

# The curse of three dimensions: Why your brain is lying to you

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General Terms: Design, Human factors, Experimentation

Additional Key Words and Phrases: Statistical graphics, optical illusion, perceptual theory, visualization

## ACM Reference Format:

VanderPlas, S., Hofmann, H., and Cook, D. 2014. The Curse of Three Dimensions: Why Your Brain is Lying to You. *ACM Trans. Appl. Percept.* 1, 1, Article 1 (January 2014), 14 pages.

DOI = 10.1145/0000000.0000000 <http://doi.acm.org/10.1145/0000000.0000000>

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DOI 10.1145/0000000.0000000 <http://doi.acm.org/10.1145/0000000.0000000>

## 1. INTRODUCTION

*motivate power of graphics first?*

One of the basic principles of statistical graphics is that a chart should accurately reflect the data. **XXX what are statistical graphics, and how are they different from ‘visualizations’/sketches?**

Tufte’s “Lie Factor”, for instance, is calculated based on the ratio of the effect size shown in the graphic to the effect size in the data [Tufte 1991]. If there is a systematic difference, the lie factor will be notably different from 1 (values between .95 and 1.05 are typically acceptable). While Tufte’s lie factor is an effective measurement of the mapping from data to graphics, it does not give us any insight about the mapping from the chart to the brain. Ideally, charts not only represent the data accurately, but also allow readers to draw accurate conclusions from the graph. While in many cases these two goals can be achieved simultaneously, there are instances in which human perception produces a psychological distortion that interferes with the interpretation of charts, even if the charts are accurate portrayals of the numerical data. In this paper, we examine situations in which low-level human perceptual processes interfere with making accurate judgements from displays and suggest an experimental methodology for estimating the psychological “lie factor” of a chart.

## 2. THE CURSE OF THREE DIMENSIONS

The human visual system is largely optimized for perception of three dimensions. Biologically, binocular vision ensures that we have the necessary information to construct a functional mental representation of the three-dimensional world, but even in the absence of binocular information the brain uses numerous heuristics to parse otherwise ambiguous two-dimensional retinal images into meaningful three-dimensional information. Predictably, however, these heuristics are not without drawbacks; the same two-dimensional neural representation might correspond to multiple three-dimensional objects, as in the well-known Necker Cube (Gregory 1968; shown in figure 1). Additionally, the same three-dimensional object often has infinitely many two-dimensional representations, for instance, when viewed from different angles. Optical illusions that demonstrate this phenomenon are the so-called “impossible objects” such as the Penrose triangle Gregory 1968. In the case of the Necker cube, the two different figures are equally salient, and thus the brain does not prefer one interpretation over the other and instead continuously switches between interpretations. In many other instances, experience with the real world informs the choice between multiple possibilities of rendering a three-dimensional object from the same two-dimensional representation. This indicates that processing occurs “top down” in that our previous experience influences our current perceptions. Without this top-down influence, the brain would not be able to map the picture back to a three-dimensional object. One of the most well studied examples of the influence of top-down processing is the Müller-Lyer illusion, shown in figure 2.

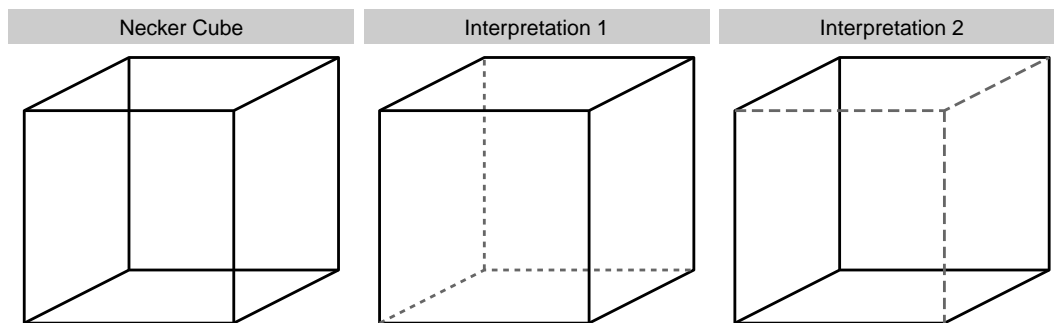


Fig. 1. The Necker Cube is a so-called “ambiguous object” because two different transparent objects produce the same retinal image (and thus the same perceptual experience). Commonly, the image seems to transition instantaneously from one possible mental representation to the other.

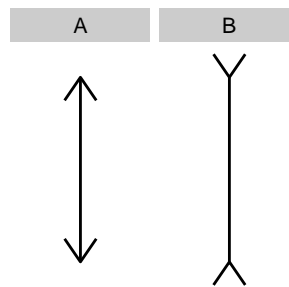


Fig. 2. The Müller-Lyer illusion. The central segment of figure A is perceived as shorter than the central segment of figure B, even though the two are actually the same length.

XXX define what the illusion is before you explain how it happens. One explanation for the Müller-Lyer illusion XXX reference is that the brain interprets the ambiguous lines as a common three-dimensional object: corners of a room: figure 2A occurs when viewing the outside corner of a rectangular prism, figure 2B occurs when viewing the prism from the inside. In regions which do not commonly have rectangular buildings, the illusion is significantly less pervasive [Ahluwalia 1978]. Figure 3 provides one possible context that would lead to the Müller-Lyer effect. This real-world experience carries with it an inferred perspective - when the arrows point inward, the object is typically closer than when the arrows point outward, which causes the brain to interpret the outward-pointing figure as larger, even though the retinal size is identical for the two objects. This inferred “depth cue” [Gregory 1968] is reasonably consistent across individuals. A similar effect can also be found in the Necker Cube - whichever face appears to be furthest away also seems larger, even though any two parallel faces are equally sized in the image. This approach has proved to be very advantageous for real world scenarios [Gregory 1968], as pictures of real objects are seldom ambiguous. This strategy also allows for high performance with limited neural bandwidth.

