Sixth International Symposium on Physics of Fluids (ISPF6)
International Journal of Modern Physics: Conference Series
Vol. 42 (2016) 1660174 (12 pages)

DOI: 10.1142/S2010194516601745

CALCULATION OF TAKEOFF AND LANDING PERFORMANCE UNDER DIFFERENT ENVIRONMENTS

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Published 22 June 2016

The takeoff and the landing of an aircraft are the most dangerous phases of flight, so the study of takeoff performance and landing performance is of great importance for aircraft design and safety. In the paper, a high-precision numerical integration algorithm for evaluating takeoff and landing performance is developed, which establishes and solves the equations of dynamics during takeoff and landing process. The proposed method is capable of calculating takeoff and landing performance under different environments. To verify the current approach, all-engine operating takeoff distance, one engine failure takeoff distance, takeoff stop distance and landing distance of a certain type of plane with four engines are analyzed. It shows that the developed calculation method is reliable and effective. Based on this algorithm, a detailed analysis of the main factors affecting the takeoff and landing performance is given. The influences of altitude, temperature, takeoff and landing weight and wind speed are analyzed. Through theoretical derivation and examples verification, the proposed method will be reference for the study of takeoff and landing performance.

Keywords: takeoff performance, landing performance, equations of dynamics

1. Introduction

Takeoff and landing time is a very short proportion of the total flight time, but it is the accident-prone phase of the mission. Research about the flight characteristic of takeoff and landing is of great importance to the safety of airplane and can provide some reference for aircraft design. At present, these methods: analytical method, numerical integration and energy method can be used to calculate takeoff and landing performance. Among these methods, numerical integration, with a high precision, can well describe the process of takeoff and landing[1]. This paper analyzes the process of takeoff and landing in details and develops an algorithm based on numerical integration to calculate and contrastive analyze takeoff and landing performance under different environments.

2. Aircraft Takeoff Performance Calculation Method

2.1. *Takeoff speeds and takeoff distance*

When an aircraft takes off, one or more engines may fail suddenly and it may encounter a variety of situations. There are four basic situations which should be researched for calculating the takeoff performance of an aircraft, including:

(i) takeoff with all engines operating(AEO),
(ii) takeoff with one engine inoperative(OEI),
(iii) rejected takeoff with all engines operating,
(iv) rejected takeoff with one engine inoperative.

Figure 1 shows the above four takeoff situations and marks several important takeoff speeds.

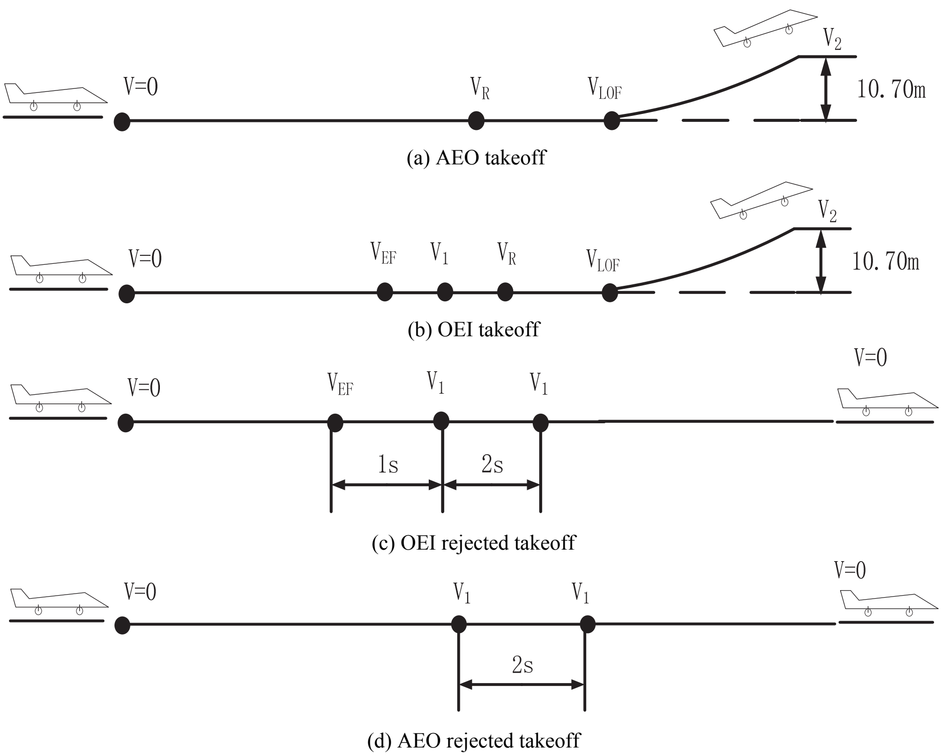


Fig. 1. Aircraft takeoff process.

