Time Series Analysis
Homework Assignment 5

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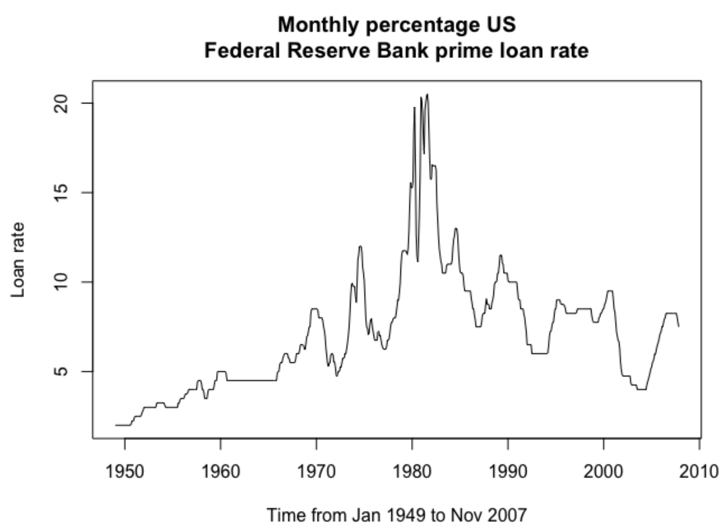
ARIMA and GARCH

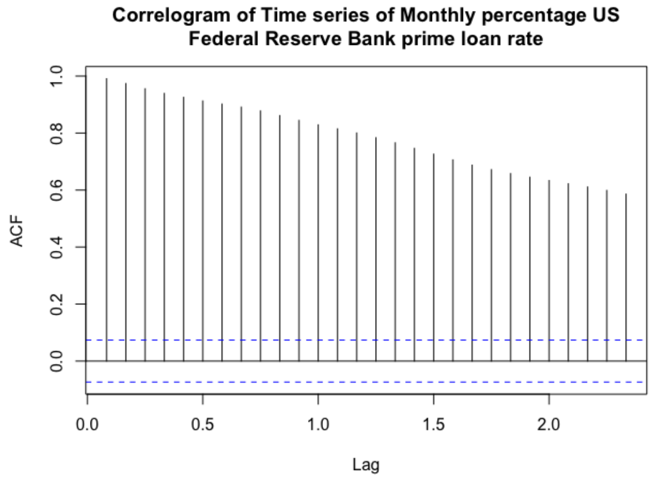
Exercise 1.

The data in mprime.txt are of the monthly percentage US Federal Reserve Bank prime loan rate, courtesy of the Board of Governors of the Federal Reserve System, from January 1949 until November 2007.

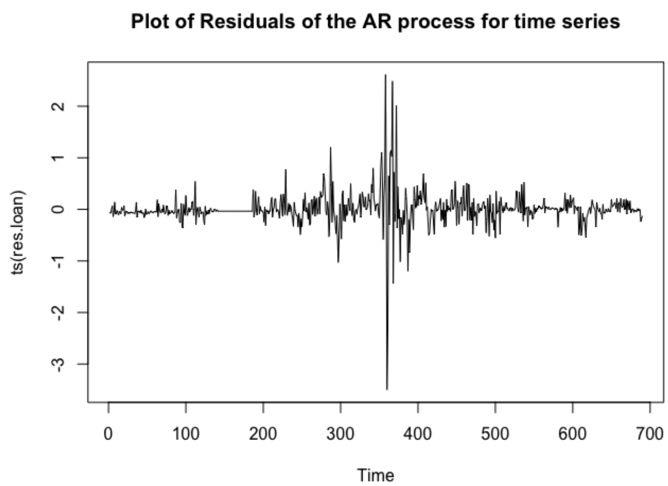
We load the text file and the required libraries and the time series object is created.

(a) Plot the time series {xt}. Use acf to plot the correlogram of {xt}.