Figure 3 shows not only the real-world context, but that the perspective cues which contribute to the illusion allow for an accurate neural representation of the object in context. XXX the previous sentence is grammatically correct, but it's a hard read - chop it in two?



Fig. 3. Real-world context that gives rise to the Müller-Lyer illusion

### 3. THREE DIMENSIONAL CONTEXT OF THE SINE ILLUSION

While the classic Müller-Lyer illusion is seldom a factor in statistical charts, there are other illusions caused by the interpretation of a two-dimensional stimulus in the context of three-dimensional objects, leading to a distortion in the mental representation of the original stimulus. The sine illusion (also known as the line width illusion [XXX cite Day and stecher, and ours](#)) is one example of this phenomenon which occurs frequently in statistical graphics. Like the Müller-Lyer illusion, it is pervasive and very difficult to “un-see” or mentally correct. [XXX what is the sine illusion in words?](#) Figure 4 shows the sine illusion in its original form, and figure 5 shows the same illusion in one of the charts from Playfair’s Statistical Atlas [Playfair 1786]. This illusion, which is also known as the line-width illusion, has also been documented in parallel sets plots [Hofmann and Vendettuoli 2013]. [XXX factors that play into the sine illusion: large changes in the slope of a trend; extent of slope is measured with respect to the local coordinate system in which the chart is displayed, i.e. it is influenced by the aspect ratio.](#)

Cleveland and McGill [1984] determined that comparison of the vertical distance between two curves is often inaccurate, as “the brain wants to judge minimum distance between the curves in different regions, and not vertical distance”. While they do not explain a reason for this tendency, introspection does readily confirm their explanation: we judge the distance between two curves based on the shortest distance between them, which geometrically is the distance along the line perpendicular to the tangent line of the curve. Day & Stecher [1991] suggest that the sine illusion is similar in principle to the Müller-Lyer illusion, attributing it to the perceptual compromise between the vertical extent and the overall dimensions of the figure. The sine illusion is similar to the Müller-Lyer illusion in another way, as well – there are three-dimensional analogues of the two-dimensional image that may influence the perceptual context. One of these contexts is shown in figure 6, generated from the same function shown in the two-dimensional analogue, figure 4, but with the length projected onto a third dimension. While the images do not match exactly, the

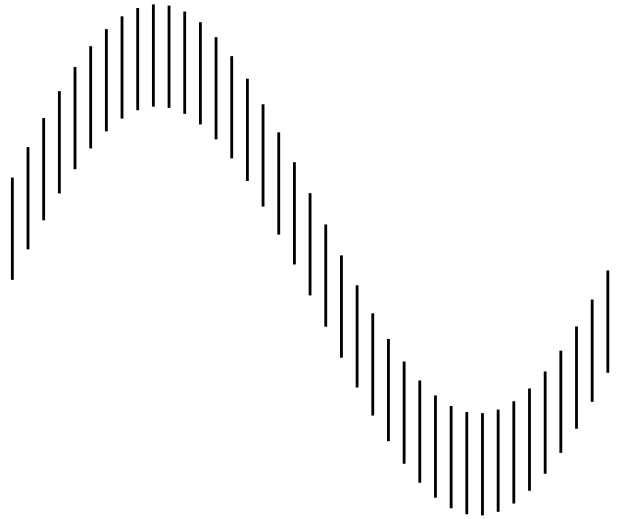


Fig. 4. The classic sine illusion. Each vertical line has the same length, though the lines at the peak and trough of the curve appear longer.

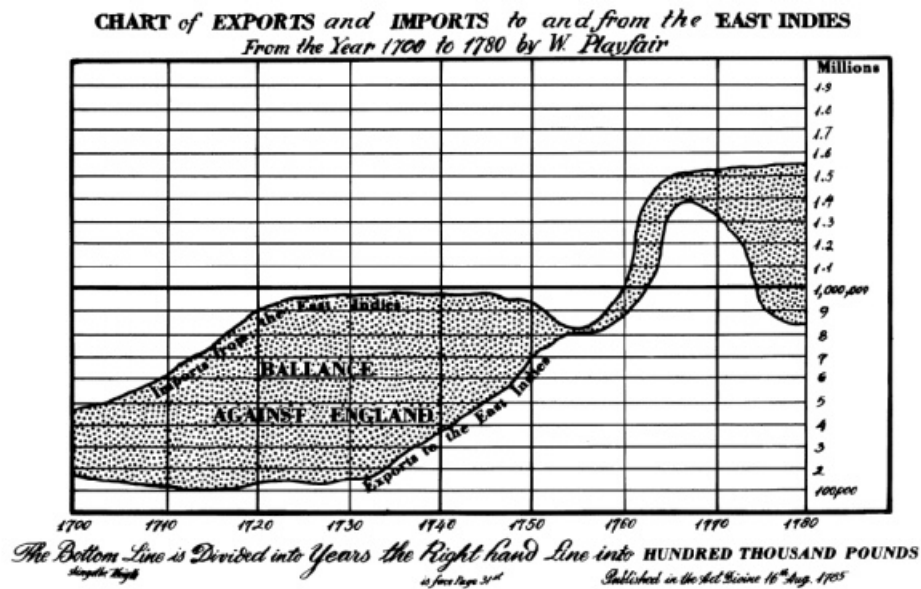


Fig. 5. Playfair's chart of trade between the East Indies and England, 1700-1780. The trade balance is influenced by the sine illusion: the difference between imports and exports in 1763 does not appear to be the same size as that in 1745, though the vertical distance is approximately the same.