V_{EF} is Engine failure speed, at which the critical engine is assumed to fail. V_{EF} must be selected by the applicant during calculating OEI takeoff performance. V_1 is also selected by the applicant; but V_1 may not be less than V_{EF} plus the speed gained with critical engine inoperative during the time interval between the instant at which the critical engine is failed, and the instant at which the pilot recognizes and reacts to the engine failure, as indicated by the pilot's initiation of the first action to stop the airplane during accelerate-stop tests. Civil aircraft usually take one second as the time interval between V_{EF} and V_1. V_R is rotation speed and V_{LOF} is lift-off speed at which the airplane first becomes airborne. V_2 is takeoff safety speed and is the velocity at the end of 10.7 meter climb[2,3]. V_{EF}, V_1, V_R, V_{LOF} andV_2 are important takeoff speeds and CCAR-25 prescribes limits to them as:

$$\begin{cases} V_{EF} \geq V_{MCG} \\ V_1 \leq V_R \\ V_R \geq 1.05V_{MC} \\ V_{LOF} \geq 1.1V_{MU}\ , & AEO \\ V_{LOF} \geq 1.05V_{MU}\ , & OEI \\ V_2 \geq 1.1V_{MC} \\ V_2 \geq 1.2V_S \end{cases} \tag{1}$$

where V_{MCG} is the minimum control speed on the ground, V_{MC} is the minimum control speed airborne, V_{MU} is the minimum unstick velocity, and V_S is stall speed. V_{MCG}, V_{MC} and V_{MU} is known when calculating the takeoff performance for a certain aircraft. V_S occurs at the maximum aircraft lift coefficient which is also known and is defined as:

$$V_S = \sqrt{\frac{2(G/S)}{\rho C_{L\ MAX}}} \tag{2}$$

where G is aircraft weight, S is aircraft wing area and ρ is atmosphere air density.

Aircraft takeoff distance and accelerate-stop distance are related to runway condition. A dry runway and a wet runway are different when calculating distances. This article only deals with dry runway. TOD_{dry} is short for takeoff distance on a dry runway[4].

$$TOD_{dry} = Max\left\{TOD_{N-1}, 1.15TOD_N\right\} \tag{3}$$

In Eq. (3), TOD_{N-1} is the horizontal distance along the takeoff path, with one engine inoperative, from the start of the takeoff to the point at which the airplane is 10.7 meters above the takeoff surface; TOD_N is the horizontal distance along the takeoff path, with all engines operating, from the start of the takeoff to the point at which the airplane is 10.7 meters above the takeoff surface.

ASD_{dry} is short for accelerate-stop distance on a dry runway[4].

$$ASD_{dry} = Max\left\{ASD_{N-1}, ASD_N\right\} \tag{4}$$

In Eq. (4), ASD_{N-1} is the sum of the distances below.

(i) Accelerate the airplane from a standing start with all engines operating to V_{EF} ;
(ii) Allow the airplane to accelerate from V_{EF} to the highest speed reached during the rejected takeoff, assuming the critical engine fails at V_{EF} and the pilot takes the first action to reject the takeoff at the V_1 for takeoff from a dry runway;
(iii) Come to a full stop on a dry runway from V_1;
(iv) A distance equivalent to 2 seconds at the V_1 for takeoff from a dry runway.

ASD_N is the sum of the distances below.

(i) Accelerate the airplane from a standing start with all engines operating to V_1 ;
(ii) With all engines still operating ,come to a full stop from V_1;
(iii) A distance equivalent to 2 seconds at the V_1 for takeoff from a dry runway.

2.2. *Theory analysis of aircraft takeoff*

Aircraft takeoff process can be divided into three parts: ground roll, rotation and lift-off to safety height[5]. Through force analysis to solve the dynamic equation and combining kinematics equations, the horizontal distance of aircraft at each takeoff part can be calculated.

