A Guide to Designing Experiments to Test Statistical Graphics

Emily Robinson Heike Hofmann Susan Vanderplas

2025-03-14

In this paper, we discuss considerations and methods for experimentally testing visualizations. We discuss levels of user engagement with graphics, common issues when developing a sampling or data generation model, the importance of pilot testing, and data analysis methods. Along the way, we also provide recommendations of how to avoid some of the unique pitfalls of human testing in statistical and visualization research.

1 Introduction

Data visualizations are a critically important tool for communicating scientific information to the public in what creators hope is an easy-to-digest, visually attractive form. There are many strategies for creating charts and graphs, from Tufte-esque minimalism (Tufte, 1991) to charts designed with extra imagery and aesthetic appeal that draw the viewer's attention and persist in memory (Cairo, 2012). For a specific type of data, there are also usually many different chart forms to display that data: for instance, if we have a set of categorical data and we wish to show the relative proportions of each category, we could do so using a stacked bar chart or the polar equivalent, a pie chart. There have been several attempts to list out all of the types of charts (Ribecca, 2022), create a taxonomy of charts (Bertin & Berg, 1983; Desnoyers, 2011), and even to create charts using a domain-specific grammar of graphics (Wilkinson, 1999) that is also useful for classification. One extremely useful reference is from Data to Viz (https://www.data-to-viz.com/), which uses a decision tree to show different visualizations compatible with the data; R, python, D3.js, and React code are provided to demonstrate how to create those visualizations. With all of the different design choices available, how are chart creators to know what is the best approach for communicating data to the appropriate audience?

While there are heuristics, rules-of-thumb, and guidelines (Allen & Erhardt, 2016; Few, 2006; Haemer, 1948; "Joint Committee on Standards for Graphic Presentation," 1915; Kosslyn, 2006) for creating useful and visually attractive data displays, the best way to establish the efficacy of various design decisions is to test the visualization on humans, evaluating different variants under controlled conditions (Cleveland et al., 1988; Cleveland & McGill, 1985). Empirical assessments of visualizations, when carefully designed, allow statisticians to determine which representation of the same data is most effective along one or more dimension(s) of interest: estimation or prediction accuracy, within or between group comparisons, response time, and ability to draw real-world decisions are common goals for charts.

It is extremely challenging to design studies which strike the right balance between experimental control (i.e. internal validity) and generalizability to a wider context (i.e. external validity). Simply asking people to read quantities off of a graph may not generalize beyond the questions asked or the data used in the chart (Croxton, 1932; Croxton & Stryker, 1927; Eells, 1926; Huhn, 1927), but designing a study that is sufficiently robust to those issues requires manipulation or control of so many factors that the amount of participants and trials quickly becomes daunting or unaffordable. In this paper, we attempt to distill the experience gained from conducting several different types of graphics experiments (Hofmann et al., 2012; Robinson, 2022; VanderPlas et al., 2019; Vanderplas et al., 2024; VanderPlas & Hofmann, 2015, 2017), discussing the use of different testing methods (Vanderplas et al., 2020), the process of designing a graphical experiment, and analysis of the resulting empirical data. It is our hope that this paper will lower the barriers that exist for conducting empirical graphics research and reduce the probability of costly mistakes.

Section 2 discusses different methods for testing graphics, and which methods best address different levels of user engagement. In Section 3, we discuss the process of developing a data-generating model used to control the statistical features of data in the tested visualizations. Model development is a nuanced and iterative process that ultimately determines the success and generalizability of the experimental results. We explore different experimental design considerations in Section 4 and then move to the importance of pilot testing in Section 5. Finally, we provide some common analysis strategies in Section 6, including strategies for handling the unexpected data features which are so common in graphical testing experiments.

2 Testing Methods and User Engagement

There are many different testing methods used to empirically assess statistical graphics. This paper uses studies conducted online without additional equipment as primary examples, though many of the same considerations apply to in-person experiments conducted using additional equipment, including 3D printed charts, eye-tracking equipment, and interactive data displays. Online experiments have lower overhead, offer relatively fast data collection, and provide useful results for well-designed experiments. The toolkit used for these experiments is R-based (R Core Team, 2022), and includes ggplot2 (Wickham, 2016) and Shiny (Chang et al., 2021) as primary components. In many experiments, we customized the Shiny interface with JavaScript and d3 (Bostock et al., 2011), enabling interactive graphics, use of svgs, and other useful extensions. While we prefer this set of tools, most of the observations described here apply to a wide variety of different workflows for graphical experimentation, including in-person experiments.

It is important to consider the level of user engagement which is necessary to complete a particular visual or graphical task. For instance, testing whether someone can detect an effect such as a linear trend in noisy data is a perceptual question. Perceptual questions are often examined experimentally using methods which allow the user to interact with the data on a basic visual level: users are presented with a visual stimulus and answer yes/no questions to indicate whether the effect is detected. Numerical estimation is another common task when testing graphics: in these experiments, the participant views a chart, estimates the requested numerical quantity, and enters the estimate into the application through a numerical input, slider, or other form element. Sometimes, it is possible to set up a scenario where the user adjusts the plot using a set of controls designed to provide a fixed set of interactive operations. This type of user engagement was used to assess the strength of the sine illusion (VanderPlas & Hofmann, 2015): users adjusted the strength of a transformation designed to correct the illusion until the lines appeared to be the same length, as shown in Figure 1, providing a direct measure of the magnitude of the sine illusion's effect. In other situations, it may be preferable to have the user directly interact with the visual stimulus. In Vanderplas et al. (2024), participants were asked to rotate and interact with a 3D rendered bar chart; the application recorded user interactions and corresponding rotation matrices, providing insight into the visual comparisons the user may have been performing. This information was used as a supplement to the explicitly provided estimates, providing some contextual information as well as the ability to identify the level of participant engagement with the questions. When experiments are conducted as part of classroom experiential learning, it is sometimes helpful to be able to separate the low-effort participants from those who were fully intellectually engaged in the task. Interactive graphics provide another level of user engagement that can be much more open-ended. With interactive graphics, researchers can ask participants to directly annotate plots, toggle aesthetics, and highlight groups and plot features. Careful implementation of the experiment application may allow for each of these interactions to be recorded and analyzed, producing a rich, if messy, set of data that may allow researchers to tease apart visual estimation error from common shortcuts such as rounding used during direct numerical estimation.

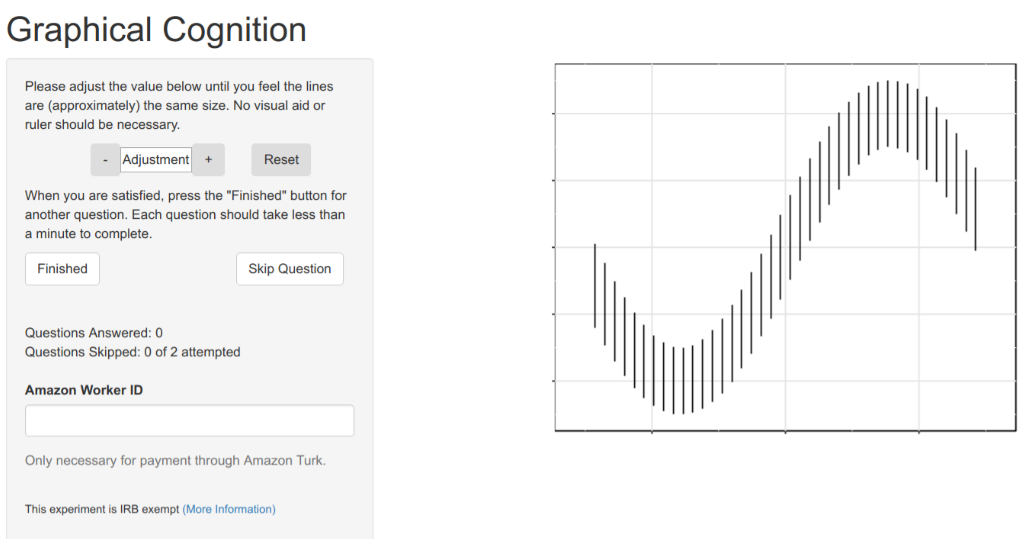

Figure 1: Direct adjustment of a plot in a perceptual task. In this experiment, designed to assess the strength of the sine illusion, the user adjusts the plot using - and + buttons, which control the strength of a transformation designed to correct the effect of the sine illusion. When the user is satisfied that the lines are of equal length, they select the 'Finished' button to move to the next task. The experiment used a psychophysics experimental design, the method of adjustment, but leveraged the interactive Shiny interface to record the entire sequence of adjustments made by the user for each trial. A demo version of this application can be found at https://shiny.srvanderplas.com/sine-illusion/.

