Time Series Analysis
Homework Assignment 3
Submitted by Sruthy Annie Santhosh,312213

Autoregressive Processes

Exercise 1:

Identify each of the following as specific ARIMA models and state whether or not they are stationary. (In each case {Zt} denotes white noise.)

We know that from the slides, an ARIMA(p,d,q) process has the form

$$\Phi(B)(1-B)^d X_t = \Theta(B) Z_t,$$

where Φ(B) and Θ(B) are polynomials of orders p and q, respectively.

(a) $X_t = X_{t-1} + Z_t - Z_{t-1} + 0.25Z_{t-2}$.

Considering the polynomial, Φ(x) = 1 - x
The solutions of the polynomial Φ(x) = 0 are $x_1 = x_2 = 1+0i$
The roots of the function can be found using the polyroot() function in R.
Here the roots equal to 1 are on the complex unit circle.

```
> polyroot(c(1, -1))
[1] 1+0i
```

Hence Φ(B) is the polynomial of order p = 0, and d = 1.

Now considering the polynomial , $\Theta(x) = 1 - x + 0.25 x^2$
The solutions of the polynomial are $x_1 = x_2 = 2$. Here both the roots are outside the complex unit circle.

```
> polyroot(c(1,-1,0.25))
[1] 2+0i 2-0i
```

Hence Θ(B) is the polynomial of order q = 2.

Thus the process {Xt} is an ARIMA(0,1,2) model. It is not stationary.

(b) $X_t = 1.3X_{t-1} - 0.3X_{t-2} + Z_t$.

Considering the polynomial, $\Phi(x) = 1 - 1.3x + 0.3x^2$
The solutions of the polynomial $\Phi(x)$ are $x_1 = 1$, $x_2 = 3.333$
Here one root is on the complex unit circle and one root is outside the complex unit circle

```
> polyroot(c(1,-1.3,0.3))
[1] 1.000000+0i 3.333333-0i
```

Hence $\Phi(B)$ is the polynomial of order p = 1, and d = 1.

Now considering the polynomial , $\Theta(x) = 1$
Hence $\Theta(B)$ is the polynomial of order q = 0.

Thus the process {Xt} is an ARIMA(1,1,0) model. It is not stationary.

(c) $X_t = 1.2X_{t-1} - 0.36X_{t-2} + Z_t + 0.8Z_{t-1}$.

Considering the polynomial, $\Phi(x) = 1 - 1.2x + 0.36x^2$
The solutions of the polynomial $\Phi(x)$ are $x_1 = x_2 = 1.666667$
Here both the roots are outside the complex unit circle.

```
> polyroot(c(1,-1.2,0.36))
[1] 1.666667-0i 1.666667+0i
```

Hence $\Phi(B)$ is the polynomial of order p = 2, and d = 0.

Now considering the polynomial, $\Theta(x) = 1 + 0.8x$.
The solutions of the polynomial are $x = -1.25$. Here the root is outside the complex unit circle.

```
> polyroot(c(1,0.8))
[1] -1.25+0i
```

Hence $\Theta(B)$ is the polynomial of order q = 1.

Thus the process {Xt} is an ARIMA(2,0,1) model. It is stationary.

(d) $X_t = 0.4X_{t-1} + 0.6X_{t-2} + Z_t - Z_{t-1} + 0.16Z_{t-2}$.

Considering the polynomial, $\Phi(x) = 1 - 0.4x - 0.6x^2$
The solutions of the polynomial $\Phi(x)$ are $x_1 = 1$, $x_2 = -1.666667$
Here one of the roots is on the complex unit circle and the other is outside the complex unit circle.

```
> polyroot(c(1,-0.4,-0.6))
[1]  1.000000-0i -1.666667+0i
```

Hence Φ(B) is the polynomial of order p = 1, and d = 1.

Now considering the polynomial, $\Theta(x) = 1 - x + 0.16\,x^2$
The solutions of the polynomial are $x_1 = 1.25$, $x_2 = -5$. Here the roots are outside the complex unit circle.

```
> polyroot(c(1, -1, 0.16))
[1] 1.25+0i 5.00-0i
```

Hence Θ(B) is the polynomial of order q = 2.