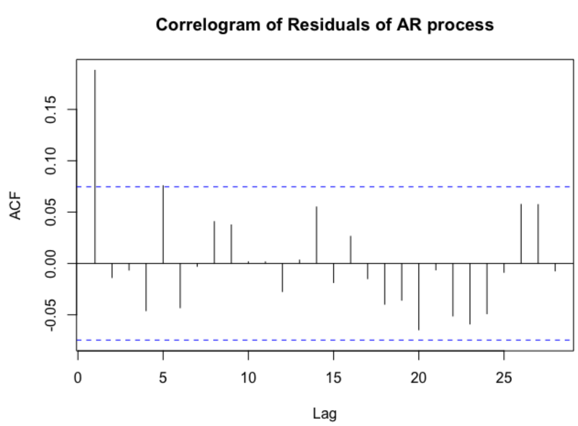




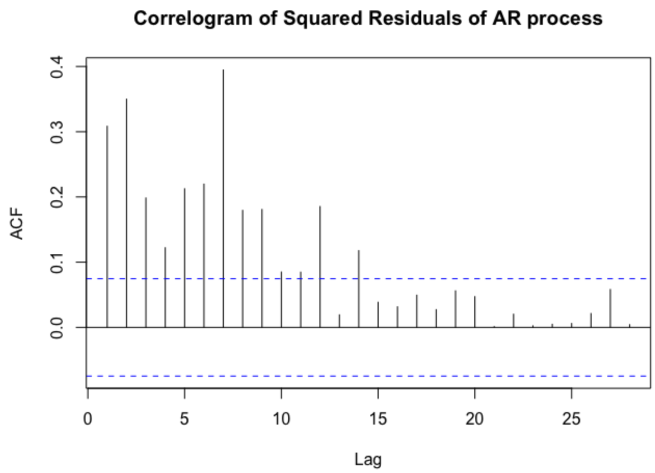
(b) Fit an AR(p) process for {xt}. Let p be chosen automatically. Plot the correlograms of residuals and squared residuals of the fitted AR(p) process. Comment.



Correlograms of Residuals of fitted AR(p) process



Correlograms of squared residuals of fitted AR(p) process




```
ar(x = loan.ts)

Coefficients:
      1        2        3        4        5        6        7        8        9       10
 1.4644  -0.5512   0.0161  -0.0288   0.2079  -0.2759   0.1665   0.1194  -0.0705  -0.1081
     11       12       13       14       15       16       17
 0.0397  -0.0274   0.1112   0.0206  -0.1491   0.1349  -0.0833

Order selected 17  sigma^2 estimated as  0.1563
```

The coefficients of the fitted AR process is as shown above. The order selected is 17 and variance is 0.1563. For the squared residual, the number of significant lags are higher(many), while in the correlogram of residual, there is only one significant lag.

(c) Fit the following GARCH modes to the residuals in step (b), and select the best-fitting model: GARCH(0,1), GARCH(1,0), GARCH(1,1). Give the estimated parameters of the best-fitting model.

After fitting the residuals to the three different GARCH modes given, we use AIC to find the best fitting model.

```
> #Fitting GARCH modes to residuals
> AIC(loan.garch1)
[1] 16.57766
> AIC(loan.garch2)
[1] 414.7042
> AIC(loan.garch3)
[1] -367.3791
>
```

We observe that the GARCH(1,1) has the least AIC of -367.3791. Hence that is the best fitting model.

We find the estimated parameters of the best fitting model by using summary(). They are as follows:

```
> #Estimated params of best fitting model
> summary(loan.garch3)

Call:
garch(x = res.loan, order = c(1, 1), trace = F)

Model:
GARCH(1,1)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.4021 -0.4707 -0.1141  0.4060  4.6217 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)    
a0 0.0012069   0.0001496    8.065 6.66e-16 ***
a1 0.2170622   0.0217351    9.987  < 2e-16 ***
b1 0.7867688   0.0164320   47.880  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
	Jarque Bera Test

data:  Residuals
X-squared = 449.25, df = 2, p-value < 2.2e-16


	Box-Ljung test

data:  Squared.Residuals
X-squared = 0.0082608, df = 1, p-value = 0.9276
```