Visual inference (Buja et al., 2009; Wickham et al., 2010) is another useful testing tool for perceptual questions such as "which chart displays this data more clearly" (Hofmann et al., 2012) while simultaneously assessing the statistical significance of the graphical finding in a chart. Visual inference charts are often called "lineups" in analogy to the criminal procedure where the suspect is placed in a line with several other individuals with similar characteristics. In a graphical lineup procedure, there is a target plot containing the real data, embedded in an array of innocent "null" plots generated through resampling or simulation. If viewers consistently pick the target plot at a higher rate than any of the null plots, the target plot is said to be visually significant (Loy & Hofmann, 2013; Majumder et al., 2013) and a "see" value, the visual analogue of a p-value (Chowdhury et al., 2020), can be calculated using the vinference R package or the process described in VanderPlas et al. (2021). The details of this calculation are beyond the scope of this broader discussion of how to test charts, but more detail on visual inference is provided in <insert citation to visual inference WIRE article under development>.

In another variation of the statistical lineup procedure, data generated from two models are compared, with target plots from each model embedded in the array of K total plots. The $K - 2$ null plots are constructed from a mixture model that blends the two competing models (VanderPlas & Hofmann, 2017). Viewers are asked to select the panel(s) which are most different, and the primary source of information are trials in which viewers selected the target from one model but not the other, indicating that the display method used allowed viewers to differentiate one model's data (but not the other) from the nulls created through a mixture model. This variation allows the experimenter to assess graphical design choices to determine whether they effectively emphasize structural differences in the data (VanderPlas & Hofmann, 2017).

One advantage of the visual inference technique is that the experimenter can ask a very general question, such as "which of these plots is the most different?", rather than a specific question about the displayed data which may require more quantitative sophistication. All of the necessary information to make the decision is embedded in the choice of the model used to generate the null plots. This feature is extremely convenient when conducting the experiment and even allows small children to complete the task. The downside is that as a result, visual inference experiments do not allow experimenters to assess the viewer's understanding of the information shown in the chart. In most cases, visual inference experiments remove any contextual information from the charts, including axis labels and values, plot titles, and so on, in order to encourage participants to make decisions based solely on the graphical presentation. This lack of context is a double-edged sword: visual inference can involve participants who do not have any mathematical training or instincts (including children), but researchers also cannot use this technique to assess higher levels of engagement with a chart, such as estimation, prediction, or reasoning based on displayed information.

To assess the viewer's understanding of information shown in a chart, we must ask questions and allow the user to provide feedback. User feedback may be collected on a numerical scale or through the use of written comments, recorded "think-aloud" processes, and other more qualitative interaction methods. In some studies, asking users to interpret a chart within a larger scenario can be effective, as in Figure 2, while in others it is more helpful to ask users to explain answers. In visual inference studies, asking users why a specific panel was chosen has been demonstrated to provide rich insight into otherwise confusing numerical results (VanderPlas & Hofmann, 2017).

Think-aloud methods ask the viewer to narrate their internal thought process, either during or after completing a task (Haak et al., 2003). These recordings (or transcripts) can provide valuable insights into conscious cognition, and are often used when conducting usability studies. While we have not to date recorded users talking out loud about what they are seeing during a study, think aloud methods could easily be implemented within a Shiny application, with audio recordings saved to the server for transcription and analysis (Dunbar, 1995; Kirschenbaum, 2003; Trafton et al., 2000). It is even possible that these recordings could be automatically transcribed using speech-to-text models. We have used think-aloud methods informally during pilot studies to "harden" graphical experiments and verify the selection of parameters used in an experiment. The success of this approach, combined with the few studies which used think-aloud to assess charts (Haider et al., 2021; Kulhavy et al., 1992; Lee et al., 2016), suggests that think-aloud methods are an often-overlooked but useful tool for assessing data visualizations.

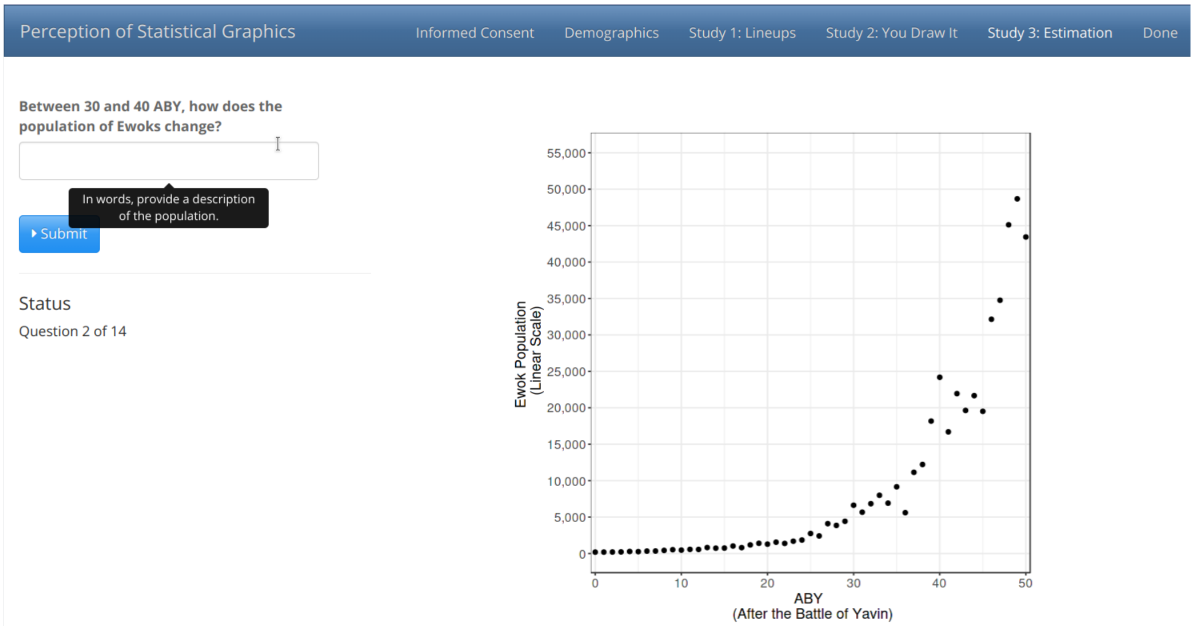

Figure 2: This question asks users to write out a description of how the population of Ewoks changes over time, without any further cues, to determine whether participants default to multiplicative or additive language descriptions.

Of course, in an online, asynchronous experiment, every user interaction with the testing materials (typically hosted on a web page) can also be recorded along with time stamps, mouse positions, browser size and screen resolution, and other information. While we have not used this type of information heavily in our experimental analyses thus far, in most experiments we collect time stamp data in order to assess how long participants spend on each question. Typically, the first round of test questions takes the longest for participants to complete. Additional replicates do not usually affect accuracy (i.e. there is no immediate learning effect) until after 'too many' tests cognitive fatigue proves to be detrimental to accuracy (Chowdhury et al., 2018). This sweet spot between replicates and fatigue depends on the cognitive burden in each test and should factor into designing the experiment. In some experiments, we have provided participants with supportive tools, such as "scratch pads" and calculators built into the Shiny application to support the complex calculations required to answer higher-level numerical estimation questions (Figure 3). In order to be supportive, the tools must be easy to use, but assuming this bar is met, the tools can reduce participant cognitive load while recording a wealth of information. This information provides real insight into how participants were looking at the data, what strategies they tried and discarded for reading the chart, and what visual estimation methods were used. While systematic analysis and modeling of this data may be difficult, as it is usually messy and often must be manually coded, the insights provided can be extremely useful. However, unless participants are required to use these tools, it is difficult to gather comprehensive information - those participants that don't use supportive tools likely differ in meaningful ways from those who do. As a result, the information gathered from supportive tools likely does not generalize to the entire sample.