Assume that the aircraft takes off with a constant wind speed V_W and Set the direction against the wind is positive. During the part of ground roll, the dynamic equation and kinematics equations is:

$$\begin{cases} V = \dfrac{dX}{dt} + V_W \\ a = \dfrac{dV}{dt} \\ \dfrac{G}{g} a = T\cos(\alpha + \psi_T) - \dfrac{1}{2}\rho V^2 S C_D - \mu(G\cos\Phi - \dfrac{1}{2}\rho V^2 S C_L) - G\sin\Phi \end{cases} \tag{5}$$

where V is flight-path velocity, X is horizontal distance, a is acceleration, g is gravitational acceleration, T is thrust, Φ is runway slope, α is attack angle, Ψ_T is thrust deflection angle, C_D is coefficient of drag, C_L is coefficient of lift and μ is friction coefficient.

The equation to calculate the ground roll is shown below[6], which can be solved through numerical integration method.

$$\begin{cases} V = \dfrac{g}{G}\displaystyle\int \left[T\cos(\alpha + \psi_T) - \dfrac{1}{2}\rho V^2 S C_D - \mu(G\cos\Phi - \dfrac{1}{2}\rho V^2 S C_L) - G\sin\Phi \right] dt \\ X = \displaystyle\int (V - V_W)\, dt \end{cases} \tag{6}$$

During the part of rotation, aircraft rotated at a given rate and the height of aircraft is nearly zero[7]. Because the aircraft doesn't leave the ground, the track angle is zero. The distance during rotation is:

$$\begin{cases} V = \dfrac{g}{G}\displaystyle\int\left[T\cos(\alpha+\psi_T) - \dfrac{1}{2}\rho V^2 SC_D - \mu(G\cos\Phi - \dfrac{1}{2}\rho V^2 SC_L) - G\sin\Phi\right]dt \\ X_r = \displaystyle\int (V - V_W)dt \\ \dfrac{d\theta}{dt} = \dfrac{d(\theta+\alpha)}{dt} = \dfrac{d\alpha}{dt} = C \end{cases} \tag{7}$$

Because the time of this part is very short, Eq. (7) can be simplified as:

$$\begin{cases} X = \dfrac{(V_R - V_W) + (V_{LOF} - V_W)}{2}\cdot t \\ t = \dfrac{\alpha_{LOF} - \alpha_R}{C} \end{cases} \tag{8}$$

where α_R is the attack angle when aircraft speed is rotation speed and the α_{LOF} is the attack angle when the aircraft speed is lift-off speed.

During the part of lift-off to safety height, the dynamic equation and kinematics equations is:

$$\begin{cases} m\dfrac{dV}{dt} = T\cos(\alpha+\Psi_T) - \dfrac{1}{2}\rho V^2 SC_D - G\sin\theta \\ mV\dfrac{d\theta}{dt} = T\sin(\alpha+\Psi_T) + \dfrac{1}{2}\rho V^2 SC_L - G\cos\theta \\ \dfrac{dX}{dt} = V\cos\theta - V_W \\ \dfrac{dH}{dt} = V\sin\theta \end{cases} \tag{9}$$

Eq. (9) are ordinary differential equations of the first order, which can be solved by the widely used fourth-order Runge-Kutta Method. The initial conditions of it are velocity, height, track angle and attack angle when airplane lifts off, and when H=10.7m the calculation will terminate.

Before using above method to calculate takeoff distance and accelerate-stop distance, VR should be determined first. The procedure to determine VR is as follows.

(i) According to Eq. (1), choose the max of $1.05V_{MC}$ and V_{MCG} as the initial value of V_R.
(ii) Solve the rotation part with OEI using Eq. (7) and get V_{LOF}.
(iii) If V_{LOF} satisfies Eq. (1), go to the next step; if not, increase V_R and go to the former step.
(iv) Solve the lift-off part with OEI using Eq. (9) and get V_2.
(v) If V_2 satisfies Eq. (1), the V_R is needed; if not, increase V_R and go to step (i).

3. Aircraft Landing Performance Calculation Method

Generally, the landing phase is split into four segments: approach, flare, transition and full braking segment[8,9]. Figure 2 shows the total process of landing.