similarities are striking. Additionally, the tendency to judge vertical distance using the extant width noted in Cleveland and McGill [1985] corresponds to the measurement of depth in the three-dimensional image. One main difference between the first three dimensional image shown in figure 6 and the original image is that the lines connecting the top and bottom sections of the curve are slightly angled in the three-dimensional version; this is due to the perspective

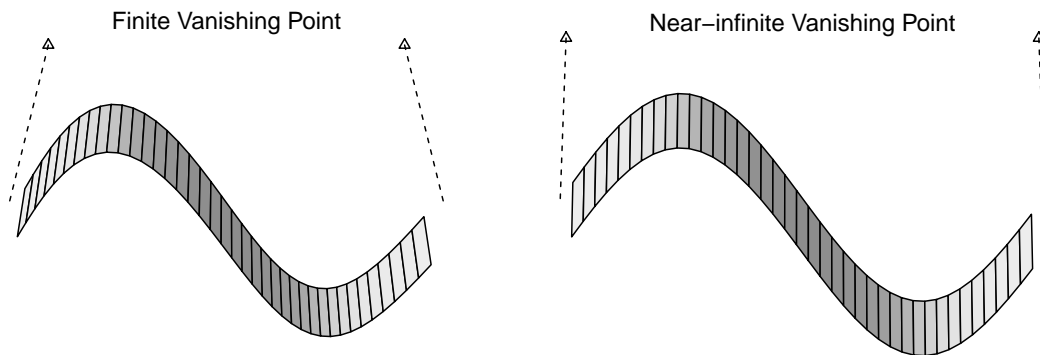


Fig. 6. Three dimensional context for the sine illusion. The second figure has a vanishing point closer to infinity, and very closely resembles the form of the classic sine illusion.

projection used to create the image and the corresponding angles of rotation chosen such that the entire surface is visible.

As the vanishing point moves further away from the viewer and the 3d projection decreases in strength, the three-dimensional reconstruction of the image converges to figure 4. The second image in figure 6 shows a weaker 3-dimensional projection that is much closer to figure 4, however, the three-dimensional contextual information provided by the shading removes much of the illusion’s distortions. This is similar to the Müller-Lyer illusion, as figure 3 is not at all ambiguous because the contextual depth information provided by the rest of the surface of the house is sufficient to remove the illusion that the closer corner is in fact larger due to the perspective.

One difference between the sine illusion and the Müller-Lyer illusion that may influence the tendency to see a three-dimensional “ribbon” instead of the two-dimensional sine curve is that the vertical lines in the sine illusion are ambiguously oriented - there is an entire plane of possible three-dimensional reconstructions for each line, and each possible rotation leads to a line of different length. It is this facet of the image that we believe partially contributes to the ambiguity of the image, though it is not a necessary feature for the illusion to persist, as the illusion also can be found in scatterplots and in “ribbon plots” such as figure 5.

The psychological mechanisms which force three-dimensional context onto two-dimensional stimuli are useful adaptations to a three-dimensional world [Gregory 1968], but they do have disadvantages when applied to abstract two-dimensional stimuli, such as statistical charts. In order to estimate the psychological lie factor that occurs due to this illusion, we assessed the strength of the illusion experimentally. XXX define the psychological lie factor somewhere.

## 4. MEASURING THE PSYCHOLOGICAL LIE FACTOR EXPERIMENTALLY

### 4.1 Study Setup

In order to determine the amount of the psychological distortion, participants were presented with different stimuli consisting of six subplots, similar to figure 7. These sets of charts were constructed such that line lengths along a curve were varied to different degrees to counteract the illusion. Figure 8 shows the amount of line correction used in each of the sub-plots above. Participants were asked to answer the question: “In which graph is the size of the curve most consistent?”. The phrasing ‘size of the curve’ was chosen deliberately so as not to bias participants to explicitly measure line lengths.

The amount of correction in each sub-plot was chosen such that each stimuli set contained a selection of curves corrected to various degrees using correction factors determined previously in an internal pilot study. The correction factor we use for extending the length of the line segment at location  $x$  is given as

