$R_2 \to R_2 + R_1$

$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 2 & 6 \\ -1 & -2 & -1 & -7 \end{bmatrix}$

$R_3 \to R_3 + R_1$

$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & -2 & -6 \end{bmatrix}$

$R_2 \to R_2/2$

$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -2 & -6 \end{bmatrix}$

$R_3 \to R_3 + 2R_2$

$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_1 \to R_1 + R_2$

$\uparrow \qquad \uparrow$

~~Basis~~: $\cancel{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}, \cancel{\begin{bmatrix} -1 \\ 3 \end{bmatrix}}$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 + 4x_4 = 0$$

$$x_3 + 3x_4 = 0$$

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -4 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}.$$

$$x_1 = -2x_2 - 4x_4.$$

$$x_3 = -3x_4.$$

3. $$S = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -2 \\ 5 \end{bmatrix} \right)$$

$$c_1 \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 6 \\ 3 \\ 9 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 3 \\ 5 \\ 4 \end{bmatrix} + c_4 \begin{bmatrix} 2 \\ 3 \\ -2 \\ 5 \end{bmatrix}$$

$$c_1 + 3c_2 + c_3 + 2c_4 = 0$$
$$2c_1 + 6c_2 + 3c_3 + 3c_4 = 0$$
$$c_1 + 3c_2 + 5c_3 - 2c_4 = 0$$
$$3c_1 + 9c_2 + 4c_3 + 5c_4 = 0$$

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 3 & 3 \\ 1 & 3 & 5 & -2 \\ 3 & 9 & 4 & 5 \end{bmatrix}$$

$R_2 \to -2R_1 + R_2$ $\begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & -1 \\ 1 & 3 & 5 & -2 \\ 3 & 9 & 4 & 5 \end{bmatrix}$

$R_3 \to -R_1 + R_3$ $\begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 4 & -4 \\ 3 & 9 & 4 & 5 \end{bmatrix}$

$R_4 \to -3R_1 + R_4$ $\begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$R_3 \to -4R_2 + R_3$ $\begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$R_4 \to -R_2 + R_4$ $\begin{bmatrix} 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_1 \to -R_2 + R_1$ $\begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\uparrow \qquad \uparrow$

$c_1 + 3c_2 + 3c_4 = 0$

$c_3 - c_4 = 0$

$c_1 = -3c_2 - 3c_4$

$c_3 = c_4$

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 5 \\ 4 \end{bmatrix} \right\}.$

$$A_2 = \begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

4. $x_1 + x_2 + 3x_3 = 4$ — ①

$x_1 + 2x_2 + 4x_3 = 5$ — ②

$\langle 1\ 1\ 3 \rangle = a \qquad \langle 1, 2, 4 \rangle = b$

$$a \times b = \begin{vmatrix} i & j & k \\ 1 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = \cancel{(4-6)i}$$

$$i\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} - j\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$

$(4-6)i - j(4-3) + k(2-1) = -2i - j + k$

$$\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \langle -2, -1, 1 \rangle$$

put $x_3 = 0$; solve for x_1 & x_2.

$x_1 + x_2 = 4$ (from ①)

$x_1 + 2x_2 = 5$ (from ②)

$-x_2 = -1 \Rightarrow x_2 = 1$

$x_1 + 1 = 4$

$x_1 = 4 - 1 = 3 \qquad x_1 = 3$

$\langle 3, 1, 0 \rangle$: co-ord. pt.

cross prod : $\langle -2, -1, 1 \rangle$

$$\langle [3-2(t)]\,\cancel{x_1}\,i,\ [1-(-1)t]\,\cancel{x_2}\,j,\ [0+1(t)]\,\cancel{x_3}\,k \rangle$$

$$\langle (3-2t)\,i,\ [1+t]\,j,\ [t\,k] .$$

$$(3-2t)i + (1+t)j + (tk)$$

$$\begin{cases} x_1 = 3-2t \\ x_2 = 1+t \\ x_3 = t \end{cases}$$

↳ forms parametric eq.

5. (a) $A_1 = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

$|A_1| = 6 - 4 = 2$ [Inverse exists]

$\cancel{A_1^{-1} = \frac{1}{|A_1|} \times adj(A_1)}$

$$\left[\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$$

$$Adj(A_1) = \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2} \times \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1/2 & 3/2 \end{bmatrix}$$

(b) $A_2 = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 2 \\ 4 & 7 & 4 \end{bmatrix}$

~~cofactor matrix~~ $\cancel{= \left[\begin{vmatrix} 5 & 2 \\ 7 & 4 \end{vmatrix} \; - \begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} \right]}$

~~det (A) =~~ $\cancel{a_{11} \mid a_{22} \; a_{23}}$

$$1(20-14) - 3(8-8) + 1(14-20)$$

$$= 1(6) - 3(0) + 1(-6) = \underline{\underline{0}}$$

NO inverse

(c) $A_3 = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 1 & -1 \\ -1 & 0 & -3 \end{bmatrix}$