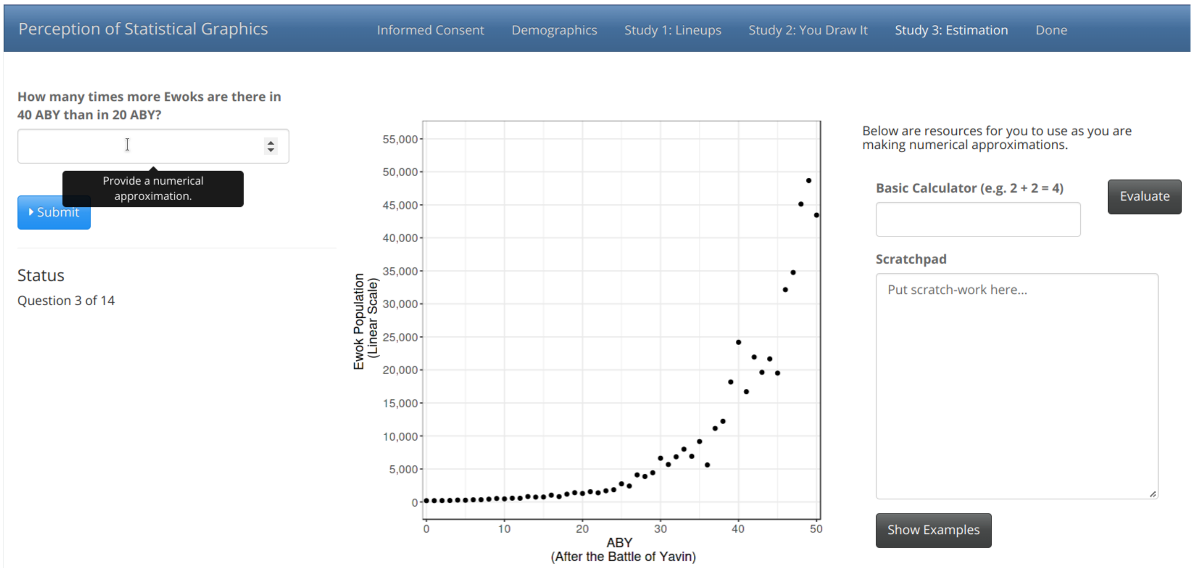

Figure 3: This question asks participants for a numerical estimate, but provides a basic calculator and scratchpad. All user interactions with the calculator and scratchpad are logged, providing insight into the user's thought process and estimation strategy.

One of the most difficult components of designing an experiment which asks users to directly estimate information from a chart using a full scenario (background information, etc. as well as contextual details from the chart) is that the questions must be extremely carefully constructed. Mathematics education researchers provide guidelines for selecting different levels of questioning in order to assess graph comprehension: literal reading of the data, reading between the data, and reading beyond the data (Curcio, 1987; Friel et al., 2001; Glazer, 2011; Wood, 1968). In a recent study, we identified questions based on this framework to evaluate direct estimates and extend those estimates to make comparisons between two points.

Even when great care is taken with the construction of the question, participant answer accuracy is fundamentally limited by the fact that many participants do not read and interpret the question with the care and precision that it was written. Questions that ask participants to e.g. estimate the multiplicative change in a quantity at two time points may be misunderstood as asking for an estimate of the additive difference, and the resulting estimates are then one or more orders of magnitude off of the correct answer. This is one area where lineup methods are convenient - they do not depend on participants to understand the nuances of language or scenarios built around the chart under investigation. However, in some situations it may be sufficient to ask participants to estimate direct numerical quantities that have little contextual information, as done in (VanderPlas et al., 2019) when assessing the accuracy of framed plots re-created from the Statistical Atlas.

Another useful measurement strategy is to require participants to engage directly with an interactive visualization. This is useful in a directed task, where users are asked to interact with the chart in a specific way and the result is recorded, but it is also possible to use interactive visualizations in an open-ended task, recording how users engage with the graphic in an exploratory (as opposed to goal-directed) manner. In one recent experiment, we asked participants to forecast an exponential trend, with data presented on either a linear or log scale. Using JavaScript code modified from New York Times interactive graphics "You Draw It" features (Katz, 2017), we had users draw trend lines with their computer mouse and make forecasts directly on interactive charts, with the data and user-drawn predictions recorded to our database. With interactive graphics rendered using JavaScript (or other web libraries), the only limit to the types of questions one can ask in testing graphics is one's ability to write code to interact with the visualization library. This type of testing method can be extremely natural for participants, but it also is hard to generalize when discussing testing methods because of the potential range of applications where it might be employed.

Whichever testing method is chosen should be appropriate to the type of question under investigation and the level of visual and cognitive engagement required to answer that question. While lineups are excellent tools for assessing perceptual questions, they cannot address questions aimed at understanding how people use charts within the wider context of a story or practical task; this requires more direct methods with higher ecological validity.

All of the testing methods described here require significant work to develop a strategy for data generation appropriate for testing the underlying question. For instance, when testing the perception of exponential growth, we had to develop a model which would generate data with varying growth rates, but where the data had a pre-specified domain and range. The data generating model is particularly critical when using lineups, as the null sampling model must replicate the important visual features in the data. Each testing method has specific requirements, but it is important to carefully calibrate the model parameters to allow for some variability, but not too much, and to ensure that participants can succeed at the task and do not feel like they are being made to analyze random noise. This Goldilocks-style problem is the focus of the next section.

3 Developing a Model

Once the graphical task has been identified, it is necessary to develop a model which can be used to explore the graphical features of interest in a precise manner. This is the single longest part of the entire experimental design and execution process, in part because choosing a model that replicates important visual features of the data is extremely complex (Cook et al., 2021; Hullman & Gelman, 2021; VanderPlas, 2021).

There are two main options when developing a statistical model for graphical testing: start with a large data set and sample from that data set (Hofmann et al., 2012), or start from a model and sample data from that model generating process (Robinson, 2022; VanderPlas & Hofmann, 2015, 2017). This decision is largely determined by the availability of a large data set containing the requisite features of interest and the qualities being manipulated in the experiment. For instance, Hofmann et al. (2012) used samples of different sizes from a pre-existing data set to manipulate the amount of signal in each comparison; with a small sample, there is less signal and the same amount of noise, making the true plot harder to spot. In many situations, though, a convenient data set with the right properties is harder to acquire, and it becomes necessary to develop a sampling model to generate data for user evaluation.

The tools we discuss in the remainder of this section can be applied both to pre-existing data sets and to model-based sampling methods.

3.1 Screening Parameters with Simulation

The choice of the parameter space used in testing is crucial to gain insight from a study without putting too much burden on participants with overlong studies. Choosing an appropriate space for testing parameters is a well-known problem in psychometric testing: the space considered should cover the area between 'only some activation' to 'almost full activation' of an appropriate psychometric function (Schьtt et al., 2016; Valentin et al., 2024). When testing charts, visual assessment is obviously key, but researchers can make use of statistical indices related to the testing condition to narrow the parameter space to a reasonable and efficient subset from which maximal information can be acquired.
These statistical indices may also serve as quantitative proxies for the difficulty of the visual task.

To identify a statistical proxy for visual difficulty that may help with narrowing the parameter space, it can be useful to consider numerical measures used to estimate the same types of visual information that will be assessed in the experiment. For instance, we have used

- R^2 as a measure of the strength of a linear relationship
- Gini inequality as a measure of the strength of clustering
- lack-of-fit statistics to assess the amount of curvature in an exponential relationship (shown in Figure 4)

Then, a wide range of potential combinations of parameter values or sampling strategies can be explored and summarized graphically; if the numerical statistic cannot differentiate between the null and target under a condition, it is reasonable to expect that a visual inspection of the data may also not show significant results. As with any measure, it is important that difficulty levels span a range from easy to hard; we do not learn anything from finding out that everyone can distinguish all of the combinations.