$$(1 - w) + w \cdot (1 / \cos(\tan^{-1}(|f'(x)|)))$$

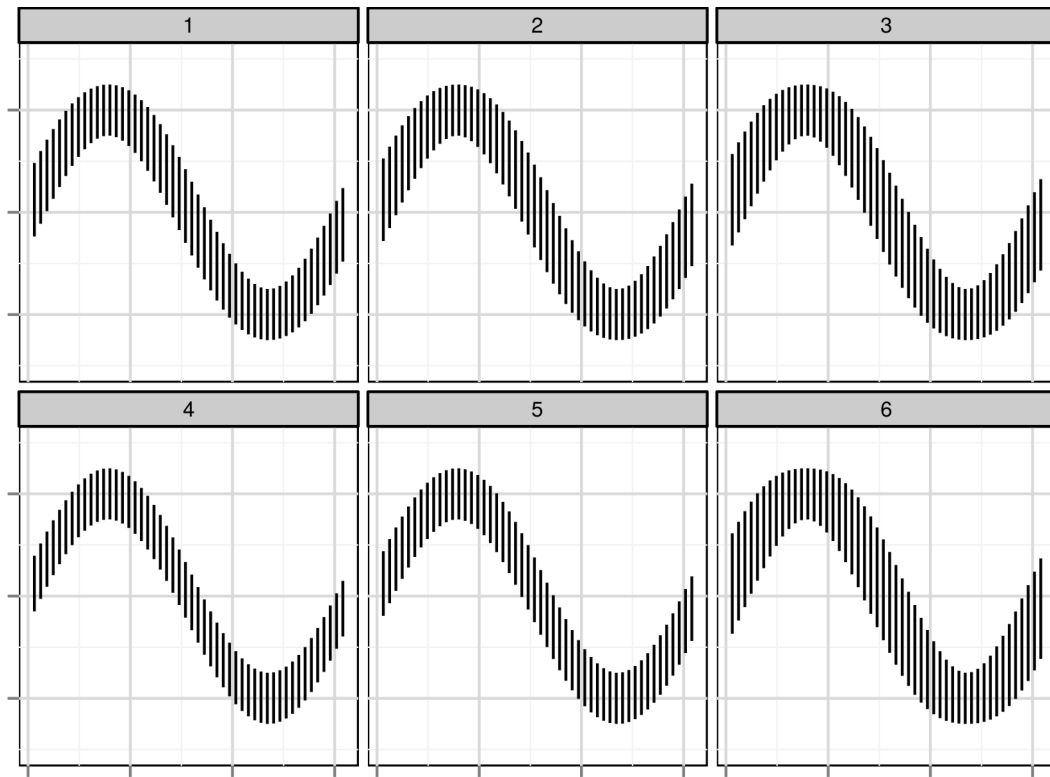


Fig. 7. One of the charts presented to participants through Amazon Mechanical Turk. Figure 8 shows the actual differences in line lengths. XXX which of the weights in table I correspond to this chart? Note, that weighting is done in a random order - e.g. plot #1 corresponds to a weight of 0, plot #2 to ???, and plot #3 to ???

where  $w$  is varied between 0 (for no correction) and 1.5 (strong over-correction) - XXX ah, slow down! that's two aspects in one sentence.  $w$  is a weighting factor - which gives 0 and 1 special meaning. the choice of 1.5 is already a design choice for the experiment - that needs a separate sentence. From a pilot study, we suspected values around  $w = 0.8$  to be promising XXX promising for what?, but did not know whether this would generalize to those outside of the statistical graphics community. XXX sorry, I'm not following - how did this lead to twelve sets? Do you mean that we picked twelve sets of stimuli? - describe the process for picking a set. This led to twelve stimuli sets with correction factors as shown in table I. XXX rephrase caption text and repeat in the writeup. The difficulty level of each of the twelve stimuli sets was determined by the similarity of the six plots shown. XXX extra sentence for the difficulty. why do we need it?

Three underlying mean functions were tested using these weight values:  $\sin(x)$ ,  $\exp(x)$ , and  $1/x$ .

XXX show these functions and discuss why they are good choices.

Each participant was presented with eleven sets of graphs (each “set” consisting of 6 separate plots), consisting of one “easy” test set, five stimuli sets of difficulty level 1 through 5 with the sine curve as the underlying function, and another five graph sets (also of difficulty levels 1 to 5) with either the exponential or the inverse curve as the underlying function. After the easy introductory chart, which was presented first, the order of the plots was randomized across difficulty level as well as function type. The test chart consisted of a set of six sine curves with a very low level

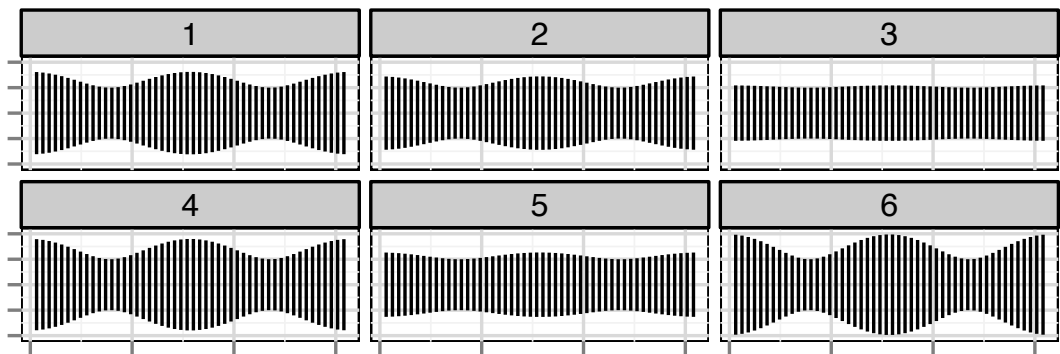


Fig. 8. Line lengths (without the trend) for figure 7, demonstrating the effective lie factor for each sub-plot. XXX I'm not sure that the order in these plots is the same as in the stimulus chart. also, be a bit more precise - what exactly can be seen in this chart? - it's the detrended version of fig 7 - how does that show the 'effective' lie factor? - 'effective lie factor isn't defined, so make sure that you describe precisely what you mean by that.

Table I. Ordered weight factors and corresponding line factors for the sine curve stimuli sets. XXX what was the underlying concept behind the choice of weights?  $D$  needs to be defined before this table is referred to - at least give a reference to  $D$ s definition. Is  $D$  really the same for all three stimuli?

set	difficulty	Weight ( $w$ )						Lie Factor ( $D$ )					
		sub-plot						sub-plot					
		1	2	3	4	5	6	1	2	3	4	5	6
1	test	0.00	0.20	0.40	0.80	1.25	1.40	1.00	1.18	1.35	1.71	2.11	2.24
2	test	0.00	0.15	0.35	0.80	1.20	1.40	1.00	1.13	1.31	1.71	2.06	2.24
3	1	0.00	0.20	0.40	0.60	0.80	1.00	1.00	1.18	1.35	1.53	1.71	1.88
4	1	0.10	0.30	0.50	0.70	0.90	1.10	1.09	1.27	1.44	1.62	1.80	1.97
5	2	0.05	0.30	0.50	0.65	0.80	1.00	1.04	1.27	1.44	1.57	1.71	1.88
6	2	0.10	0.30	0.55	0.70	0.85	1.00	1.09	1.27	1.49	1.62	1.75	1.88
7	3	0.40	0.60	0.70	0.80	0.90	1.05	1.35	1.53	1.62	1.71	1.80	1.93
8	3	0.35	0.65	0.75	0.85	0.95	1.05	1.31	1.57	1.66	1.75	1.84	1.93
9	4	0.35	0.50	0.60	0.70	0.80	0.95	1.31	1.44	1.53	1.62	1.71	1.84
10	4	0.40	0.55	0.65	0.75	0.85	1.00	1.35	1.49	1.57	1.66	1.75	1.88
11	5	0.50	0.65	0.75	0.80	0.90	1.00	1.44	1.57	1.66	1.71	1.80	1.88
12	5	0.50	0.60	0.70	0.75	0.85	1.00	1.44	1.53	1.62	1.66	1.75	1.88

difficulty level, and was used as an introduction to the testing procedure. Participants were asked to select a single plot out of the 6 plots presented as having lines which were the most even.