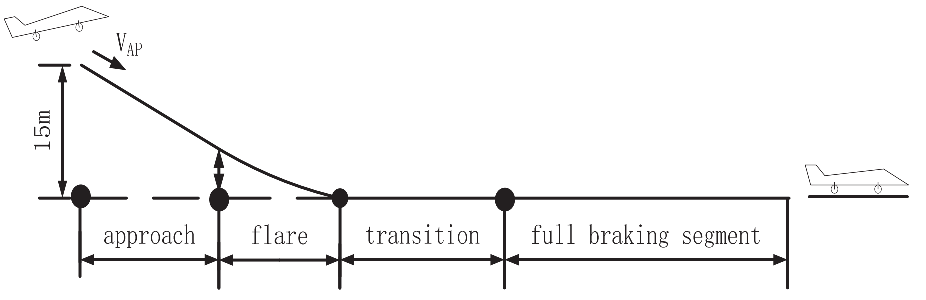


Fig. 2. Aircraft landing process.

V_{AP} is approaching speed and satisfies Eq. (10).

$$\begin{cases} V_{AP} \geq 1.23 V_S \\ V_{AP} \geq V_{MCL} \end{cases} \tag{10}$$

where VMCL is the minimum control speed during approaching and landing with all engines operating. This speed is kwon when calculating performance.

When an aircraft glides down, its track angel is constant and is usually set as -3 degree. The thrust is idle power. The dynamic equation and kinematics equations of approach segment are:

$$\begin{cases} \dfrac{G}{g}\dfrac{dV}{dt} = T_{idle}\cos(\alpha + \Psi_T) - \dfrac{1}{2}\rho V^2 S C_D - G\sin\theta \\ T_{idle}\sin(\alpha + \Psi_T) + \dfrac{1}{2}\rho V^2 S C_L - G\cos\theta = 0 \\ T_{idle}\sin(\alpha + \Psi_T) + \dfrac{1}{2}\rho V^2 S C_L - G\cos\theta = 0 \\ \dfrac{dX}{dt} = V\cos\theta - V_W \\ \dfrac{dH}{dt} = V\sin\theta \\ \theta = -3^\circ \end{cases} \tag{11}$$

where idle power and $\alpha_{glide} + \Psi_T$ is very small, $T_{idle}\sin(\alpha_{glide} + \Psi_T) \approx 0$.

When an aircraft finishes gliding and start to flare up, the dynamic equation and kinematics equations of flare segment are:

$$\begin{cases} \dfrac{G}{g}\dfrac{dV}{dt} = T_{idle}\cos(\alpha_f + \Psi_T) - \dfrac{1}{2}\rho V^2 S C_D - G\sin\theta \\ \dfrac{G}{g}V\dfrac{d\theta}{dt} = \dfrac{1}{2}\rho V^2 S C_L - G\cos\theta \\ \dfrac{dX}{dt} = V\cos\theta - V_W \\ \dfrac{dH}{dt} = V\sin\theta \\ \dfrac{d\theta}{dt} = -1^\circ / s \end{cases} \tag{12}$$

Use fourth-order Runge-Kutta Method to solve Eqs. (11) and (12). The procedure to determine the height (H_f) in which an aircraft starts to flare up is as follows.

(i) Assume initial H_f is 5 meters.

(ii) Solve Eq. (11). The approaching speed is the initial speed and H_f is terminal height.

(iii) Solve Eq. (12). The initial calculation height is H_f and terminal height is zero.

(iv) Calculate touchdown sinking speed according to Eq. (13). If it is not less than the minimum touch-down speed, the height H_f is needed. If it is, increase H_f and go to step (ii).

$$V_{TDS} = -V_{\mathrm{y}}\Big|_{H=0}\sin\theta \tag{13}$$

An aircraft decelerates through its braking system, spoilers and thrust reverser during ground roll of landing after touchdown.

$$\begin{cases} X = \int (V - V_W)dt \\ V = \dfrac{g}{G}\int\left[T_{idle}\cos(\alpha + \Psi_T) - \dfrac{1}{2}\rho V^2 S C_D - \mu(G\cos\Phi - \dfrac{1}{2}\rho V^2 S C_L) - G\sin\Phi\right]dt \end{cases} \tag{14}$$

Eq. (14) can be used to solve the distance of transition and full braking segment. However, the friction coefficients before and after braking system works are different values. The lift and drag coefficient before and after spoilers work are different values too. When thrust reverser plays a part in landing, it should be taken into consideration.