$1(-3+0) - 0(\quad) + 4(-1 \cancel{-0})$

$\cancel{1(-3) - 0 + 4(0)}$

$\cancel{= -3}$

$1(-3) - 0 + 4(1)$

$-3 + 4 = 1$

Cofactor ~~Adj.(A3)~~ $= \begin{bmatrix} -3 & -(3-1) & 0+1 \\ 0 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix}$

$\text{Cofactor}^T = \begin{bmatrix} -3 & 0 & -4 \\ -2 & 1 & -3 \\ 1 & 0 & 1 \end{bmatrix} = \text{adj.}(A_3)$

$A^{-1} = \dfrac{\text{Adj. } A_3}{|A_3|} = \begin{bmatrix} -3 & 0 & -4 \\ -2 & 1 & -3 \\ 1 & 0 & 1 \end{bmatrix}$

6.
(a) $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = 10 - 12$

$= -2$

(b) $\begin{vmatrix} 2 & 3 & 1 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{vmatrix}$

$2(0-1) - 3(6-3) + 1(3-0)$

$= 2(-1) - 3(3) + 1(3)$

$= -2 - 9 + 3 = \underline{\underline{-8}}$

(c) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 8 \\ 1 & 1 & 2 & 3 \\ 1 & 3 & 5 & 8 \end{vmatrix}$

6(c)

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 8 \\ 1 & 1 & 2 & 3 \\ 1 & 3 & 5 & 8 \end{vmatrix} \quad R_2 \to R_2 - R_1$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 3 & 5 & 8 \end{vmatrix}$$

$R_3 \to R_3 - R_1$
$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & -1 & -1 & -1 \\ 1 & 3 & 5 & 8 \end{vmatrix}$$

$R_4 \to R_4 - R_1$
$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 2 & 4 \end{vmatrix}$$

$R_2 \to R_2/2$
$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 2 & 4 \end{vmatrix}$$

~~$R_3 \to R_3 - R_2$ 1 2 3 4 0~~

$R_2 \to -R_2$
$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -1 & -2 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 2 & 4 \end{vmatrix}$$

$R_3 \to R_3 - R_2$
$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 4 \end{vmatrix}$$

$R_2 \to -R_2$
$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 4 \end{vmatrix} \Rightarrow R_4 \to R_4 - R_2 \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix}$$

$R_2 \to 2R_2$
$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix}$$

$$(-)\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{vmatrix}$$

$-(1\times 2\times 1\times 1)$

6(d)
$$\begin{vmatrix} 0 & 0 & 0 & 3 & 0 \\ -2 & 0 & 0 & 2 & 0 \\ 8 & -1 & 0 & -7 & 2 \\ -1 & 2 & 2 & 3 & 2 \\ 2 & 2 & 3 & 6 & 4 \end{vmatrix}$$

$R_1 \leftrightarrow R_2$
$$= -\begin{vmatrix} -2 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 8 & -1 & 0 & -7 & 2 \\ -1 & 2 & 2 & 3 & 2 \\ 2 & 2 & 3 & 6 & 4 \end{vmatrix}$$

~~$4R_1 + R_3 \to R_3$~~

$R_3 \to 4R_1 + R_3$

$R_4 \to R_4 - 0.5R_1$

$R_5 \to R_1 + R_5$

$$-\begin{vmatrix} -2 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & -1 & 0 & 1 & 2 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 2 & 3 & 8 & 4 \end{vmatrix}$$

$R_2 \leftrightarrow R_3$
$$\begin{vmatrix} -2 & 0 & 0 & 2 & 0 \\ 0 & -1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 3 & 10 & 8 \end{vmatrix}$$

$R_3 \leftrightarrow R_4$
$$-\begin{vmatrix} -2 & 0 & 0 & 2 & 0 \\ 0 & -1 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 10 & 8 \end{vmatrix}$$

$R_5 \to R_5 - 1.5R_3$
$$-\begin{vmatrix} -2 & 0 & 0 & 2 & 0 \\ 0 & -1 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 & -1 \end{vmatrix}$$

$R_5 \to R_5 - \frac{4}{3}R_4$
$$-\begin{vmatrix} -2 & 0 & 0 & 2 & 0 \\ 0 & -1 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$-(-2\times -1\times 2\times 3\times -1) = 12$

(7) $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{vmatrix}$

$|A| = (2t^2 - 4t) - (t^2 - t) + (4 - 2)$

$= 2t^2 - 4t - t^2 + t + 2$

$= t^2 - 3t + 2 \Rightarrow (t-2)(t-1) = 0$

$|A| = 0 \Rightarrow$ A is not invertible.

$\cancel{t(t-3) + 2 = 0}$

$\cancel{t(t-3) = -2}$

$t = 2,\ t = 1,$

Matrix is invertible iff $|A| \neq 0$

$t \notin \{2, 1, 0\}$ ~~...~~

for all real & comp. val. of t other than $\{2, 1, 0\}$.