4.2 Demographics

Participants for the study were recruited through the Amazon Turk web service. Responses from 128 users at 123 unique IP addresses were collected. XXX payout procedure? how many responses total?

Though participants were asked to participate in 11 total trials, some participants continued to provide feedback beyond the eleven trials required to receive payment through Amazon. For any subsequent responses we randomly selected one of the 32 possible stimuli. This approach allowed us to collect some data in which a single participant provided responses to all three underlying functions.

- XXX overview of number of responses for each stimulus?
- XXX how many of the test charts did people fail? Did you have to clean up data in any other way?
- XXX how many people started experiment, but did not complete it?



XXX why is it necessary for the analysis to know about individuals? In order to more accurately identify unique individuals, data were grouped by IP address rather than by user-entered identification code, as it is common for individuals to have multiple accounts and somewhat less likely for two Amazon Mechanical Turk participants to share the same IP address and complete the same study in a reasonably short window. We treated responses from the same IP addresses as additional responses from the same user.

XXX What other demographics did you collect? age? gender? - the section header ‘demographics’ seems out of place, if you only talk about the problem of identifying individuals.

XXX What data did you collect - exactly? Maybe a screenshot of the website would be good somewhere and a discussion of what participants had to do - on a click level basis.

### 4.3 Analysis

*Psychological “Lie Factor”.* As the strength of the correction varies across the horizontal range of the curve, we quantify the psychological distortion as the ratio of the maximum line length to the minimum line length for each plot  $j$ :  $D_j = l_{\max}/l_{\min}$ , and denote the chosen ratio as  $D^*$ .

XXX is  $l_{\min}$  always equal to 1 in our experiment?

XXX I think  $D^*$  needs a bit more formality. Let’s call it  $D_{ij}^*$  for participant’s  $i$  choice on the  $j$ th set. You have 36? different sets, so  $1 \leq j \leq 36$  and  $1 \leq i \leq 128$ . For each participant you have at least 11 responses, more for some. It might be beneficial to even have a third subscript  $k$  and introduce this as  $D_{ijk}^*$  for  $k \in \{1, 2, 3\}$ ,  $1 \leq j \leq 12$  and  $1 \leq i \leq 128$ .

Without any correction for the sine illusion, this factor is, like Tufte’s lie-factor, equal to one. Values above one indicate that at least in some areas of the curve line segments are extended.

XXX why only extension? - That was just our choice. We could also have worked the correction by averaging across lines to keep the average at 1.

We compute this quantity for each sub-plot in each stimulus presented to the participant. The participant’s choice therefore provides us with an estimate of what value of  $D$  constitutes the most consistent line length (out of the set shown). As each set of 6 plots is not guaranteed to contain a plot with  $w = 0$ , corresponding to constant length, choosing a plot with  $D = 1.4$  indicates more distortion if there is a sub-plot with  $w = 0$  ( $D = 1$ ) present than if least distorted sub-plot present has  $w = 0.4$  ( $D = 1.2$ ). Correcting for this bias, the set of  $D^* = \{D_1, \dots, D_6\}$  that is available to choose from produces an estimate of the overall psychological ‘lie factor’ as

$$P = D^* / \min_{1 \leq j \leq 6} D_j \quad (1)$$

for each plot and each participant.

XXX previously,  $D^*$  was defined differently.

XXX the way that  $P$  is defined, is as a set of ratios, because  $D^*$  is a set. I’m not sure that that’s what you want, and it’s not what you describe afterwards.

XXX Generally - I don’t quite see why we need to divide by the minimum. You have a problem with the choice of plot, if the minimal or maximal  $D$  was picked. But due to the discrete nature of the choice, we have a similar problem with any choice of plot: we only know the answer up to the interval of  $D_j$ s shown. At this point I’m not quite buying into the transformation to  $P$ .

XXX Could you make a picture of raw responses in interval coding? - i.e. if we have a choice of  $D$  as 1.14 and the next two answers are 1.0 and 1.4, you shade the area between 1 and 1.4 with more weight towards 1.14 ... - these would be densities that are specific to a stimuli set.

That is,  $P$  is the ratio of the lie factor of the chosen plot to the smallest lie factor available in the set of available plots. When the set of available plots contains an uncorrected plot,  $P = D$ ; when an uncorrected plot is not presented,  $P < D$ , since each  $D_j \geq 1$ ,  $j = 1, \dots, 6$ . This transformation is a conservative approach to estimating the lie factor, but allows us to show a variety of scaled transformations and estimate the effect with more granularity around  $w = 0.8$ .

By considering each participant's answers for the plot with the most consistent line length, we can obtain an estimate of the psychological distortion from the sine illusion on an individual level. Estimating distortion factors for each participant facilitates comparison of these estimated values to determine whether the illusion is a product of an individual's perceptual experience or whether there is a possible underlying perceptual heuristic for the sine illusion common across the majority of participants. If the illusion is a learned misperception rather than an underlying perceptual "bug", we would expect there to be considerable variability in the estimated individual lie factor  $P_i$  for each unique participant  $i$ ,  $1 \leq i \leq 123$ , as it is likely that personal experience varies more widely than perceptual heuristics and their underlying neural architecture.