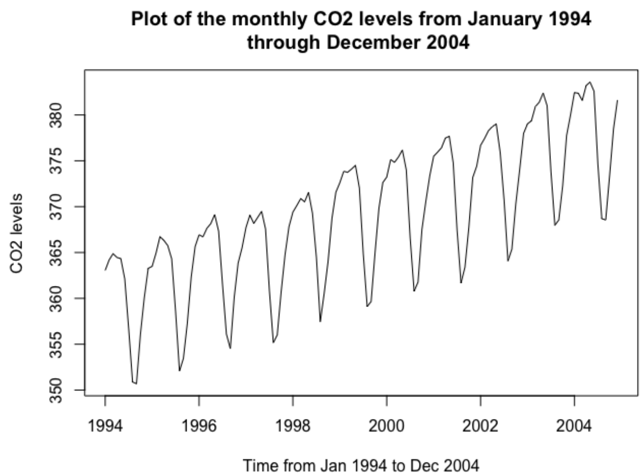
Exercise 2:

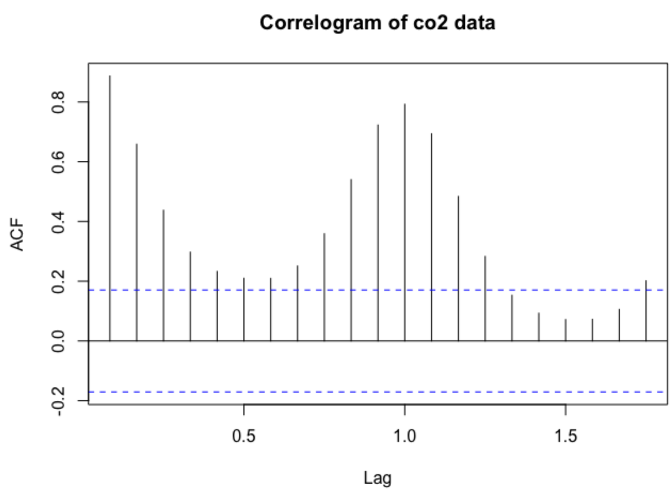
The statistics of the monthly CO2 levels from January 1994 through December 2004 can be obtained in R by using "library(TSA)" and "data(co2)".

After importing the required libraries and data files, time series object is created.

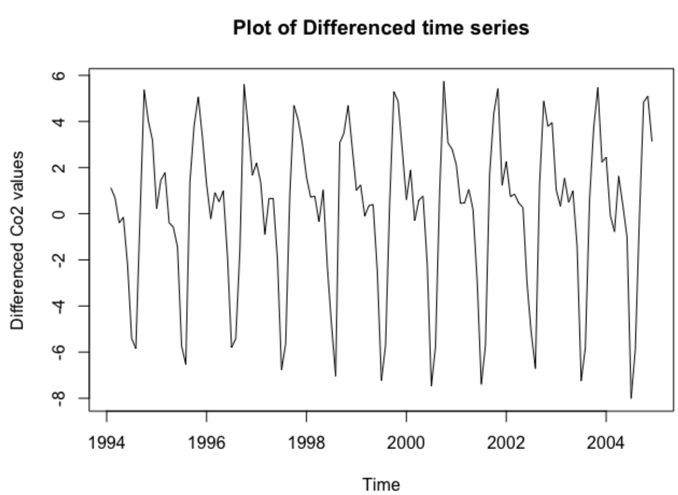
(a) Plot the time series {xt}. Produce the correlogram of {xt}.