While this method is certainly more critical for model-based sampling methods, it is also important when data are generated by sampling from a larger data set. When sampling from a larger dataset, parameters are more often sample size and stratification methods, but it is still important to iteratively assess the data generating procedure through simulation. Using numerical proxies for visual characteristics of data displays such as curvature, linearity, scatter, dispersion can assist with identifying optimal parameter settings to use across different experimental conditions. Even with this strategy, it is still critical to fine-tune the parameter choices with visual calibration and pilot testing.

3.2 Fine-Tuning Parameter Choices

Once an appropriate set of parameters are identified using the numerical screening method, it is important to calibrate these parameter selections visually. No numerical statistic is a perfect measure of what we actually see: at best, they are approximations of what we might potentially see. We have found it to be useful to have one experimenter calibrate the model parameters at a gross level, and then have another experimenter narrow in on the parameters which are visually reasonable within the selected range. Then, both examiners visually inspect a large number of plots generated using those parameters to get a sense for how difficult the task at hand is (this strategy is also described by Lu et al. (2022)). At some point, all experimenters become so visually saturated with the nuances of the data generating mechanism that it may become necessary to "sanity check" the protocol with family members, friends, and colleagues. These informal surveys provide

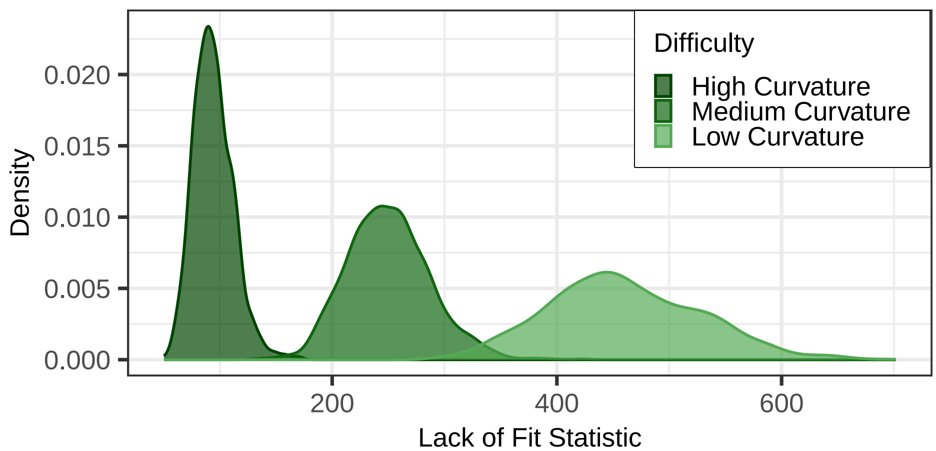

Figure 4: Density plot of the lack of fit statistic showing separation of selected difficulty levels: High (obvious curvature), Medium (noticeable curvature), and Low (almost linear). Each density plot is the result of 1000 simulations from a model $y_i = \alpha \cdot e^{\beta \cdot x_i + \epsilon_i} + \theta$, where $\epsilon \sim N(0, \sigma^2)$. α and θ were selected after manipulation of β and σ to ensure that all data generated had similar y ranges so as not to provide visual cues about model differences outside of the plot curvature.

extremely useful feedback and can help to counteract the visual saturation of being immersed in the design of a visualization experiment for months at a time.

3.3 Visual Assessment is Critical

We cannot overstate the importance of visual assessment of your model stimuli, preferably with fresh eyes. We highly recommend performing several rounds of think-aloud pilot testing before deploying an experiment. In support of this assessment, we offer up a cautionary tale of our own experience: that of VanderPlas & Hofmann (2017), where we designed an experiment to test which plot aesthetics promoted discovery of linear trends and/or clusters.

The experiment was a full 2x3x3 factorial exploration of three data generating parameters, with 3 replicates at each parameter combination (54 data sets) and 10 aesthetic combinations (for a total of 540 lineups). Each lineup had 20 different sub-panels, so we should have carefully visually inspected some 10,800 different panels. As is evident from the fact that we're telling this story as a cautionary tale, we missed a critical problem with our data-generating mechanism: when clusters were assigned to randomly generated data after the fact, we didn't control the cluster size, leading to clusters of one or two points in relatively few sub-panels. This became particularly noticeable when bounding ellipses were added to the plot, as the method used to generate those ellipses required at least 3 points in the cluster. The missing boundary ellipse in the corresponding sub-panels escaped our notice during the stimuli proof-reading phase of the experiment, but did not escape the notice of our participants, who only needed to examine about 10 lineups each (around 200 panels). An example of one of the problematic lineups is shown in Figure 5: many participants selected panel 16 because of the missing ellipse; not a wrong choice, but certainly not the effect we intended to test.

One reason why it is so difficult to generate sampling models for visual explorations is that our visual system is optimized for identifying differences between groups. This ability can interfere with the natural to use the null sampling models that might be used in equivalent numerical tests when running experiments that use visualizations. We re-ran the experiment using a different clustering method that controlled the number of points in each group. Instead of noticing the number of ellipses, participants instead used the differences in size and shape of the ellipses formed when clustering after the data generating procedure. That is, participants could still detect the artificial nature of the induced clusters using other features. While it can be difficult to get the data generating method right, it is essential to conducting visual experiments that generalize well beyond the effects shown in a single data set or phenomenon. The time and effort invested in this step at the outset of the experiment pays dividends when it allows for clear generalization of the experimental results to an entire statistical concept rather than a single data set.

4 Experimental Design Considerations

It would be difficult to develop a full data generating model without some idea of the experimental design: the basic structure of the parameters which are to be manipulated, how the users will be tested, and so on. These experimental design factors are fairly natural for scientists to accumulate over the course of imagining and planning an experiment. When conducting graphical tests, however, there are additional considerations beyond those taught in a standard experimental design course.

4.1 Cognitive Load

The primary human design factor to be aware of is that visual tasks and assessing statistical graphics can be extremely cognitively taxing. In our experience, it is difficult to expect participants to evaluate more than about 15 charts in one sitting. If users are asked to deeply engage and answer multiple questions about each chart, this limit may be lower (8-10), but even with a relatively simple task, as in lineup methods, it is hard for participants to accurately evaluate more than about 15 charts. Tasks which are more interactive, such as ‘You Draw It’, may be somewhat easier for participants, but it is unlikely that participants would be willing to complete more than about 20 tasks in one sitting even with tasks that require fewer decisions and more engagement. At some point, participants’ interest in accuracy will decline, and the researcher is better off using a larger number of participants completing fewer tasks within the window where participants are attentive and engaged.

4.2 Participant Instructions

It can be extremely helpful to include “practice” demonstrations of the task to show the basic process, logic, and reasoning. While it is tempting to make these tasks fully representative of the type of judgement which will be required of participants, practice tasks which are too close to the experimental task may bias participants; we have found that it works well to have a relatively