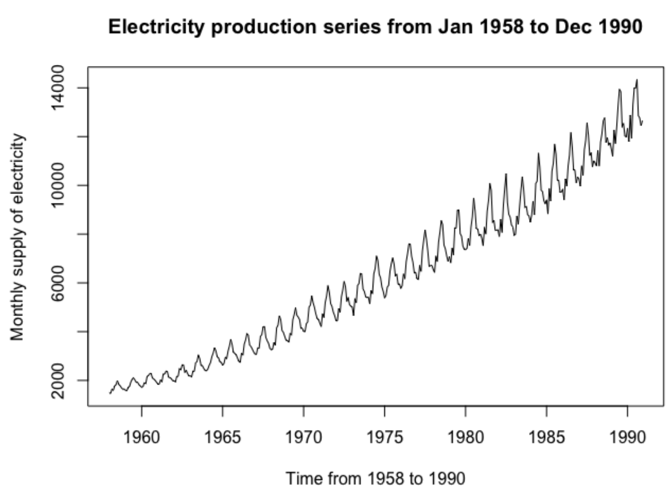
Thus the process {Xt} is an ARIMA(1,1,2) model. It is not stationary.

Exercise 2:

The statistic of monthly supply of electricity (millions of kWh), beer (Ml), and chocolate-based production (tonnes) in Australia from January 1958 to December 1990 is given in the data file named "cbe.dat". Each of the following problems should only be considered for the electricity production series $\{x_t\}$. Fit a seasonal ARIMA model to the time series $y_t := \log x_t$. Proceed as follows:

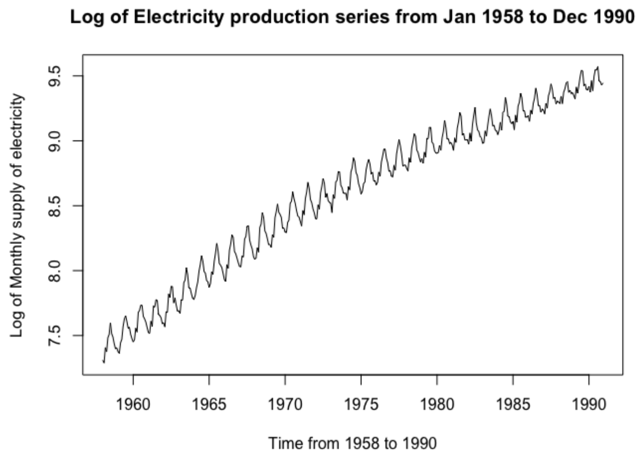
After importing the required libraries and data files, time series object is created. Then I plotted the electricity time series as below:

Plot of the electricity time series



Then the log of the time series is found and then plotted again:

Plot of the logarithm time series



1. With D = 1, choose d using the KPSS test for (non)-stationarity

```
> kpss.test(diff(Elec.log, lag=12))

        KPSS Test for Level Stationarity

data:  diff(Elec.log, lag = 12)
KPSS Level = 2.5816, Truncation lag parameter = 5, p-value = 0.01
```

From the above test, we choose d as 1.

2. Choose p, q, P, Q ∈ {0, 1} according to the best AIC for the logarithm of the original series

From the list of AIC values obtained we observe that the smallest AIC is -1873.532 . Hence the seasonal ARIMA model (0,1,1) (1,1,1) gives the best fit.