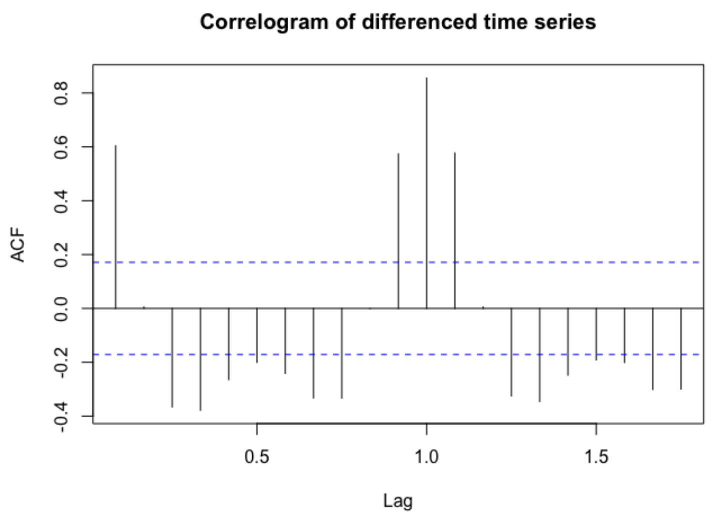
The time series object was plotted as below. Then the correlogram was plotted using acf.





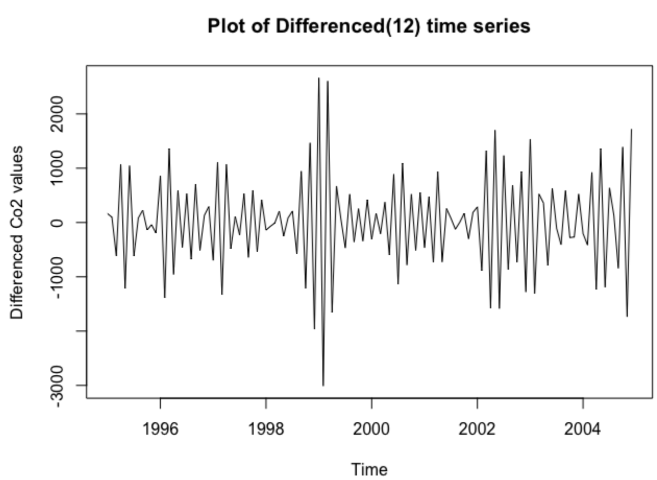
(b) Plot the correlogram of the differenced time series yt = xt − xt−1. Comment.



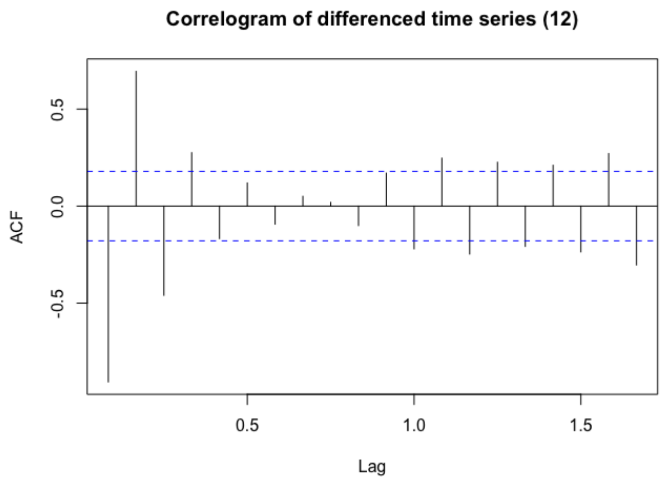


We see that there is some autocorrelation for the differenced time series. Thus we can fit Arima Model to the time series

(c) Plot the differenced time series zt = yt−yt−12, and plot the correlogram of {zt}. Comment.



Here also we see some autocorrelation, even though it is lesser than the previous differenced time series. Hence we can fit Arima model to the timeseries.