4. Validation Example

To verify and analyze the calculation method established by the paper, this section accomplishes the takeoff and landing performance calculation of a certain aircraft with four engines, called A for short. The result shows that the calculation method is reliable. And influence factors of takeoff distance and landing distance under different environments are also analyzed.

4.1. *Calculation results of A-type aircraft takeoff and landing performance*

A-type aircraft has four engines. Its wing area is 167.235m², its takeoff weight is 53500 kilos and its landing weight is 40000 kilos. Assume that the altitude of the airport is zero, wind speed is zero, the temperature is 15 centigrade degree and runway slope is zero. Calculate takeoff and landing performance using the programmed algorithm in this paper. The results are as follows.

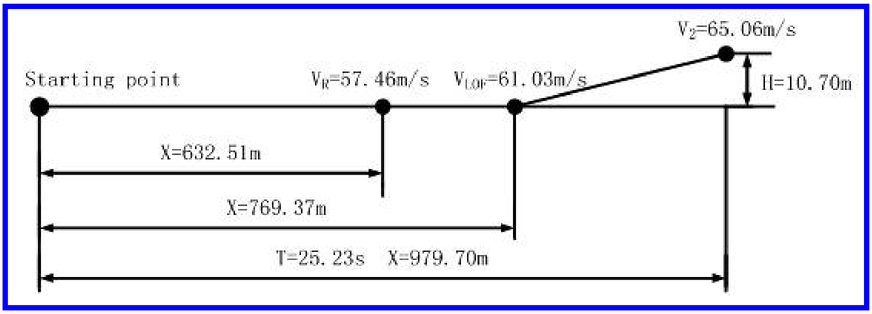


Fig. 3. Aircraft AEO takeoff performance.

Figure 3 gives horizontal distance and time of takeoff with all engines operating. V_R, V_{LOF} and V_2 are also shown in the figure.

The distance and time of other takeoff situations are listed in Table 1.

Table 1 Distance and time of OEI takeoff, rejected takeoff with all engines operating and rejected takeoff with one engine inoperative.

V_1/V_R	TOD_{N-1} (m)	t_1 (s)	ASD_N (m)	t_2 (s)	ASD_{N-1} (m)	t_3 (s)
0.8	1154.12	28.5	947.05	33.0	920.76	32.5
0.85	1115.59	27.7	1063.72	35.1	1034.85	34.6
0.9	1074.30	27.0	1188.69	37.3	1160.31	36.8
0.95	1041.65	26.4	1328.31	39.6	1297.93	39.1
1.00	1029.79	26.2	1481.22	42.1	1446.83	41.5

In Table 1, t1 is the time of OEI takeoff, t2 is the time of rejected takeoff with all engines operating and t3 is the time of rejected takeoff with one engine inoperative. V1/VR is a given value.

Takeoff distance and accelerate-stop distance on a dry runway is in Table 2.

Table 2 Takeoff distance and accelerate-stop distance.

V_1/V_R	TOD_{dry} (m)	ASD_{dry} (m)
0.8	1154.12	947.05
0.85	1126.66	1063.72
0.9		1188.69
0.95		1328.31
1.00		1481.22

The balance field length of A-type aircraft is 990.24 meters and the corresponding speed is 47.74 meters per second.

The landing performance of A-type aircraft is shown in Figure 4. In Fig. 4, X is the horizontal distance of the whole landing and T is the corresponding time.