Each set of  $w$  values as defined in table I corresponds to a value of  $P$  as defined in equation 1. We test for only a set of discrete values of  $w$ , which is reflected directly in the number of different values of  $P$  we can observe. This approach allows us to use a finite set of stimuli for testing, so that we can explicitly control the range of  $w$  displayed in each set of plots. To mathematically model a continuous quantity (the real domain of possible  $P$  values) using discrete data, we employ a Bayesian approach to model an overall psychological lie factor  $\theta$  and individual participant lie factors  $\theta_i$ .

Plots used in the experiment have factors  $P$  ranging between 1 and 2.5, so we can use a truncated normal data model for participant  $i$  viewing plot  $j$ , with  $P = p_{ij} \sim N(\theta_i, \sigma)$  and independent flat priors  $\pi(\theta) = 0.4$  and  $\pi(\sigma) = 2.5$  for  $\sigma \in [.1, .5]$ . These prior distributions  $\pi(\cdot)$  represent our expectations of the values of  $\theta$  and  $\sigma$  before the experiment; assigning them constant values indicates that we had little useable knowledge about the joint or marginal distributions of  $\theta$  and  $\sigma$  before the experiment was conducted. Using Bayesian estimation, we can then obtain posterior distributions for  $\theta_i$  and  $\theta$ , the individual and overall mean lie factors. We are not particularly interested in the actual values of  $\sigma$ , but the additional parameter is a useful tool to better estimate possible values for  $\theta$ .

#### 4.4 Results

The posterior density of  $\theta$  for each function is shown in figure 9, along with separate posterior densities for each individual  $\theta_i$ .  $\theta$  is reasonably similar for all three functions, suggesting that while function type may moderate the size of the effect, the illusion occurs regardless of function type. Individual curves have different variability due to the number of trials completed, and are necessarily more spread out as there is less data with which to estimate the individual posterior distributions.

On an individual level, figure 10 shows the posterior density for  $\theta_i$  for four of the participants who completed at least 6 trials in each category. While in many cases, the most probable  $\theta_i$  is similar across trials, individuals do seem to have been somewhat more affected by the illusion when the underlying function was sinusoidal, though this may reflect a discrepancy in the number of trials rather than a stronger illusion. Alternately, as the illusion depends on variable slope, it is possible that the monotonic exponential and inverse stimuli induced a weaker three-dimensional context. **XXX the individuals you picked are among the most extreme ones.**

**XXX what do you mean by 'appropriately compare intervals'? Oh, ok, those are 95% credible intervals - say that first. Are 100 enough or is that choice triggered by computational restrictions?** In order to appropriately compare intervals for each participant's  $\theta_i$ , we simulated 11 new "data points" from our model (thus enforcing a uniform 11 trials per participant for each function type) to get a single new estimate of  $\hat{\theta}_i$ . For each participant, we generated 100 of these  $\hat{\theta}_i$  and used these simulated values to calculate the 95% credible intervals shown in figure 11. These intervals will allow us to consider the variability in  $\theta_i$  due to participant preference rather than the number of trials a participant completed during the study. Removing this additional variability provides us with the opportunity to consider whether the sine illusion stems from an individual's perceptual experiences or from a lower-level perceptual heuristic.

Posterior predictive intervals for  $\theta_i$  as shown in figure 11 suggest that overall, the  $\theta_i$  are similar across individuals. Very few **XXX how many exactly?** of the intervals overlap the region (1, 1.05), which corresponds to an "acceptable" lie factor according to Tufte. This indicates significant distortion for most participants in our experiment, and the marked overlap of the intervals for each participant provides evidence consistent with a common magnitude of distortion. This suggests that there may be some common psychological strategy that is misapplied to the perception of these stimuli.

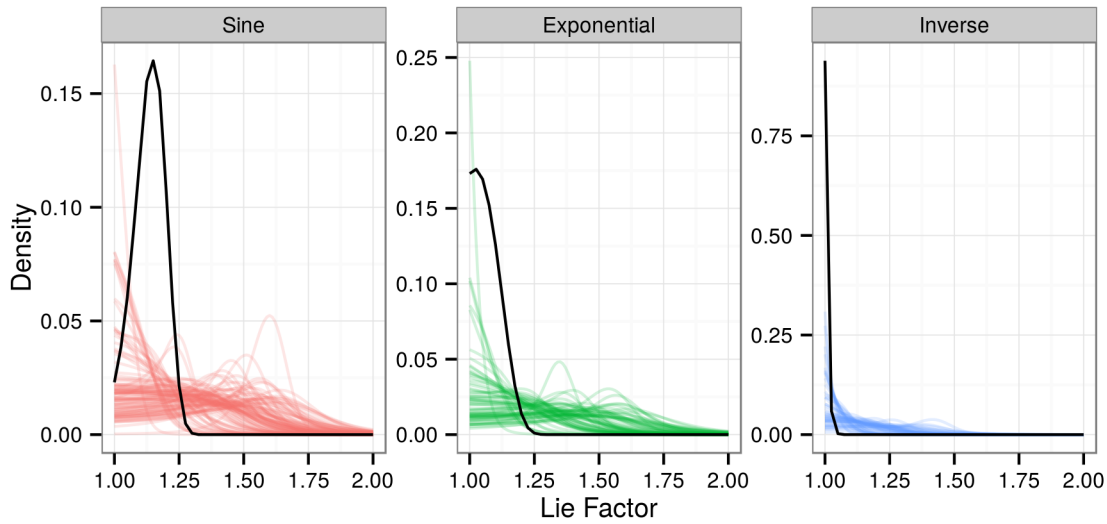


Fig. 9. Estimated densities for  $\theta_i$ , shown in color, with the estimated overall density for  $\theta$  shown in black. Individuals have extremely similar posterior distribution of  $\theta_i$ , and even different functions have similar  $\hat{\theta}$ , suggesting a common underlying mental distortion. XXX shouldn't the posterior distribution be somewhat closer to the densities of individual  $\theta_i$ s?