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10.
(a) $A_1 = \begin{bmatrix} 3 & -5 \\ 2 & -3 \end{bmatrix}$

① $\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

② $A - \lambda I$ $\begin{bmatrix} 3 & -5 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 3-\lambda & -5 \\ 2 & -3-\lambda \end{bmatrix}$

③ $|A - \lambda I| = [(3-\lambda)(-3-\lambda)] - [-10]$

$= -9 - 3\lambda + 3\lambda + \lambda^2 + 10$

$= \lambda^2 + 1$

④ $\lambda^2 + 1 = 0$

$\lambda^2 = -1$

$\lambda = i, -i$

Eigenvectors for $\lambda = i$

$\begin{bmatrix} 3 & -5 \\ 2 & -3 \end{bmatrix} - i \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-i & -5 \\ 2 & -3-i \end{bmatrix}$

Row-echlon.

$R_1 \to \left[\frac{3}{10} + \frac{i}{10}\right] R_1$

$= \begin{bmatrix} 1 & -\frac{3}{2} - i\frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$x + \left[-\frac{3}{2} - \frac{i}{2}\right] y = 0$

$x = -\left[-\frac{3}{2} - \frac{i}{2}\right] y$

$\lambda = +i$ $\begin{bmatrix} \frac{3}{2} + i\frac{1}{2} \\ 1 \end{bmatrix}$

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$\lambda = -i$

$$\begin{bmatrix} 3 & -5 \\ 2 & -3 \end{bmatrix} - (-i)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3+i & -5 \\ 2 & -3+i \end{bmatrix}$$

$$R_2 \to R_2 - \left[\frac{3}{5} - \frac{i}{5}\right]R_1 = \begin{bmatrix} 3+i & -5 \\ 0 & 0 \end{bmatrix}$$

Row echlon

$$R_1 \to \left[\frac{3}{10} - \frac{i}{10}\right]R_1$$

$$= \begin{bmatrix} 1 & -\frac{3}{2}+\frac{i}{2} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$x + \left(-\frac{3}{2} + \frac{i}{2}\right)y = 0$$

$$x = -\left(-\frac{3}{2} + i\frac{1}{2}\right)y.$$

$y = 1$

$$\begin{bmatrix} \frac{3}{2} - \frac{i}{2} \\ 1 \end{bmatrix}$$

10 (b) $\begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$\lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$A - \lambda I \begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} 4-\lambda & -4 & 2 \\ 2 & -2-\lambda & 2. \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$$|A + \lambda I| = 4-\lambda\left(-2(2+\lambda) * (1-\lambda)\right) + 4\left(2(1-\lambda)\right) + 2(0)$$

$$4-\lambda(-2-\cancel{\lambda}-1+\cancel{\lambda}) + 8 - 8\lambda$$

$$4-\lambda(-3) + 8 - 8\lambda$$

10 (b)

$$\begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\begin{vmatrix} 4-\lambda & -4 & 2 \\ 2 & -2-\lambda & 2 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$4-\lambda(\lambda^2+\lambda-2) - (-4)(2-2\lambda) + 2 = 0$$

$$= (\lambda-2)(\lambda-1)\lambda = 0$$

$$\boxed{\lambda = 2 \; ; \; \lambda = 1 \; ; \; \lambda = 0}$$

$\boxed{\lambda = 2}$

$$\begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -4 & 2 \\ 2 & -4 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_2 \to R_2 - R_1 = \begin{bmatrix} 2 & -4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_3 \to -R_3 \quad \begin{bmatrix} 2 & -4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_2 \leftrightarrow R_3 \quad \begin{bmatrix} 2 & -4 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \to \frac{R_1}{2} \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x - 2y = 0$

$z = 0$

$$v = \begin{bmatrix} 2y \\ y \\ 0 \end{bmatrix} \quad \text{Let } y = 1 \quad \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = 1}$

$$\begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 2 \\ 2 & -3 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2/3 R_1$
$$\begin{bmatrix} 3 & -4 & 2 \\ 0 & -1/3 & 2/3 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to -3R_2$
$$\begin{bmatrix} 3 & -4 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 + 4R_2$
$$\begin{bmatrix} 3 & 0 & -6 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to \frac{R_1}{3}$
$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x = 2z$

$y = 2z$

Let $z = 1$ $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

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$$\boxed{\lambda = 0}$$

$$\begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ Row echlon.

$R_2 \to R_2 - \frac{R_1}{2}$ $$\begin{bmatrix} 4 & -4 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 - 2R_3$ $$\begin{bmatrix} 4 & -4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ $R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 4 & -4 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_1 \to \frac{R_1}{4} \Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x - y = 0$ | $z = 0$

$z = 0$ | $x = y$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$ $\therefore$ Eigen vectors

$$\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$