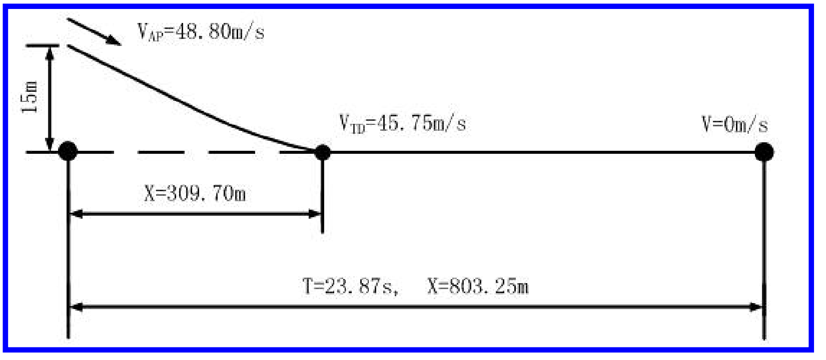
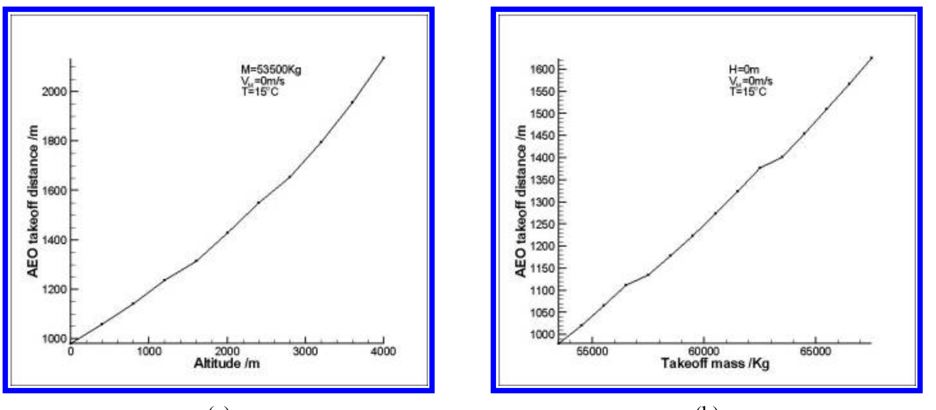


Fig 4. Aircraft landing performance

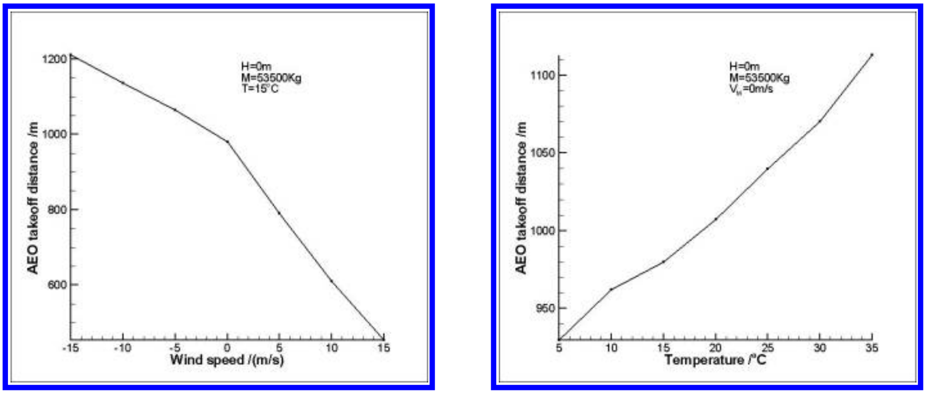
4.2. *Analysis of takeoff and landing performance influence factors*

An aircraft usually needs to accomplish takeoff and landing in different kinds of environments. The different environments studied in this paper means different airport attitudes, takeoff and landing weights, wind speeds and temperatures. To A-type aircraft, this section calculates the AEO takeoff and landing distances under different environments and analyzes some factors which may influence them.

Figure 5 gives 4 relation curves about how airport attitude, takeoff weight, wind speed and temperature influence AEO takeoff distance.



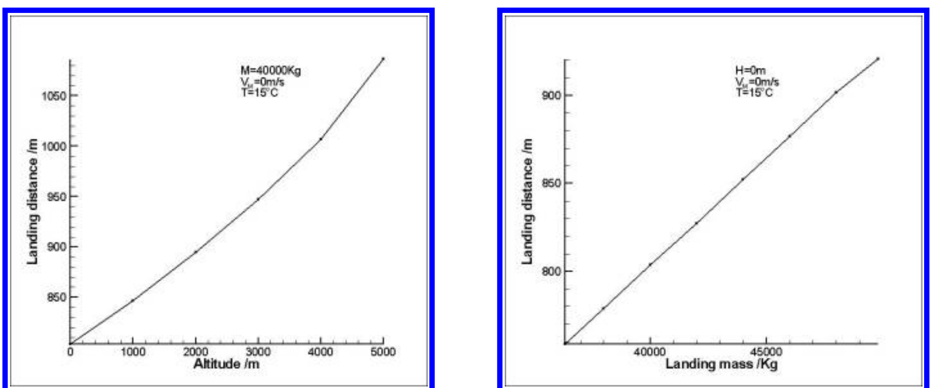
(a) (b)