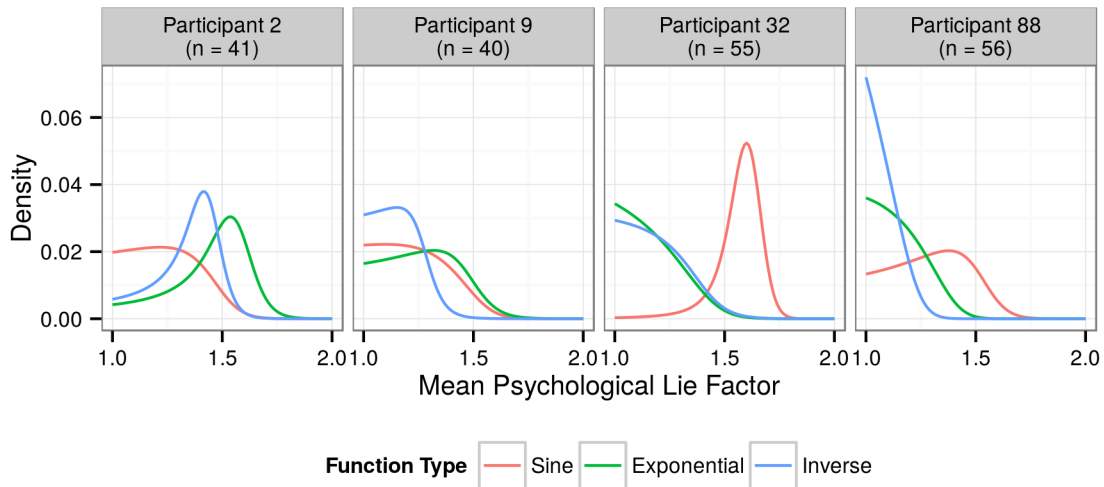


Fig. 10. Posterior distributions for  $\theta_i$  for four of the participants who completed at least 6 trials of each of the three function types.

*Comparison of the Preferred Stimuli.* Estimates of  $\hat{\theta} = E[\theta]$  for each function are 1.29, 1.13, and 1.3 respectively for exponential, inverse, and sine functions, suggesting a similar psychological distortion even for very different functions, though it seems as if the inverse function causes somewhat less distortion, possibly because the correction factor is not as proportionately large. Credible intervals can be found in Table II. As all three of the credible intervals exclude 1.05, there is evidence that a psychological distortion is occurring; that is, there is evidence of a significant psychological lie factor. In addition, the method of adjusting the estimated lie factor we have used here is conservative; it is likely that because most stimuli contain a sub-plot with  $w \geq 1$  our estimate of  $\theta$  for the inverse function is low (as the lie

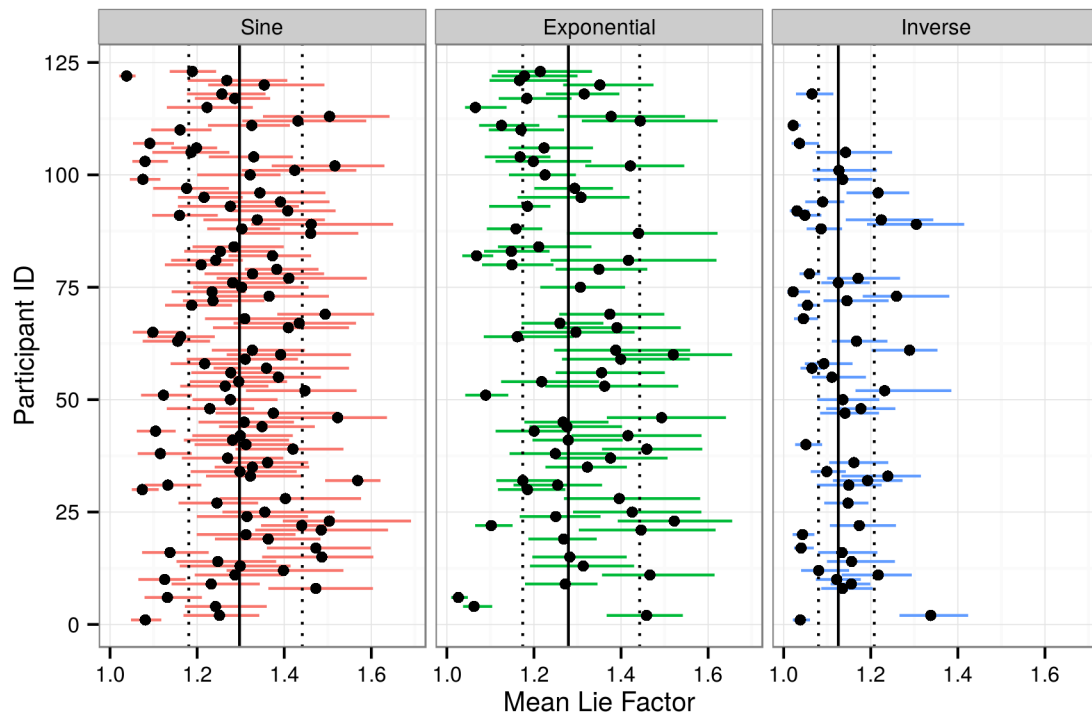


Fig. 11. 95% posterior predictive intervals for  $\theta_i$ , calculated for each stimulus type. Vertical lines indicate the median estimate of the overall  $\theta$  with a 95% credible interval.