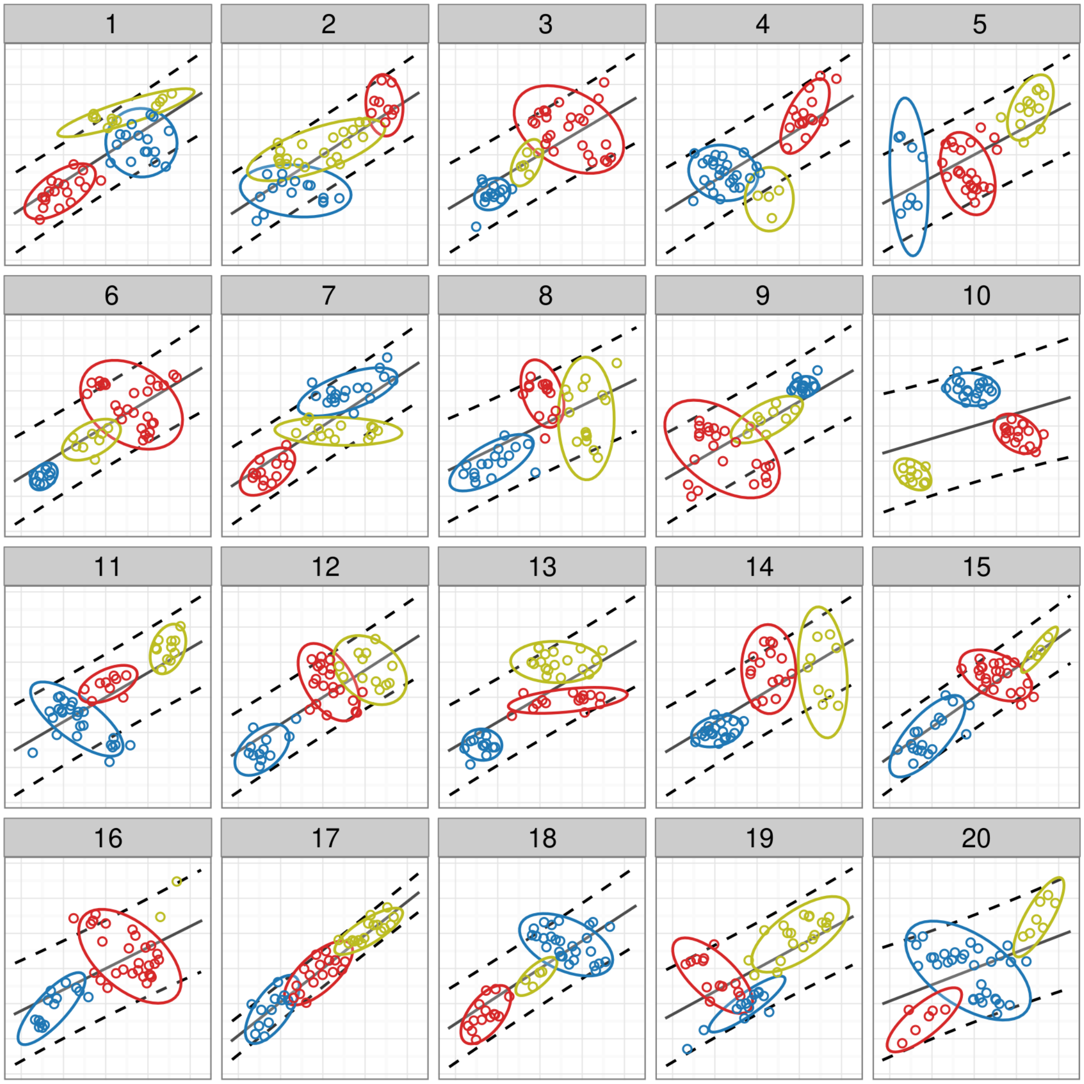

Figure 5: A lineup from Vanderplas & Hofmann (2017). Panel 10 shows the clustered target data and panel 17 shows the target data with a strong linear relationship; either of these target panels was the expected choice. Unfortunately, panel 16 has only two bounding ellipses shown, which is an unintentional difference that resulted from a faulty method for assigning clusters to null plots; many participants selected this panel instead of one of the target panels.

easy practice task which utilizes a slightly different type of plot and/or type of data than what will be tested in the experiment. In cases which require interactivity, gif animations of the task being carried out are useful, as are additional visual cues, such as the yellow box used in the 'You Draw It' task[1] to indicate that there were points which were not completed.

4.3 Accounting for Participant Variability

While studies have found some relationship between lineup performance and demographic factors (VanderPlas & Hofmann, 2016), these differences are relatively small when participants are recruited from online testing platforms like Amazon Mechanical Turk or Prolific, or when participants are recruited from a university student population. Studies have also not found a strong relationship between visual task performance and participant recruitment method (VanderPlas et al., 2019), perhaps in part because demographics which use social media sites and demographics completing online research tasks for pay overlap heavily. However, when participants are recruited using a statistically representative sample of the wider population, education becomes an extremely useful predictor of ability to effectively read and draw conclusions about a chart (Rice et al., 2024). In almost every situation, though, we find that some participants are very good at evaluating graphics and some participants are not; as a result, it can be extremely helpful to use random effects models with an effect for individual participants.

4.4 Demographic Data

It may be useful to ask a few more demographic questions about STEM education level for studies which ask more of participants from a mathematical standpoint; while lineup studies have not found strong associations with those variables, lineup studies also do not require participants to engage with the data presented in a chart in a way that requires higher-order mathematical reasoning. This has allowed us to make the argument to the ethics committee (institutional review board, or IRB) that our research is exempt, as we do not collect enough demographic information to identify participants, however, collecting reduced demographic information occasionally comes at a cost. One recent study examining the use of log scales and exponential data was conducted using Prolific, which recruits participants from around the world; we required only that participants were fluent in English to participate. It was only after the experiment was completed that we realized that different countries introduce logarithms as a concept at different points during primary and secondary education; it might be that individuals in some countries have much more experience with log scales than those educated in the United States. We mention this only to point out that while every experiment contains a few missed opportunities, it is worth giving careful thought to the demographic questions asked of participants and what information may be helpful during the analysis stage.

4.5 Trial Allocation to Participants

In statistical design terms, most of our studies involve some type of balanced incomplete block design, where participants are assigned to a subset of experimental conditions which allow for

[1] See a gif of testing with 'You Draw It' here

estimation of the full range of effects specified in the model. The particular structure of these designs depends heavily on the factorial structure of the study, but we typically arrange participants' trials to ensure that participants see the same data set only once (where possible) and see as many different experimental conditions as possible. It is also important to reduce the impact of order effects using either e.g. Latin square designs or randomization where possible, but we recognize that this is not always feasible due to the need to maintain participant naivete during some portions of the experiment.

4.6 Data Collection Platforms and Infrastructure

We have conducted visualization experiments using a wide variety of tools: custom web PHP servers running interactive forms for data collection, generic web survey platforms (Qualtrics, Google Forms) for static graphics, and Shiny applications that control every part of the experimental process from instructions to providing completion codes for payment while rendering interactive graphics and generating fully randomized data for each participant. In our research, Shiny has provided the right balance between control over the experimental setting, procedure, etc. and the intricate details of web server management, however, this balance is likely different for every group and potentially for every experiment. Likewise, we have recruited participants using services such as Amazon Mechanical Turk and Prolific, on social media sites (Reddit, Twitter(X), Mastodon, and BlueSky), and by email; each recruitment method has trade-offs between cost, convenience, number of people in the sampling frame, and control over participant demographics. Rice et al. (2024) recruited participants using nationally representative panel samples and found that conclusions from fully representative samples of the population can be very different from volunteer samples recruited using other services, as many people are unmotivated to engage with charts, don't know how to read charts, or impose pre-formed conclusions onto visual displays. These individuals are unlikely to participate in most ad-hoc volunteer-based sampling methods. While using representative panel sampling products offered by organizations such as NORC or Gallup is considerably more expensive than samples from Amazon MTurk or Prolific, these results suggest that results from sampling methods used online may not generalize to the broader population. People who participate in studies via online platforms are more technologically sophisticated and educated than the general population, and this bias may significantly impact the conclusions.

If conducting an online experiment, we recommend consideration of panel-based sampling methods as well as online testing platforms. While Amazon MTurk was once the platform of choice for this type of research (Heer & Bostock, 2010), in 2025 Prolific, which is aimed at academic experiments, seems to provide better options that ensure that experiments are completed by people rather than AI than MTurk. It seems likely that the preferred platform for participant recruitment will continue to evolve over time, thus, we will recommend that experimenters consider available options, compare pricing structures (as these vary widely), and consider whether add-on fees for e.g. demographically representative samples are worth the additional cost. An additional consideration when choosing between panel samples and online recruitment platforms is the constraints placed on the types of questions and graphics which can be used. In our experience, it is much easier to run an experiment using the online recruitment platforms, which place many fewer constraints on both the number and types of questions which can be asked and the types of stimuli which can be used in the experiment.