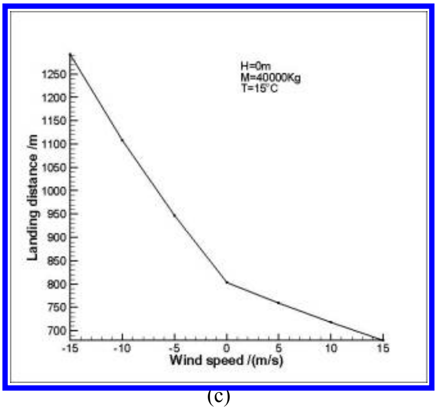
(c) (d)

Fig. 5. Relation curves between AEO takeoff distance and influence factors:
(a) altitude, (b) takeoff mass, (c) wind speed and (d) temperature.

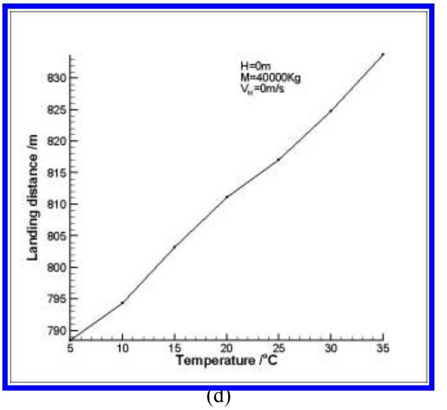
Figure 5(a) illustrates AEO takeoff distance increases linearly as the altitude increases below the altitude of 1600 meters. But above the altitude the linear relation is broken and the higher the altitude is, the faster the increasing rate of AEO takeoff distance. It's indicated that takeoff in plateau has a higher requirement for the length of airport runway compared with flatlands. Figure 5(b) shows that the increase of takeoff weight will lead to longer AEO takeoff distance and the two are nearly linear. In figure 5(c), the positive speed is dead wind and the negative speed is tail wind. It's clear in the figure that dead wind is good for takeoff and AEO takeoff distance changes linearly with both dead wind speed and tail wind speed, but their linear relation is different. The curve shape of figure 5(d) is similar to that of figure 5(c) and the almost same conclusions can be drawn. However, the influence of temperature on AEO takeoff distance is limited because the distance only increases 200 meters from 5 to 35℃.



(a) (b)



(c)



(d)

Fig. 6. Relation curves between landing distance and influence factors:

(a) altitude, (b) landing mass, (c) wind speed and (d) temperature.

Figure 6 is the relation curves between landing distance and its influence factors. The graphs in figure 6 are similar to the corresponding graphs in Figure 5, so the same conclusions can be drawn. However, the influence of these factors to landing distance is much smaller than the influence to AEO takeoff distance.

5. Conclusions

The paper researches takeoff and landing process by force analysis and establishes the dynamic equations of each segment of takeoff and landing. Aimed at solving the equations, an algorithm based on high-precision numerical integration is developed, which can evaluate takeoff and landing performance under different environments.

Takeoff and landing performance of an aircraft with four engines under different environments is calculated and the relevant influence factors are analyzed. The conclusions drawn from the analysis are as follows.

(i) Increasing altitude, takeoff and landing weight or temperature will lead to worse takeoff and landing performance; dead wind is good for takeoff and landing while tail wind not.

(ii) There are some linear relations between influence factors and AEO takeoff or landing distance.

(iii) The influence of temperature on takeoff and landing performance is smaller than other influence factors; the influence of four factors on takeoff performance is larger than landing performance.

It's worthy of note that the real environment of takeoff and landing may be more complex. Except altitude, takeoff and landing mass, wind speed and temperature, the conditions of runway such as wet, contaminated and frozen runway will also influence takeoff and landing performance. These problems will be subjected to a detailed study in the authors' ongoing work.

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