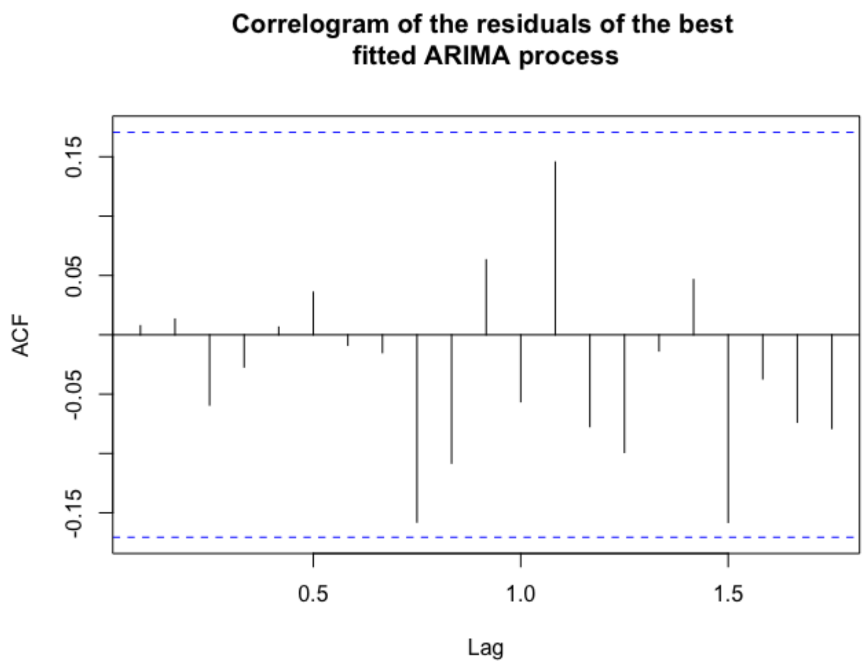
(d) Choose the best-fitting Seasonal ARIMA model for the time series {xt} with p, q, P, Q ∈ {0, 1} and d and D chosen by the results of parts (b) and (c). Base your choice on the AIC and comment on the correlogram of the residuals of the best-fitting model.

We choose d as 1

```
> AIC(arima(co2, order = c(0,1,0), seas = list(order = c(0,1,0), 12)))
[1] 373.4504
> AIC(arima(co2, order = c(0,1,0), seas = list(order = c(0,1,1), 12)))
[1] 316.0014
> AIC(arima(co2, order = c(0,1,0), seas = list(order = c(1,1,0), 12)))
[1] 343.4749
> AIC(arima(co2, order = c(0,1,0), seas = list(order = c(1,1,1), 12)))
[1] 317.1216
> AIC(arima(co2, order = c(0,1,1), seas = list(order = c(0,1,0), 12)))
[1] 332.519
> AIC(arima(co2, order = c(0,1,1), seas = list(order = c(0,1,1), 12)))
[1] 285.0769
> AIC(arima(co2, order = c(0,1,1), seas = list(order = c(1,1,0), 12)))
[1] 305.2464
> AIC(arima(co2, order = c(0,1,1), seas = list(order = c(1,1,1), 12)))
[1] 287.0684
> AIC(arima(co2, order = c(1,1,0), seas = list(order = c(0,1,0), 12)))
[1] 335.1776
> AIC(arima(co2, order = c(1,1,0), seas = list(order = c(0,1,1), 12)))
[1] 292.9968
> AIC(arima(co2, order = c(1,1,0), seas = list(order = c(1,1,0), 12)))
[1] 309.6512
> AIC(arima(co2, order = c(1,1,0), seas = list(order = c(1,1,1), 12)))
[1] 294.9961
> AIC(arima(co2, order = c(1,1,1), seas = list(order = c(0,1,0), 12)))
[1] 330.8405
> AIC(arima(co2, order = c(1,1,1), seas = list(order = c(0,1,1), 12)))
[1] 287.0383
> AIC(arima(co2, order = c(1,1,1), seas = list(order = c(1,1,0), 12)))
[1] 305.8601
>
```

We see that the lowest AIC value is for the model ARIMA(0,1,1)(0,1,1) which is 285.0769. Thus this model is the best fitting seasonal ARIMA model for the time series.

Correlogram of the residuals of the best fitted ARIMA model



We see that there is no significant lag here. This shows that none of the correlations for the autocorrelation function of the residuals are significant.Thus the model meets the assumption that the residuals are independent.