4.7 Ethical Review of Human Subjects Experiments

In almost any research environment, there are requirements that experiments involving human participants undergo review by an ethics board to ensure that participants are not harmed by participating in the study. We are most familiar with regulations in the United States and can only speak to that environment. Most graphics experiments conducted in the US fall under the "exempt" category of experiments which require only basic review and approval. These experiments record only basic demographic information, ask participants to complete tasks that involve no risk, and experimenters record information which would not be embarrassing to participants if exposed. In order to ensure that our experiments fall into this category, we will often ensure that demographic information is collected and stored separately from any identifiable information, such as e.g. user IDs from the participant recruitment platform that allow us to monitor task completion and pay participants for their time. Researchers should also ensure that they comply with privacy laws such as the European General Data Protection Regulation (GDPR); complying with this law while collecting data which is not identifiable can be complicated. We recommend consulting with your ethics board and institutional recommendations in order to maintain legal compliance and safeguard participant privacy.

5 Pilot Testing and Quality Assurance

Once the data generating model is set, the experiment is designed, and the charts have been developed, the next step is an extensive round of pilot testing. The goal of pilot testing is to ensure that the experiment is set up properly and that no issues have been overlooked. Pilot testing also provides an opportunity to ensure that directions are clear, participants know what they are supposed to be doing, and to estimate task completion time (which determines how much participant will be paid on many online testing platforms). Our studies usually go through 2-3 rounds of pilot testing, with at least one of those rounds involving any relatives and friends who are less technologically savvy. We also purposely include talented individuals who can accidentally crash any testing applications. A final round of testing usually includes any and all coworkers and friends who may be available for 10-15 minutes during the work day. Pilot study samples do not need to be representative of any particular population, though we would caution against using exclusively visualization researchers or statisticians in your pilot sample because some issues are less likely to show up with knowledgeable participants. In some studies, each participant may see a new set of data, depending on how the study is designed. In these cases, we highly recommend saving all generated data to a database as well, so that it is possible to go back and examine exactly what happened and/or how things went wrong. Hard drive space is extremely cheap relative to almost any other cost in an experiment; saving all of the data is a sensible measure.

One highly useful (but not strictly essential) component of an experiment that can be set up during the pilot testing stage is a basic analysis script which summarizes all data collected to date visually. We have used such scripts in the past to produce automatically updating dashboards or web pages, allowing for real time or near-real time monitoring of data collection efforts. This provides an easy way to summarize completion of the experiment so that individuals can receive credit (if using services like MTurk or Prolific), but also allows interested participants to see the data if recruiting participants who are more likely to be interested in the results, as sometimes happens on social media sites. As data collection online can also happen extremely quickly (300 participants in <2h in our most recent Prolific experiment), this can provide an illusion of control over the deluge of data, allowing any issues to be spotted and resolved relatively quickly. If server load is a potential issue, it may also help to release batches of trials over a longer period of time in order to minimize the chance of having to make participants wait for others to complete the task before the server can handle additional connections[2].

6 Analyzing the Data

One constant with data analysis of these types of experiments is that no matter how carefully the experiment is planned, designed, and executed, there will be surprises. This is the dual curse and blessing of studying human perception: the visual system never quite works the way that we expect that it will, which provides endless fodder for science and occasionally complicates the data analysis.

6.1 Generalized Linear Mixed Models

The see-value approach (Chowdhury et al., 2020; Majumder et al., 2013; VanderPlas et al., 2021) is extremely useful for single lineups, but when a series of lineups that are part of a designed experiment are used, generalized linear mixed models are a much simpler way to summarize the overall effect of various manipulated factors (Hofmann et al., 2012; Robinson, 2022; VanderPlas & Hofmann, 2017). This approach also works extremely well for psychometric experiments (VanderPlas & Hofmann, 2015), as psychometric models can be easily fit into the framework of a generalized linear mixed model (Ju et al., 2024) that has higher power than the Rasch models (Andrich, 1988; Lu et al., 2022) which were historically recommended for these experiments. In addition, similar model structures with different link functions can be used to model accuracy, response time, and confidence, if all three types of information are collected from participants during the experiment.

6.2 Numerical Estimation

There are additional considerations that should be expected when asking participants to estimate numerical quantities. Anchoring and rounding cause participant responses to cluster in ways that can bias statistical estimators, requiring methods designed for these types of data (Heitjan & Rubin, 1991; Tourangeau et al., 2000; Ushakov & Ushakov, 2017). An alternative approach is to analyze the data graphically, as shown in Figure 6, which uses a density plot with rug annotations to show individual participant point estimates. Rounding effects can clearly be seen in the rug plot, but a kernel density calculated using an appropriately selected bandwidth shows clear visual differences between linear and log scale charts. In addition, a smaller second mode can be seen in both linear and log conditions that corresponds to the underlying model value; this suggests that a minority of participants fit a mental regression model to the data and use that model to estimate rather than estimating based off of the closest point. When charts are carefully constructed to account for the experimental structure and participant estimation strategies, displaying both individual and aggregate responses, it is possible to demonstrate a measurable difference between conditions

[2] This is the one major drawback to our preferred solution of self-hosting a Shiny server to handle data collection: the free version of Shiny server is limited to about 15 connections at any given time. Prolific has recently added rate-limiting functions to the experimental control platform, which makes controlling the number of active jobs much easier.

without the need for complicated statistical modeling which corrects for rounding and anchoring effects.

While Figure 6 shows point estimates, the same approach can be modified to account for experimental methods which generate participant response curves. In these cases, rather than adding a rug plot to a density plot showing aggregate estimates, it may be more helpful to create spaghetti plots of individual estimates with a superimposed consensus estimate and relevant annotations showing e.g. anchor points.

The advantage of this approach is that it can accommodate extremely messy data without requiring the extensive data cleaning, modeling of heuristics like rounding and anchoring, and elimination of nonsensical responses that might be necessary to fit a statistical model. While statistical models are undoubtedly beneficial in many situations, we have often found that graphical displays of experimental results are at least as useful for analyzing and presenting the results of graphical experiments.

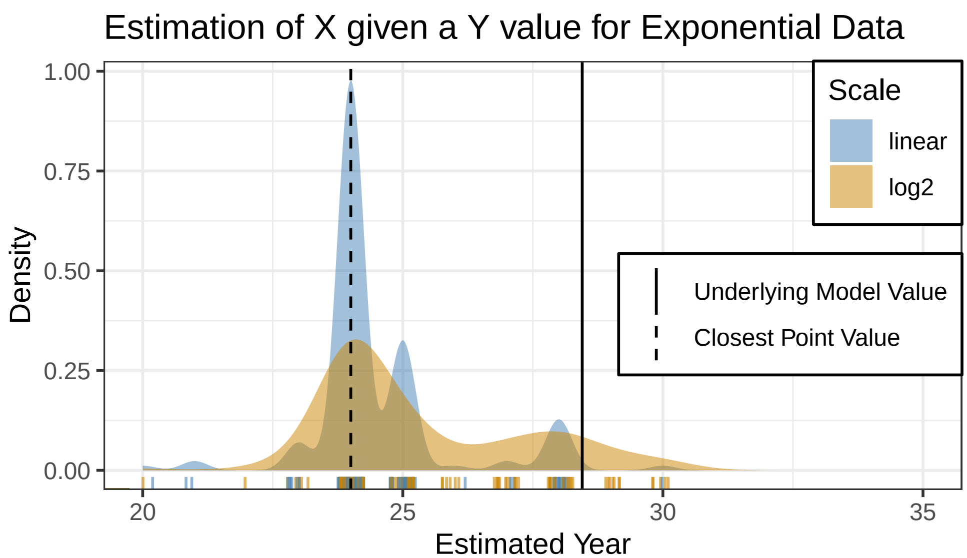

Figure 6: Density of participant estimates for the year in which the population reaches 4000. Colors are associated to scale - linear (blue) and log (orange) - and vertical lines indicate the true value based on the underlying model equation (black solid) and closest point value based on the simulated data set (black dashed). A jittered rug plot along the x-axis shows where participant estimates were made. The plot shows anchoring occurred to the closest point as shown by an increase in density around the dashed line. Density peaks occurred at whole values indicating rounding errors.