So we find the coefficients as:

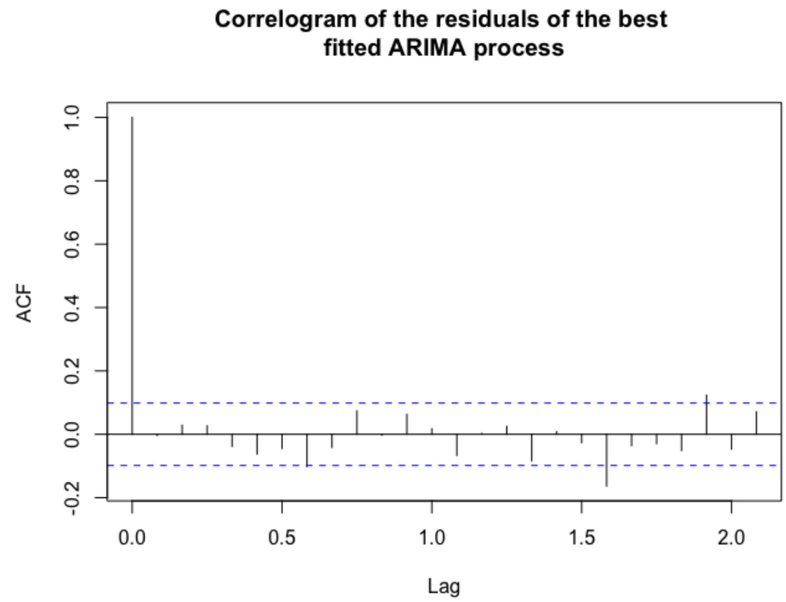
```
arima(x = Elec.log, order = c(0, 1, 1), seasonal = list(order = c(1, 1, 1),
    12))

Coefficients:
          ma1    sar1     sma1
      -0.6530  0.1567  -0.7656
s.e.   0.0434  0.0719   0.0481

sigma^2 estimated as 0.0004212:  log likelihood = 940.77,  aic = -1873.53
```

All the different combinations of AIC values :

```
> AIC(arima(Elec.log, order = c(0,1,0), seas = list(order = c(0,1,0), 12)))
[1] -1625.263
> AIC(arima(Elec.log, order = c(0,1,0), seas = list(order = c(0,1,1), 12)))
[1] -1738.92
> AIC(arima(Elec.log, order = c(0,1,0), seas = list(order = c(1,1,0), 12)))
[1] -1660.195
> AIC(arima(Elec.log, order = c(0,1,0), seas = list(order = c(1,1,1), 12)))
[1] -1750.762
> AIC(arima(Elec.log, order = c(0,1,1), seas = list(order = c(0,1,0), 12)))
[1] -1763.833
> AIC(arima(Elec.log, order = c(0,1,1), seas = list(order = c(0,1,1), 12)))
[1] -1870.802
> AIC(arima(Elec.log, order = c(0,1,1), seas = list(order = c(1,1,0), 12)))
[1] -1814.401
> AIC(arima(Elec.log, order = c(0,1,1), seas = list(order = c(1,1,1), 12)))
[1] -1873.532
> AIC(arima(Elec.log, order = c(1,1,0), seas = list(order = c(0,1,0), 12)))
[1] -1721.034
> AIC(arima(Elec.log, order = c(1,1,0), seas = list(order = c(0,1,1), 12)))
[1] -1828.401
> AIC(arima(Elec.log, order = c(1,1,0), seas = list(order = c(1,1,0), 12)))
[1] -1764.072
> AIC(arima(Elec.log, order = c(1,1,0), seas = list(order = c(1,1,1), 12)))
[1] -1836.322
> AIC(arima(Elec.log, order = c(1,1,1), seas = list(order = c(0,1,0), 12)))
[1] -1761.975
> AIC(arima(Elec.log, order = c(1,1,1), seas = list(order = c(0,1,1), 12)))
[1] -1868.915
> AIC(arima(Elec.log, order = c(1,1,1), seas = list(order = c(1,1,0), 12)))
[1] -1813.542
> AIC(arima(Elec.log, order = c(1,1,1), seas = list(order = c(1,1,1), 12)))
[1] -1871.537
```

3. Plot the correlogram of the residuals of the best fitted ARIMA process. Comment on that.

Finding the correlogram of the residuals of the best fitted ARIMA process using acf.
Then we plot that as below:



From this correlogram we see that the residuals of fitted model ARIMA(0,1,1)(1, 1, 1) are approximately white noise. This suggests that it could be the best-fitting seasonal ARIMA model.