ESO207A: Data Structures and Algorithms

Theoretical Assignment 3

Due Date: 30th October 11:59pm, 2025

Total Number of Pages: 2 | Total Points 100

Instructions-

1. For submission typeset the solution to each problem and compile them in a single pdf file. Hand-written solutions will not be accepted. You can use LaTeX or Word for typesetting.
2. Start each problem from a new page. Write down your Name, Roll number and problem number clearly for each problem.
3. For each question, give the pseudo-code of the algorithm with a clear description of the algorithm. Unclear description will receive less marks. Less optimal solutions will receive only partial marks.
4. Please refrain from attempting problems without proper consideration. In case of uncertainty, clearly indicate "I Don't Know", 10% of the total marks will be awarded for such responses.
5. Assume that sorting would have $O(n \log n)$ complexity.

Question 1. Restoring Galactic Communication Network

The **Galactic Federation** maintains a communication network consisting of n planets, numbered from 1 to n, connected by m bidirectional communication channels. After a severe solar storm, parts of the network have been damaged, leaving several planets disconnected.

To restore communication, engineers can construct new channels between any two planets i and j. However, the construction cost is proportional to the square of the difference in their identification numbers:

$$\text{Cost}(i, j) = (i - j)^2$$

The goal is to ensure that **Planet 1** (the command center) and **Planet** n (the outer colony) are connected—either directly or indirectly—by adding at most two new channels. The objective is to determine the minimum total cost required to achieve this connectivity.

(a) (25 points) Design an efficient algorithm to compute the minimum total cost needed to make Planet 1 and Planet n connected by adding at most two new edges. Clearly describe the main steps of your approach and justify any data structures used.

(b) (15 points) Discuss why the algorithm guarantees the minimum possible cost and provide pseudo code of your approach from part (a).

(c) (5 points) Analyze the time and space complexity of your code from part (b).

Question 2. **The Postal Routes of Valoria**

In the ancient kingdom of **Valoria**, there are n towns connected by m two-way trade routes. Each route connects two distinct towns and can be traveled in both directions.

The King wishes to establish a grand postal route for royal messengers. The rules for the route are as follows:

- Every trade route must be used **exactly once** during the journey.
- A messenger may travel along the routes in any direction, but cannot reuse any route.
- The postal routes starts and ends in different towns.

(a) (15 points) Design an efficient algorithm to decide whether a valid royal postal route exists in the kingdom of Valoria. If it exists, also provide such a route. Explain your approach.

(b) (5 points) The royal messengers have now requested you to find a route that starts and ends in the same town. Mention the changes you need to make in your algorithm in part (a).

(c) (5 points) Analyze the time and space complexity of your algorithm for both the parts (a) and (b).

Question 3. **The Sky Trams of Aetheria**

In the floating city of **Aetheria**, hundreds of sky trams glide endlessly between levitating islands. Each tram follows a fixed circular route, repeating the same sequence of islands forever. There are n tram routes, where the i^{th} route is represented by a list of atmost m distinct island identifiers that the tram visits in order. For example, if `route[0] = [1, 2, 3, 4]`, it means that the first tram travels as

$$1 \to 2 \to 3 \to 4 \to 1 \to 2 \to 3 \to 4 \to \dots$$

A messenger begins their journey at island `source` and wishes to reach island `target`. The messenger may board or exit a tram only at an island that the tram visits. Two trams are said to **intersect** if they both stop at the same island, allowing the messenger to switch from one tram to another.

Your task is to determine the smallest number of trams the messenger must use to travel from `source` to `target`. If it is impossible to reach the target island, report that as well.

(a) (5 points) Formulate the problem using a graph. Explain your formulation and mention what do the nodes and edges represent in your graph.

(b) (10 points) Design an efficient algorithm to compute the minimum number of trams required to travel from `source` to `target`. Describe your approach.

(c) (10 points) Provide a pseudocode to implement your approach.

(d) (5 points) Analyze the time and space complexity of your algorithm.