6.3 Direct Interactions

If participants are making predictions and/or fitting visual statistics, we have had success analyzing these responses by comparing the responses to results from a statistical model to determine how visual statistics differ from the numerical quantities derived mathematically. For instance, we calculate the deviation between participant responses and the linear regression in 'You Draw It' experiments, then fitted generalized additive mixed models to summarize the results across different experimental conditions to assess how user-drawn predictions deviated from the statistical estimates. In other direct interactions, it may be useful to compare participant selections or annotations to closest points on the chart to assess anchoring behavior; for discrete selections, methods discussed in numerical estimation may also be useful.

6.4 Qualitative Responses

In many cases, it is helpful to combine participants' qualitative reasoning with their quantitative responses to designed graphical experiments. This approach provides useful context as to what participants use to make their decisions, and can be useful when assessing why unexpected responses occurred. We have used word clouds to show overall themes in participant responses successfully in (VanderPlas & Hofmann, 2017); when paired with an appropriate linear model it became clear that participants were fixating on unequal cluster size as a visual cue. In cases where participants are provided with additional utilities such as calculators and scratchpads, it can be useful to select responses from individual participants which illustrate the different types of calculations performed (but analyzing this data quantitatively can be difficult).

7 Conclusion

Testing features of visualization graphically using online platforms provides an incredibly powerful and efficient way to establish empirical guidelines for statistical graphics and visualization. There are nearly endless ways to combine web graphics, user interactions, and data collection to get insight into perception and use of graphics in practical settings. We have been continually surprised at the richness of the data collected in these experiments and the ability to combine qualitative and quantitative assessment to support conclusions that are both nuanced and of practical use when deciding how to design and present data using visualizations.

In this paper, we have attempted to contextualize and motivate the logic behind the process we use to design empirical graphics experiments. We've discussed model development, experimental design considerations, pilot testing, and data analysis methods that have been honed over many successful and less-than-optimal experiments. While no experiment involving humans ever goes exactly to plan, following this process helps to avoid some of the most likely mishaps, ensuring that each new experiment's "bonus" findings have only minimal impacts on the study's overall utility.

While it can be difficult to conduct empirical tests of different visualizations, the results of these experiments support guidelines for graphical design and communication of statistical findings in an accessible and explainable way. Many chart design guidelines and recommendations are based on heuristics, but as scientists, we should prefer guidelines which are based on empirical, experimentally derived results over opinions. Testing statistical graphics and developing empirically supported guidelines for chart creation promises to support better scientific communication, which is critical for educating the public about topics like climate change, public health, the risk of severe weather and more.

8 References

Allen, E. A., & Erhardt, E. B. (2016). Visualizing Scientific Data. In J. T. Cacioppo, L. G. Tassinary, & G. G. Berntson (Eds.), Handbook of Psychophysiology (4th ed., pp. 679—697). Cambridge University Press. https://doi.org/https://doi.org/10.1017/9781107415782.031

Andrich, D. (1988). Rasch models for measurement (1st ed.). SAGE Publications.

Bertin, J., & Berg, W. J. (1983). Semiology of graphics: Diagrams, networks, maps (Vol. 1). University of Wisconsin press Madison.

Bostock, M., Ogievetsky, V., & Heer, J. (2011). D3 Data-Driven Documents. IEEE Transactions on Visualization and Computer Graphics, 17(12), 2301—2309. https://doi.org/https://doi.org/10/b7bhhf

Buja, A., Cook, D., Hofmann, H., Lawrence, M., Lee, E.-K., Swayne, D. F., & Wickham, H. (2009). Statistical inference for exploratory data analysis and model diagnostics. Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, 367(1906), 4361—4383.

Cairo, A. (2012). The Functional Art: An introduction to information graphics and visualization. New Riders.

Chang, W., Cheng, J., Allaire, J., Sievert, C., Schloerke, B., Xie, Y., Allen, J., McPherson, J., Dipert, A., & Borges, B. (2021). Shiny: Web application framework for r. https://CRAN.R-project.org/package=shiny

Chowdhury, N. R., Cook, D., Hofmann, H., & Majumder, M. (2018). Measuring Lineup Difficulty By Matching Distance Metrics With Subject Choices in Crowd-Sourced Data. Journal of Computational and Graphical Statistics, 27(1), 132—145. https://doi.org/https://doi.org/10.1080/10618600.2017.1356323

Chowdhury, N. R., Diehl, H., Broderick, T., & Stein, A. (2020). The See Value App: Visual Decision Making for Drug Development. https://rinpharma.com/publication/rinpharma_183/.

Cleveland, W. S., McGill, M. E., & McGill, R. (1988). The Shape Parameter of a Two-Variable Graph. Journal of the American Statistical Association, 83(402), 289—300. https://doi.org/https://doi.org/10.1080/01621459.1988.10478598

Cleveland, W. S., & McGill, R. (1985). Graphical Perception and Graphical Methods for Analyzing Scientific Data. Science, 229(4716), 828—833. https://doi.org/https://doi.org/10.1126/science.229.4716.828

Cook, D., Reid, N., & Tanaka, E. (2021). The Foundation Is Available for Thinking About Data Visualization Inferentially. Harvard Data Science Review, 3(3). https://doi.org/https://doi.org/10.1162/99608f92.8453435d

Croxton, F. E. (1932). Graphic Comparisons by Bars, Squares, Circles, and Cubes. Journal of the American Statistical Association, 27(177), 54—60.

Croxton, F. E., & Stryker, R. E. (1927). Bar Charts Versus Circle Diagrams. Journal of the American Statistical Association, 22(160), 473—482. https://doi.org/https://doi.org/10.2307/2276829

Curcio, F. (1987). Comprehension of mathematical relationships expressed in graphs. Journal for Research in Mathematics Education, 18(5), 382—393.

Desnoyers, L. (2011). Toward a Taxonomy of Visuals in Science Communication. 58(2), 16.

Dunbar, K. (1995). How scientists really reason: Scientific reasoning in real-world laboratories. The Nature of Insight, 18, 365—395.

Eells, W. C. (1926). The Relative Merits of Circles and Bars for Representing Component Parts. Journal of the American Statistical Association, 21(154), 119—132. https://doi.org/https://doi.org/10.2307/2277140

Few, S. (2006). Information Dashboard Design: The Effective Visual Communication of Data.