Table II. Credible intervals for the overall  $\theta$  for exponential, inverse, and sine stimuli.

Function	95% Credible Interval for $\theta$	Median
Sin	(1.1808, 1.4415)	1.2973
Exp	(1.1742, 1.4428)	1.2791
Inv	(1.08, 1.208)	1.1255

factor for fully corrected inverse plots is greater than the corresponding lie factors for the other two functions used in this experiment).

‘optimal’ is a tricky property - optimality needs a formal proof for back up. Don’t go there. It’s an estimated weight, or participant based weight. The optimal weight values corresponding to these  $\theta$  are shown in figure 12. In all three cases, the optimally corrected plots appear less distorted than the uncorrected plots.

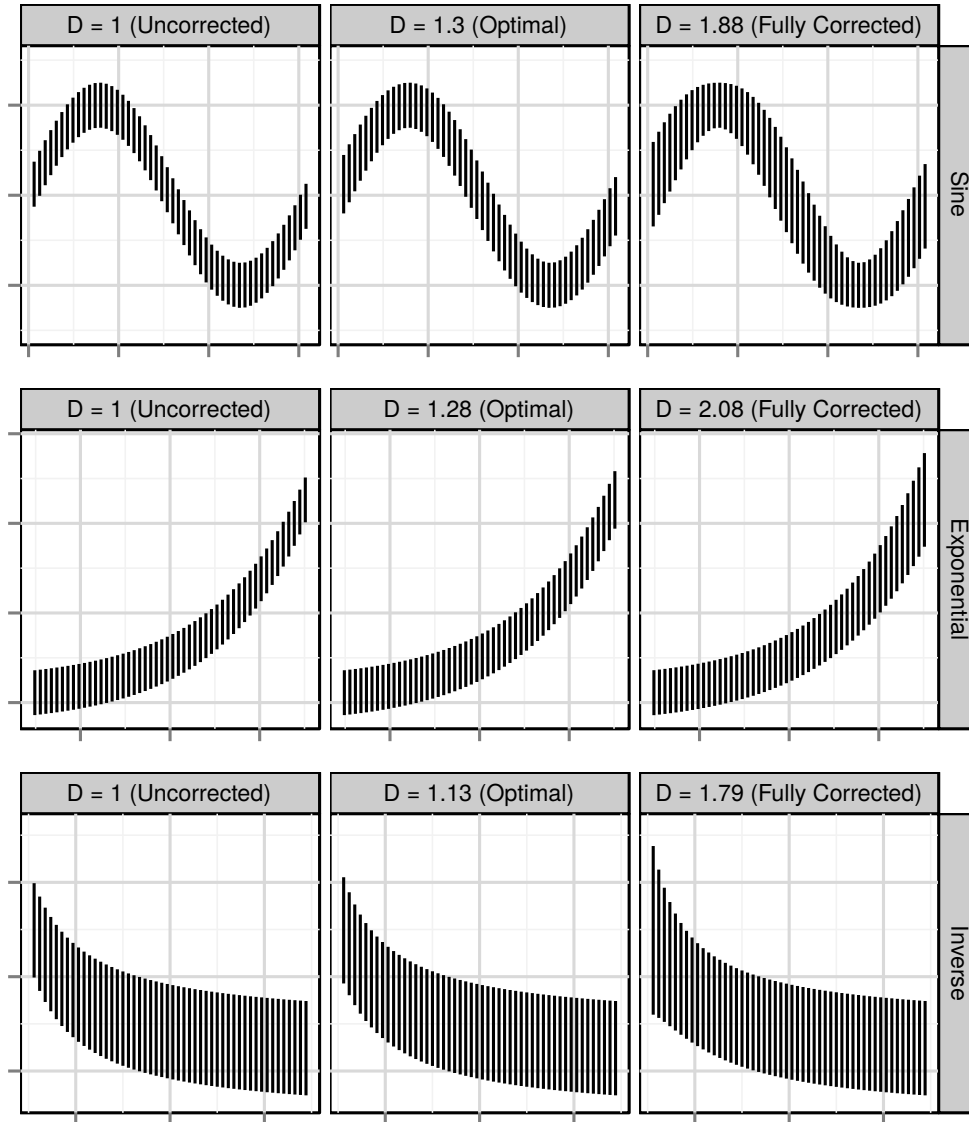


Fig. 12. Uncorrected, optimally corrected (according to the median value of  $\theta$ ), and fully corrected stimuli for all three underlying functions used in the experiment. The corrected value shown here is equivalent to the distortion factor  $D$  defined as  $D = \ell_{max}/\ell_{min}$ .

## 5. CONCLUSIONS

The sine illusion arises from misapplication of three-dimensional context to a two-dimensional stimulus which results in nearly unavoidable perceptual distortions. These distortions impact the inferences made from charts which are similar to three-dimensional figures, even when we are not consciously aware that this context exists. We have estimated that the illusion produces a distortion of about 135%. This distortion occurs entirely between the retinal image and the mental representation of the object; it is not due to the chart, rather, it is an artifact of our perceptual system.

As Tufte advocated for charts that showed the data without distortion, our goal is to raise awareness of perceptual distortions that occur within the brain itself due to misapplied heuristics. While applying corrections to the data to remove these distortions is somewhat radical, the persistence of the illusion despite awareness of its presence presents a challenge to those seeking to display data visually. In addition, many graph types can induce this illusion (scatterplots, ribbon plots, parallel sets plots), so avoiding a specific type of graph is not an effective solution. The best solution to this problem is to raise awareness: to demonstrate that optical illusions occur within statistical graphics, and to understand how these illusions arise.

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