O'Reilly Media, Incorporated.
Friel, S., Curcio, F., & Bright, G. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. Journal for Research in Mathematics Education, 32(2), 124—158.
Glazer, N. (2011). Challenges with graph interpretation: A review of the literature. Studies in Science Education, 47(2), 183—210.
Haak, M., De Jong, M., & Schellens, P. (2003). Retrospective vs. Concurrent think-aloud protocols: Testing the usability of an online library catalogue. Behaviour & IT, 22, 339—351. https://doi.org/10.1080/0044929031000
Haemer, K. W. (1948). Double Scales are Dangerous. The American Statistician, 2(3), 24—24. https://doi.org/https://doi.org/10.1080/00031305.1948.10501588
Haider, J. D., Pohl, M., Beecham, R., & Dykes, J. (2021). Strategies for detecting difference in map line-up tasks. In C. Ardito, R. Lanzilotti, A. Malizia, H. Petrie, A. Piccinno, G. Desolda, & K. Inkpen (Eds.), Human-computer interaction — INTERACT 2021 (pp. 558—578). Springer International Publishing. https://doi.org/10.1007/978-3-030-85613-7_36
Heer, J., & Bostock, M. (2010). Crowdsourcing graphical perception: Using mechanical turk to assess visualization design. Proceedings of the SIGCHI Conference on Human Factors in Computing Systems, 203—212. https://doi.org/10.1145/1753326.1753357
Heitjan, D. F., & Rubin, D. B. (1991). Ignorability and Coarse Data. The Annals of Statistics, 19(4), 2244—2253. https://doi.org/https://doi.org/10.1214/aos/1176348396
Hofmann, H., Follett, L., Majumder, M., & Cook, D. (2012). Graphical tests for power comparison of competing designs. IEEE Transactions on Visualization and Computer Graphics, 18(12), 2441—2448. https://doi.org/10/f4fwkz
Huhn, R. von. (1927). Further Studies in the Graphic Use of Circles and Bars: A Discussion of the Eell's Experiment. Journal of the American Statistical Association, 22(157), 31—39. https://doi.org/https://doi.org/10.2307/2277346
Hullman, J., & Gelman, A. (2021). Designing for Interactive Exploratory Data Analysis Requires Theories of Graphical Inference. Harvard Data Science Review, 3(3). https://doi.org/https://doi.org/10.1162/99608f92.3ab8a587
Joint committee on standards for graphic presentation. (1915). Publications of the American Statistical Association, 14(112), 790—797. http://www.jstor.org/stable/2965153
Ju, W. (Will)., Vanderplas, S., & Hofmann, H. (2024). One Model that fits them All: Psychometrics with Generalized Linear Mixed Effects Models. Electronic Imaging, 1—9. https://doi.org/10.2352/EI.2023.35.1.VDA-A01
Katz, J. (2017). You Draw It: Just How Bad Is the Drug Overdose Epidemic? The New York Times. https://www.nytimes.com/interactive/2017/04/14/upshot/drug-overdose-epidemic-you-draw-it.html
Kirschenbaum, S. S. (2003). Comparative Cognitive Task Analysis: The Cognition of Weather Forecasting. Proceedings of the Human Factors and Ergonomics Society Annual Meeting, 47, 473—477. https://doi.org/https://doi.org/10.1177/154193120304700347
Kosslyn, S. M. (2006). Graph Design for the Eye and Mind. Oxford University Press.
Kulhavy, Pridemore, & Stock. (1992). Cartographic experience and thinking aloud about thematic maps. Cartographica, 29(1), 1—9. https://doi.org/10.3138/H61J-VX35-J6WW-8111
Lee, S., Kim, S.-H., Hung, Y.-H., Lam, H., Kang, Y.-A., & Yi, J. S. (2016). How do people make sense of unfamiliar visualizations?: A grounded model of novice's information visualization sensemaking. IEEE Transactions on Visualization and Computer Graphics, 22(1), 499—508. https://doi.org/10.1109/TVCG.2015.2467195
Loy, A., & Hofmann, H. (2013). Diagnostic tools for hierarchical linear models. Wiley Interdisciplinary Reviews: Computational Statistics, 5(1), 48—61. https://doi.org/https://doi.org/10.100

2/wics.1238
Lu, M., Lanir, J., Wang, C., Yao, Y., Zhang, W., Deussen, O., & Huang, H. (2022). Modeling Just Noticeable Differences in Charts. IEEE Transactions on Visualization and Computer Graphics, 28(1), 718—726. https://doi.org/https://doi.org/10.1109/TVCG.2021.3114874
Majumder, M., Hofmann, H., & Cook, D. (2013). Validation of visual statistical inference, applied to linear models. Journal of the American Statistical Association, 108(503), 942—956. https://doi.org/https://doi.org/10/f5gntt
R Core Team. (2022). R: A language and environment for statistical computing. R Foundation for Statistical Computing. https://www.R-project.org/
Ribecca, S. (2022). The data visualisation catalogue. https://datavizcatalogue.com.
Rice, K., Hofmann, H., du Toit, N., & Mulrow, E. (2024). Testing Perceptual Accuracy in a U.S. General Population Survey Using Stacked Bar Charts. Journal of Data Science, 1—18. https://doi.org/10.6339/24-JDS1121
Robinson, E. A. (2022). Human Perception of Exponentially Increasing Data Displayed on a Log Scale Evaluated Through Experimental Graphics Tasks [Doctoral, University of Nebraska, Lincoln]. https://github.com/earobinson95/EmilyARobinson-UNL-dissertation/raw/main/EmilyRobinson-final-dissertation.pdf
Schьtt, H. H., Harmeling, S., Macke, J. H., & Wichmann, F. A. (2016). Painfree and accurate bayesian estimation of psychometric functions for (potentially) overdispersed data. Vision Research, 122, 105—123. https://doi.org/10.1016/j.visres.2016.02.002
Tourangeau, R., Rips, L. J., & Rasinski, K. (2000). The psychology of survey response. Cambridge University Press. https://doi.org/https://doi.org/10.1017/CBO9780511819322
Trafton, G. J., Kirschenbaum, S. S., Tsui, T. L., Miyamoto, R. T., Ballas, J. A., & Raymond, P. D. (2000). Turning pictures into numbers: Extracting and generating information from complex visualizations. International Journal of Human-Computer Studies, 53(5), 827—850. https://doi.org/10.1006/ijhc.2000.0419
Tufte, E. (1991). The Visual Display of Quantitative Information (2nd ed.). Graphics Press.
Ushakov, N. G., & Ushakov, V. G. (2017). Statistical analysis of rounded data: Recovering of information lost due to rounding. Journal of the Korean Statistical Society, 46(3), 426—437. https://doi.org/https://doi.org/10.1016/j.jkss.2017.01.003
Valentin, S., Kleinegesse, S., Bramley, N. R., Seriиs, P., Gutmann, M. U., & Lucas, C. G. (2024). Designing optimal behavioral experiments using machine learning. eLife, 13. https://doi.org/10.7554/eLife.86224
VanderPlas, S. (2021). Designing Graphics Requires Useful Experimental Testing Frameworks and Graphics Derived From Empirical Results. Harvard Data Science Review, 3(3). https://doi.org/https://doi.org/10.1162/99608f92.7d099fd0
Vanderplas, S., Blankenship, E., & Wiederich, T. (2024). Escaping Flatland: Graphics, Dimensionality, and Human Perception. In H. Mori & Y. Asahi (Eds.), Human Interface and the Management of Information (pp. 140—156). Springer Nature Switzerland. https://doi.org/10.1007/978-3-031-60114-9_11
Vanderplas, S., Cook, D., & Hofmann, H. (2020). Testing Statistical Charts: What Makes a Good Graph? Annual Review of Statistics and Its Application, 7(1). https://doi.org/https://doi.org/10.1146/annurev-statistics-031219-041252
VanderPlas, S., Goluch, R., & Hofmann, H. (2019). Framed! Reproducing and Revisiting 150 year old charts. Journal of Computational and Graphical Statistics. https://doi.org/https://doi.org/10.1080/10618600.2018.1562937
VanderPlas, S., & Hofmann, H. (2015). Signs of the Sine Illusion–Why We Need to Care. Journal of Computational and Graphical Statistics, 24(4), 1170—1190. https://doi.org/https://doi.org/10.1080/10618600.2014.951547

VanderPlas, S., & Hofmann, H. (2016). Spatial Reasoning and Data Displays. IEEE Transactions on Visualization and Computer Graphics, 22(1), 459—468. https://doi.org/https://doi.org/10.1109/TVCG.2015.2469125

VanderPlas, S., & Hofmann, H. (2017). Clusters Beat Trend!? Testing Feature Hierarchy in Statistical Graphics. Journal of Computational and Graphical Statistics, 26(2), 231—242. https://doi.org/10.1080/10618600.2016.1209116

VanderPlas, S., Rцttger, C., Cook, D., & Hofmann, H. (2021). Statistical significance calculations for scenarios in visual inference. Stat, 10(1). https://doi.org/https://doi.org/10.1002/sta4.337

Wickham, H. (2016). ggplot2: Elegant graphics for data analysis. Springer-Verlag New York. https://ggplot2.tidyverse.org

Wickham, H., Cook, D., Hofmann, H., & Buja, A. (2010). Graphical inference for infovis. IEEE Transactions on Visualization and Computer Graphics, 16(6), 973—979. https://doi.org/https://doi.org/10.1109/TVCG.2010.161

Wilkinson, L. (1999). The grammar of graphics. Springer. http://public.ebookcentral.proquest.com/choice/publicfullrecord.aspx?p=3085765

Wood, R. (1968). Objectives in the teaching of mathematics. Educational Research, 10(2), 83—98.